Robust Direct Adaptive Fuzzy Control of Switched Constrained Manipulators with Unknown Dynamics

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Summary/Abstract

In this article, we address the problem of controlling robots with arbitrarily-switched constraints and unknown dynamics. Switching between different constraints of a robot would result in a switched nonlinear system that does not inherit the behavior of its individual subsystems. In order to guarantee stable performance of robots with arbitrarily switched constraints and unknown dynamics, we propose a Robust Adaptive Fuzzy Control (RAFC) strategy that can guarantee global stable performance under such challenging conditions. The suggested control strategy relies on the synergy of the Sliding Mode Control (SMC) that adds robustness against possible dynamics parameters drift, finding a Common Lyapunov Function (CLF) that guarantees stability under arbitrary constraints switching, and Direct Adaptive Fuzzy System (DAFS) that relaxes the need for knowing the precise robot dynamics. Experiments are performed on a KUKA Lightweight Robot (LWR) doing camshaft caps assembly of an automotive powertrain. The given robotic assembly process falls in the category of switched constrained robots and the efficiency of the suggested RAFC strategy in controlling such a robotic task will be shown.

1 Introduction

Robots are considered nowadays the most important tools in automating many processes and their control was attracted by research and industrial institutions. One of the earliest control scheme of robots was reported in 1973 by Markiewicz when he suggested a computed torque and inverse dynamics control strategy for robotic manipulators [1]. Conventional PD and PID controllers were successfully employed in controlling different types of robots with acceptable performance [2]. Despite the design simplicity of PD and PID controllers, they can perform well only within a small region of operation that results in a reduced robustness against possible drift in the point of operation. In order to enhance the control performance of robots, output feedback control schemes were proposed and an improved stable control strategy, over a wider mode of operation for the robotic systems, was obtained [11, 19]. Adaptive control strategies were successfully used in improving the robots control performance when we have unknown parameters in the robot dynamics and excellent tracking performance was reported [5, 6, 8, 10]. Further improvements were achieved through using the sliding mode control strategy that can accommodate possible parameters drift in the robot dynamics and a more robust performance was resulted [7, 12]. In order to accommodate more uncertainty in the robots dynamics, universal approximators, like neural networks and fuzzy systems, were successfully employed in developing control strategies that relax the need for knowing the dynamics of the robot [14, 15, 18, 21].

The majority of the robot systems applications involve interactions between the robots and their environment resulting in constrained motion robots. Different control strategies were proposed to handle the constrained motion robot systems, and hybrid position/force control is considered the most prominent scheme in dealing with such systems through controlling both the position and force of interaction between the end effector and the environment [3, 9, 22]. Likewise to the unconstrained robots, the dynamics certainty was also considered through using fuzzy logic control for both holonomic and nonholonomic constrained robots (see for example [13, 23, 25] and the references therein). In [17], it was shown that for switched control systems, switching between different stable subsystems can cause unstable or undesirable performance. In many constrained robots applications, we have multiple constraints switched from one to another. For instance, in robotic assembly operations, different contact phases could result between the manipulated object and the environment that makes different constraints, possibly one at a time, to be inserted in the overall dynamics of the robotic process which would result in a switched nonlinear system. Furthermore, in many situations the parameters of the robot dynamics are not precisely known that would add more challenges in controlling such robotic systems. In [28, 29], the authors proposed control strategies that handles the transient switching in the constraints and excellent tracking performance was obtained. However, the whole dynamics parameters are required to be precisely known which is unfortunately not the case for many industrial robots.

In this paper, we address the problem of controlling a robot manipulator with switched constraints and unknown dynamics. We suggest a Robust Adaptive Fuzzy Control (RAFC) strategy in controlling such interesting robots. The RAFC strategy is a synergy of finding a Common Lyapunov Function (CLF), the Sliding Mode Control (SMC), and fuzzy logic approximation.
Therefore, the main contribution of this paper is to solve the control problem for the switched constrained robots with unknown dynamics. In order to validate the suggested control strategy, we use it in controlling a KUKA Lightweight Robot (LWR) doing a camshaft caps of a powetrain of an automotive engine.

The rest of the paper is organized as follows. In section 2, we describe the dynamics of the switched constrained robots and present our control problem. Section 3 will lodge several preliminary concepts like fuzzy logic approximators along with basic definitions, assumptions, and properties. The suggested control strategy is presented in section 4. In section 5, we explain the experimental test stand with the results obtained when using the RAFC strategy in controlling the considered switched constrained robotic system and section 6 summarizes the concluding remarks.

2 Problem Formulation

The dynamics of a switched constrained robotic system can be described by [28, 29]:

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + f_\sigma(q) \tag{1} \]

with:

\[ f_\sigma(q) = J^T(q)D^T_\sigma(\alpha)\lambda \tag{2} \]

Where \( q \in \mathbb{R}^n \) is the links position vector, \( M(q) \in R^{n\times n} \) is the inertia matrix, \( C(q, \dot{q})\dot{q} \in \mathbb{R}^n \) is the centripetal and Coriolis vector, \( G(q) \in \mathbb{R}^{n\times n} \) is the gravity vector, \( \tau \in \mathbb{R}^n \) is the torque vector actuating the links, \( J(q) \in \mathbb{R}^{n\times 6} \) is the Jacobian of the manipulator, \( \lambda \in \mathbb{R}^m \) is the vector of Lagrange multipliers, \( D(\alpha) \) is the gradient of the task space constraints, and \( \alpha = (x, y, z, \Theta, \Psi, \Phi)^T \) is the pose (the Cartesian position and orientation) of the manipulated object with \( x, y, \) and \( z \) are the Cartesian position and \( \Theta, \Psi, \) and \( \Phi \) are the roll, pitch and yaw respectively, \( \sigma \) is the index of the constraints \( (\sigma = 1, 2, ..., P) \), and \( P \) is the total number of the constraints.

Free space robot motion can be viewed as a special case of (1) with \( f_\sigma(q) = 0 \). Suppose that \( M(q), C(q, \dot{q}), \) and \( G(q) \) are unknown. Then equation (1) would be an unknown switched nonlinear system, and the objective of this paper is to propose a direct adaptive fuzzy control strategy that can guarantee global stable performance of the robot under arbitrary unknown constraints switching behavior with unknown robot dynamics.

3 Preliminaries

Before presenting the main control strategy suggested in this paper, we will explain the concept of fuzzy logic approximators along with other preliminary concepts, properties and assumptions.

3.1 Fuzzy Logic Approximators

One of the vital applications of the fuzzy set theory is the functions approximation. It gives a feasible way of approximating unknown smooth functions through the use of T-S fuzzy models. Suppose that we desire to approximate the control action of (1), and consider that \( (q_1, q_2, ..., q_n, q_n) = (u_1, u_2, ..., u_{2n}) \). Let’s assume that the output of each mapping, that will be approximated, is \( y_f \). Such approximation would be feasible in the context of fuzzy If-Then rules as:

Controller Rule i:

\[ \text{If } u_1 \text{ is } A_{i1} \text{ and } u_2 \text{ is } A_{i2} \text{ and... and } u_{2n} \text{ is } A_{i2n} \text{ Then } y_f = y^i_f \tag{3} \]

where \( i = 1, 2, ..., L \) is the total number of the If-Then rules, \( A_{ij} (i = 1, 2, ..., L; j = 1, 2, ..., m) \) are the premise fuzzy sets, and \( y^i_f \) is crisp output of the \( k^{th} \) rule. Through using a singleton fuzzifier along with product inference, the overall output for the fuzzy system above can be computed as [13, 16]:

\[ y_f = \theta^T h(u) \tag{4} \]

with:

\[ \mu_i(u) = \prod_{j=1}^{2n} A_{ij}(u_j) \]

\[ h(u) = \left[ \frac{\mu_1(u)}{\sum_{i=1}^{L} \mu_i(u)}, \frac{\mu_2(u)}{\sum_{i=1}^{L} \mu_i(u)}, ..., \frac{\mu_L(u)}{\sum_{i=1}^{L} \mu_i(u)} \right] \]

\[ \mu_i(u) \geq 0 \]

\[ \sum_{i=1}^{L} \mu_i(u) > 0 \]

and:

\[ \theta = (y^1_f, y^2_f, ..., y^L_f) \]

Hence, the control action \( \tau \) of (1) can be approximated through a fuzzy logic controller \( \tau_f = (y^1_f, ..., y^m_f) \) and this is called a direct fuzzy control strategy [16]. That is:

\[ \tau_f(q, \dot{q}, \theta) = \theta^T h(q, \dot{q}) \tag{5} \]

Next, we will explain several concepts, properties, and assumptions necessary for the derivation of the suggested control strategies.

3.2 Properties and Assumptions

Below properties are common between robot manipulators [20]:

\textbf{P1.} For all robot manipulators, \( M(q) \) is a positive definite and symmetric matrix.

\textbf{P2.} For all robot manipulators, the matrix \( \dot{M}(q) - 2C(q, \dot{q}) \) is a skew symmetric matrix, that is for all \( \dot{x} \in \mathbb{R}^n \) and \( \dot{x} \neq 0 \), we have \( \dot{x}^T(\dot{M}(q) - 2C(q, \dot{q}))\dot{x} = 0 \).

Define the joints error vector to be:

\[ \ddot{q} = q - q_d \tag{6} \]
and consider the joints filtered error vector to be described as:

\[ s = \ddot{q} + \gamma \dddot{q} \]  

(7)

with \( \gamma > 0 \). (7) can be rewritten as:

\[ s = \ddot{q} - \dot{\gamma} \]  

(8)

where:

\[ \dot{\gamma} = \dot{\gamma}_d - \gamma \ddot{q} \]  

(9)

**Note 1.** It has been shown that the filtered error described by (7) has the following properties: (i) the equation \( s(t) = 0 \) defines the time-varying hyperplane in \( R^n \), on which the tracking error vector \( \ddot{q} \) decays exponentially to zero,(ii) if \( \ddot{q}(0) = 0 \) and \(|s(t)| \leq \epsilon \) with constant \( \epsilon \), then \( \ddot{q}(t) \in \Omega_\epsilon = \{ \ddot{q} \mid \ddot{q} \leq 2^{i-1} \gamma i \} \) for \( \forall t \geq 0 \) and (iii) if \( \ddot{q}(0) \neq 0 \) and \(|s(t)| \leq \epsilon \) then \( \ddot{q}(t) \) will converge to \( \Omega_\epsilon \) within a time constant of \( \frac{\epsilon(\alpha - 1)}{\gamma} \) [4].

Taking the time derivative of (8), we obtain:

\[ \ddot{s} = \dddot{q} - \dot{\gamma} \]  

(10)

Despite the robustness of the SMC, a possible chattering may deteriorate the control performance and may even drive the system to be unstable. Therefore, a modified filtered error [27] is introduced that can be expressed as:

\[ s_\varepsilon = s - \varepsilon \text{tanh}(\frac{\varepsilon}{\varepsilon}) \]  

(11)

For bounded values of \( q, f_\sigma(q) \) is bounded. That is:

\[ |f_\sigma(q)| \leq b_\sigma \]  

(12)

Furthermore, consider that the bound of \( b_\sigma \) for all values of \( \sigma \) is \( B \), that is:

\[ B = \sup_\sigma(b_\sigma) \]  

(13)

We will design the control strategy relying on the modified filtered error (11). However, before we proceed in explaining the suggested control strategy, we will present two assumptions that need to be satisfied:

**A1.** The signals \( q, \dot{q}, \ddot{q} \) are assumed to be available for measurement.

**A2.** The signals \( q, \dot{q}_d, \ddot{q}_d \) are assumed to be bounded and piecewise continuous.

## 4 Robust Adaptive Fuzzy Control (RAFC) Design

In [29], the authors derived the certainty equivalence control strategy for the switched constrained robots with known dynamics to be:

\[ \tau_m^* = C(q, \dot{q})(\varepsilon \text{sat}(\frac{s}{\varepsilon}) + \ddot{\gamma}_r) + M(q)\dddot{q}_r + G(q) \]  

(14)

For the case of the unknown robot dynamics, (14) can’t be computed. Therefore, we will use a fuzzy logic controller \( \tau_f \), that was explained in section 3.1, in approximating \( \tau_m^* \) defined in (14). Suppose that the approximation error between \( \tau_f \) and \( \tau_m^* \) is:

\[ w = \tau_f(q, \dot{q}|\theta) - \tau_m^* \]  

(15)

The minimum approximation \( w^* \) error is defined to be:

\[ w^* = \tau_f(q, \dot{q}|\theta^*) - \tau_m^* \]  

(16)

Where \( \theta^* \) is the optimal parameter vector of \( \theta \) that is defined as:

\[ \theta^* = \arg \min_{\theta \in M_\theta}[\sup_{q \in M_q, \dot{q} \in M_q} \tau_f(q, \dot{q}|\theta^*) - \tau_m^*] \]  

(17)

and

\[ \tau_f(q, \dot{q}|\theta) = \theta^T h(q, \dot{q}) \]  

(18)

with \( M_q \) and \( M_\theta \) are the allowable sets of \( q \) and \( \dot{q} \) respectively. We will assume that \(|\dot{w}| \leq M_w \) and \(|\dot{B}| \leq M_B \), i.e. the approximation error \( \dot{w} \) and the constraint parameters \( \dot{B} \) will remain within prescribed sets. For the parameter vector \( \theta \), we will prove that it will always remain within a certain bound.

Let’s consider the control action to be composed of two terms; a fuzzy control action \( \tau_f \) and a bounding term \( \tau_b \), that is:

\[ \tau = \tau_f + \tau_b \]  

(19)

Where:

\[ \tau_b = -K_ds(t) - \Gamma(\dot{B} + \ddot{w}) \]  

(20)

with \( K_d = \text{diag}(k_{d1}, k_{d2}, ..., k_{dn}) \), \( k_{d1}, k_{d2}, ..., k_{dn} \) are positive constants, and \( \Gamma = \text{diag}(\text{tanh}(\frac{\varepsilon}{\gamma}), \text{tanh}(\frac{\varepsilon}{\gamma}), ..., \text{tanh}(\frac{\varepsilon}{\gamma})) \). Therefore, the need for knowing the robot dynamics is relaxed through the use of the control action (19). In order to guarantee a stable performance for the suggested RAFC strategy, the parameters vectors \( \dot{B}, \theta, \) and \( \ddot{w} \) are updated according to the following laws:

\[ \dot{\theta} = -\eta_2 \sigma_2^T h(q, \dot{q}) \]  

(22)

\[ \dot{w} = \begin{cases} \eta_3 |s_e| & \text{if } (|\dot{B}| < M_B) \text{ or } (|\dot{B}| = M_B \text{ and } \eta_1 |s_e| \leq 0) \\ P(\eta_1 |s_e|) & \text{if } (|\dot{B}| = M_B \text{ and } \eta_1 |s_e| > 0) \end{cases} \]  

(21)

\[ \tau = \tau_f + \tau_b \]  

(19)

where \( \eta_1, \eta_2, \eta_3 > 0 \) and \( P(.) \) is the projection function, that is:

\[ P(\eta_1 |s_e|) = |n_1 |s_e| - n_1 |s_e| (\frac{\dot{B}^T \hat{B}}{|\dot{B}|^2}) \]

\[ n_1 > 0 \text{ and } \eta_1 |s_e| \leq 0 \]  

(23)
The stability of the RAFC strategy can be ascertained through considering the Lyapunov candidate:
\[
V = \frac{1}{2} s^T \tilde{M}(q) s + \frac{1}{2 \eta_1} \tilde{\theta}^T \tilde{\theta} + \frac{1}{2 \eta_2} \tilde{\dot{\theta}}^T \tilde{\dot{\theta}} + \frac{1}{2 \eta_3} \tilde{w}^T \tilde{w}
\]
(24)

Where \( \tilde{\theta} = \theta - \theta^* \) and \( \tilde{w} = \tilde{w} - w^* \). Taking the time derivative of (24), we obtain:
\[
\dot{V} = s^T \tilde{M}(q) \dot{s} + \frac{1}{2} s^T \tilde{M}(q) s + \frac{1}{\eta_1} \tilde{\theta}^T \tilde{\dot{\theta}} + \frac{1}{\eta_2} \tilde{\dot{\theta}}^T \tilde{\dot{\theta}} + \frac{1}{\eta_3} \tilde{w}^T \tilde{w}
\]
(25)

After several mathematical simplifications and using the RAFC strategy, one can show that \( \dot{V} \) is always negative semi-definite (more specifically \( \dot{V} \leq -s^T K_d s \)) that leads to the fact that the system is stable with all closed loop signals to be bounded.

5 Experimental Results

In order to show the performance of the suggested RAFC strategy, we built a test stand that is composed of a KUKA Lightweight Robot (LWR), which is a 7-DOF industrial robot, doing a camshaft caps assembly of an engine powertrain. The key features of the KUKA LWR 4+ is detailed in [26]. For research purposes, a Fast Research Interface (FRI) is available in the robot that makes its joints control strategy customizable by the user through a C++ platform hence allowing researchers to apply their own control strategies in controlling the robot joints [24].

Furthermore, the manipulator joints can be set as a rigid or flexible according to the user requirement through the programming platform. Figure 1 shows the test stand of our experiment as per doing the camshaft caps assembly process. From Figure 1, we can see that during the assembly task execution, the robot passes through different switched phases; starting from the free space motion (Figure 1.b), then the constrained motion of Figure 1.c, 1.d, 1.e, and 1.f with different constraints in each phase. Hence, the robotic system depicted in Figure 1 falls in the category of the switched constrained robots. The RAFC strategy was programmed through C++ in a remote PC. The features of the PC that we used is of Intel (R) Core (TM) i5-2540 CPU with 2.6 GHz speed and 4 GB RAM running under a Linux environment. The rate of the communication between the remote PC and the robot, through the FRI, is 100 Hz. We used the suggested RAFC strategy in commanding the robot for doing the assembly process depicted in Figure 1 for four times so that we have a thorough evaluation of the strategy performance.

Figure 2.a through Figure 2.f show the desired pose, say \( \alpha_d = (x_d, y_d, z_d, \Theta_d, \Psi_d, \Phi_d)^T \), of the manipulated object for performing the given assembly. The corresponding desired joints position \( (q_d) \) and velocity \( \dot{q}_d \) are shown in Figure 3. Figure 4.a-g show the control action when using the RAFC control strategy in commanding the joints for doing the given task.

The RAFC strategy was used with the following details:
\[
K_d = \text{diag}(60, 30, 30, 45, 15, 9, 6)^T
\]
\[
\epsilon^T = [0.005, 0.07, 0.06, 0.05, 0.04, 0.09, 0.04]^T
\]
\[
C^T = [80, 80, 80, 50, 12, 12, 5]^T
\]
\[
M^T = [0.025, 0.08, 0.03, 0.17, 0.045, 0.006, 0.005]^T
\]
\[
M_w = [0.002, 0.017, 0.0025, 0.002, 0.00045, 0.0007, 0.0005]^T
\]
\[
\eta_1 = 0.001, \eta_2 = 0.01, \text{ and } \eta_3 = 0.0001.
\]

Gauss membership functions of the form:
\[
A_{i,j}(u_j) = \exp\left(-\frac{(u_j - c)^2}{2\sigma^2}\right)
\]
(26)

are used in the premise of the \( i^{th} \) If-Then rule of the RAFC. \( c \) and \( \sigma \) are the center and width of the gauss membership function. The points position and velocity, say \( q \) and \( \dot{q} \) respectively, are considered the input variables for the fuzzy logic controller, and each one of those state variables is assigned with two membership functions for the premise part of the if-then rules. For simplicity, we will describe each Gauss membership function described by (26), with an ordered pair \((c, \sigma)\). Below are the parameters of the fuzzy sets of the variables considered in the RAFC, say \( q \) and \( \dot{q} \):

\( q_1 = (-0.7, 0.0849) \) and \((-0.9, 0.0849)\)
\( q_2 = (-0.4, 0.0849) \) and \((-1.0, 0.0849)\)
\( q_3 = (0.3, 0.0849) \) and \((0.2, 0.0849)\)
\( q_4 = (1.6, 0.0849) \) and \((1.2, 0.0849)\)
\( q_5 = (-0.1, 0.0849) \) and \((-0.4, 0.0849)\)
\( q_6 = (-0.7, 0.0849) \) and \((-1.1, 0.0849)\)
\( q_7 = (-2.0, 0.0849) \) and \((-2.3, 0.0849)\)
\( q_8 = (0.14, 0.0849) \) and \((-0.1, 0.0849)\)
\( q_9 = (0.6, 0.0849) \) and \((-0.5, 0.0849)\)
\( q_3 = (0.05, 0.0849) \) and \((-0.06, 0.0849)\)
\( q_3 = (0.3, 0.0849) \) and \((-0.3, 0.0849)\)
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\( q_3 = (0.4, 0.0849) \) and \((-0.6, 0.0849)\)
\( q_3 = (0.6, 0.0849) \) and \((-0.2, 0.0849)\)

Figure 4.h-n show the position error signals and Figure 4.a-u show the velocity error signals for all joints when using the suggested control strategy. We can see that the RAFC is having an excellent tracking performance despite the uncertain and unknown robot dynamics. Figure 2.g-l show the corresponding error signals in the task space, i.e. the pose error signals, and we can notice that the excellent joint space performance is significantly reflected to the task space performance. Figure 6.a-g show the parameters vector \( \hat{B}_1, \hat{B}_7 \) and Figure 6.h-n show the vector \( \hat{w}_1, \hat{w}_7 \). From the results of the RAFC strategy, we can notice that:

1. If unknown uncertainty is added to the dynamics, then we would have \( M_{eq}(q) = M(q) + \Delta M(q) \), \( C_{eq}(q, \dot{q}) = C(q, \dot{q}) + \Delta C(q, \dot{q}) \), and \( G_{eq}(q) = G(q) + \Delta G(q) \). In this case, the RAFC strategy can easily accommodate such unknown uncertainty, since the control action (19) does not rely on the robot model, provided that the resultant robot dynamics, say \( M_{eq}(q), C_{eq}(q, \dot{q}), \) and \( G_{eq}(q) \), satisfy properties \( \text{P1} \) and \( \text{P2} \).
Figure 1: Camshaft caps assembly process: (a) Camshaft caps assembly as a double peg-in-hole process; (b) Phase 1 (free space); (c) Phase 2; (d) Phase 3; (e) Phase 4; (f) Phase 5.

Figure 2: The manipulated object signals: (a) $x$ (in mm); (b) $y$ (in mm); (c) $z$ (in mm); (d) $\Theta$ (in degree); (e) $\Psi$ (in degree); (f) $\Phi$ (in degree); (g) $e_x$ (in mm); (h) $e_y$ (in mm); (i) $e_z$ (in mm); (j) $e_\Theta$ (in degree); (k) $e_\Psi$ (in degree); (l) $e_\Phi$ (in degree).

2. The RAFC strategy is a synergy of considering switching in the constraint, accommodating unknown dynamics, accommodating unknown dynamics, and considering the nonlinear feature of the robot. Consequently, we had excellent joints position and velocity tracking performance.

3. For the parameters update laws (21) and (23), we can see that the first line is $\eta_1 |s_c| \geq 0$. This would make the parameters vectors $\hat{B}$ and $\hat{w}$ to increase and the use of the projection function is necessary in this case to prevent the proliferation of those parameters and avoid possible instability.

6 Conclusion

Robust Adaptive Fuzzy Control (RAFC) was suggested for robot manipulators with arbitrarily switched constraints and unknown dynamics. The RAFC strategy is a synergy of the concepts of the Sliding Mode Control (SMC), the fuzzy logic approximation, and finding a Common Lyapunov Function (CLF). The use of the SMC adds robustness against possible parameters drifts, the fuzzy logic approximation accommodates the robot dynamics anonymity and uncertainty, and the CLF guarantees the stable performance under arbitrary switching. Experiment is performed on a KUKA Lightweight Robot (LWR) doing a camshaft caps assembly of a powertrain. Such a robotic system falls in the category of the switched constrained robots and the experimental results show the
efficient performance of the RAFC strategy despite the dynamics anonymity and arbitrary constraints switching.

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References


Figure 5: Joints filtered error and modified filtered error signals: (a) $s_1$; (b) $s_2$; (c) $s_3$; (d) $s_4$; (e) $s_5$; (f) $s_6$; (g) $s_7$; (h) $s_{c1}$; (i) $s_{c2}$; (j) $s_{c3}$; (k) $s_{c4}$; (l) $s_{c5}$; (m) $s_{c6}$; (n) $s_{c7}$.

Figure 6: (a) $\hat{B}_1$; (b) $\hat{B}_2$; (c) $\hat{B}_3$; (d) $\hat{B}_4$; (e) $\hat{B}_5$; (f) $\hat{B}_6$; (g) $\hat{B}_7$; (h) $\hat{w}_1$; (i) $\hat{w}_2$; (j) $\hat{w}_3$; (k) $\hat{w}_4$; (l) $\hat{w}_5$; (m) $\hat{w}_6$; (n) $\hat{w}_7$.


