On Sugeno Integral as an Aggregation Function

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Abstract

Aggregation refers to the process of combining numerical values $x_1, \ldots, x_m$ into a single one $M^{(m)}(x_1, \ldots, x_m)$, so that the final result of aggregation takes into account all the individual values. In decision making, values to be aggregated are typically preference or satisfaction degrees and thus belong to the unit interval $[0, 1]$.

This paper aims at investigating the Sugeno integral which can be regarded as an aggregation function. In particular, we show that any Sugeno integral is a weighted max-min function, that is, setting $X = \{1, \ldots, m\}$, a function of the form

$$M^{(m)}(x_1, \ldots, x_m) = \bigvee_{T \subseteq X} [a_T \wedge (\bigwedge_{i \in T} x_i)], \quad a_T \in [0, 1],$$

where a is a set function satisfying $a_\emptyset = 0$ and $\bigvee_{T \subseteq X} a_T = 1$. Such functions are investigated in this paper. We also show that those functions can also be written as

$$M^{(m)}(x_1, \ldots, x_m) = \bigwedge_{T \subseteq X} [b_T \vee (\bigvee_{i \in T} x_i)], \quad b_T \in [0, 1],$$

(weighted min-max functions) where b is a set function satisfying $b_\emptyset = 1$ and $\bigwedge_{T \subseteq X} b_T = 0$. The correspondance formulae $b = b(a)$ and $a = a(b)$ are given as well. For instance, we have

$$(.1 \wedge x_1) \vee (.3 \wedge x_2) \vee (x_2 \wedge x_3) = (.1 \vee x_2) \wedge (.3 \vee x_3) \wedge (x_1 \vee x_2).$$

We also propose an axiomatic characterization of this class of functions based on some aggregation properties: the increasingness and the stability for minimum and maximum with the same unit.

Most of these results are applied to the Sugeno integral. In particular, we can derive equivalent expressions and characterize the family of all the Sugeno integrals.

We also consider particular weighted max-min functions: Boolean max-min functions, weighted maximum and minimum functions, ordered weighted maximum and minimum functions, partial maximum and minimum functions, order statistics and associative medians. Of course, all these functions are Sugeno integrals.

Keywords: fuzzy measures; Sugeno integral; aggregation functions; multicriteria decision making; pseudo-Boolean functions; max-min algebra; ordinal scales.