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Characterization of some aggregation functions stable for positive linear transformations

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Abstract

Synthesizing judgments is an important part of multiple criteria decision making methods. The most typical situation concerns individuals who form quantifiable judgments about a measure of an object (weight, length, area, height, volume, importance or other attributes, for instance in the framework of a hierarchy)(see [3,4]) or quantifiable judgments on pairs of alternatives along each criterion. In the latter case, the judgments are very often expressed with the help of fuzzy preference relations (see [8,9]).

In order to reach a consensus (overall opinion) on these judgments, classical aggregation functions have been proposed: arithmetic means, geometric means, root-power means and many others. Of course, given such an aggregation function, we can ask for a motivation of its use, i.e. for natural, reasonable assumptions which lead to this function. Conversely, we can specify some assumptions (called axioms or properties) and determine all the aggregation functions satisfying these. This is the topic with which we deal here.

This paper aims at describing the family of all aggregation functions fulfilling three specific properties. The first two are increasing monotonicity and stability for the same transformations of interval scales in the sense of the theory of measurement (see [15]), i.e. stability for positive linear transformations (we refer to the corresponding functional equation in [5,6] where the arithmetic mean is characterized). The third property is chosen among well-known algebraic properties such as associativity, decomposability and bisymmetry.

We make a distinction between aggregation functions having a fixed number of arguments (aggregation m-functions) and aggregation functions defined for all number of arguments (aggregation operators or aggregators).

References

[1] J. Aczél (1948), *On mean values*, Bulletin of the American Math. Society, **54**: 392-400.

[2] J. Aczél (1966), *Lectures on Functional Equations and Applications*, (Academic Press, New-York).

[3] J. Aczél (1984), *On weighted synthesis of judgements*, Aequationes Mathematicae **27**: 288-307.

[4] J. Aczél and C. Alsina (1987), *Synthesizing judgments: a functional equations approach*, Math. Modelling, **9**: 311-320.

[5] J. Aczél and F.S. Roberts (1989), *On the possible merging functions*, Math. Social Sciences, **17**: 205-243.

[6] J. Aczél, F.S. Roberts and Z. Rosenbaum (1986), *On scientific laws without dimensional constants*, Journal of Math. Analysis and Appl., **119**: 389-416.

[7] J. Fodor, J.-L. Marichal (1995), *On nonstrict means*, Aequationes Mathematicae, submitted.

[8] J. Fodor, J.-L. Marichal and M. Roubens (1994), *Characterization of some aggregation functions arising from MCDM problems*, Proceedings of IPMU'94, Paris: 1026-1031.

[9] J. Fodor and M. Roubens (1994), *Fuzzy Preference Modelling and Multicriteria Decision Support*, Kluwer, Dordrecht.

[10] A.N. Kolmogoroff (1930), *Sur la notion de la moyenne*, Accad. Naz. Lincei Mem. Cl. Sci. Fis. Mat. Natur. Sez.,**12**: 388-391.

[11] C.H. Ling (1965), *Representation of associative functions*, Publ. Math. Debrecen, **12**: 189-212.

[12] J.-L. Marichal and P. Mathonet (1996), *A characterization of the ordered weighted averaging functions based on the ordered bisymmetry property*, IEEE Trans. Fuzzy Syst., submitted.

[13] J.-L. Marichal and M. Roubens (1993), *Characterization of some stable aggregation functions*, Proc. Intern. Conf. on Industrial Engineering and Production Management, Mons, Belgique: 187-196.

[14] M. Nagumo (1930), *Über eine Klasse der Mittelwerte*, Japanese Journal of Mathematics, **6**: 71-79.

[15] F.S. Roberts (1979), *Measurement Theory with Applications to Decision-making, Utility and the Social Sciences*, Addison-Wesley Pub., Reading.

[16] W. Silvert (1979), *Symmetric summation: A class of operations on fuzzy sets*, IEEE Trans. Systems, Man Cybernet., **9**: 667-659.