

CONSENSUS WITH ORDINAL DATA

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We consider a finite set of potential actions that are compared using different points of view. The evaluation of each action for a given point of view is measured on a qualitative (ordinal) scale. We suppose that “utilities” are associated to each measure related to a given qualitative scale in such a way that all these monotonic utilities are commensurable.

The utilities are thus observed on a common ordinal scale, i.e. admissible transformations are monotonic increasing functions defined on a support which is supposed to be the unit interval.

The importance degree related to each point of view and to any subset of all different points of view is expressed in terms of a Choquet capacity also called fuzzy measure (monotonic set function such that the measure of the empty set is equal to zero and the measure of the whole set is one). This degree is measured on the same ordinal scale that is used to observe the utilities.

If one accepts these rather strong assumptions, the multiple criteria decision making framework can be linked to the theory of preference between acts for decision under uncertainty. The states of the world correspond to the points of view and the set of acts corresponds to the set of potential actions.

We will consider an aggregation operator that determines a consensus among the different points of view called a Sugeno integral, i.e. a max-min combination of utilities and importance degrees.

Particular cases of Sugeno integrals are Boolean max-min operators (in that case, Choquet capacities are supposed to be equal either to zero or one) and the weighted max (Choquet capacities are then identified to possibility measures).

In the case of Boolean Choquet capacities, the Sugeno integral and the corresponding Choquet integral are confounded.

Sugeno integrals and weighted max can be easily expressed in terms of the median of the utilities and some importance degrees. Clearly the median is the qualitative counterpart to the averaging operators (classical weighted means and Choquet integrals) underlying the cardinal expected utility theory (Bernoulli (1938), von Neumann and Morgenstern (1944), Savage (1954), Jaffray (1992), Sarin and Wakker (1992)).

The first characterization of qualitative consensus functions is due to Sabbadin (1998) in the spirit of the work of Savage.

The Sugeno integral consensus function has been axiomatized in an equivalent way by Marichal (1998) using the following axioms : continuity, idempotence, comparison meaningfulness : the linear order on the set of actions induced by the consensus function is preserved for any continuous monotonic transformations of the common ordinal scale used to evaluate the actions for all criteria and the importance degrees, extension principle.

These conditions characterize consensus functions which are the Sugeno integrals.

Both approaches will be presented with the support of some practical examples.