Impact of Residual Transceiver Impairments on MMSE Filtering Performance of Rayleigh-Product MIMO Channels

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Abstract— **Recent studies have demonstrated the presence of residual transceiver hardware impairments even after employing calibration and compensation techniques in different wireless systems. The effect of these impairments becomes more severe in the systems involving a large number of inexpensive Radio Frequency (RF) chains such as massive Multiple Input Multiple Output (MIMO) systems due to the requirement of cost-efficient implementation. However, most of the existing studies consider ideal transceivers without incorporating the effect of residual hardware impairments. In this regard, this paper studies the impact of additive residual transceiver hardware impairments on the Minimum Mean Square Error (MMSE) filtering performance of Rayleigh-Product (RP) MIMO channels. Using principles from Random Matrix Theory (RMT), the MMSE filtering performance of the RP channels is analyzed and a tight lower bound is derived by taking the effects of residual additive transceiver impairments into account. Moreover, some useful insights on the performance of the considered system with respect to various parameters such as the transmit Signal to Noise Ratio (SNR), the number of scatterers and the severity of impairments on both the transmit and receive sides are provided.**

I. Introduction

Recently, massive Multiple-Input Multiple-Output (MIMO) has been considered as a candidate technology for the Fifth Generation (5G) of wireless communications due to its benefits in terms of higher system throughput, increased energy efficiency and reduced end to end latency [1]. However, one of the main requirements for the low-cost implementation of this technology is inexpensive transceiver hardware, which is easily prone to impairments. Therefore, it is crucial to take the effect of transceiver hardware impairments into account while designing wireless transceivers, especially for the large antenna systems demanding low-cost hardware components.

Most of the existing MIMO related works are based on the assumption of rich scattering wireless environment which allows to consider the signal transmitted from each transmit antenna to appear highly uncorrelated and to have a unique spatial signature at the receiver [2]. However, in practice, several scenarios with spatial fading correlation, rank-deficiency, pinhole or keyhole effects, and limited scattering exist [2, 3], where the underlying environment is not sufficiently scattered to hold this assumption. In this context, double scattering channels, which model the aforementioned aspects by utilizing the geometry of the underlying propagation environment, have received important attention [2–5]. The existence of such channels has been experimentally verified [4] and these channels have also been recently studied for the case of massive MIMO systems [6]. The RP channel considered in this paper is a special case of double scattering channels with the identity correlation matrices for the transmit-side antenna, receive-side antenna and the scatterers, and captures the effect of rank-deficiency caused by insufficient scattering or a keyhole phenomenon [7].

Several works in the literature have studied RP MIMO channels in different settings [7–9]. Authors in [7] studied the outage probability of MIMO beamforming in RP channels by considering three special cases of this channel. Furthermore, the contribution in [8] analyzed the performance of RP MIMO channels with linear Minimum Mean-Square-Error (MMSE) and Zero-Forcing (ZF) receivers, and characterized the achievable diversity-multiplexing tradeoffs. Moreover, the recent work in [9] derived the closed-form expressions for the ergodic mutual information and the average symbol error rate of the RP channels. However, all the aforementioned works are based on the assumption of ideal transceiver hardware, which is unrealistic in practice.

In practice, hardware impairments may result due to various factors such as sampling rate and carrier frequency offset, In-phase/Quadrature (I/Q) imbalance, and oscillator phase noise [10]. Even after employing suitable compensation algorithms at the receiver or calibration schemes at the transmitter, a certain level of distortion still exists due to inevitable residual hardware impairments [12]. In this regard, recent works have studied the effect of residual transceiver impairments on the performance of various wireless systems [10–15] considering ergodic capacity as a performance metric in most of the cases. In the context of RP channels, the only work which studied the impact of transceiver impairments is the recent work in [16], however, only optimal linear receivers were considered, which are complex to realize in practice.

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MMSE receivers are much more practical to implement than other classes of receivers such as successive interference cancellation receivers, which require sequential user processing. On the other hand, the MMSE receiver only requires the joint filtering of the received signals followed by individual user detection and decoding [17]. In contrast to the widely-used conventional ergodic capacity metric [10–13], the average MMSE criterion provides meaningful insights on the performance of single-user receivers after performing multi-user MMSE filtering [18]. In this regard, the contribution in [18] analyzed the MMSE filtering performance of Dual-Hop Amplify and Forward (DH-AF) multiple access channels and the authors in [15] extended this for the case of DH-AF massive MIMO relay systems with additive residual transceiver impairments. In this paper, we analyze the MMSE filtering performance of the RP channels in the presence of residual additive transceiver impairments. Using principles from Random Matrix Theory (RMT), we derive a tight lower bound for the average MMSE of the RP channels by taking the effects of residual additive hardware impairments into account. Furthermore, we validate our theoretical analysis with the help of numerical results and provide useful insights on the impact of residual hardware impairments on the RP channels.

The remainder of this paper is structured as follows: Section II provides the system model of RP MIMO channels with the residual additive transceiver hardware impairments. Section III provides theoretical analysis for the MMSE filtering performance of the considered MIMO channels while Section IV evaluates and discusses the effect of residual hardware impairments with the help of numerical results. Finally, concluding remarks are provided in Section V.

Notations: Throughout this paper, the notations $(\cdot)^{\mathsf{T}}$, $(\cdot)^{\mathsf{H}}$, and $\mathrm{tr}(\cdot)$ represent transpose, Hermitian transpose, and trace operators, respectively. The notation $\mathbb{E}[\cdot]$ denotes the expectation operator, and the notations $\mathcal{C}^M$ and $\mathcal{C}^{M\times N}$ refer to complex M-dimensional vectors and $M \times N$ matrices, respectively, $\mathrm{diag}\{\cdot\}$ represents a diagonal matrix, and $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ denotes a circularly symmetric complex Gaussian with zero-mean and covariance matrix $\mathbf{\Sigma}$.

II. System and Signal Models

We consider a flat fading point to point MIMO channel with M number of transmit antennas, N number of receive antennas and K number of scatterers. In Fig. 1(a), we present the system model for the conventional RP MIMO system with the ideal transceiver hardware, and in Fig. 1(b), we illustrate the system model for the considered RP MIMO channels with additive transceiver hardware impairments. Although the hardware impairments may impact the signal model either in an additive or a multiplicative way, for the sake of analytical tractability, we consider the case with additive residual impairments, i.e., the residual impairments are modeled as additive distortion noises as in [13–15]. In the following, we start with the signal model for the conventional case and then write the signal model for the considered case with the additive residual transceiver hardware impairments.

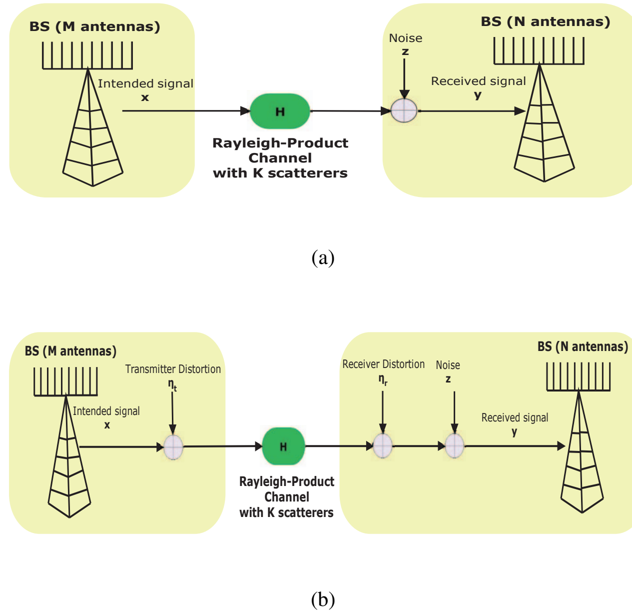

Fig. 1. Illustrations of (a) conventional RP MIMO system with K scatterers and ideal transceiver hardware. (b) RP MIMO system with K scatterers and residual additive transceiver hardware impairments.

In terms of mathematical characterization, a double scattering channel is realized by using a matrix product which involves two independent complex Gaussian matrices and three deterministic matrices to represent transmit, receive and scatterer correlation matrices [3]. As mentioned earlier in Section I, double scattering channels with the identity transmit-side antenna, receive-side antenna and scatterer correlation matrices are defined as the RP channels. Thus, after neglecting the spatial correlation of transmit antennas, receive antennas and scatterers, an RP channel can be mathematically written as the product of two statistically independent complex Gaussian matrices [7].

Following the above discussion, the RP MIMO channel $\mathbf{H} \in \mathcal{C}^{N\times M} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N \otimes \mathbf{I}_M)$ with K number of scatterers in the propagation environment is defined as [3]

$$\mathbf{H} = \frac{1}{\sqrt{K}}\mathbf{H}_1\mathbf{H}_2, \tag{1}$$

where $\mathbf{H}_1 \in \mathcal{C}^{N\times K} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N \otimes \mathbf{I}_K)$ and $\mathbf{H}_2 \in \mathcal{C}^{K\times M} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_K \otimes \mathbf{I}_M)$ are random matrices.

For the conventional system model (Fig. 1(a)), the received signal can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \tag{2}$$

where $\mathbf{x} \in \mathcal{C}^{M\times 1}$ is the zero-mean transmit Gaussian vector with the covariance matrix $\mathbb{E}[\mathbf{x}\mathbf{x}^{\mathsf{H}}] = \mathbf{Q} = \frac{\rho}{M}\mathbf{I}_M$, and $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ denotes the Additive White Gaussian Noise (AWGN) vector at the receiver.

Let $\boldsymbol{\eta}_{\mathrm{t}}$ and $\boldsymbol{\eta}_{\mathrm{r}}$ denote the distortion noises resulted from the residual impairments at the transmitter-side and receiver-side, respectively, as depicted in Fig. 1(b). With the help of measurements in [10], it has been shown that the transmit and receive distortion noises caused due to additive transceiver hardware impairments are Gaussian distributed with their average power being proportional to the average signal power. Besides, the aggregate impact of many impairments can also be modeled as circularly-symmetric complex Gaussianity [16]. Following this approach (similar to [15, 16]), the distortion noises $\boldsymbol{\eta}_{\rm t}$ and $\boldsymbol{\eta}_{\rm r}$ can be defined as

$$\begin{aligned} \boldsymbol{\eta}_{\rm t} &\sim \mathcal{CN}(\mathbf{0}, \delta_{\rm t}^2 \mathrm{diag}\,(q_1, \ldots, q_M)), \\ \boldsymbol{\eta}_{\rm r} &\sim \mathcal{CN}(\mathbf{0}, \delta_{\rm r}^2 \,\mathrm{tr}\,(\mathbf{Q})\, \mathbf{I}_N), \end{aligned} \tag{3}$$

where $\delta_{\rm t}^2$ and $\delta_{\rm r}^2$ are the proportionality parameters which indicate the severity of the residual additive transceiver hardware impairments in the transmit and receive sides, respectively, and $\mathbf{Q}$ represents the transmit covariance matrix with the diagonal elements $(q_1, \ldots, q_M)$. By substituting the form of $\mathbf{Q}$ in (3), we obtain: $\boldsymbol{\eta}_{\rm t} \sim \mathcal{CN}(\mathbf{0}, \delta_{\rm t}^2 \frac{\rho}{M} \mathbf{I}_M)$, and $\boldsymbol{\eta}_{\rm r} \sim \mathcal{CN}(\mathbf{0}, \delta_{\rm r}^2 \rho \mathbf{I}_N)$.

After including the effect of residual additive hardware impairments in (2), the received signal corresponding to the considered system model in Fig. 1(b) can be written as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}(\mathbf{x} + \boldsymbol{\eta}_{\rm t}) + \boldsymbol{\eta}_{\rm r} + \mathbf{z} && (4)\\ &= \mathbf{h}_m x_m + \sum_{i=1, i\neq m}^{N} \mathbf{h}_i x_i + \mathbf{H}\boldsymbol{\eta}_{\rm t} + \boldsymbol{\eta}_{\rm r} + \mathbf{z}, && (5) \end{aligned}$$

where x_m denotes the signal transmitted from the mth transmit antenna. It can be noted that (5) reduces to the ideal case in (2) for $\delta_{\rm t} = \delta_{\rm r} = 0$.

It should be noted that the parameters $\boldsymbol{\eta}_{\rm t}$ and $\boldsymbol{\eta}_{\rm r}$ are related to the Error Vector Magnitude (EVM), which is a commonly used metric in quantifying signal distortion caused due to residual hardware impairments [10, 13]. In practical scenarios, its typical value can be determined either from the minimum EVM requirements specified on the published standards or from the publications related to the previous implementations. For example, the EVM requirements for Long Term Evolution (LTE) system correspond to $\delta_t \in [0.08 \;\; 0.175]$ [13, 19].

III. MMSE Receiver Performance Analysis

In this section, we investigate the MMSE filtering performance of RP MIMO channels by utilizing some important principles from the asymptotic RMT such as free probability theory. One of the main motivations behind using asymptotic RMT principles in our analysis is that the random results converge quickly to the deterministic ones when the dimensions of the considered channel matrices go to infinity but with a fixed ratio. In addition to the simplicity of obtaining closed-form expressions with this method, the results obtained using this analysis are also sufficiently valid for the finite number of antennas as depicted later in Section IV. For the simplicity of MMSE filtering performance analysis in this paper, we consider linear processing and assume $N = M$.

For the considered RP MIMO channel with residual additive hardware impairments, the average MMSE, let us denote by $\mathrm{MMSE}_{\rm avg}$, can be expressed as

$$\begin{aligned} \mathrm{MMSE}_{\rm avg} &= \mathbb{E}\left[\frac{1}{M}\sum_{m=1}^{M} \mathrm{MMSE}_m\right] \\ &= \mathbb{E}\left[\frac{1}{N}\sum_{m=1}^{M}\left[\left(\mathbf{I}_M + \frac{\rho}{M}\mathbf{H}^{\rm H}\mathbf{R}^{-1}\mathbf{H}\right)^{-1}\right]_{m,m}\right] \\ &= \mathbb{E}\left[\frac{1}{M}\mathrm{tr}\left\{\left(\mathbf{I}_M + \frac{\rho}{KM}\mathbf{H}_2^{\rm H}\mathbf{H}_1^{\rm H}\mathbf{R}^{-1}\mathbf{H}_1\mathbf{H}_2\right)^{-1}\right\}\right] \\ &= \mathbb{E}\,[\,\frac{1}{N}\mathrm{tr}\,\{\,\left(\mathbf{I}_K + \tilde{f}_1 \mathbf{H}_2\mathbf{H}_2^{\rm H}\mathbf{H}_1^{\rm H}\mathbf{H}_1\right)^{-1} \\ &\times \left(\mathbf{I}_K + \tilde{f}_2 \mathbf{H}_2\mathbf{H}_2^{\rm H}\mathbf{H}_1^{\rm H}\mathbf{H}_1\right)\,\}\,]\,, \end{aligned} \tag{6}$$

where $\mathbf{R} = \frac{\rho\delta_{\rm t}^2}{KM}\mathbf{H}_1\mathbf{H}_2\mathbf{H}_2^{\rm H}\mathbf{H}_1^{\rm H} + \left(\rho\delta_{\rm r}^2 + 1\right)\mathbf{I}_N$, $\tilde{f}_1 = \frac{\tilde{\delta}_{\rm t}^2\rho}{KM\tilde{\delta}_{\rm r}^2}$ and $\tilde{f}_2 = \frac{\delta_{\rm t}^2\rho}{KM\tilde{\delta}_{\rm r}^2}$ with $\tilde{\delta}_{\rm t}^2 = 1 + \delta_{\rm t}^2$, $\tilde{\delta}_{\rm r}^2 = 1 + \rho\delta_{\rm r}^2$, and $[\mathbf{A}]_{k,k}$ denotes the kth diagonal element of a square matrix $\mathbf{A}$. For the notational simplicity, we define $\tilde{\mathbf{K}} = \mathbf{H}_2\mathbf{H}_2^{\rm H}\mathbf{H}_1^{\rm H}\mathbf{H}_1$.

In our analysis, we consider the values of M, N, and K going to infinity with the fixed ratios $\beta = \frac{K}{M}$ and $\gamma = \frac{N}{K}$. Subsequently, the average MMSE in (6) can be written as

$$\mathrm{MMSE}_{\rm avg} = \lim_{K,M,N\to\infty} \mathbb{E}\left[\frac{1}{N}\mathrm{tr}\left\{\left(\mathbf{I} + \tilde{f}_1\tilde{\mathbf{K}}\right)^{-1}\left(\mathbf{I} + \tilde{f}_2\tilde{\mathbf{K}}\right)\right\}\right]. \tag{7}$$

Using the trace inequality for the matrix product from [20] in (7), the lower bound for the average MMSE of the RP MIMO channel in the presence of residual additive transceiver impairments can be expressed as

$$\begin{aligned} \mathrm{MMSE}_{\rm avg} &\geq \lim_{K,M,N\to\infty} \mathbb{E}\left[\frac{1}{N}\sum_{n=1}^{N}\frac{1 + \tilde{f}_2\lambda_{K-k+1}\left(\tilde{\mathbf{K}}\right)}{1 + \tilde{f}_2\lambda_k\left(\tilde{\mathbf{K}}\right)}\right] \\ &\to \int_0^1 \frac{1 + F_{\tilde{\mathbf{K}}}^{-1}(1-x)}{1 + \tilde{f}_2 F_{\tilde{\mathbf{K}}}^{-1}(x)}\mathrm{d}x, \end{aligned} \tag{8}$$

where $\lambda_i\left(\mathbf{X}\right)$ denotes the ith ordered eigenvalue of matrix $\mathbf{X}$, and $F_{\mathbf{X}}^{-1}$ is the inverse of the asymptotic cumulative distribution function of $\mathbf{X}$. To compute this, we need to find the corresponding asymptotic probability density function (a.e.p.d.f.), i.e., $f_{\tilde{\mathbf{K}}}^{\infty}(x)$, which can be computed by using the following procedure.

In the RMT literature, one of the widely used methods to obtain the a.e.p.d.f of a function is by means of Stieltjes transform [21]. For the simplicity of the analysis, we define the following notations

$$\tilde{\mathbf{N}}_1 = \mathbf{H}_1^{\rm H}\mathbf{H}_1 \tag{9}$$

$$\tilde{\mathbf{N}}_2 = \mathbf{H}_2\mathbf{H}_2^{\rm H} \tag{10}$$

$$\tilde{\mathbf{K}} = \mathbf{H}_2\mathbf{H}_2^{\rm H}\mathbf{H}_1^{\rm H}\mathbf{H}_1 = \tilde{\mathbf{N}}_2\tilde{\mathbf{N}}_1. \tag{11}$$

Herein, the matrix $\tilde{\mathbf{K}}$ has the same form as the one in [16] and we obtain the Stieltjes transform of $\tilde{\mathbf{K}}$ following the same method, which is described below for the sake of completeness. Using the multiplicative free convolution property of asymptotically free deterministic matrices $\tilde{\mathbf{N}}_1$ and $\tilde{\mathbf{N}}_2$, the Σ transform of $\tilde{\mathbf{K}}$ can be written as

$$\Sigma_{\tilde{\mathbf{K}}}(x) = \Sigma_{\tilde{\mathbf{N}}_2}(x)\Sigma_{\tilde{\mathbf{N}}_1}(x) \Longleftrightarrow \quad (12)$$
$$\left(-\frac{x+1}{x}\right)\eta_{\tilde{\mathbf{K}}}^{-1}(x+1) = \frac{1}{1+\beta x}\frac{1}{\gamma+x}.$$

Next, the inverse of the η transform is calculated using Definition 2 (see Appendix) and (12). Then after employing Lemma 2, the Stieltjes transform $\mathcal{S}_{\tilde{\mathbf{K}}}$ of the asymptotic distribution of eigenvalues of $\tilde{\mathbf{K}}$ can be obtained for any $x \in \mathbb{C}$ by solving the following cubic polymonial

$$\beta x^2 \mathcal{S}_{\tilde{\mathbf{K}}}^3 - (x(1+\beta(\gamma-2)))S_{\tilde{\mathbf{K}}}^2$$
$$-(x + (\beta-1)(\gamma-1))S_{\tilde{\mathbf{K}}} - 1 = 0. \quad (13)$$

Finally, the a.e.p.d.f. of $\tilde{\mathbf{K}}$ is obtained by evaluating the imaginary part of the Stieltjes transform $\mathcal{S}_{\tilde{\mathbf{K}}}$ for the real arguments as in [21, Eq. 2.45]

$$f_{\tilde{\mathbf{K}}}^{\infty}(x) = \lim_{y\to 0^+} \frac{1}{\pi}\mathrm{Im}\{\mathcal{S}_{\tilde{\mathbf{K}}}(x+\mathrm{j}y)\}. \quad (14)$$

IV. Numerical Results

In this section, we illustrate the MMSE filtering performance of RP MIMO channels in the presence of residual hardware impairments with the help of numerical results. In order to illustrate the variation of the average MMSE metric defined in Section III with respect to the transmit SNR, we plot the average MMSE versus ρ in Fig. 2 considering the cases without impairments, i.e., $\delta = 0$ and with impairments for $\delta = 0.08$ and $\delta = 0.15$. For this analysis, the impairment values on both the transmit and receive sides are considered to be the same, i.e., $\delta = \delta_t = \delta_r$. From the figure, it can be noted that the analytical results provide tight lower bounds over the considered SNR range for all the values of the residual transceiver impairments. Furthermore, it can be depicted that in the absence of residual hardware impairments, i.e., in the ideal scenario, the average MMSE decreases with the increase in the value of transmit SNR (ρ). On the other hand, in the presence of residual transceiver impairments, i.e., $\delta_t = \delta_r \neq 0$, the average MMSE first decreases with the increase in the value of ρ and then saturates after a certain value of ρ. Moreover, in Fig. 2, we also illustrate the effect of different values of impairments on the average MMSE. It can be noted that the average MMSE decreases with the increase in the value of δ and the saturation point occurs earlier (i.e., at the lower value of ρ) with the increase in the value of δ.

Figure 3 shows the average MMSE performance versus the number of scatterers K in the considered double scattering environment. In this figure, we plot both theoretical and simulated results for the case without impairments, i.e., $\delta = 0$ and for the case with the additive residual hardware impairments. It can be noted that the average MMSE increases with the increase in the number of scatterers. The intuitive reasoning behind this is that the dense scattering environment results in the degradation in the MMSE filtering performance due to

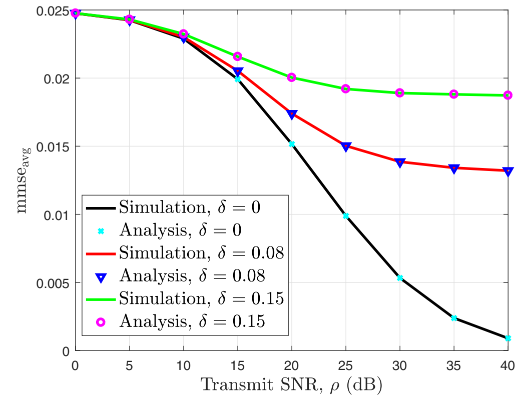

Fig. 2. Average MMSE versus the transmit SNR ρ ($M = 20$, $K = 10$, $N = 20$).

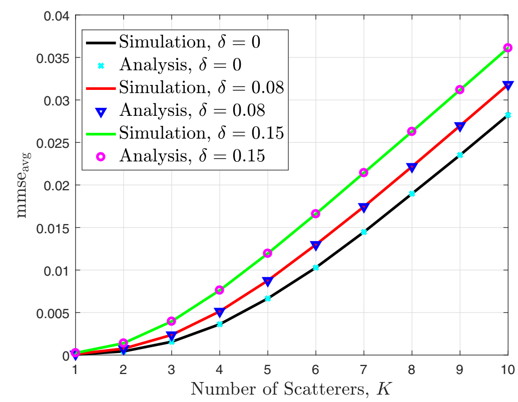

Fig. 3. Average MMSE versus the number of scatterers K ($\rho = 20$ dB, $M = 15$, $N = 15$).

the larger number of multi-path components to be resolved, thus resulting in the increase in the value of $\mathrm{MMSE}_{\mathrm{avg}}$. Besides, as expected, the value of $\mathrm{MMSE}_{\mathrm{avg}}$ increases for the higher values of impairments over all the considered range of the scatterers. It should be noted that the obtained result corresponds to the channel model considered in Section II.

Furthermore, in order to study the effect of transmit and receive impairments on the MMSE filtering performance, we plot $\mathrm{MMSE}_{\mathrm{avg}}$ versus the impairment value $\delta \in [0\ 1]$ in Fig. 4 by considering the parameters $\rho = 15$ dB, $M = 5$, $K = 4$, $N = 5$. For evaluating the effect of transmit-side impairment δ_t, the value of receive-side impairment δ_r is considered to be zero and vice versa. From the figure, it can be depicted that $\mathrm{MMSE}_{\mathrm{avg}}$ increases with the increase in the value of δ for both the cases, however, the effect of receive-side impairment δ_r on the average MMSE is found to be much severe than that of δ_t as noted for the case of DH-AF MIMO relay channels in [15].

V. Conclusions

Taking into account the fact that residual hardware impairments are inevitable in practice, this paper studied the effect of additive residual hardware impairments on the MMSE filtering performance of RP MIMO channels. The lower bound for the average MMSE metric was derived by using tools from

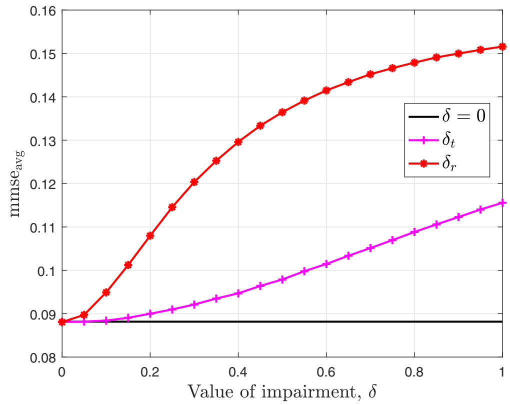

Fig. 4. Average MMSE versus the value of impairment δ for three different cases ($\rho = 15$ dB, $M = 5$, $K = 4$, $N = 5$).

random matrix theory. It has been noted that the derived lower bound is tight over the considered range of transmit SNRs as well as the number of scatterers. Furthermore, it has been concluded that MMSE filtering performance of the considered RP channels first increases with the increase in the value of SNR and then gets saturated after a certain value of SNR. Moreover, it has been shown that MMSE filtering performance degrades significantly with the increase in the number of scatterers and with the increase in the severity of transceiver impairments for all the evaluated cases.

APPENDIX I
PRELIMINARIES ON RANDOM MATRIX THEORY

Let $f_{\mathbf{X}}(x)$ denote the eigenvalue probability density function of a matrix $\mathbf{X}$. In the following, we provide some definitions and lemmas from RMT.

Definition 1 (η-transform [21, Definition 2.11]): The η-transform of the density of eigenvalues of a positive semi-definite matrix $\mathbf{X}$ is given by $\eta_{\mathbf{X}}(\delta) = \int_0^\infty \frac{1}{1+\delta x} f_{\mathbf{X}}(x) \mathrm{d}x$.

Definition 2 (S-transform [21, Definition 2.15]): The S-transform of the density of eigenvalues of a positive semi-definite matrix $\mathbf{X}$ is defined as

$$\Sigma_{\mathbf{X}}(x) = -\frac{x+1}{x}\eta_{\mathbf{X}}^{-1}(x+1). \quad (15)$$

Definition 3 (The Marčenko-Pastur law [22]): For an $M \times K$ matrix $\mathbf{H} \sim \mathcal{CN}\left(\mathbf{0}, \frac{1}{M}\mathbf{I}_M \otimes \mathbf{I}_K\right)$, as $M, K \to \infty$ with $\frac{K}{M} \to \beta$, the a.e.p.d.f. of $\mathbf{H}^{\mathsf{H}}\mathbf{H}$ converges almost surely to a non-random limiting distribution with the following density function

$$f_{\mathbf{H}^{\mathsf{H}}\mathbf{H}}^{\infty}(x) = \left(1 - \frac{1}{\beta}\right)^{+}(x) + \frac{\sqrt{(x-a)^{+}(b-x)^{+}}}{2\pi\beta x}, \quad (16)$$

where $a = (1-\sqrt{\beta})^2$, $b = (1+\sqrt{\beta})^2$, and $\delta(.)$ is a Dirac delta function. Similarly, the a.e.p.d.f. of $\mathbf{H}\mathbf{H}^{\mathsf{H}}$ can be obtained as

$$f_{\mathbf{H}\mathbf{H}^{\mathsf{H}}}^{\infty}(x) = (1-\beta)^{+}(x) + \frac{\sqrt{(x-a)^{+}(b-x)^{+}}}{2\pi x}. \quad (17)$$

Lemma 1 ([21, Eqs. 2.87, 2.88]): The S-transforms of the matrices $\mathbf{H}^{\mathsf{H}}\mathbf{H}$ and $\mathbf{H}\mathbf{H}^{\mathsf{H}}$ are given by

$$\Sigma_{\mathbf{H}^{\mathsf{H}}\mathbf{H}}(x,\beta) = \frac{1}{1+\beta x}, \Sigma_{\mathbf{H}\mathbf{H}^{\mathsf{H}}}(x,\beta) = \frac{1}{\beta + x}. \quad (18)$$

Lemma 2 ([21, Eq. 2.48]): The Stieltjes-transform of a positive semidefinite matrix $\mathbf{X}$ can be obtained by its η-transform using $\mathcal{S}_{\mathbf{X}}(x) = -\frac{\eta_{\mathbf{X}}(-1/x)}{x}$.

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