The War of Rare Earth Elements: A Dynamic Game Approach

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Abstract

Rare earth elements govern today's high-tech world and are deemed to be essential for the attainment of sustainable development goals. Since the 1990s, these elements have been predominantly supplied by one single actor, China. However, due to the increasing global relevance of their availability, other countries are now encouraged to enter the market. The objective of this paper is to analyze the strategic interactions among (potential) suppliers. In particular, we are interested in (1) the optimal timing for a newcomer (e. g. the U.S.) to enter the market, (2) the incumbent's (i. e. China's) optimal behavior, and (3) the cost-efficiency of cooperative vs. competitive market relations. By setting up a continuous-time dynamic game model, we show that (1) the newcomer should postpone the production launch until its rare earth reserves coincide with those of the incumbent, (2) the incumbent should strive for a late market entry and therefore keep its monopolistic resource extraction at the lowest possible level, (3) compared to the payoffs under competition, cooperation leads to a Pareto improvement when started at an early stage. The findings of our model are particularly relevant for the rational strategic positioning of the two great powers, America and China.

Key words: rare earth elements, dynamic games, open-loop strategic Nash equilibria

JEL Classification: C61, C7, Q3

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1 Introduction

In December 2017, the U.S. Department of the Interior published a list of "critical" mineral commodities, that is, minerals that are "essential to the economic and national security of the United States" and whose "supply chain is vulnerable to disruption" (Federal Register, 2017). Among the 35 critical minerals that are listed in this executive order, nearly half correspond to so-called rare earth elements (REEs). Since America predominantly depends on REEs from China, the country recently announced its plan to invest in the development of its national production capacities. Through the use of continuous-time dynamic game models, the present study therefore aims to define the time at which the production launch should optimally be triggered and to determine how the incumbent can best defend its current monopolistic position.

1.1 History of rare earth elements

Rare earth elements are a group of 17 chemical elements in the periodic table (Connelly et al., 2005)[1] for which no backstop technology exists.[2] Owing to their very special electrochemical, luminescent, and magnetic properties, these elements are key raw materials for advanced technologies, used worldwide in crucial fields such as the energy, military, automotive, or communication sectors (Goodenough et al., 2018). Despite their many common properties, each REE has its particular characteristics, making them unsubstitutable between each other, that is, different usages require specific REEs. For this reason, the ongoing launch and disappearance of products in the technology market also results in a continuously changing demand for each individual elements. However, matching the supply of individual REEs with their varying demand is a big challenge that can lead to significant market imbalances and thus to large price fluctuations (Schlinkert and van den Boogaart, 2015). This challenge is often referred to as the "balancing problem" (Neary and Highley, 1984; Binnemans et al., 2013) and comes from the fact that on the one hand rare earths do not occur in nature as native

[1] The 15 lanthanides (at. no. 57–71) plus scandium (at. no. 21) and yttrium (at. no. 39).

[2] Following the definition of Nordhaus et al. (1973), "the backstop technology is a set of processes that (1) is capable of meeting the demand requirements, and (2) has a virtually infinite resource base".

metals but are contained in different concentrations and proportions with other REEs in certain minerals,[3] and on the other hand the individual REEs cannot be mined separately. Meeting the demand for all REEs therefore induces supply surpluses of some elements.

Although the global demand for rare earths has so far always been met, ensuring their future availability recently became a central challenge: while the annual difference between the overall rare earth supply and demand used to be largely positive, it started decreasing as of 2005 and even became negative in 2010 (Massari and Ruberti, 2013).

A first reason for this most recent challenge is the steeply growing demand for REEs. More precisely, after their discovery in the 18th century, the demand for rare earth elements only became significant as of the mid-20th century and has been increasing ever since (Zhou et al., 2017). To date, as governments and industries worldwide strive to mitigate the devastating effects of climate change, the demand for green technologies (for example, electric and hybrid vehicles, wind turbines, solar panels, and rechargeable batteries) is expected to face tremendous growth over the coming decades (Dudley, 2017). Since the manufacturing of these sustainable products is heavily dependent on REEs,[4] the emergence of this sector is assumed to greatly contribute to the ongoing increase in the demand for rare earths (Alonso et al., 2012).

A second reason behind the availability challenge is the monopolistic structure of the rare earth supply market: from World War II until the 1960s, the demand for REEs was mainly covered by India, Brazil, and South Africa; between the 1960s and the late 1980s, the United States (U.S.) took over the lead in world production; by the 1990s, after eliminating its competitors by practicing dumping prices, China became the world's largest producer of REEs (Gordon B. Haxel and Orris, 2002) and covered 98% of global production until 2010 (Chakhmouradian and Wall, 2012). This monopolistic market situation is, however, not due to the common assumption that REEs are particularly rare by any geological measure: unlike their name suggests, these elements have an abundance in the earth's crust that is comparable to that of copper, lead, nickel, and zinc (Krishnamurthy and Gupta, 2015). Thus, although China holds most (about one third)

[3] For example, bastnaesite, monazite, xenotime, or loparite. Note that the concentration and proportion of the individual elements varies with the type of mineral and the deposit's geographical location.

[4] Especially on neodymium and dysprosium (Chu, 2010).

of the world's proven rare earth reserves, it is not the only country where these elements occur (Zhou et al., 2017). What has actually contributed to China's dominant market position so far is the lower operational costs in its rare earth industry (unregulated and unlicensed mines), as well as the environmental risks that are associated with the mining and processing of these elements, which have prevented other countries from harvesting their national deposits (Mancheri, 2015).

Despite the growing demand for REEs and the dependency on China's rare earth exports are not new phenomena, they were globally ignored until 2010, when China withheld exports to Japan, resulting in extensive media coverage and market panic. Indeed, this was not the only time that the country imposed restrictions on its exports (Mancheri, 2015). While China argues that their export cuts are meant to preserve the environment and its natural resources (Trujillo, 2015), others believe that they serve the purpose of encouraging producers that rely on REEs to relocate their facilities to China (Mancheri, 2015). In 2012, the U.S., the European Union, and Japan therefore officially filed a complaint with the World Trade Organization (WTO), claiming that China's export restraints violate WTO rules. Two years later, the WTO Appellate Body ruled in favor of the prosecutors and concluded that China had broken free trade agreement. As a consequence, China had to remove its export tariffs and quotas (Trujillo, 2015). Nevertheless, till today, their exports rely on licenses and remain quite unstable.

This precarious market situation recently triggered certain countries (for example, the U.S.) to declare REEs as critical to national interests and to contemplate the possibility of resuming their own end-to-end manufacturing lines (Chu, 2010; Chapman et al.; British Geological Survey, 2015; Federal Register, 2017).

1.2 Related literature

In the present paper, we analyze the strategic alignment of (potential) suppliers in the rare earth market, where the incumbent's monopolistic position is threatened by another country's entry plans. More specifically, we aim to define (1) the optimal time for a newcomer to trigger its resource extraction; (2) the incumbent's optimal reaction to the newcomer's entry announcement; and (3) whether or not the countries can be better off by immediately putting their resources together and starting to cooperate.

In the existing literature on strategic games in natural resource markets, there exists, to the best of our knowledge, no work on REEs, let alone on the competition between two countries for these "critical" resources. With the present study, we try to fill this gap. So far, it is typical for models in the related literature to consider the existence and ownership of Nordhaus' concept of a backstop technology (Nordhaus et al., 1973). For example, Gilbert et al. (1978) analyze the optimal price strategy of a resource monopolist who is threatened by the potential entry of competitors that own a backstop technology. This means that under their setting, the potential competitors do not have the exhaustibility constraint of the monopoly. In the rare earth market, however, it is very likely that the newcomer holds smaller initial reserves than the incumbent. In related work, Gilbert (1978) offers an intertemporal von Stackelberg equilibrium by setting up a model where one dominant firm determines the price, taking into account the other firms' reactions. In this market, all firms supply a homogeneous resource, and the reserve as well as the costs of the backstop technologies are perfectly known. In a more recent contribution, Harris et al. (2010) study nonzero-sum Cournot differential games resulting from resource depletion in a competitive framework. To approximate the impact of the finite resources on the Cournot market price and production, the authors introduce, inter alia, a backstop technology that is owned by the suppliers. Although our model setting relies on the latter literature, due to the non-existence of a backstop technology for REEs, we do not include this concept in our analysis. Furthermore, our work is closely related to a series of publications by Stiglitz (1976), Stiglitz and Dasgupta (1981a), and Dasgupta et al. (1982), in which the authors analyze the effects of different market structures on the rate of exploitation of an exhaustible natural resource, respectively on the substitute's timing of innovation. Finally, we also follow Stiglitz and Dasgupta (1981b, 1982), who study how uncertainty about the discovery date of a substitute affects the rate of resource extraction under diverse market structures. In contrast to their settings, given the rare earth market structure, we are unable to consider perfectly competitive markets.

In our work, we set up a continuous-time model with two periods and two players and search for open-loop strategic Nash equilibria. Similarly to the setting of Loury (1986), at the beginning of the second period, where the duopolistic competition starts, each country commits itself irrevocably to a supply strategy that maximizes its discounted profit in consideration of the other country's strategy. Working backwards in time, we then define the monopolistic supply commitment of the incumbent in the first time period.

Our results show that (1) the incumbent must first shrink its rare earth stock to the same level as that of the potential new supplier before it becomes profitable for the newcomer to enter the market; (2) unlike the newcomer, the incumbent does not benefit from an early entry and is hence most likely to postpone the timing of the entry by adopting a conservative extraction behavior during its monopolistic market position; and (3) when the two countries agree to cooperate from the moment of the entry announcement, they will both be better off.

As concerns the structure of the article, it is split into two main sections: Section 2, which covers the non-cooperative game, and Section 3, which outlines the cooperative game. Both sections start with a description of the model set-up, followed by the deviation of the countries' rare earth extraction rate, the market price, and the countries' payoffs. Section 3 completes the study with a comparison of each country's non-cooperative and cooperative revenues. In Section 4, we discuss the topicality of our study. The last section concludes and presents strategy recommendations.

2 Non-cooperative game

In this section, we give a detailed description of the continuous-time dynamic model where players are contenders that act selfishly. Beyond that, we illustrate the solutions to the optimal control problems and draw some initial conclusions.

2.1 The model setting

Consider country C the monopolistic supplier in the rare earth market in a first time period $I = [0, T^*)$, where country A announces at time 0 that it will enter the market and C commits to a supply strategy that cannot be reconsidered or changed, and where T^* is the optimal time for the entry. At time T^*, which marks the beginning of the second time period $II = [T^*, +\infty)$, the market changes from a monopolistic structure to a duopolistic one, and each country chooses a supply path that is the optimal response to its competitor's path.

Note that in this game, we focus on open-loop strategic Nash equilibria (OLSNE) and not on Markovian ones. This modelling choice can be explained as follows: (1) although Nash equilibria in Markovian strategies are subgame perfect, they are independent of the initial state of the game. However, in our model, the initial state of the countries' reserves is essential as it is likely to affect their supply strategies; (2) in order to get Markovian strategic Nash equilibria, the players must, at any time in the game, know their competitors' reaction to a change in the value of the reserves and consider this information to optimally revise their supply strategies. In the rare earth supply game, however, it is plausible to assume that market participants are either not willing or not able to do so, for two reasons. Firstly, when rare earth suppliers commit to a supply path right at the beginning of the game and stick to that strategy, this leads to a stable investment environment, which keeps the importing countries from looking for alternative resources and thus prevents the suppliers' reserve-value from dropping. Secondly, Markovian strategies are costly in time and money: not only must analysts observe and assess the opponents' strategic behavior before defining the optimal reaction, but the countries must also be able to quickly adapt their supply paths, which is only possible under a high efficiency of their logistics and supply chain management; (3) Harris et al. (2010) proved that if no backstop technology and its ownership are explicitly introduced in the setting, the application of Markovian strategic Nash equilibria may lead to a situation where the existence of a solution cannot be guaranteed. The introduction of a backstop technology is, however, beyond the scope of the present study.

Following Stiglitz (1976), we consider a price function of the form

$$P(t) = a\,Q^{\alpha-1}(t), \quad \alpha \in (0,1), \tag{1}$$

where $\frac{1}{1-\alpha}$ is the price elasticity of demand, a is a positive constant, and where at time t, the total market supply $Q(t)$ is the sum of A's supply $q_A(t)$ and C's supply $q_C(t)$. It is also assumed that at time t, the supply of country $i \in \{A, C\}$ equals the amount of resources that is extracted from its remaining reserves $R_i(t)$ and that $R_C(0) >> R_A(0)$.

Other than that, we observe that country A will not:

- enter the market at $T^* = 0$. In this case, A's resource extraction starts at the beginning of the game. However, based on the findings of Loury (1986) and Harris et al. (2010), the country with the smallest reserve will be the first to exhaust its resources—meaning that C can take advantage of its larger reserve. More specifically, during the period of duopolistic competition, C can increase its rare earth supply to lower the market price P. This results in a decrease in both countries' revenue π_i. Then, after the resource exhaustion has forced A to exit the market, C can compensate for its "losses" in the duopolistic period by again decreasing its supply to raise the market price P. Manifestly, under this scenario, the payoff of country A cannot be optimal;

- enter the market at $T^* = +\infty$. This means that country A never enters the market, so that its resources remain of no value. As a result, A's payoff $\pi_A = 0$, which is also not optimal;

- wait to enter the market until country C has exhausted its entire resources in finite time, that is $T^* < +\infty$ and $R_C(T^*) = 0$. Indeed, if this is the optimal strategy for A, then there is no reason for C to exhaust its resources first. Instead, C rather stops its market supply before $R_C(t) = R_A(0)$, which leads to the failure of A's strategy.

The above observations lead us to assume that country A starts to supply the market with REEs at some time $T^* \in (0, +\infty)$, at which time the reserve of C is $R_C(T^*) = R_C^* > 0$ and the market price is $P(T^*) = P^*$. Based on these assumptions, we will demonstrate the existence of T^* and define its exact value in the following sections.

2.2 Rare earth extraction and market prices under competition

In order to determine the optimal entry condition for country A, we start with the second period. Here the non-cooperative suppliers' revenue is

$$\pi_i^{II}(R_C^*, T^*) = \max_{q_i^{II}(t)} \int_{T^*}^{+\infty} e^{-rt} P^{II}(Q^{II}(t))\, q_i^{II}(t)\, dt,$$

subject to

$$\int_{T^*}^{+\infty} q_i^{II}(t)\, dt \leq R_i(T^*) = \begin{cases} R_C^*, \text{ for } i = C \\ R_A(0) \text{ given, for } i = A \end{cases}, \quad q_i^{II}(t) \geq 0,$$

and

$$\dot{R}_i(t) = -q_i^{II}(t)\,, \quad t \in [T^*, +\infty),$$

where e^{-rt} is a time-preference factor with rate r.

The Lagrangian function of country i is set up as follows:

$$\mathcal{L}_i^{II}\left(q_i^{II}(t), \lambda_i^{II}(t), \alpha_i^{II}\right) = P^{II}(Q^{II}(t))q_i^{II}(t) - \lambda_i^{II}(t)q_i^{II}(t) - \alpha_i^{II}\left(R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\, d\tau\right),$$

where $\lambda_i^{II}(t)$ is country i's shadow price at time $t \in [T^*, +\infty)$, that is, the change in its discounted revenue resulting from an extra unit of its remaining reserves $R_i(t)$, and where α_i^{II} is the static Lagrange multiplier. From the standard first-order conditions (FOCs), we get that the shadow price $\lambda_i^{II}(t)$ grows at interest rate r, which is compounded continuously:

$$\lambda_i^{II}(t) = \lambda_i^{II}(T^*)e^{r(t-T^*)}, \tag{2}$$

and this corresponds to the marginal revenue:

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = \lambda_i^{II}(t).$$

Additionally, we find that the derivative of country i's instantaneous revenue function $RV_i^{II}(t) = P^{II}(t)q_i^{II}(t)$ with respect to its optimal supply path $q_i^{II}(t)$ is

$$\lambda_i^{II}(t) = a\left(Q^{II}(t)\right)^{\alpha-1}\left(1 - \frac{(1-\alpha)q_i^{II}(t)}{q_A^{II}(t) + q_C^{II}(t)}\right) = \alpha P^{II}(t). \tag{3}$$

Using Equation (3) allows us to express $q_i^{II}(t)$ as a function that depends in particular on the second-period's initial shadow price $\lambda_i^{II}(T^*)$. To find this price, we consider that over $[T^*, +\infty)$, each country will entirely exhaust its initial reserves $R_i(T^*)$ because from an economical viewpoint, it cannot be optimal to leave some REEs in the deposit (no market value). Thus, integrating the supply functions $q_i^{II}(t)$ over $[T^*, +\infty)$ yields

$$\lambda_A^{II}(T^*) = \frac{R_C^* + \alpha R_A(0)}{R_A(0) + \alpha R_C^*}\,\lambda_C^{II}(T^*), \tag{4}$$

and

$$\lambda_C^{II}(T^*) = a\alpha\left(\frac{r}{1-\alpha}\left(R_A(0) + R_C^*\right)\right)^{\alpha-1}. \tag{5}$$

Then, from Equations (4) and (5), both countries' optimal choice and the market price can easily be deduced. The ensuing proposition presents our results.

Proposition 1 *Suppose that country A enters the market at time $T^* \in (0, +\infty)$, when the market price is $P(T^*) = P^*$ and C's reserve is $R_C(T^*) = R_C^* > 0$. Then, for any $t \geq T^*$, the second-period OLSNE*[5] *supply of C and A are given, respectively, by*

$$q_C^{II}(t) = \frac{r}{1-\alpha}\,R_C^*\,e^{\frac{r(t-T^*)}{\alpha-1}}, \tag{6}$$

and

$$q_A^{II}(t) = \frac{r}{1-\alpha}\,R_A(0)\,e^{\frac{r(t-T^*)}{\alpha-1}}. \tag{7}$$

It follows that the OLSNE market price is

$$P^{II}(t) = a\left(\frac{r}{1-\alpha}\left(R_A(0) + R_C^*\right)\right)^{\alpha-1} e^{r(t-T^*)}. \tag{8}$$

The detailed proof of this proposition can be found in the appendix.

[5] OLSNE stands for open-loop strategic Nash equilibrium.

The latter proposition shows that the ratio of the two countries' supply is constant and determined only by their initial stocks $R_i(T^*)$:

$$\frac{q_A^{II}(t)}{q_C^{II}(t)} = \frac{R_A(0)}{R_C^*}, \quad \forall t \geq T^*. \tag{9}$$

Since the right-hand side (RHS) of Equation (9) is constant over time, so is the left-hand side (LHS). Furthermore, Equations (6) and (7) show that at time T^*, the supply of both countries $q_i^{II}(T^*)$ depends positively on their initial reserves $R_i(T^*)$. That is to say that the smaller the country's reserve, the less it is willing to extract REEs. This behavior remains unchanged over time, as the supply is a decreasing function of t. Moreover, with the market price being a decreasing function of the supply, the opposite effects can be observed in Equation (8). This equation also gives country C's trigger reserve R_C^* as a decreasing function of the trigger price P^*:

$$R_C^*(P^*) = \frac{1-\alpha}{r}\left(\frac{P^*}{a}\right)^{\frac{1}{\alpha-1}} - R_A(0), \tag{10}$$

so that the later A enters the market, the higher is P^* and the lower will be R_C^*.

To determine C's trigger reserve R_C^*, we rely on Equation (3) and find that

$$\lambda_A(t) = \lambda_C(t). \tag{11}$$

By combining Equations (2) and (4), Equation (11) is satisfied if and only if $R_C^* = R_A(0)$. We can therefore conclude that

Corollary 1 *Country A enters the rare earth supply competition at time T^*, at which time the market price is $P^{II}(T^*) = P^*$ and C's reserve is*

$$R_C(T^*) = R_C^* = R_A(0).$$

The above analysis mathematically proves the observation that A enters the competition at some time $T^* \in (0, +\infty)$. Besides that, it shows that country A should hold off its market entry until C's supply has shrunk its stock to the same level as that of A. This, however, also implies that A has no influence on the time of its entry but totally depends on the monopolist's extraction behavior. In other words, the quicker country C extracts its REEs in the first time period, the faster the initial reserves of C and A coincide and the earlier country A enters the market. For this reason, let us now turn to the monopolistic supplier and study the impacts of its extraction behavior on the optimal entry timing T^*.

2.3 Rare earth extraction and market prices under monopoly

This subsection defines the monopolist's unalterable extraction commitment on the basis of how the production launch of country A affects C's supply at time T^*. Actually, it may be that country C is incapable of changing its supply volume at the moment of A's entry. In this case, C's supply is continuous at T^*: $q_C^I(T^*) = q_C^{II}(T^*)$. Of course, there can be many reasons for such volume inflexibility. For instance, one may think of a situation where C is unable to rapidly adjust the workforce or machinery utilization within its manufacturing plants, or where inelastic supply and distribution networks complicate the modification of product procurement and delivery. Either way, whenever A enters the game, the total offer of REEs will instantly increase, causing the market price to decrease at T^*: $P^I(T^*) > P^{II}(T^*)$. Another possibility is that C perfectly controls the different processes of its rare earth supply chain, so that the country is capable of changing its supply at A's entry. In addition, regardless of whether or not country A is present in the market, C does not value its remaining reserve any differently. This entails that C's shadow price is continuous at T^*: $\lambda_C^I(T^*) = \lambda_C^{II}(T^*)$. Furthermore, given the linear dependency between the shadow price and the market price (see Equation (3)), the market price is also continuous at the newcomer's entry. For this to be satisfied, country C must, at T^*, decrease its offer by the quantity of A's supply: $q_C^I(T^*) - q_A^{II}(T^*) = q_C^{II}(T^*)$. In the following, the two cases will be examined more closely.

In both cases, the monopolist faces the price function of Equation (1) and chooses a supply path that maximizes its revenue, which is given by

$$\pi_C^I(R_C(0) - R_C^*, T^*) = \max_{q_C^I(t)} \int_0^{T^*} e^{-rt} P^I(q_C^I(t))\, q_C^I(t)\, dt,$$

subject to

$$\int_0^{T^*} q_C^I(t)\, dt = R_C(0) - R_C^*\,, \quad R_C(0)\ \text{given}\,, \quad q_C^I(t) \geq 0,$$

and

$$\dot{R}_C(t) = -q_C^I(t)\,, \quad t \in [0, T^*).$$

By applying similar calculations as in the appendix, we find that at time $t \in [0, T^*)$, country C's shadow price is

$$\lambda_C^I(t) = \lambda_C^I(T^*) e^{r(t-T^*)}, \tag{12}$$

and that its marginal revenue is

$$\frac{\partial RV_C^I(t)}{\partial q_C^I(t)} = a\alpha \left(q_C^I(t)\right)^{\alpha-1} = \alpha P^I(t) = \lambda_C^I(t). \tag{13}$$

Substituting (12) into (13) and rearranging terms gives C's first-period supply function:

$$q_C^I(t) = \left(\frac{\lambda_C^I(T^*)}{a\alpha} e^{r(t-T^*)}\right)^{\frac{1}{\alpha-1}}. \tag{14}$$

From here, we can determine the first-period supply and market price in both cases. Henceforth, the continuous supply (resp. price) case is denoted by s (resp. p).

Case 1: Continuous supply

In the case where country C's supply volume is inflexible and thus continuous at the time of A's market entry, we get that

$$\lambda_{C,s}^I(T_s^*) = a\alpha \left(\frac{r}{1-\alpha} R_C^*\right)^{\alpha-1}. \tag{15}$$

Substituting (15) into (14) yields the results of the next proposition.

Proposition 2 *If country C is incapable of adjusting its supply when country A enters the* *market at time T_s^*, then for any $t \in [0, T^*)$, the optimal supply path of country C is given by*

$$q_{C,s}^I(t) = \frac{r}{1-\alpha}\, R_C^*\, e^{\frac{r(t-T_s^*)}{\alpha-1}}, \tag{16}$$

and the optimal market price is

$$P_s^I(t) = a\left(\frac{r}{1-\alpha}\, R_C^*\right)^{\alpha-1} e^{r(t-T_s^*)}. \tag{17}$$

Obviously, the monopolistic supply follows the same functional form as C's supply in the second time period (see Equation (6)).

To define the optimal time T_s^* for country A to enter the competition, we take integrals over $[0, T_s^*)$ on both sides of the dynamic equation $\dot{R}_C(t) = -q_{C,s}^I(t)$, which leads to the proposition below.

Proposition 3 *If country C is unable to adapt its supply at A's market entry, then the entry happens at time*

$$T_s^* = \frac{1-\alpha}{r} \ln\left(\frac{R_C(0)}{R_A(0)}\right). \tag{18}$$

Case 2: Continuous price

Whenever country C's flexible supply allows its shadow price to remain unaffected by A's market entry at T_p^*, we get:

$$\lambda_{C,p}^I(T_p^*) = \lambda_C^{II}(T_p^*) = a\alpha\left(\frac{r}{1-\alpha}\,(R_A(0) + R_C^*)\right)^{\alpha-1}. \tag{19}$$

Substituting (19) into (14) gives the results of the following proposition.

Proposition 4 *If country C has the capacity to change its supply when country A enters the market at time T_p^*, then for any $t \in [0, T^*)$, the optimal supply path of country C is given by*

$$q_{C,p}^I(t) = \frac{r}{1-\alpha}\,(R_A(0) + R_C^*)\; e^{\frac{r(t-T_p^*)}{\alpha-1}}, \tag{20}$$

and the optimal market price is

$$P_p^I(t) = a\left(\frac{r}{1-\alpha}(R_A(0)+R_C^*)\right)^{\alpha-1} e^{r(t-T_p^*)}. \tag{21}$$

Again, integrating the dynamic equation $\dot{R}_C(t) = -q_{C,p}^I(t)$ over $[0, T_p^*)$ on both sides gives us the optimal time T_p^* for country A to enter the market. The solution is illustrated in the subsequent proposition.

Proposition 5 *If country C manages to alter its supply at country A's market entry, then the entry happens at time*

$$T_p^* = \frac{1-\alpha}{r}\ln\left(\frac{R_C(0)+R_A(0)}{2R_A(0)}\right). \tag{22}$$

Comparing the results of Proposition 3 and 5 leads to the following conclusion:

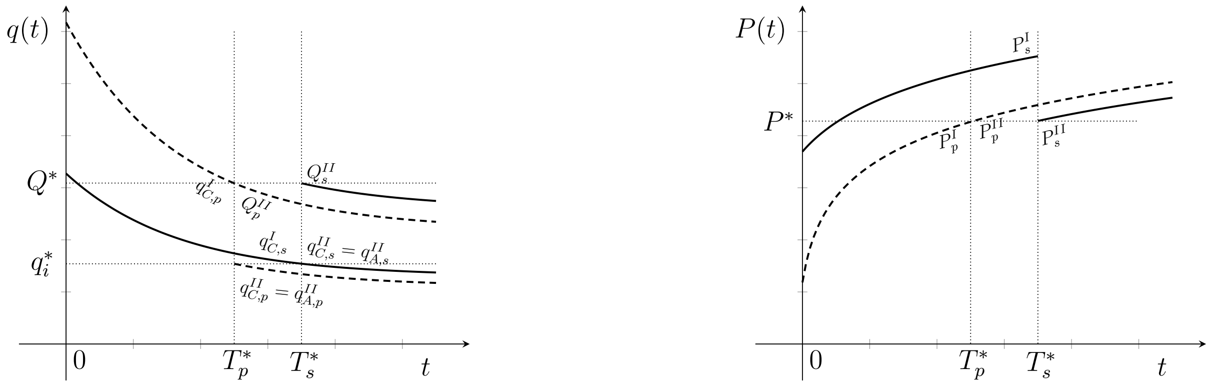
Corollary 2 *If, instead of adjusting its supply, country C keeps its supply unchanged at country A's market entry, then the entry takes place at an earlier point in time, that is, $T_s^* > T_p^*$.*

This corollary actually results from the fact that the monopolistic supplier extracts its REEs faster if the price, instead of the supply, is continuous, that is, $q_{C,s}^I(t) < q_{C,p}^I(t)$. More specifically, whenever C is unable to adjust its supply, the country keeps its offer at a moderate level to ensure that the supply growth at A's entry does not cause a collapse of the market price, and thus of its stock's value. On the other hand, if C's supply is flexible, the country can more easily balance out market fluctuations and is hence less cautious with its monopolistic extraction volume. In addition, as a consequence of the finding that $q_{C,s}^I(t) < q_{C,p}^I(t)$, the market price of the continuous price case is below that of the continuous supply case, that is, $P_s^I(t) > P_p^I(t)$. This is illustrated in Figure 1.

Remark 1 *Since the entry time T^* varies with country C's first-period supply behavior, we wish to emphasize that each T^* prior to Equation (15) must be interpreted as T_θ^*, where $\theta = \{p, s\}$.*

This remark yields that in the continuous price case, the second-period supply of both countries is below that of the continuous supply case, that is, $q_{i,s}^{II}(t) > q_{i,p}^{II}(t)$. This is because the same initial reserve $R_i(T_\theta^*)$ is depleted over a longer time period. Consequently, the opposite is true for the market price, that is, $P_s^{II}(t) < P_p^{II}(t)$. This is also portrayed in Figure 1.

Figure 1 – Monopolistic extraction behavior and consequences



2.4 Financial effects of the incumbent's reaction to a newcomer's market entry

In this subsection, we extend the above analysis and focus on the revenue of both market participants. More precisely, we study how country C's response to A's market entry affects their revenues and determine which reaction is most profitable for country i. For this purpose, i's revenue is defined in the case of continuous supply (resp. price), in both time periods.

In the first time period, the revenue of country C is given by

$$\pi^I_{C,\theta} = \int_0^{T^*_\theta} e^{-rt} P^I_\theta(t)\, q^I_{C,\theta}(t)\, dt,$$

where $\theta = \{p, s\}$. From the findings of Propositions 2 and 3, as well as Corollary 1, it follows that whenever C decides to keep its supply continuous at T^*_θ, its revenue is

$$\begin{aligned} \pi^I_{C,s} &= a \left(\frac{r}{1-\alpha}\right)^{\alpha-1} (R^*_C)^\alpha \left(e^{\frac{\alpha r T^*_s}{1-\alpha}} - e^{-rT^*_s}\right) \\ &= a \left(\frac{r}{1-\alpha} R_C(0)\right)^{\alpha-1} \left(R_C(0) - R_A(0)\right). \end{aligned} \tag{23}$$

Whereas, on the other hand, when the market entry causes a drop in C's supply, its revenue is

$$\begin{aligned}\pi^{I}_{C,p} &= a\left(\frac{r}{1-\alpha}\right)^{\alpha-1}(R_A(0)+R^*_C)^{\alpha}\left(e^{\frac{\alpha r T^*_p}{1-\alpha}}-e^{-rT^*_p}\right)\\ &= a\left(\frac{r}{1-\alpha}(R_C(0)+R_A(0))\right)^{\alpha-1}(R_C(0)-R_A(0)).\end{aligned} \tag{24}$$

Moreover, since C is the monopolistic supplier in this first period, country A's revenue is $\pi^{I}_{A,\theta}=0$. When comparing the payoffs in Equations (23) and (24), we find that the first-period revenue of country C is greater in the continuous supply case, that is, $\pi^{I}_{C,s}>\pi^{I}_{C,p}$; this seems normal as the later market entry T^*_s extends the monopolistic period.

We now turn to the second time period, where country i's revenue is given by

$$\pi^{II}_{i,\theta}=\int_{T^*_\theta}^{+\infty} e^{-rt}P^{II}_{\theta}(t)\; q^{II}_{i,\theta}(t)\; dt.$$

Here, using the findings of Proposition 1 and Corollary 1 yields that in the continuous supply case, both countries' revenue is

$$\pi^{II}_{A,s}=\pi^{II}_{C,s}=a\,R_A(0)\left(\frac{r}{1-\alpha}2R_C(0)\right)^{\alpha-1}, \tag{25}$$

and that in the continuous price case, their revenue is

$$\pi^{II}_{A,p}=\pi^{II}_{C,p}=a\,R_A(0)\left(\frac{r}{1-\alpha}\left(R_C(0)+R_A(0)\right)\right)^{\alpha-1}. \tag{26}$$

This time, the second-period revenue of Equation (25), discounted to the present value, is below that of Equation (26), that is, $\pi^{II}_{i,s}\,e^{-rT^*_s}<\pi^{II}_{i,p}\,e^{-rT^*_p}$. As a result, country A will be better off if the monopolistic supplier exploits its reserves rapidly, so that its entry is shifted further forward. For country C however, its entry-reactions have opposite effects on its first- and second-period revenue. That is why C's best reaction is determined through the aggregated revenue, which is given by

$$\Pi_{C,\theta}=\pi^{I}_{C,\theta}+\pi^{II}_{C,\theta}\,e^{-rT^*_\theta}. \tag{27}$$

Based on Equation (27), we find that the monopolistic supplier's aggregated revenue is greater in the continuous supply case, that is, $\Pi_{C,s} > \Pi_{C,p}$. Accordingly, even when C's supply volume is flexible, the country has no financial incentive to adjust its supply at T^*_θ but should rather supply the market with just enough REEs to avoid the price reaching a level at which a backstop technology could appear. The above analysis is aggregated in the following proposition.

Proposition 6 *Suppose that A's optimal entry time is* T^*_θ, *where* $\theta = \{p, s\}$. *Then*

(1) $\pi^I_{C,s} > \pi^I_{C,p}$;

(2) $\pi^{II}_{A,\theta} = \pi^{II}_{C,\theta}$;

(3) $\pi^{II}_{i,s}\, e^{-rT^*_s} < \pi^{II}_{i,p}\, e^{-rT^*_p}$, $\ i = \{A, C\}$;

(4) $\Pi_{C,s} > \Pi_{C,p}$, $\ \Pi_{A,s} < \Pi_{A,p}$, *with* $\ \Pi_{A,\theta} = \pi^{II}_{A,\theta}\, e^{-rT^*_\theta}$.

In other words, country C keeps its supply continuous at the market entry, and therefore, the non-cooperative game is suboptimal for country A. To analyze whether or not cooperation leads to a Pareto improvement, in the next section we set up a game where the countries can cooperate and coordinate their strategies.

3 Cooperative game

Consider a model[6] where countries A and C start to cooperate right at the beginning of the game, and suppose that their cooperation lasts forever. In this case, the time in the joint period $t \in [0, +\infty)$, where at time 0 both countries collectively and definitively commit themselves to a joint supply strategy $q^j(t)$ that maximizes their joint revenue RV^j. Furthermore, given that no differences in the exploitation or manufacturing processes of the two countries are considered in the model, their joint revenue RV^j is split based on the share of their initial reserves $R_i(0)$. Apart from that, the price function $P^j(t)$

[6]We keep the notations of Section 2.

of this game is identical to that in the non-cooperative game (see Equation (1)). The cooperative suppliers' joint revenue is thus given by

$$RV^j = \max_{q^j(t)} \int_0^{+\infty} e^{-rt} P^j(q^j(t))\, q^j(t)\, dt,$$

subject to

$$\int_0^{+\infty} q^j(t)\, dt = R_C(0) + R_A(0) = R^j(0) \text{ given}, \quad q^j(t) \geq 0,$$

and

$$\dot{R}^j(t) = -q^j(t)\,, \quad t \in [0, +\infty).$$

The standard process of solving optimal control problems yields that the shadow price is

$$\lambda^j(t) = \lambda^j(0)e^{rt}, \tag{28}$$

and that the marginal revenue is

$$\frac{\partial RV^j(t)}{\partial q^j(t)} = a\alpha\, (q^j(t))^{\alpha-1} = \alpha P^j(t) = \lambda^j(t). \tag{29}$$

Moreover, substituting (28) into (29) and rearranging terms leads to the joint supply function:

$$q^j(t) = \left(\frac{\lambda^j(0)e^{rt}}{a\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{30}$$

Again, if we consider that the joint reserve is completely exploited over $[0, +\infty)$, integrating the dynamic equation $\dot{R}^j(t) = -q^j(t)$ over $[0, +\infty)$ gives us the shadow price at the beginning of the game:

$$\lambda^j(0) = a\alpha \left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1}. \tag{31}$$

From here, it is enough to substitute (31) into (30) to find the optimal rare earth extraction and market price in the cooperative game. The results are presented in the ensuing proposition.

Proposition 7 *Suppose that countries A and C start their cooperation at the beginning of the* *game, and assume that it lasts forever. Then, the OLSNE joint supply is given by*

$$q^j(t) = \frac{r}{1-\alpha}(R_C(0) + R_A(0))\, e^{\frac{rt}{\alpha-1}}, \tag{32}$$

and the OLSNE market price is

$$P^j(t) = a\left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1} e^{rt}. \tag{33}$$

The results of the last proposition allow us to determine the joint revenue:

$$RV^j = a\left(\frac{r}{1-\alpha}\right)^{\alpha-1} (R_C(0) + R_A(0))^{\alpha}. \tag{34}$$

Based on the countries' initial stocks, we get from Equation (34) that the revenue of countries A and C is, respectively,

$$RV_A^j = \frac{R_A(0)}{R_A(0) + R_C(0)} RV^j = a\; R_A(0)\left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1}, \tag{35}$$

and

$$RV_C^j = \frac{R_C(0)}{R_A(0) + R_C(0)} RV^j = a\; R_C(0)\left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1}. \tag{36}$$

Now let us compare country i's cooperative revenue with that of the non-cooperative game. The following proposition states our findings.

Proposition 8 *If the cooperation between both countries starts at the beginning of the game and lasts forever, then in comparison to the non-cooperative payoffs, the cooperative game leads to a Pareto improvement:*

$$RV_i^j > \Pi_{i,\theta}\,, \quad i = \{A, C\}\,, \quad \theta = \{p, s\}.$$

The latter proposition thus states that both countries receive higher payoffs when, from the beginning of the game, they choose to not compete with each other. In other words, country A should enter the market and start to cooperate with C right at its entry announcement, where $t = 0$.

Moreover, note that the set-up of the above model can be changed such that the cooperative game starts at A's entry time T_θ^*. Similarly to the above setting, the countries' joint revenue $RV_\theta^{J^*}$ will be split based on the share of their initial reserves $R_i(T_\theta^*)$. However, with the countries' reserves being equal at T_θ^*, that is, $R_A(0) = R_C(T_\theta^*)$ (see Corollary 1), it turns out that no differences exist between their cooperative and non-cooperative revenues, such that $RV_{i,\theta}^{J^*} = \pi_{i,\theta}^{II}$. Therefore, this case will not be developed in more detail.

4 Discussion

The subject of our study is very topical in light of the importance of REEs in the ongoing digitalization and energy revolution on the one side, and the dependence on China's volatile rare earth exports on the other side, which has led global players like America, but also Australia and Japan, to make plans to enhance their domestic mining and processing capabilities.

The most recent and concrete plan, however, comes from the U.S., who despite its great need for REEs, relies almost entirely on rare earths from China, since the Mountain Pass Mine in California is currently the only active mine in the country. While this Californian mine supplied most of the world's REEs in the 1980s, nowadays it operates at just a fraction of its potential capacity, and its rare earth compounds are shipped to China for processing. However, in the context of the ongoing trade battle between the U.S. and China, America's ore shipments were saddled with higher tariffs by China. In addition, a recent visit of the Chinese President to one of the country's major rare earth facilities, as well as different Chinese reports of state-controlled entities, raised concerns about a new potential embargo on exports of REEs to America (Partington, 2019; Hornby and Sanderson, 2019). In light of these growing threats, the U.S. therefore announced its intention to reduce China's market power by building, in collaboration with Australia, a large rare earth separation plant on American soil (Hoyle, 2019).

Nonetheless, with China's rare earth reserves being much larger than those of the U.S., America's plan to re-enter the market requires careful design. Indeed, if market entry happens too early, America's resources will be exhausted before those of China, and the country will end up with a definite dependency on rare earth imports. Otherwise, if the U.S. waits too long, a third party may take advantage of the situation and supply the market with some backstop technology. Furthermore, from China's perspective it is equally important to properly react to America's entry plan, not only to keep the importing countries from looking for an alternative supplier, but also to minimize the economic losses that come with the switch to a duopolistic market structure.

5 Conclusion

The present paper studies the strategic game played by two countries that wish to position themselves optimally in the rare earth supply market. The first goal of the study is to determine a potential new supplier's optimal timing for triggering its national rare earth extraction. For this purpose, we set up a two-period continuous-time model and search for open-loop Nash equilibrium strategies. Using backward induction, we start with the second time period, where at the beginning the newcomer enters the market and both countries commit once for all to an extraction path that leads to the best possible payoff, taking into account the other country's strategy. Thereafter, we pursue our second objective, which is to determine the incumbent's optimal supply commitment in the first time period, where the country still holds a monopolistic position. Here, the monopolistic supply path is determined under two assumptions: (1) the incumbent can adapt its supply volume at the moment of the newcomer's market entry, and (2) its supply inflexibility does not allow for instantaneous volume changes. The incumbent's optimal supply reaction to the newcomer's launch plan is then defined by comparing its aggregated revenues in both scenarios. Lastly, we analyze whether a Pareto improvement occurs when both countries decide to cooperate from the moment of the entry announcement. In this case, the optimization problem is set up such that at the beginning of the first period, the countries put their reserves together and jointly choose an open-loop extraction path that maximizes their joint revenue. Later on, the competitive and cooperative revenues of each country are compared to draw conclusions.

Our findings show that the newcomer's optimal entry strategy is to hold back its production launch until its reserves coincide with those of the incumbent. It follows from this that the incumbent can actually control the entry timing through the speed of its monopolistic resource extraction: the faster (slower) its exploitation, the earlier (later) its stock is reduced to the level of the newcomer's and the entry is triggered. We further observe that the incumbent's first-period extraction rate increases with the flexibility of its supply: the country's exploitation behavior is more conservative when its supply volume cannot be adjusted promptly. Besides this, the results show that while the newcomer would benefit from an early entry, the opposite is more likely to take place as the incumbent's aggregated revenue increases with the length of the monopolistic period. This suggests that under competition, the incumbent should keep its supply levels unchanged at the moment of the newcomer's market entry, that is, it should hold back its resource extraction in the first period. In addition, we see that both countries could in fact increase their payoffs by choosing to cooperate from the moment the launch plan is announced. Therefore, unlike under competition, if the incumbent intends to cooperate, it should not delay the entry too much, otherwise the newcomer could prefer to forego the financial benefits of the cooperative game to keep the incumbent from reaching its first best solution.

Future research should focus on reformulating the countries' strategies to Markovian ones, in both time periods. In this case, one would assume that the countries do not forever commit themselves to an extraction path at the beginning of each period, but that they modify their paths with regard to time and the current value of the rare earth reserves. Besides this, one could also change the model's demand function and empirically determine its parameters. Comparing these results with those of the present paper then enables to asses, theoretically and numerically, whether or not, for the situation under study, the different assumptions about the countries' commitment approaches and the market demand lead to equivalent conclusions.

A Appendix

The second-period optimal control problem for both countries is

$$\pi_i^{II}(R_C^*, T^*) = \max_{q_i^{II}(t)} \int_{T^*}^{+\infty} e^{-rt} P^{II}(Q^{II}(t))\, q_i^{II}(t)\; dt,$$

subject to

$$\int_{T^*}^{+\infty} q_i^{II}(t)\; dt \leq R_i(T^*) = \begin{cases} R_C^*, \text{ for } \; i = C \\ R_A(0) \text{ given, for } \; i = A \end{cases} , \quad q_i^{II}(t) \geq 0,$$

and

$$\dot{R}_i(t) = -q_i^{II}(t)\,, \quad t \in \; [T^*, +\infty).$$

Since the first constraint of the optimization problem is equivalent to

$$\int_{T^*}^{t} q_i^{II}(\tau)\; d\tau \leq R_i(T^*)\,,\; t \in [T^*, +\infty),$$

the Lagrangian is set up as follows:

$$\mathcal{L}_i^{II}\Big(q_i^{II}(t), \lambda_i^{II}(t), \alpha_i^{II}\Big) = P^{II}(Q^{II}(t))q_i^{II}(t) - \lambda_i^{II}(t)q_i^{II}(t) - \alpha_i^{II}\left(R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\; d\tau\right),$$

where $\lambda_i^{II}(t)$ is the shadow price and α_i^{II} is the static Lagrange multiplier. The standard first-order conditions (FOCs) are

$$\begin{cases} \dot{\lambda}_i^{II}(t) = r\,\lambda_i^{II}(t), \\ \frac{\partial}{\partial q_i^{II}(t)}\mathcal{L}_i^{II}\big(q_i^{II}(t), \lambda_i^{II}(t), \alpha_i^{II}\big) = \frac{\partial}{\partial q_i^{II}(t)} RV_i^{II}(t) - \lambda_i^{II}(t) + \alpha_i^{II} \frac{\partial}{\partial q_i^{II}(t)} \int_{T^*}^{t} q_i^{II}(\tau)\; d\tau = 0, \\ \alpha_i^{II} \geq 0, \;\; R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\; d\tau \geq 0, \;\; \alpha_i^{II}\Big(R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\; d\tau\Big) = 0, \end{cases}$$

where the revenue of country i is $RV_i^{II}(t) = P^{II}(t)q_i^{II}(t)$. Based on the remarks in point (3) of Subsection 2, it is not optimal for country i to exhaust its resources in finite time, thus $R_i(T^*) > \int_{T^*}^{t} q_i^{II}(\tau)\; d\tau$, and hence $\alpha_i^{II} = 0$.

The first FOC yields that i's shadow price $\lambda_i^{II}(t)$ of its remaining reserve $R_i(t)$ grows at interest rate r, compounded continuously:

$$\lambda_i^{II}(t) = \lambda_i^{II}(T^*)e^{r(t-T^*)}. \tag{37}$$

The second FOC and the fact that $\alpha_i^{II} = 0$ show that the shadow price does actually correspond to the marginal revenue:

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = \lambda_i^{II}(t). \tag{38}$$

Since the revenue of country i is

$$RV_i^{II}(t) = P^{II}(t)q_i^{II}(t) = a\left(Q^{II}(t)\right)^{\alpha-1} q_i^{II}(t) = a\left(q_A^{II}(t) + q_C^{II}(t)\right)^{\alpha-1} q_i^{II}(t),$$

we obtain

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = a\left(Q^{II}(t)\right)^{\alpha-1}\left(1 - \frac{(1-\alpha)q_i^{II}(t)}{q_A^{II}(t) + q_C^{II}(t)}\right) = \lambda_i^{II}(t), \tag{39}$$

which yields that

$$\frac{1 - \frac{(1-\alpha)q_i^{II}(t)}{q_A^{II}(t)+q_C^{II}(t)}}{1 - \frac{(1-\alpha)q_j^{II}(t)}{q_A^{II}(t)+q_C^{II}(t)}} = \frac{\lambda_i^{II}(T^*)}{\lambda_j^{II}(T^*)} = \frac{\lambda_i^{II}(t)}{\lambda_j^{II}(t)}, \tag{40}$$

where $i, j \in \{A, C\}$ and $i \neq j$. After rearranging Equation (40), we find

$$q_A^{II}(t) = \frac{\lambda_C^{II}(T^*) - \alpha\lambda_A^{II}(T^*)}{\lambda_A^{II}(T^*) - \alpha\lambda_C^{II}(T^*)}\, q_C^{II}(t). \tag{41}$$

When integrating Equation (41) over $[T^*, +\infty)$ and by assuming that over $[T^*, +\infty)$ the total REE reserve is exhausted, as from an economical viewpoint it is not optimal to leave some elements in the deposit (no market value), we get

$$\frac{R_A(0)}{R_C(T^*)} = \frac{\lambda_C^{II}(T^*) - \alpha\lambda_A^{II}(T^*)}{\lambda_A^{II}(T^*) - \alpha\lambda_C^{II}(T^*)}, \tag{42}$$

that is,

$$\lambda_A^{II}(T^*) = \frac{R_C^* + \alpha R_A(0)}{R_A(0) + \alpha R_C^*}\, \lambda_C^{II}(T^*). \tag{43}$$

Equations (41) and (42) yield

$$\frac{q_A^{II}(t)}{q_C^{II}(t)} = \frac{R_A(0)}{R_C(T^*)}, \quad \forall t \geq T^*. \tag{44}$$

Since the price function in Equation (1) is

$$P^{II}(t) = a\left(q_A^{II}(t) + q_C^{II}(t)\right)^{\alpha-1} = a\left(\frac{R_A(0)}{R_C^*} + 1\right)^{\alpha-1}\left(q_C^{II}(t)\right)^{\alpha-1} \tag{45}$$

$$\left[\text{resp.} \quad P^{II}(t) = a\left(1 + \frac{R_C^*}{R_A(0)}\right)^{\alpha-1}\left(q_A^{II}(t)\right)^{\alpha-1}\right], \tag{46}$$

the revenue of country C [resp. country A] is

$$RV_C^{II}(t) = P^{II}(t) q_C^{II}(t) = a\left(\frac{R_A(0)}{R_C^*} + 1\right)^{\alpha-1}(q_C^{II}(t))^{\alpha}$$

$$\left[\text{resp.} \quad RV_A^{II}(t) = P^{II}(t) q_A^{II}(t) = a\left(1 + \frac{R_C^*}{R_A(0)}\right)^{\alpha-1}(q_A^{II}(t))^{\alpha}\right].$$

In view of (39), country i's marginal revenue is

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = \alpha P^{II}(t) = \lambda_i^{II}(t). \tag{47}$$

Equation (47) allows us to define the market price by

$$P^{II}(t) = \frac{\lambda_i^{II}(t)}{\alpha}. \tag{48}$$

When combining (45) and (47), we get

$$q_C^{II}(t) = \frac{R_C^*}{R_A(0) + R_C^*}\left(\frac{\lambda_C^{II}(t)}{a\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{R_C^*}{R_A(0) + R_C^*}\left(\frac{\lambda_C^{II}(T^*)}{a\alpha}\right)^{\frac{1}{\alpha-1}} e^{\frac{r(t-T^*)}{\alpha-1}}. \tag{49}$$

Integrating Equation (49) over $[T^*, +\infty)$, we obtain

$$\lambda_C^{II}(T^*) = a\alpha\left(\frac{r}{1-\alpha}\left(R_A(0) + R_C^*\right)\right)^{\alpha-1}. \tag{50}$$

Finally, by substituting Equation (50) into Equation (49) [resp. (48)], we find the extraction rate of country C [resp. duopoly market price of the REE]:

$$q_C^{II}(t) = \frac{r}{1-\alpha}\, R_C^*\, e^{\frac{r(t-T^*)}{\alpha-1}} \tag{51}$$

$$\left[\text{resp.}\quad P^{II}(t) = a\left(\frac{r}{1-\alpha}\,(R_A(0)+R_C^*)\right)^{\alpha-1} e^{r(t-T^*)}\right].$$

Then, by substituting Equation (51) into Equation (44), we get the extraction rate of country A:

$$q_A^{II}(t) = \frac{R_A(0)}{R_C^*}\, q_C^{II}(t) = \frac{r}{1-\alpha}\, R_A(0)\, e^{\frac{r(t-T^*)}{\alpha-1}}. \tag{52}$$

This finishes the proof. The results are presented in Proposition 1 of Section 2.

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