Lack of separation of scales

A view from reduced order modelling and homogenisation

First Benelux Workshop on damage and fracture ESIS2019

Ahmad Akbari - Lars Beex - Stéphane P.A. Bordas* - Olivier Goury - Pierre Kerfriden - Chen Li
Thierry Massart - Ludovic Noels - Soumianarayanan Vijayaraghavan

UNIVERSITÉ DU LUXEMBOURG

INDIAN INSTITUTE OF TECHNOLOGY MADRAS

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Slides can be downloaded here
http://hdl.handle.net/10993/39442

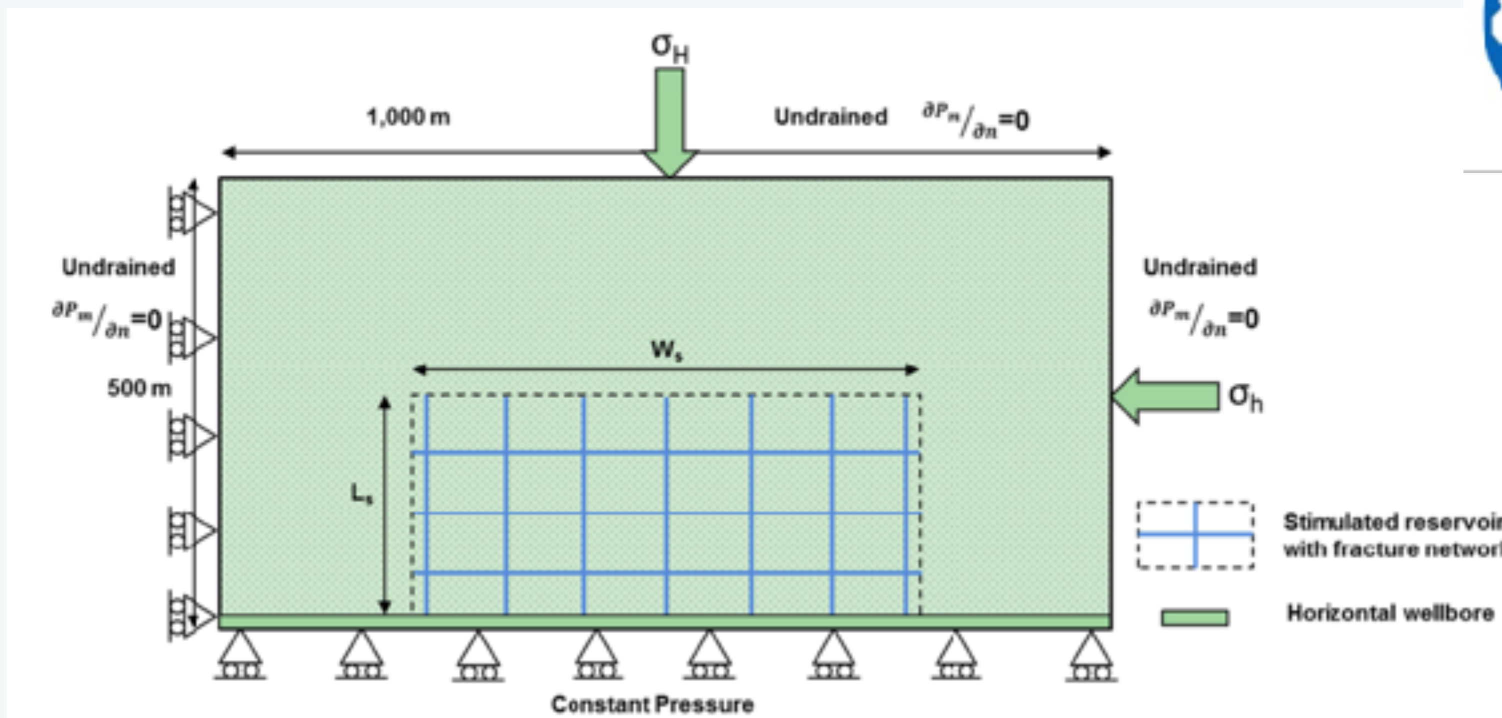
Antwerp 20190512

Computed in Luxembourg

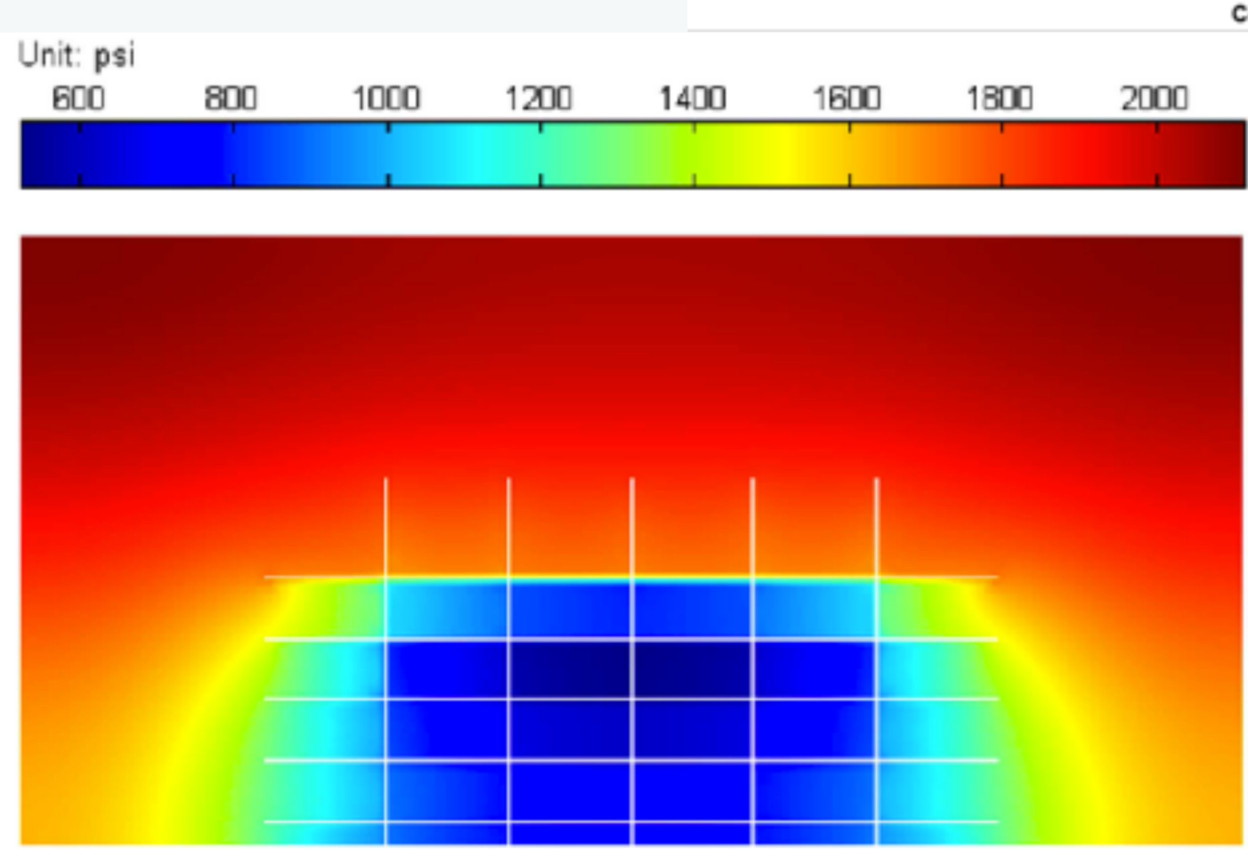
Computational Sciences Luxembourg

UNIVERSITÉ DU LUXEMBOURG

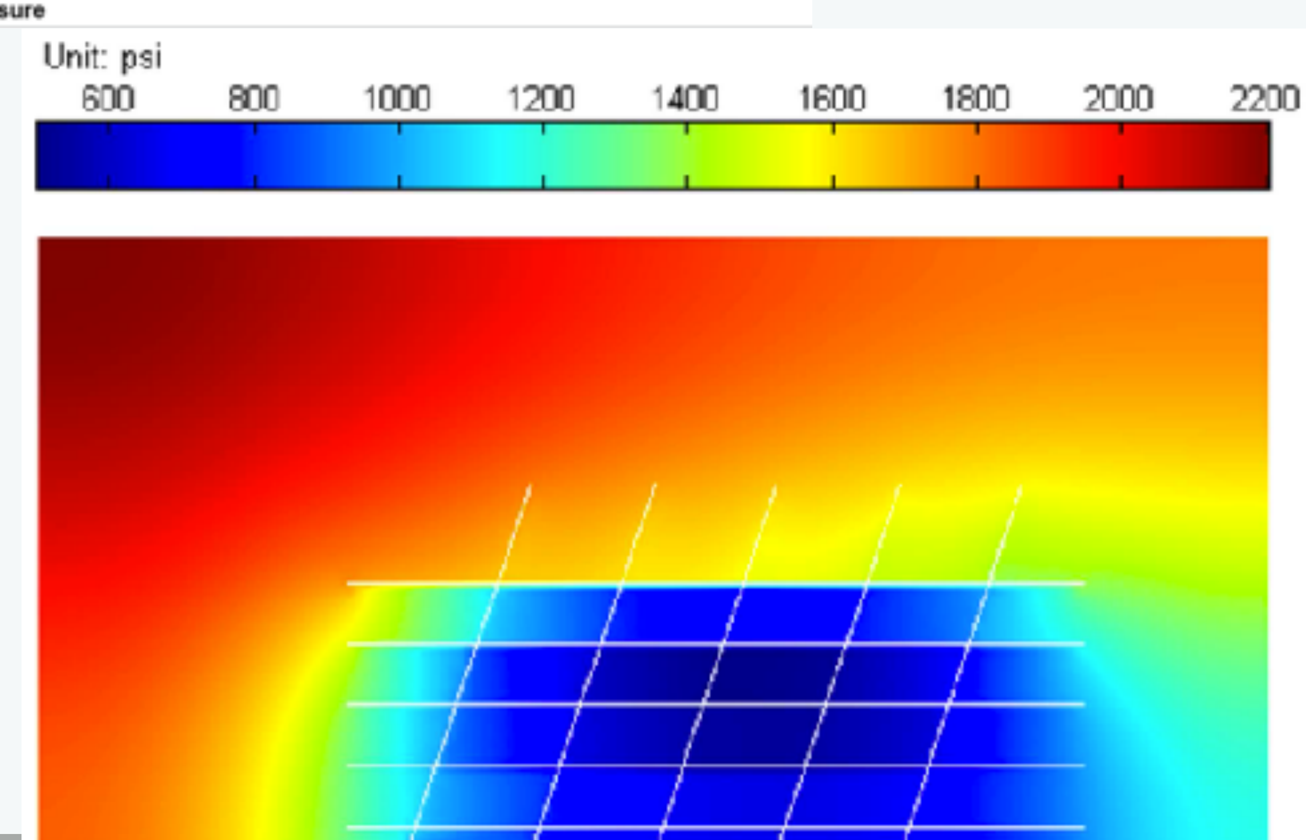
Department of Computational Engineering & Sciences



σH
1,000 m
Undrained ∂Pm/∂n=0
Undrained
∂Pm/∂n=0
500 m
Ws
Ls
Undrained
∂Pm/∂n=0
σh
Stimulated reservoir with fracture network
Horizontal wellbore
Constant Pressure

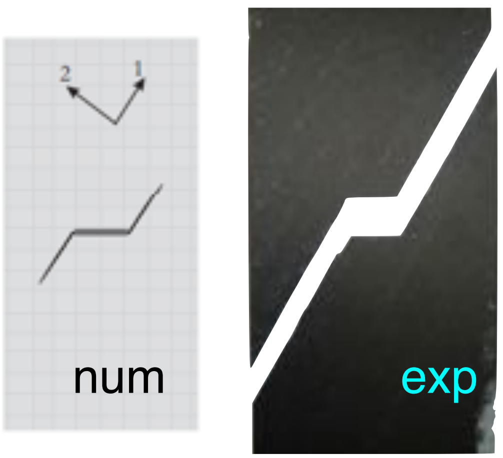


Unit: psi
600 800 1000 1200 1400 1600 1800 2000



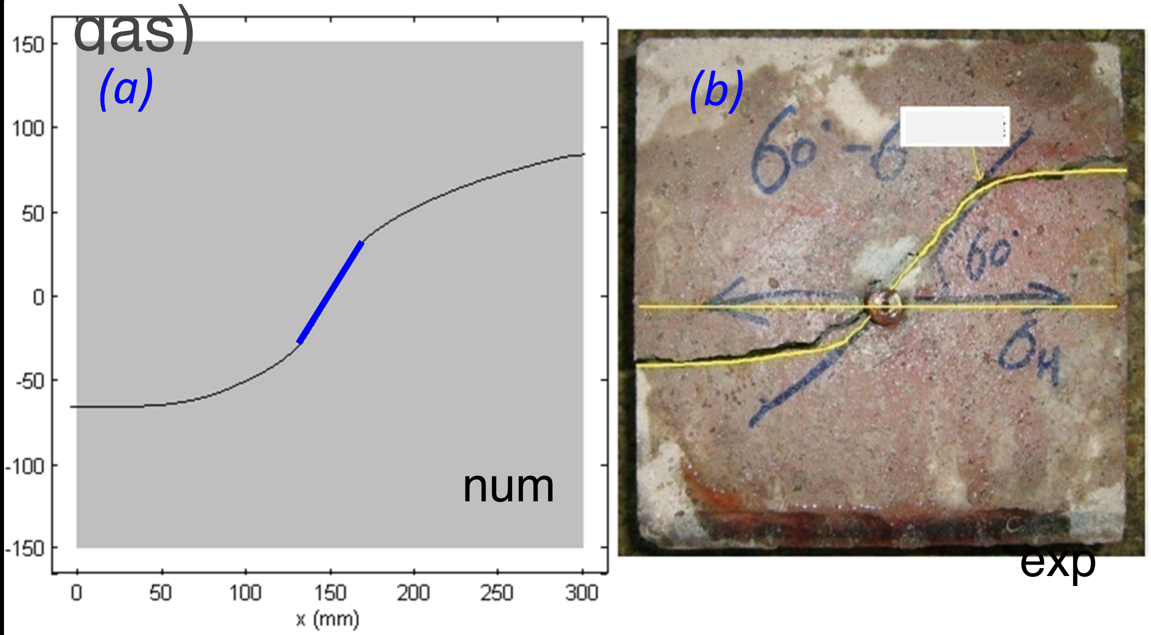
Unit: psi
600 800 1000 1200 1400 1600 1800 2000 2200

- **Limerick**: unidirectional composites

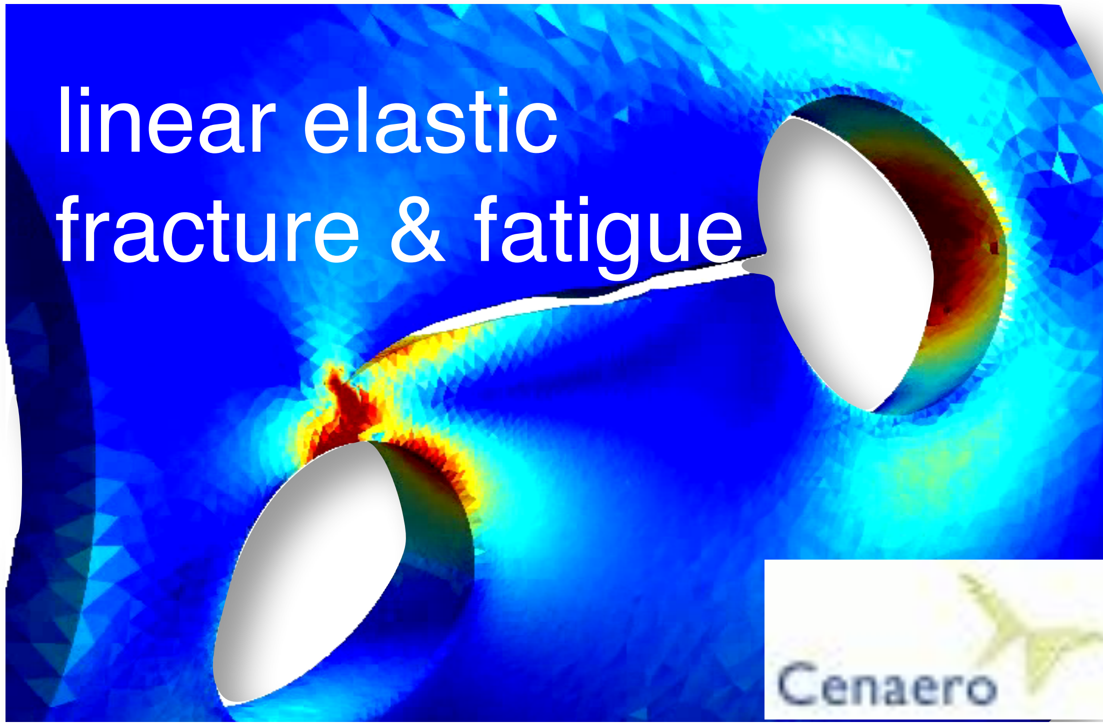


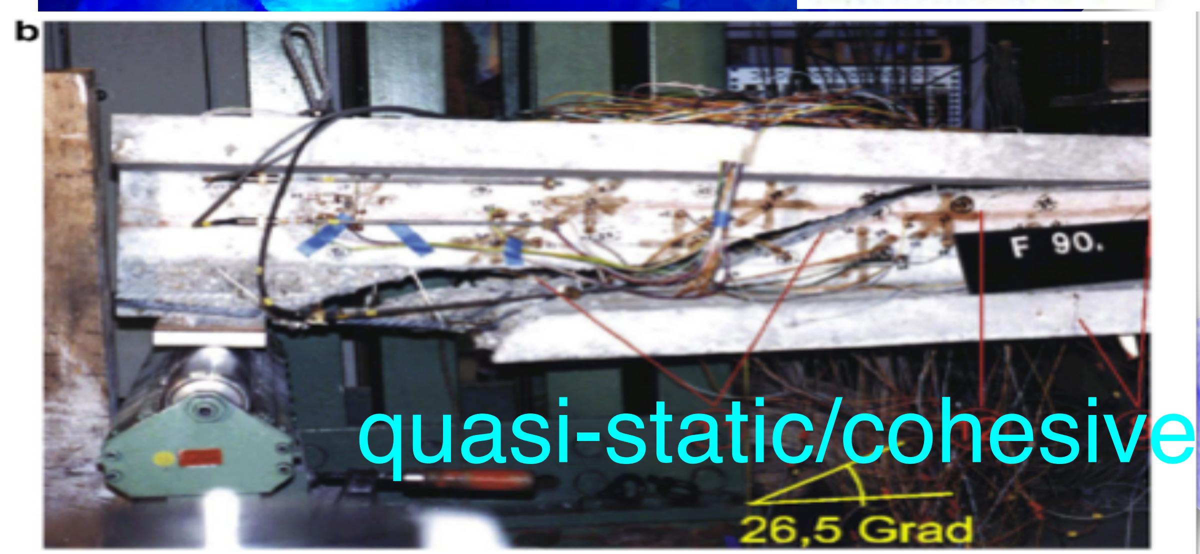
thesis L. Cahill, 2014

- **China/USA**: hydraulic fracturing (shale gas)

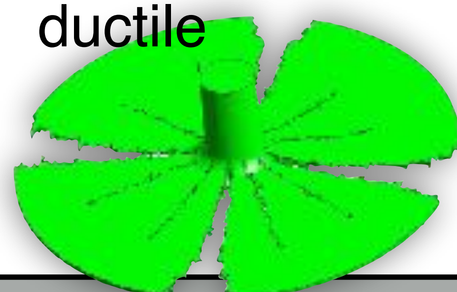


thesis M. Sheng, USA, China, 2016

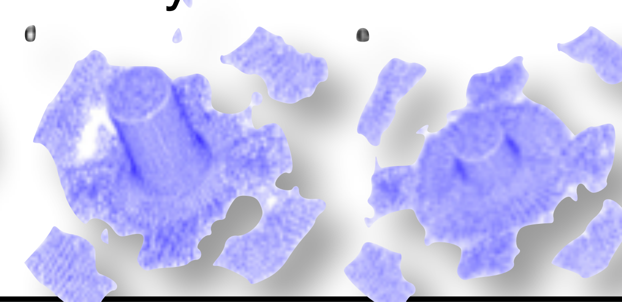




dynamics ductile

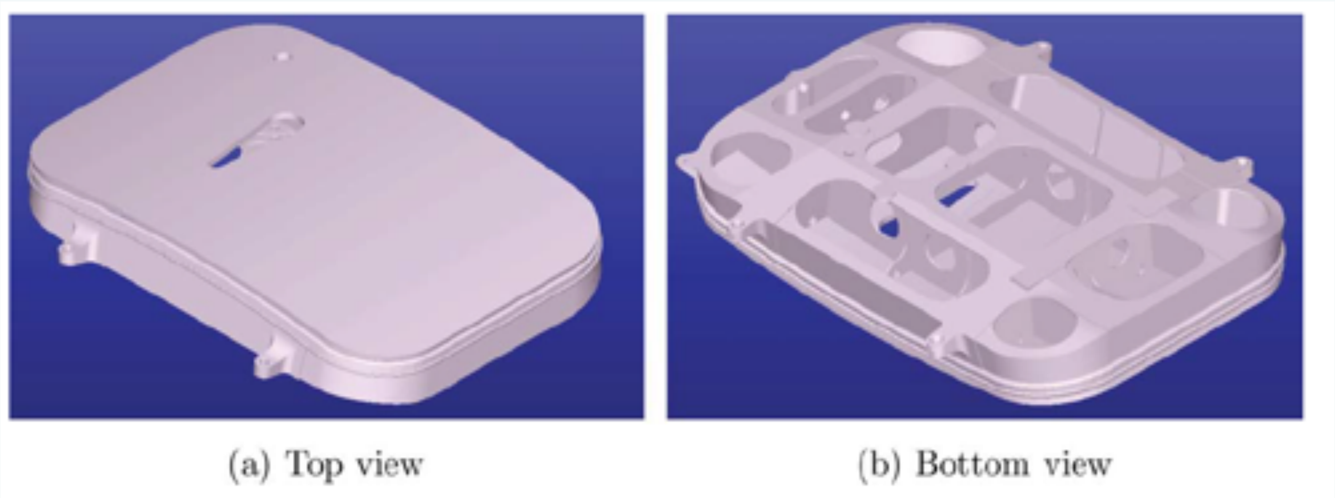


dynamics/brittle

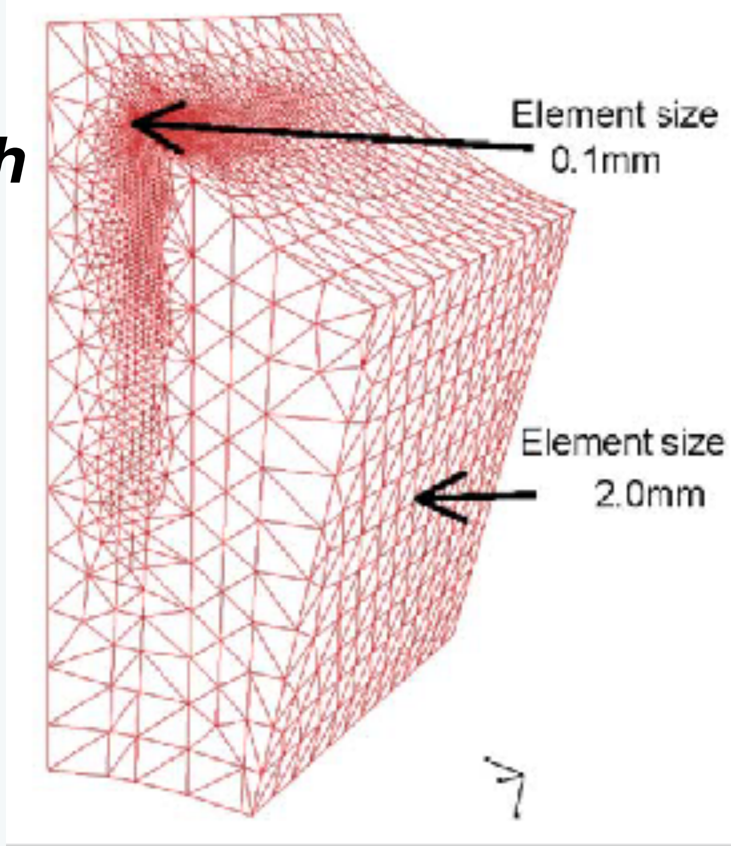


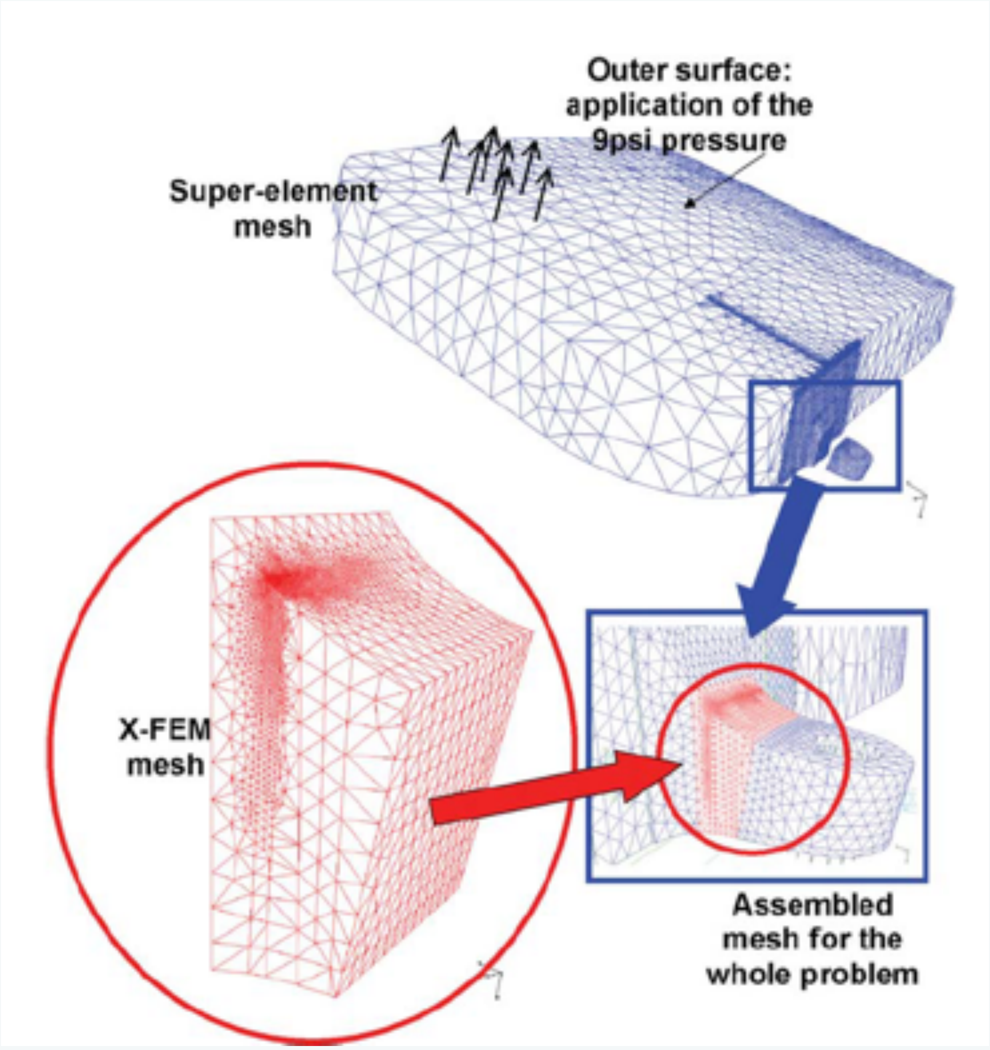
Fracture of ‘homogeneous’ materials

Question: when should a structure be inspected for flaws?



ad hoc mesh refinement

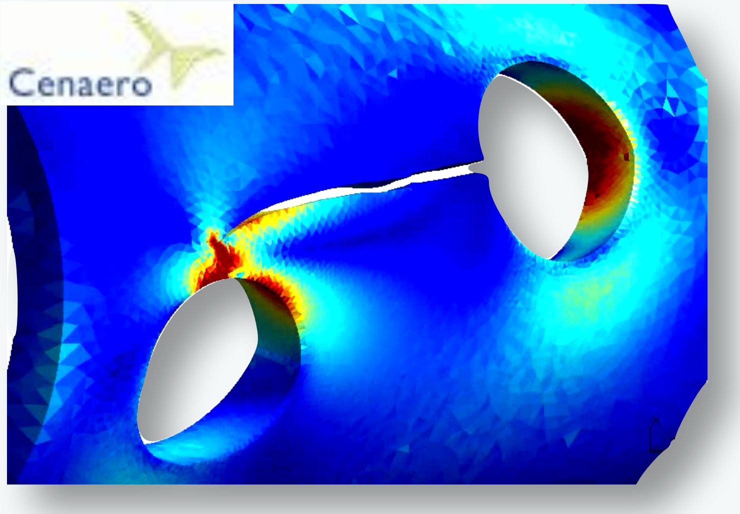


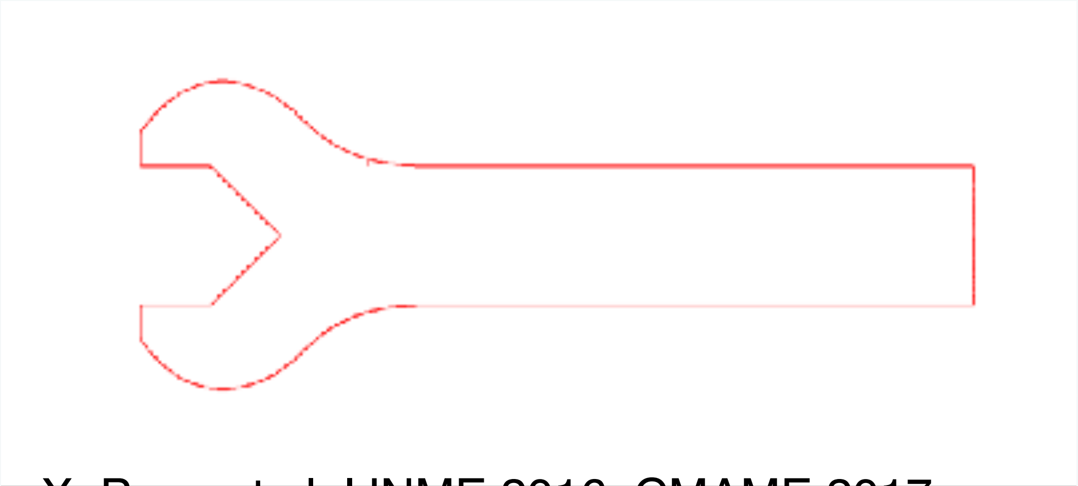


SPAB and B. Moran, Engineering Fracture Mechanics, 2006
V.P. Nguyen et al. XFEM C++ Library IJNME, 2007
Industrial applications of extended finite element methods
See also E. Wyart et al, EFM, IJNME, 2008

Fracture of homogeneous materials

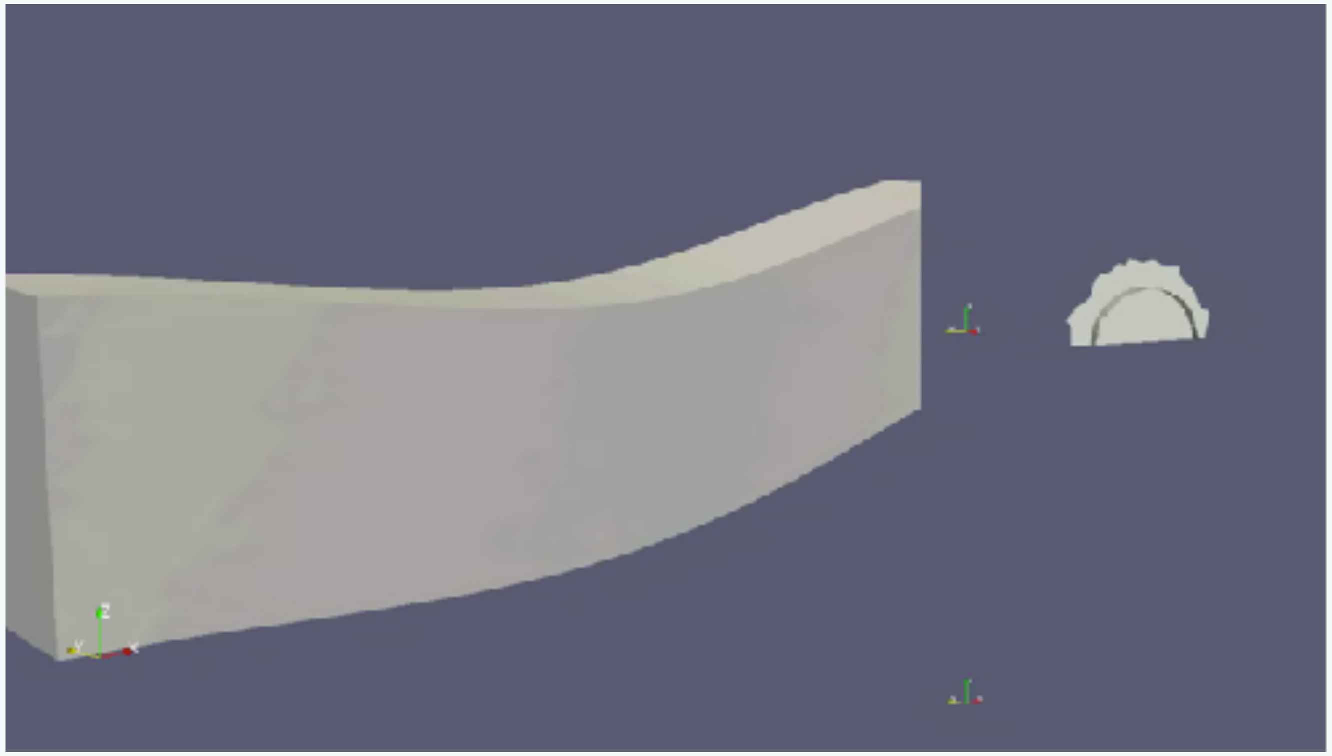
Question: How to control accuracy and simplify/avoid meshing?





X. Peng et al. IJNME 2016, CMAME 2017
Enriched Isogeometric Boundary Elements

How to avoid meshing completely for crack propagation simulations?

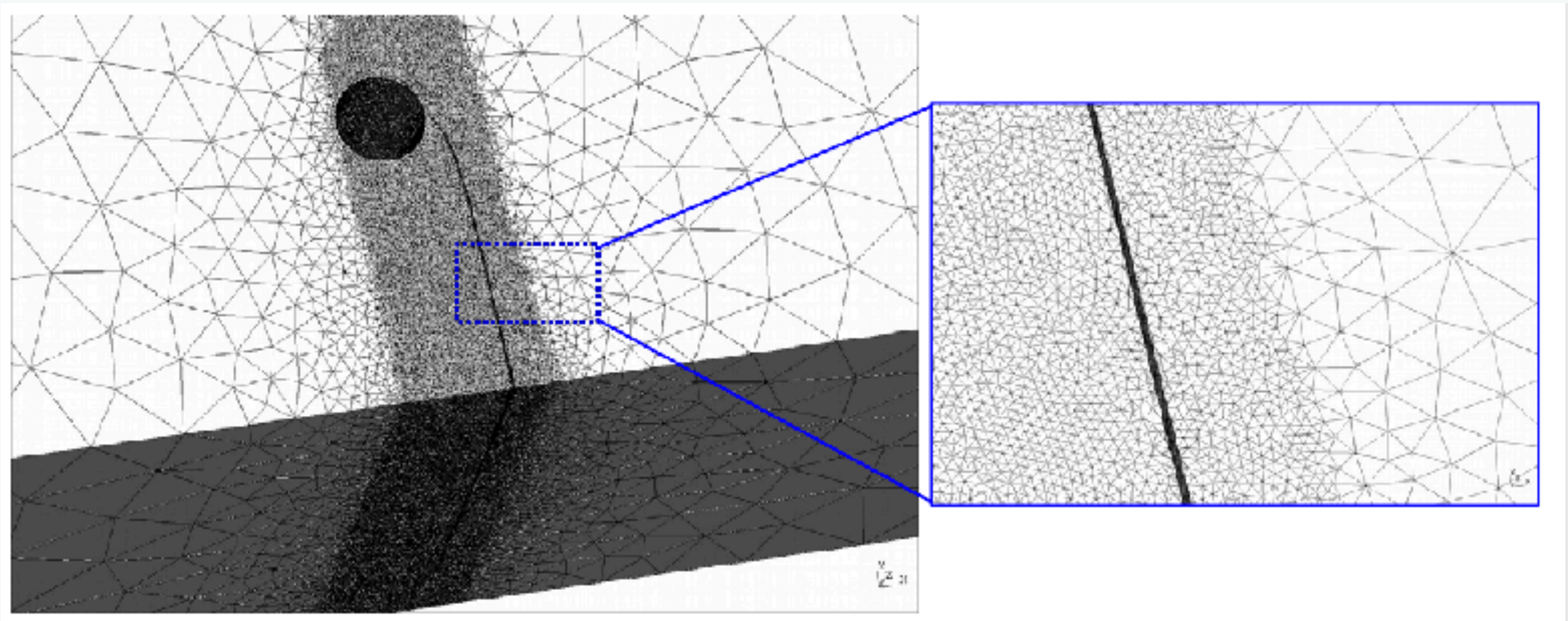


K. Agathos et al. IJNME 2016, CMAME 2016, IJNME 2017, CMAME 2017 with Eleni Chatzi and Giulio Ventura

How can we use large enrichment radii?
How can we control conditioning in large-scale enriched FEM?
How can we use higher order terms in the expansion?

(Goal oriented) adaptive computational fracture: use h-refinement

With CENAERO

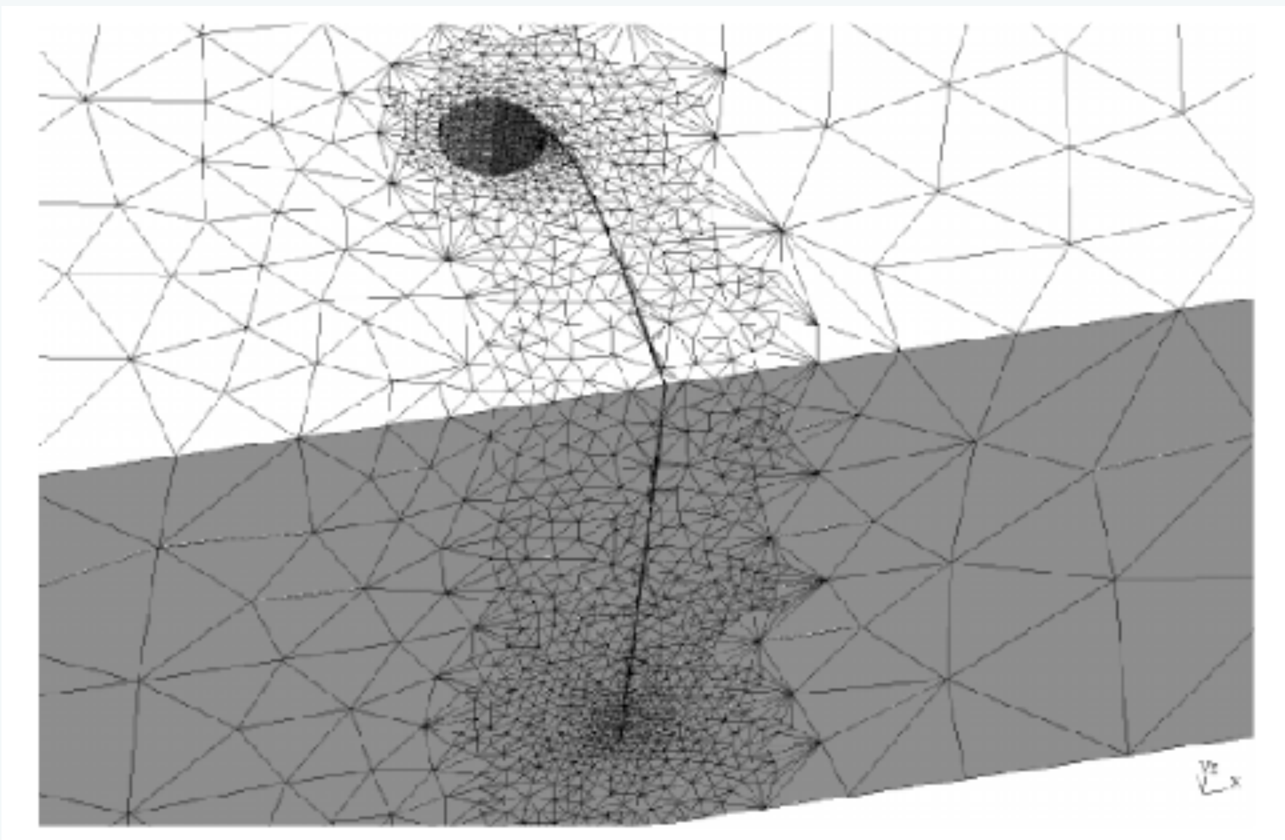


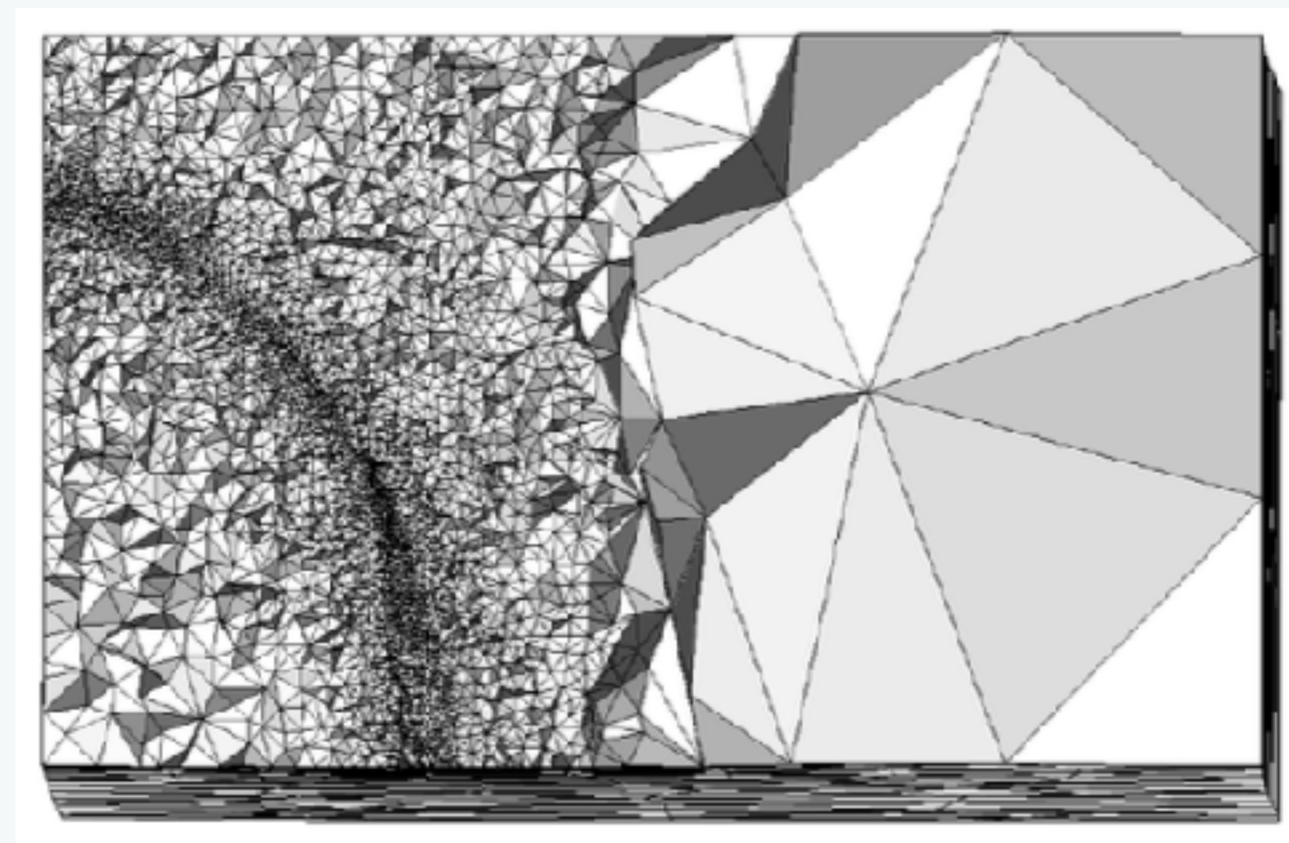
Before: mesh "finely" in the region where the crack is "expected" to propagate

Y. Jin, O. Pierard, et al. Comput. Methods Appl. Mech. Engrg. 318 (2017) 319–348
O.A. González-Estrada et al. Computers and Structures 152 (2015) 1–10
O.A. González-Estrada et al. Comput Mech (2014) 53:957–976
C. Prange et al. IJNME 91.13 (2012): 1459-1474.
M. Duflot, SPAB, IJNME 2007, CNME 2007, IJNME 2008.
J-J. Ródenas Garcia, IJNME 2007
F.B. Barros, et alIJNME 60.14 (2004): 2373-2398.
M. Rüter CMECH (2013) 1;52(2):361-76.
J. Panetier IJNME 81.6 (2010): 671-700.
P. Hild, CMECH (2010): 1-28.

Motivation

Fracture of homogeneous materials: error estimation and adaptivity with CENAERO





After: determine mesh refinement adaptively using a (goal-oriented) error estimate

Y. Jin, O. Pierard, et al. Error-controlled adaptive extended finite element method for 3D linear elastic crack propagation Comput. Methods Appl. Mech. Engrg. 318 (2017) 319–348
M. Duflot, SPAB, IJNME 2007, CNME 2007, IJNME 2008.

Discretization: XFEM

Extended Finite Element Method (XFEM)

Fracture of “XFEM” using XFEM

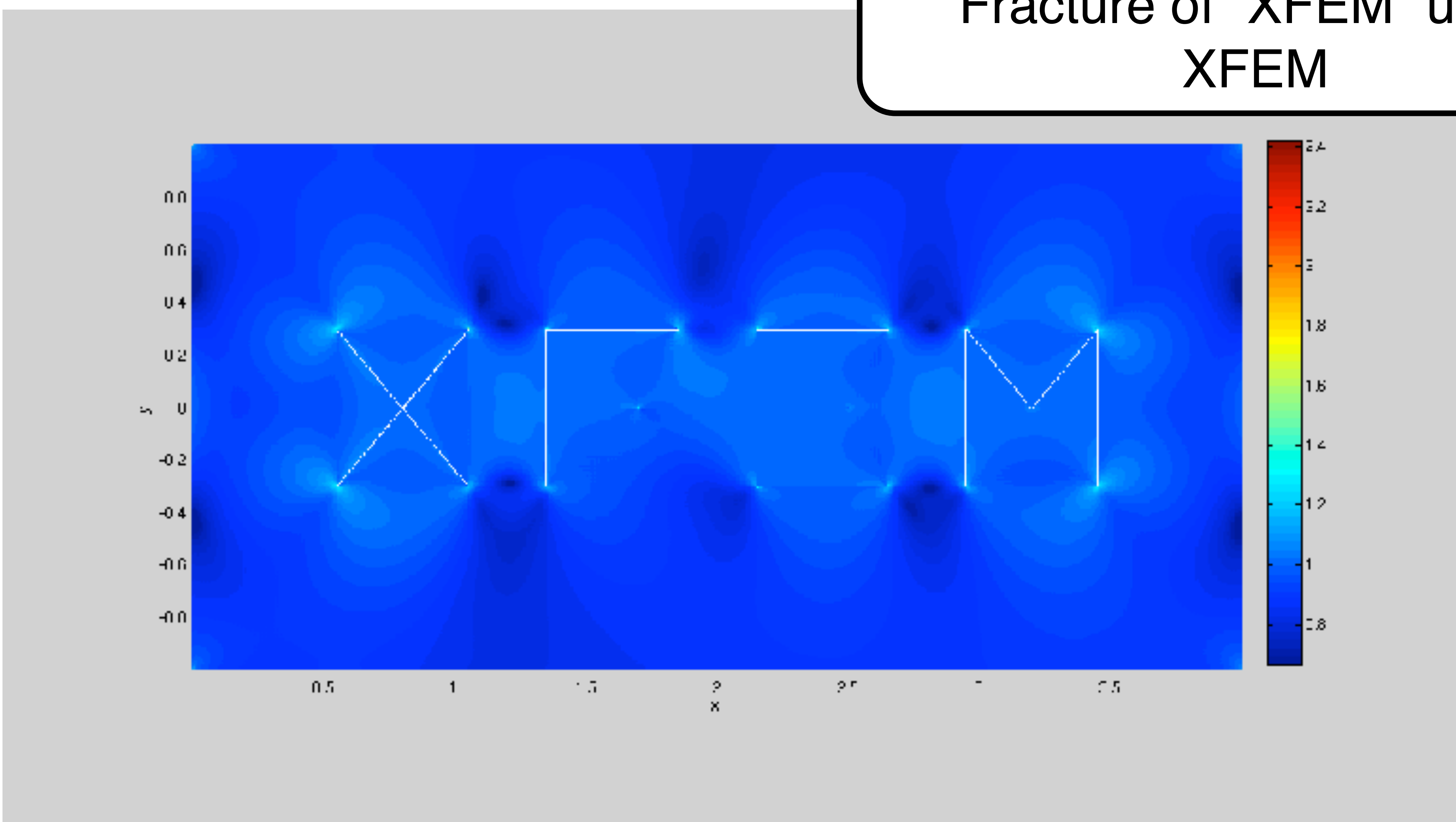
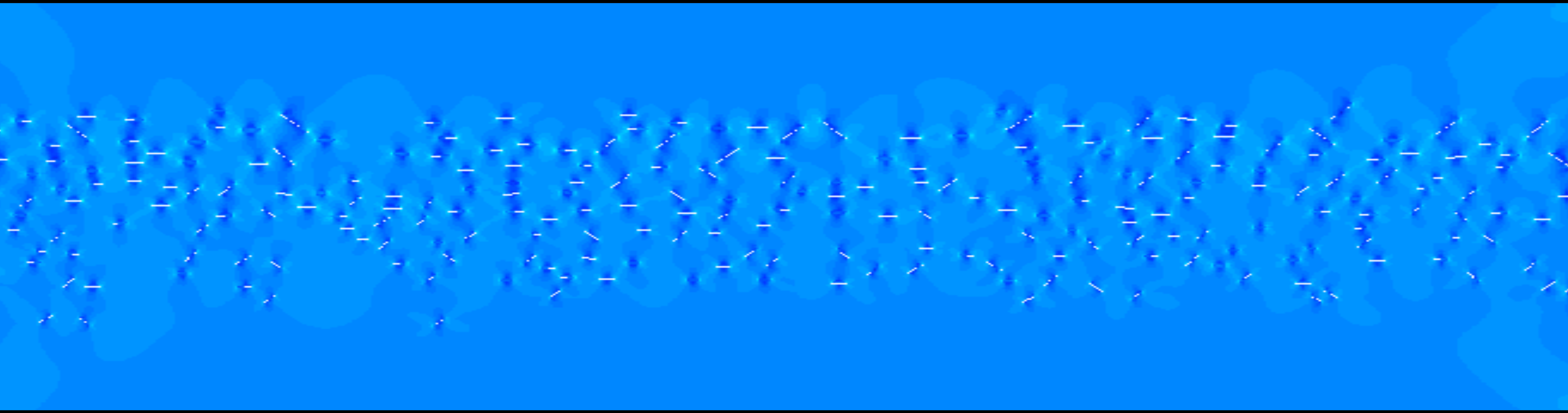


Plate with 300 cracks
vertical extension BCs

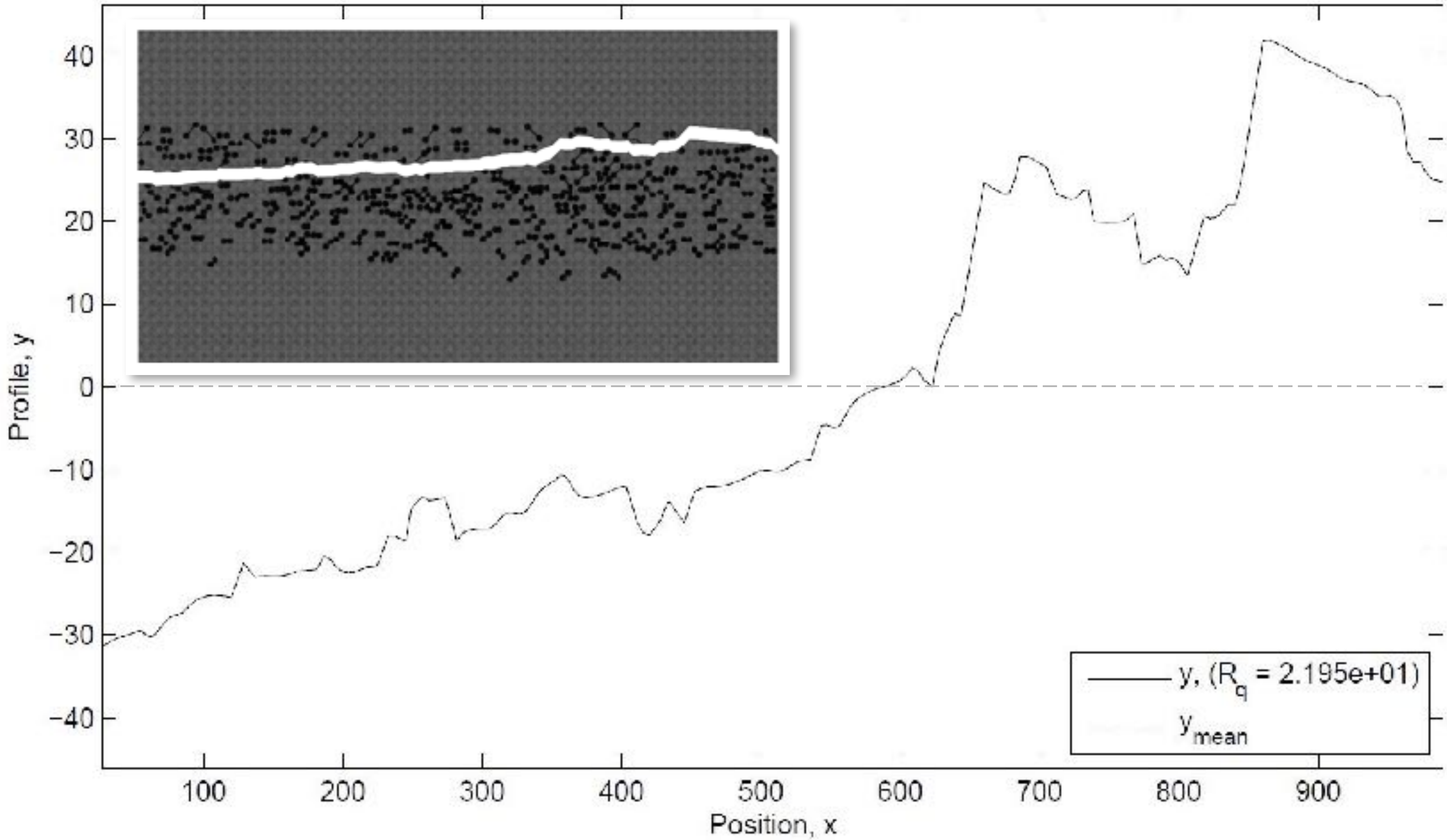
Energy-minimal crack growth using XFEM

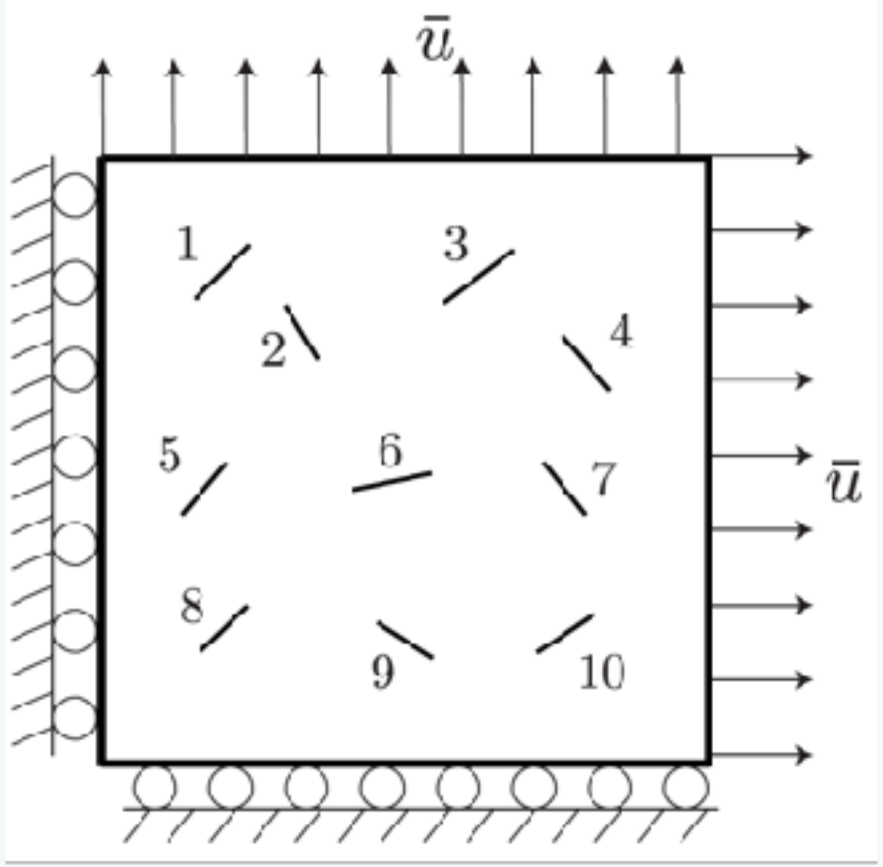


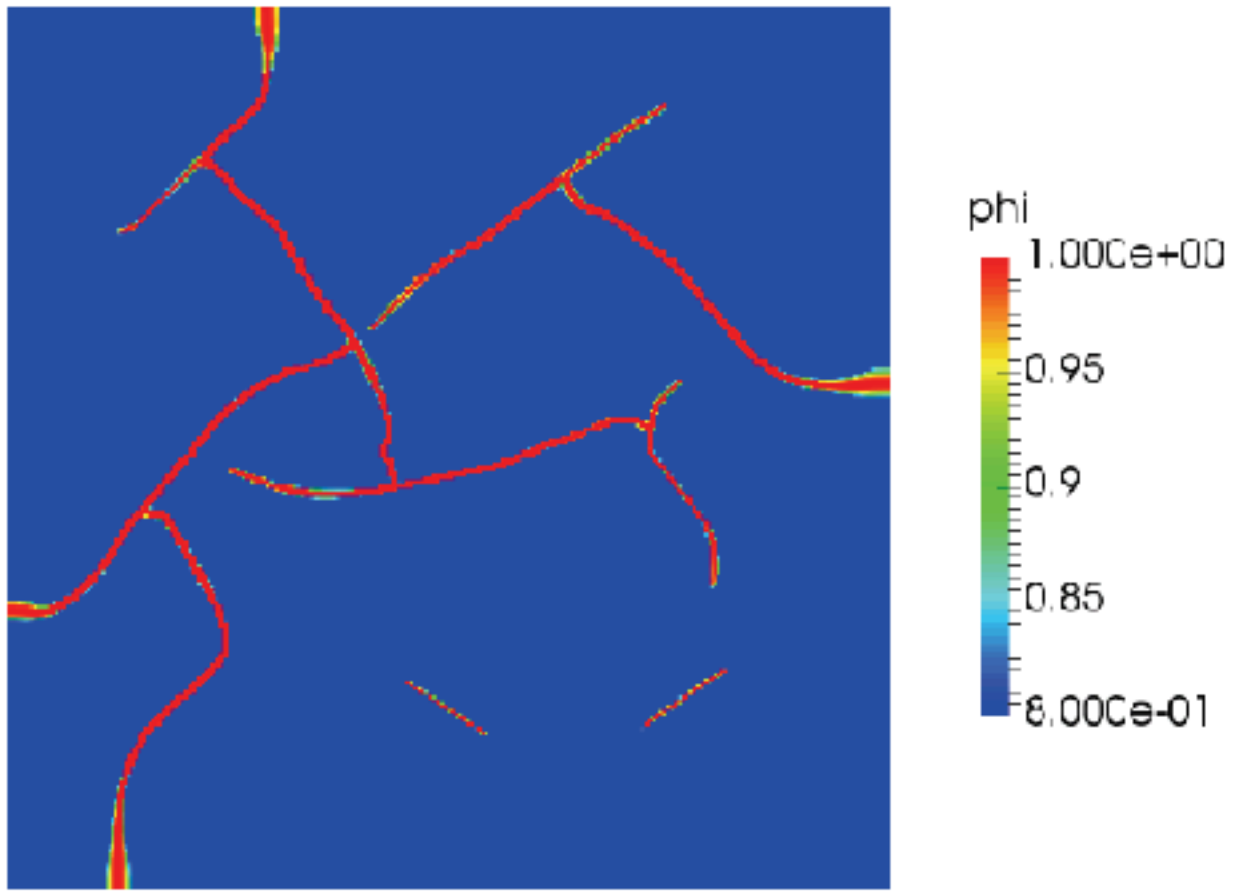
Sutula et al. Preprint of three part EFM paper at
http://hdl.handle.net/10993/29414

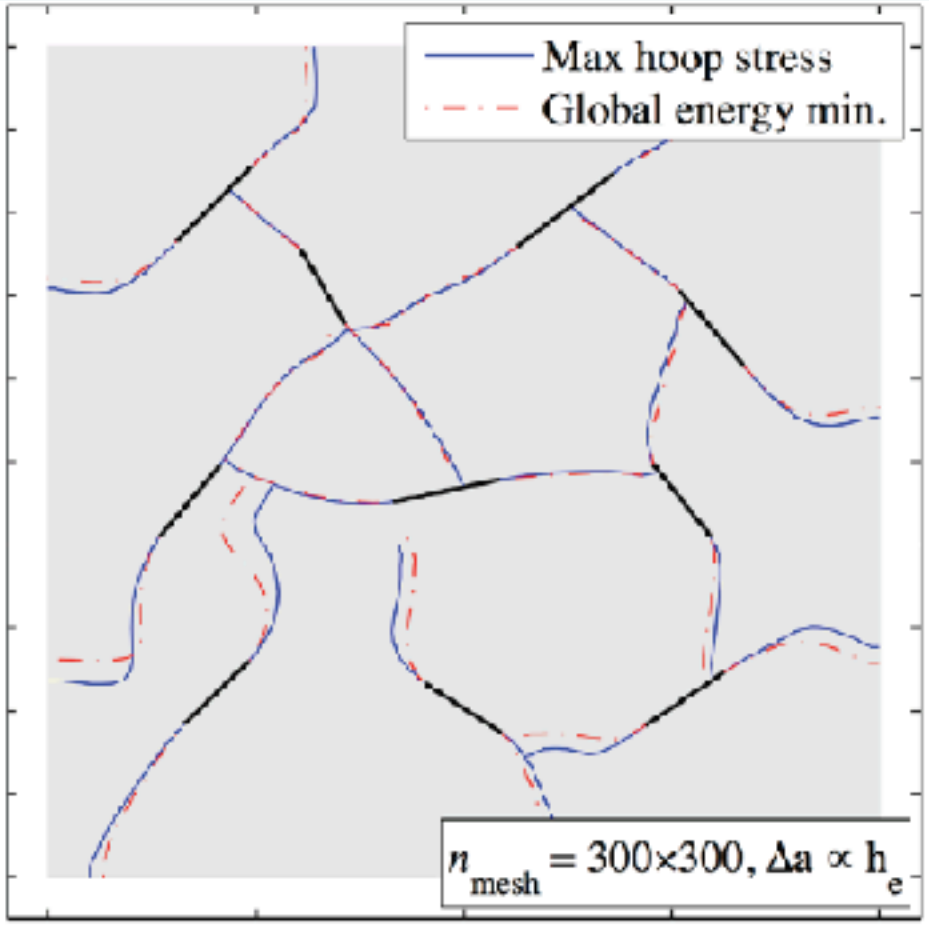
Vertical extension of a plate with 300 cracks

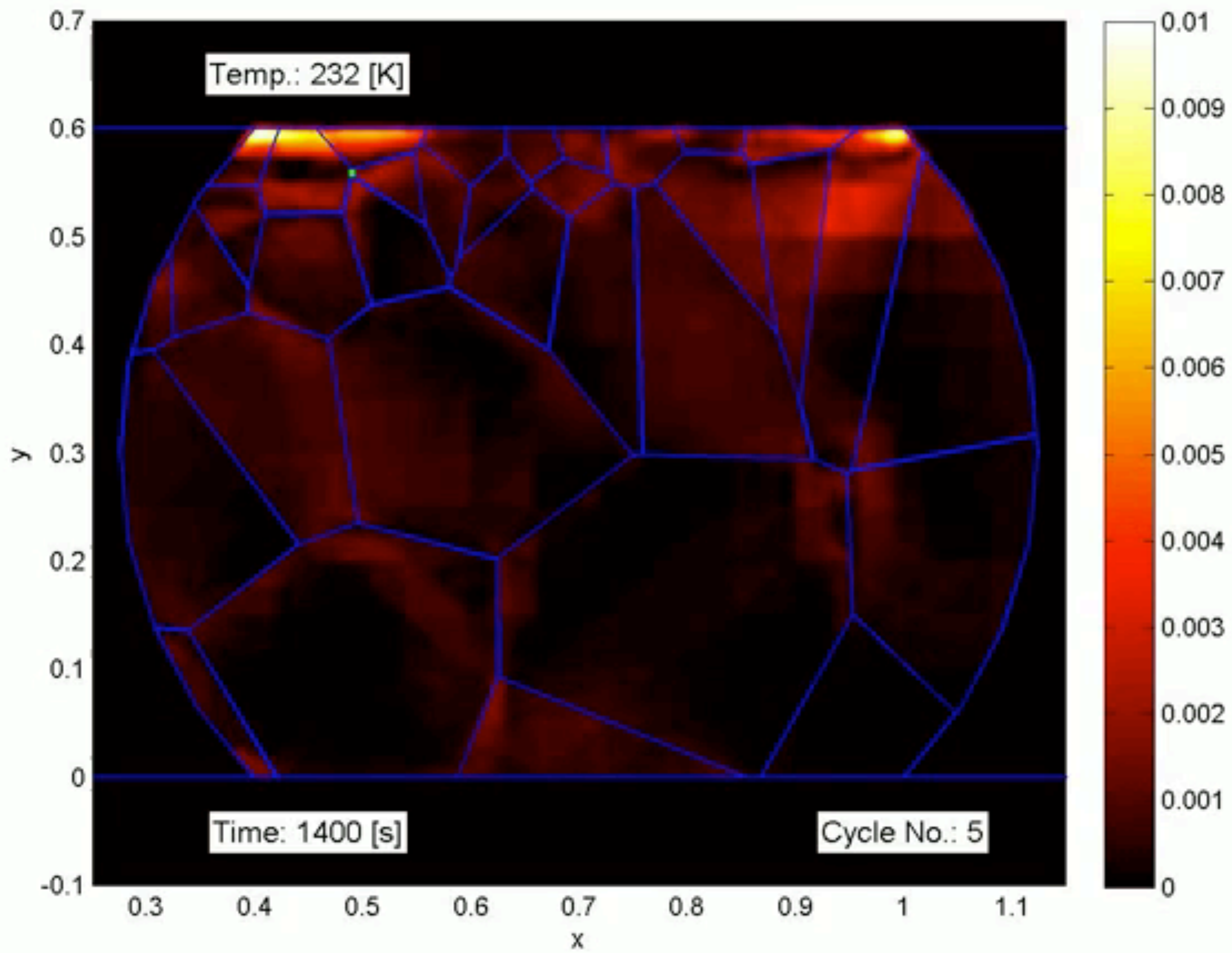
Post-split roughness




$\bar{u}$
1
2
3
4
5
6
7
8
9
10
$\bar{u}$

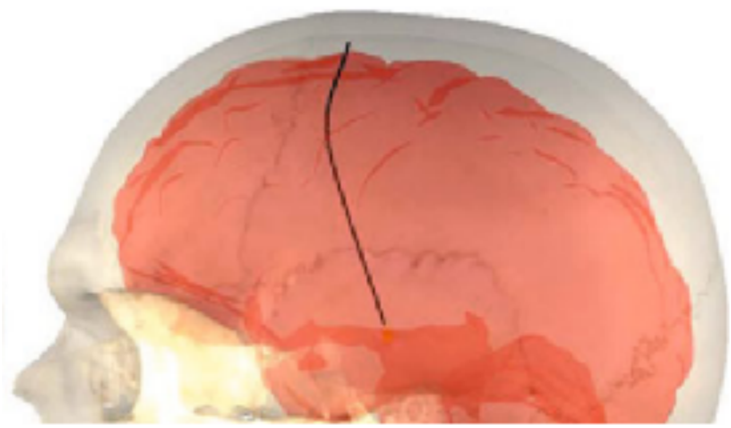

phi
1.000e+00
0.95
0.9
0.85
8.00e-01


Max hoop stress
Global energy min.
n_{mesh} = 300×300, Δa ∝ h_e

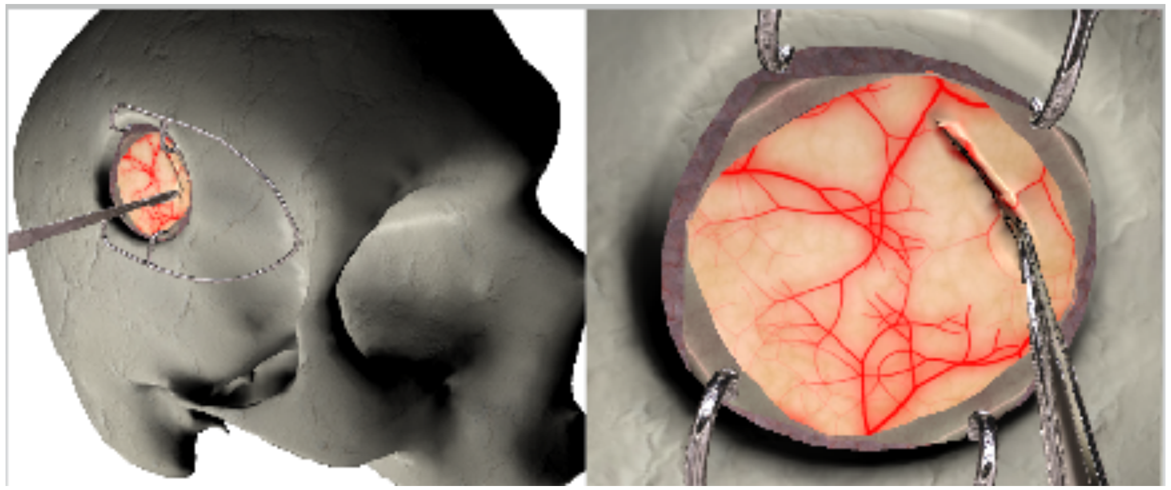

Temp.: 232 [K]
Time: 1400 [s]
Cycle No.: 5
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
y
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
x
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0

Real-time simulations with XFEM

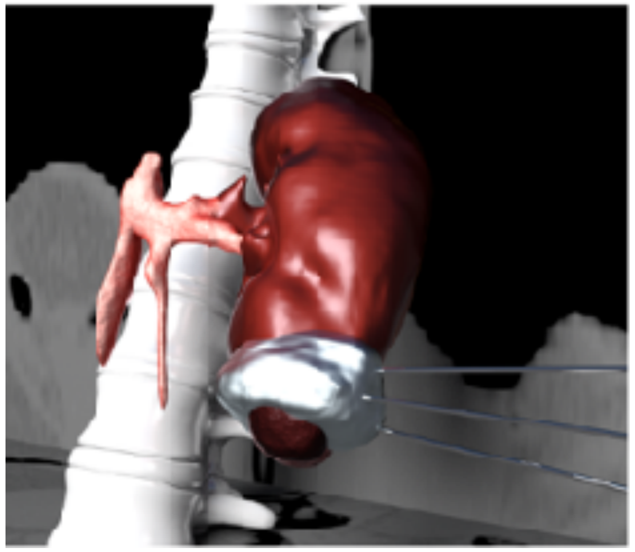
SKIP



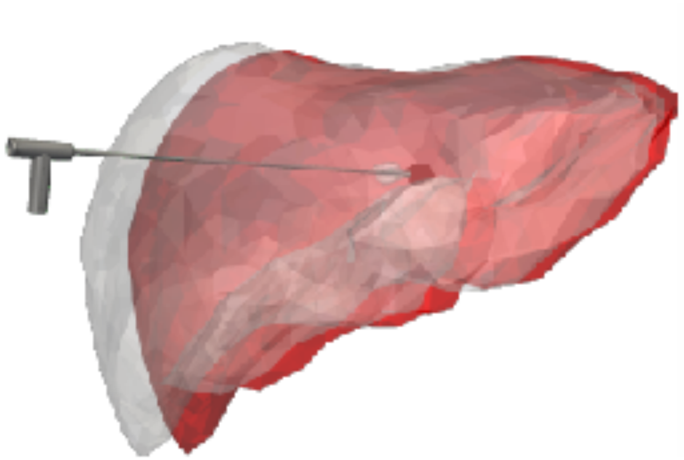
Bilger et al, MICCAI, 2011



Courtecuisse et al, Med. Image Anal., 2014



Talbot et al, SIGGRAPH, 2015

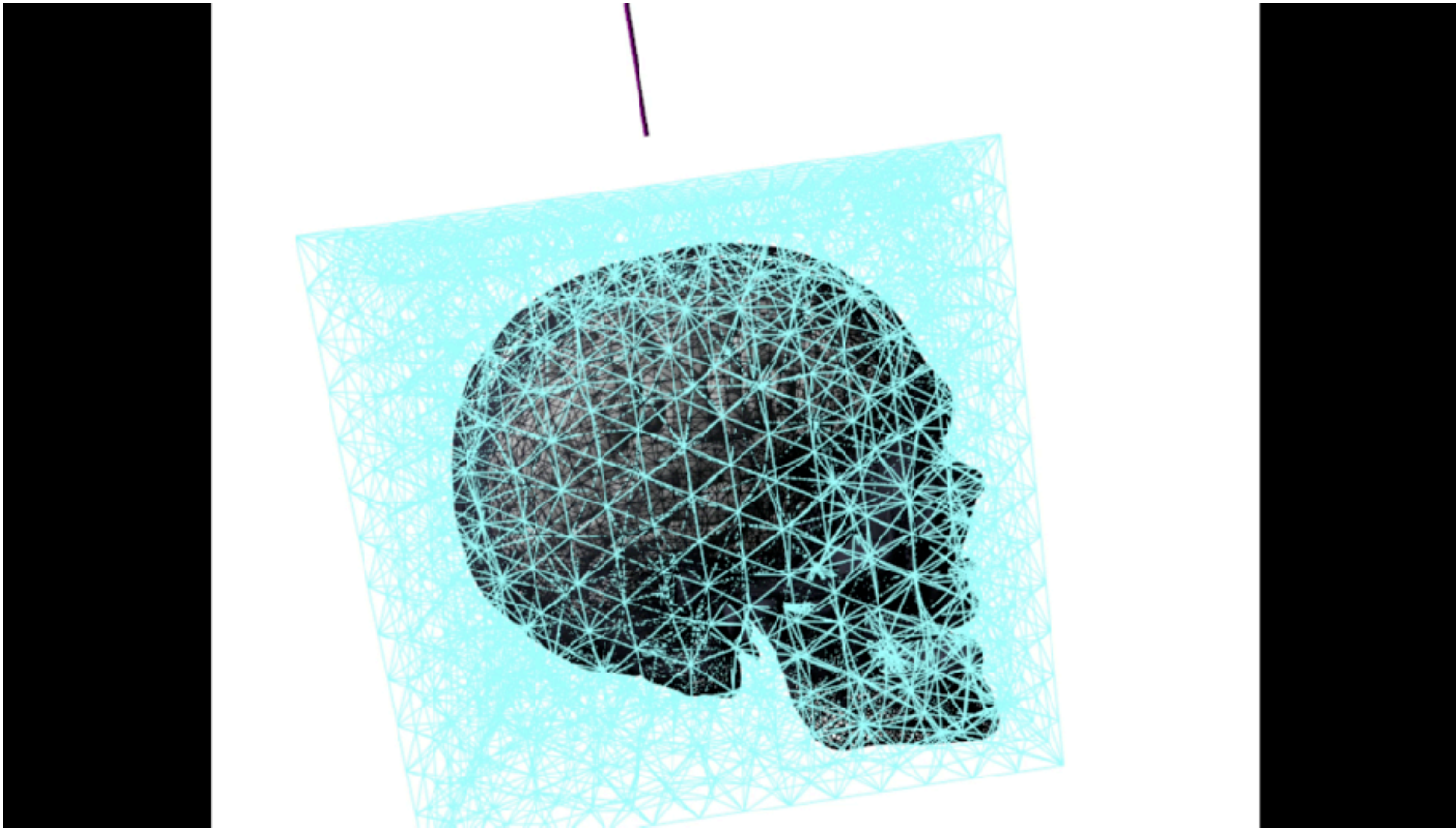


Hamzé et al, Comput. Med. Imag. Grap. 2015

H.P. Bui

RealTCut

Brain shift and electrode implantation



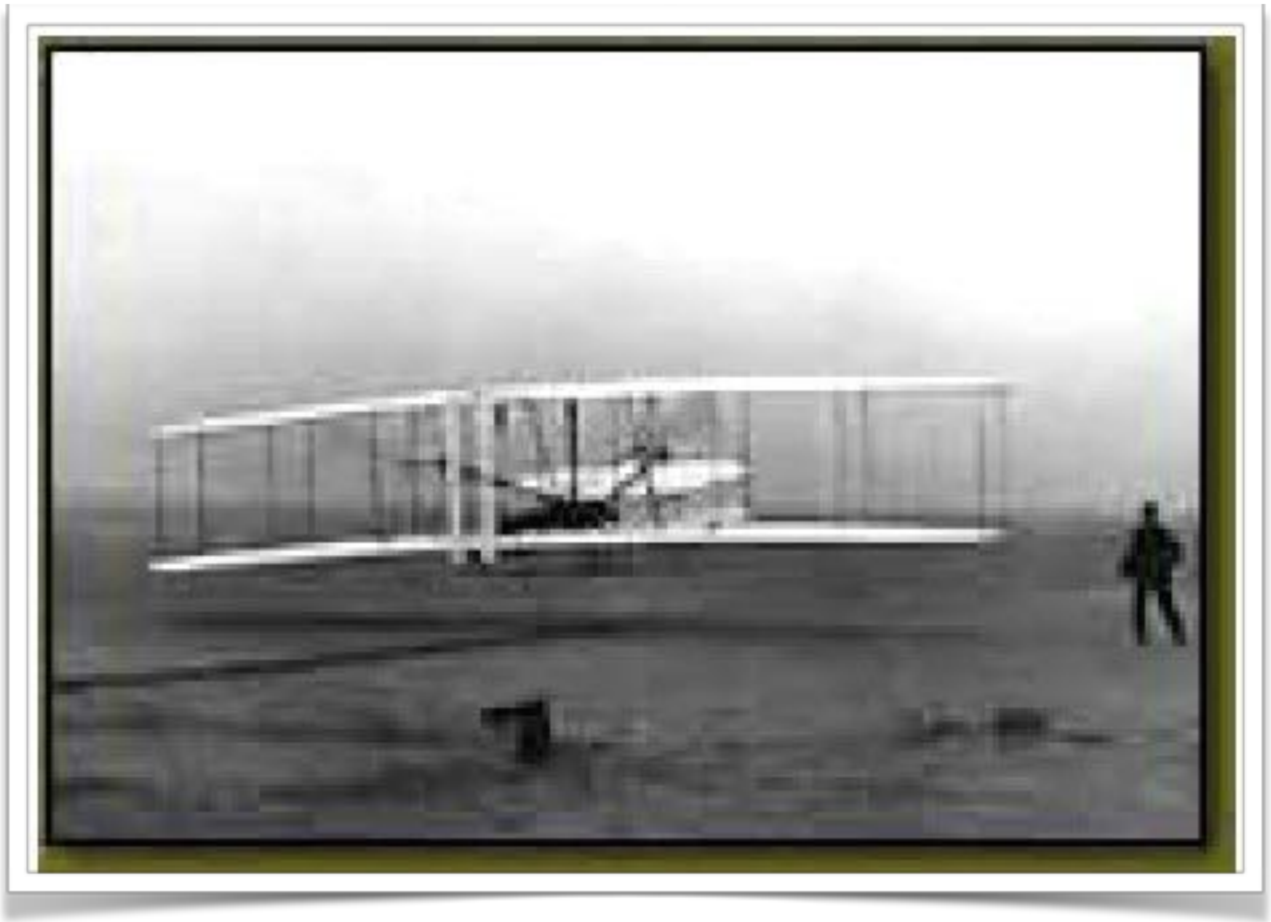
Controlling the Error on Target Motion through Real-time Mesh Adaptation: Applications to Deep Brain Stimulation, HP Bui et al, Int J Numer Meth Bio, 2017.

erc RealTCut

Wilbur and Orville Wright

Wright Flyer

10:35am Dec 17, 1903

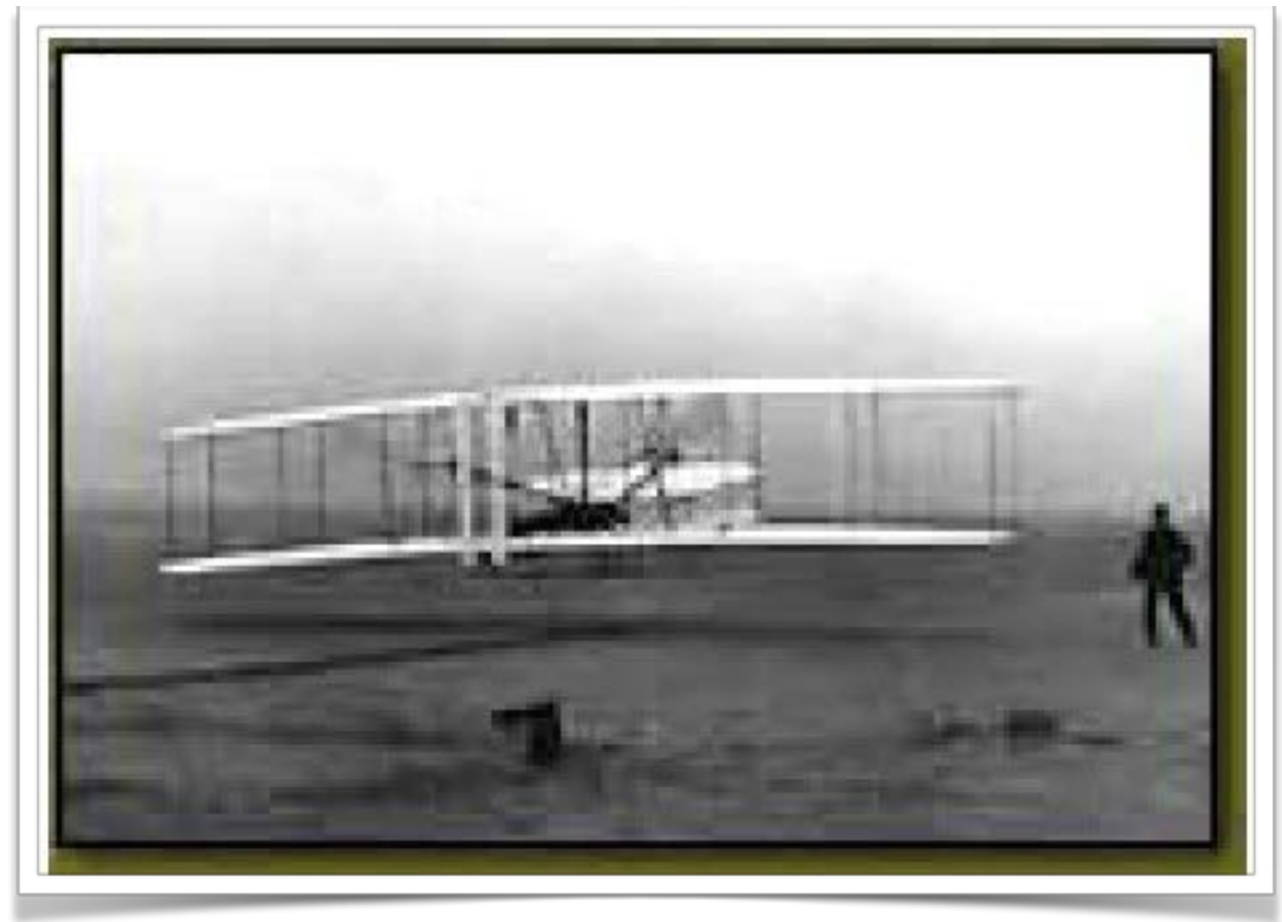


Wilbur and Orville Wright

On Dec 14 Wilbur won the coin toss, made the first attempt and stalled

Orville made the first flight on Dec. 17

12 seconds & 120 ft

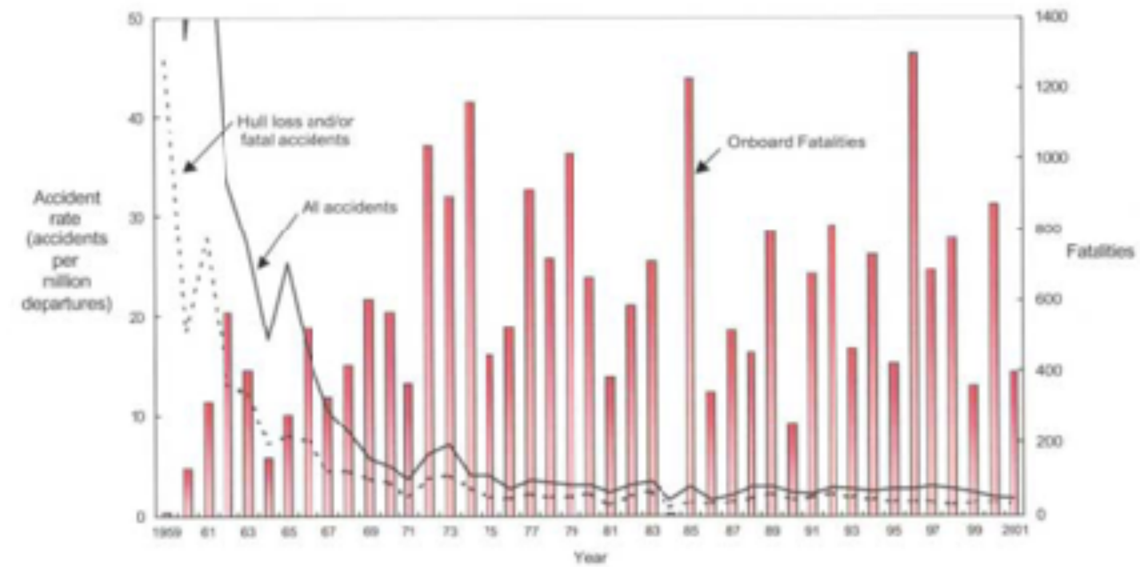


Aircraft safety

20,000 years

Worldwide statistics

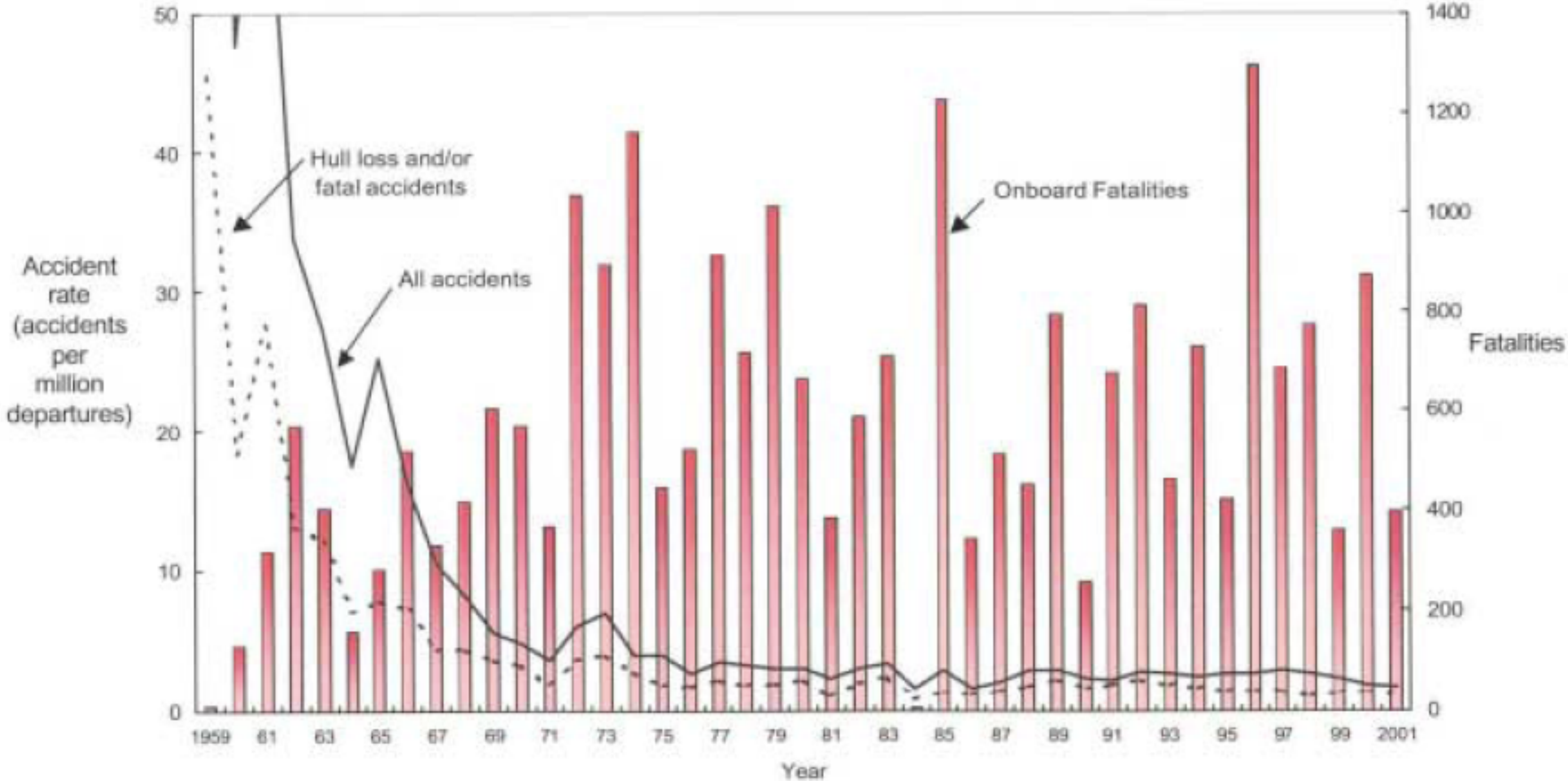
[1959-2001] 1,307 commercial jet aircraft losses



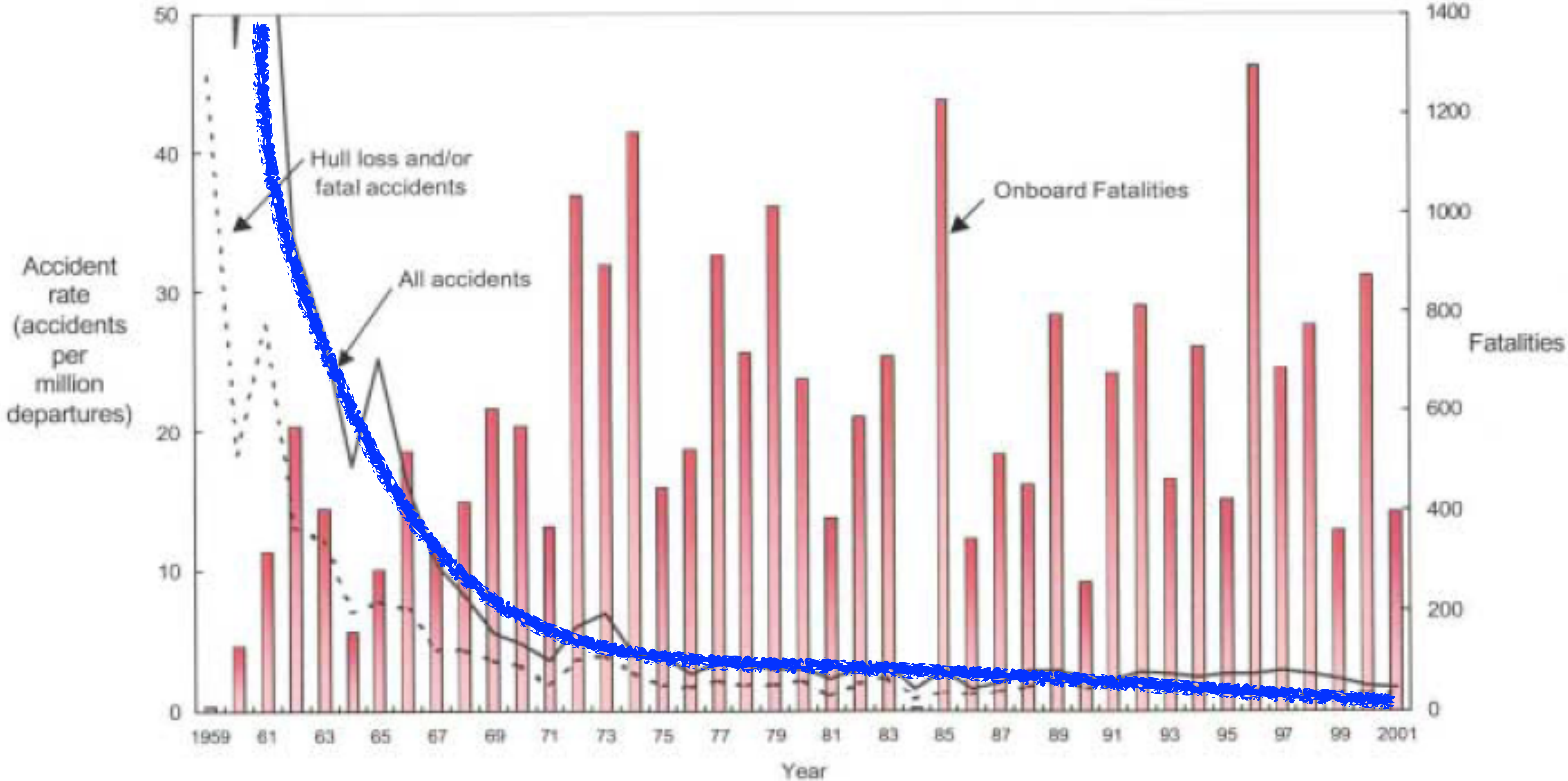
Today:

1 accident per 1,000,000 departures

Accident rates and fatalities/year

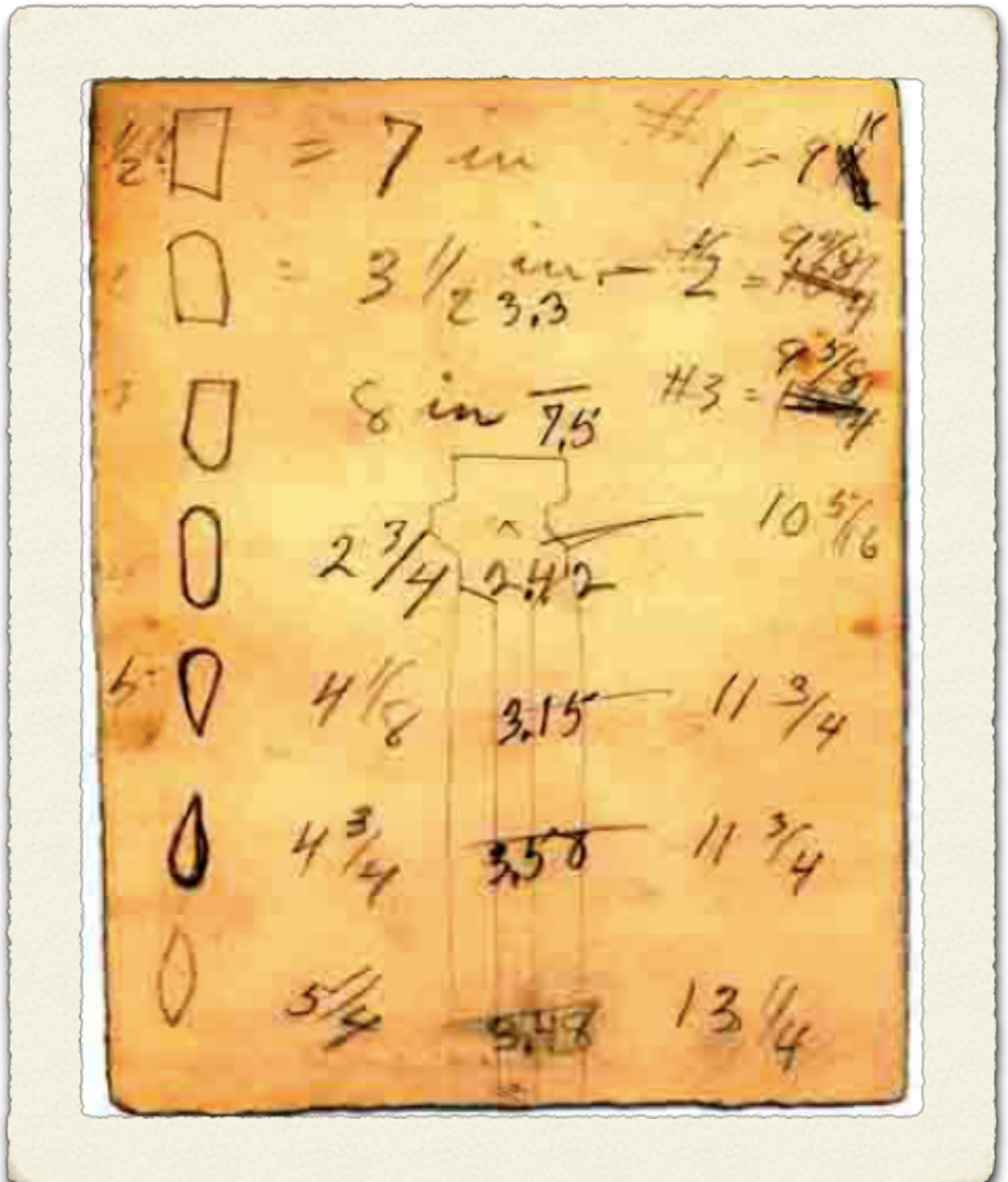


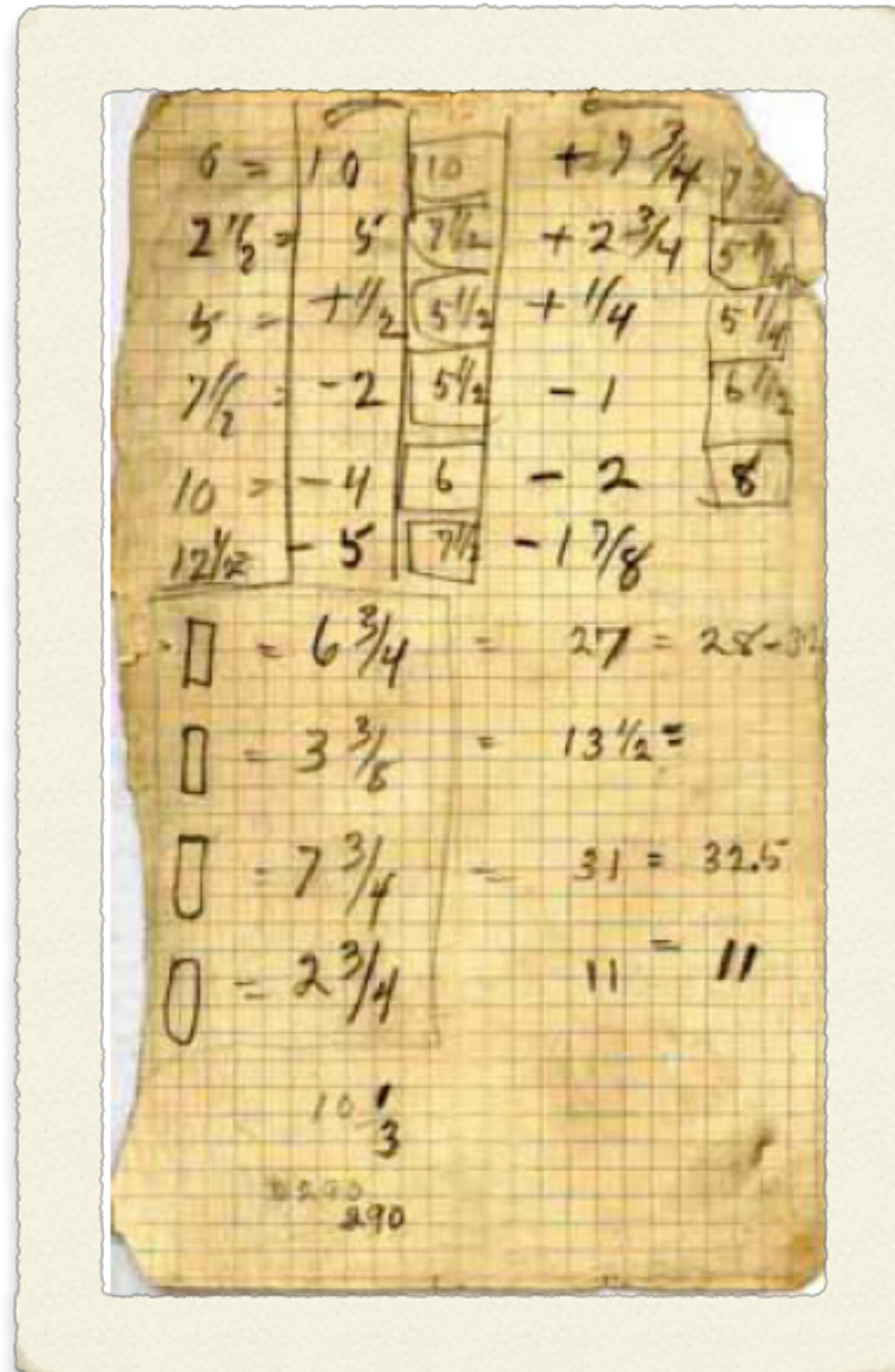
Accident rates and fatalities/year



Source: Flight Safety Foundation/Boeing Commercial Airplane Group

Learning from intuition & theory





Franklin Institute Science Museum. Wilbur Wright's handwriting

Learning from experience

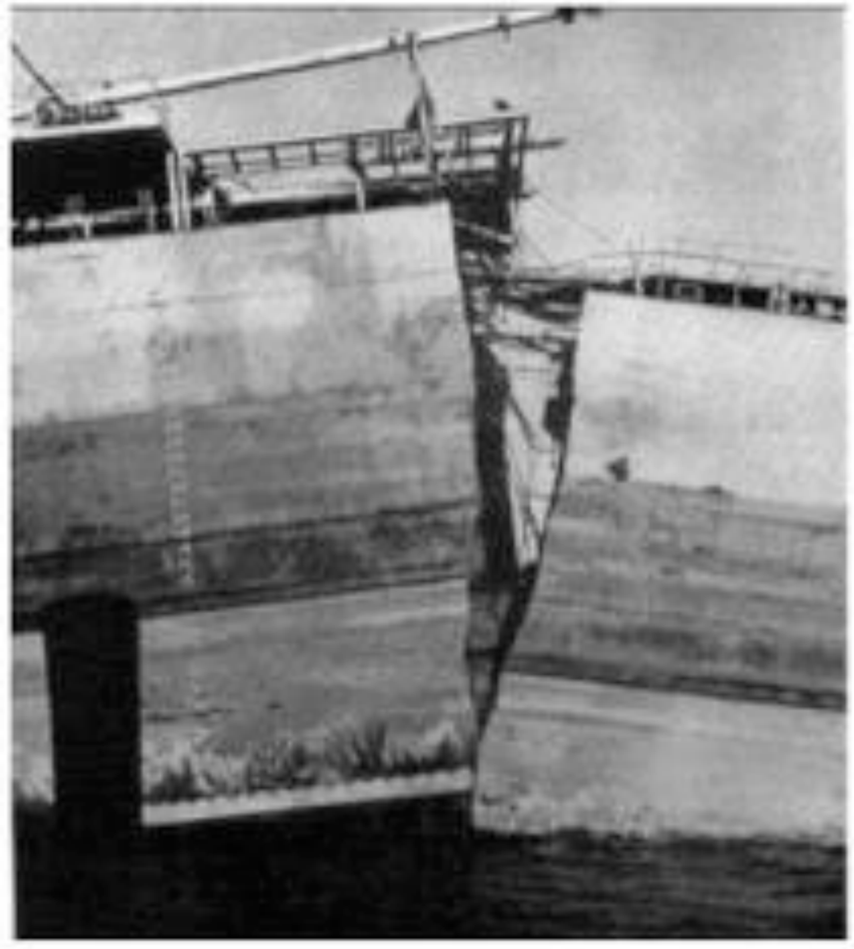
Increased practical understanding of mechanics — in particular fracture and fatigue

Aloha airlines accident - fatigue cracks at corners

Novel convertible aircraft

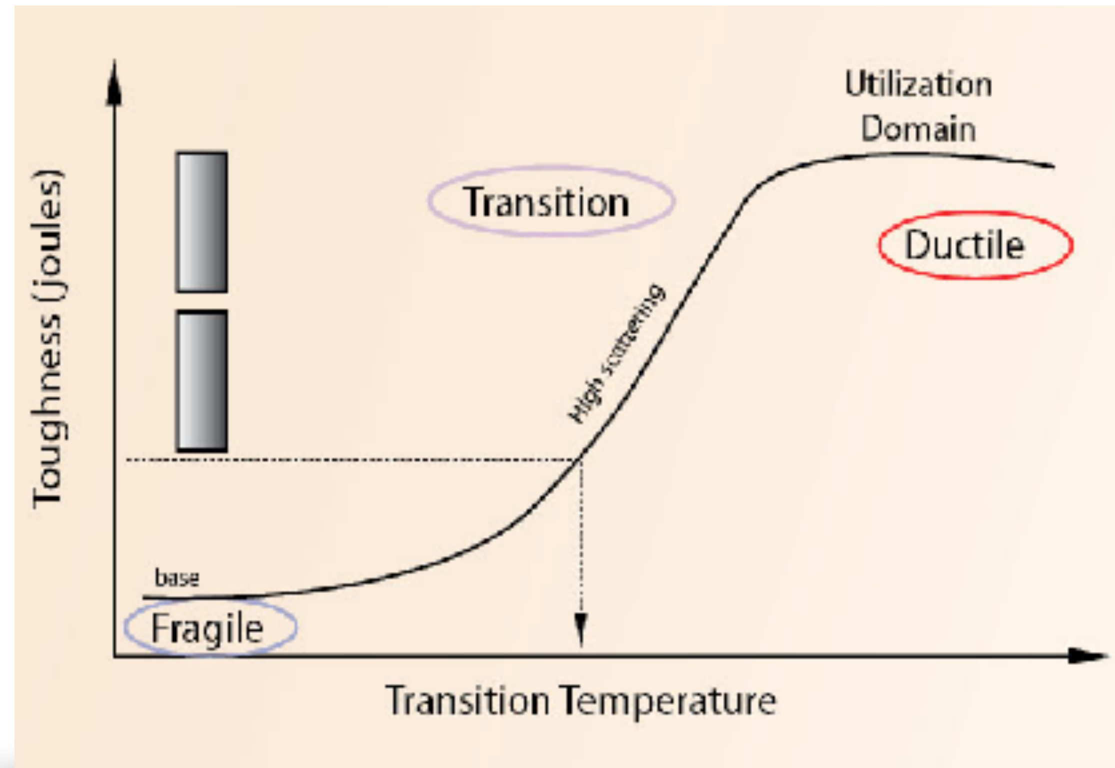
Learning from experience

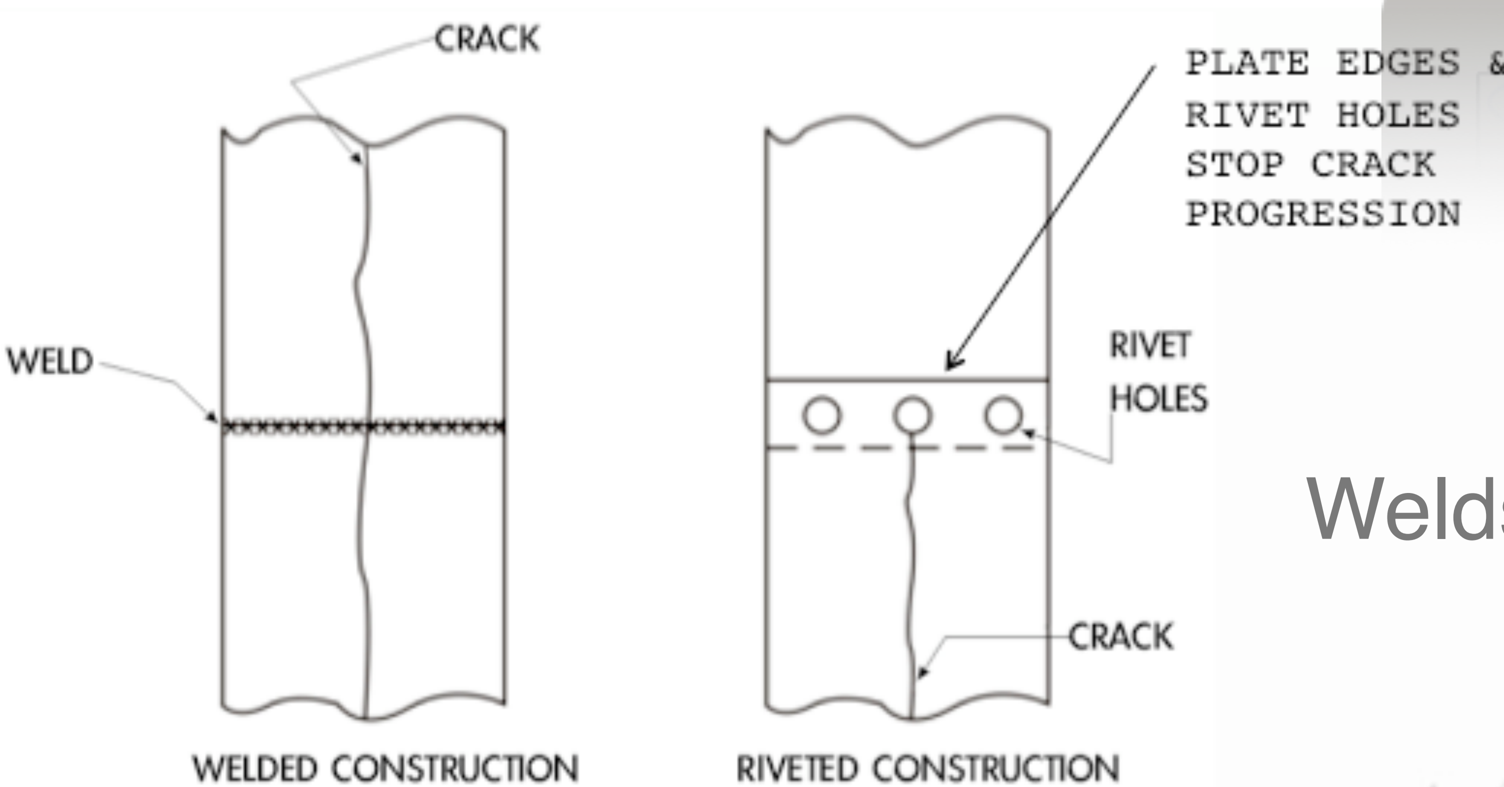
The Liberty Ships



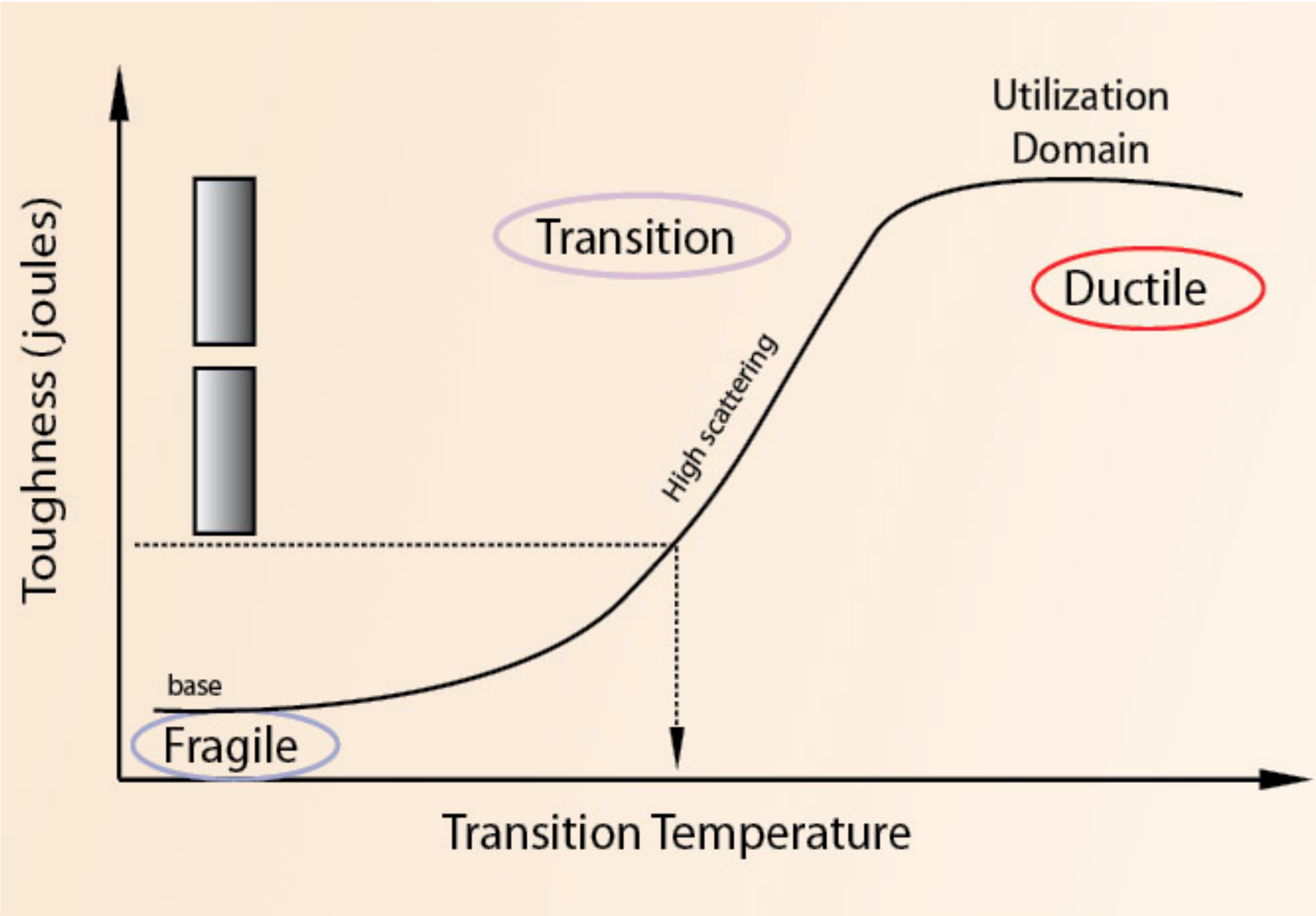
Learning from experience

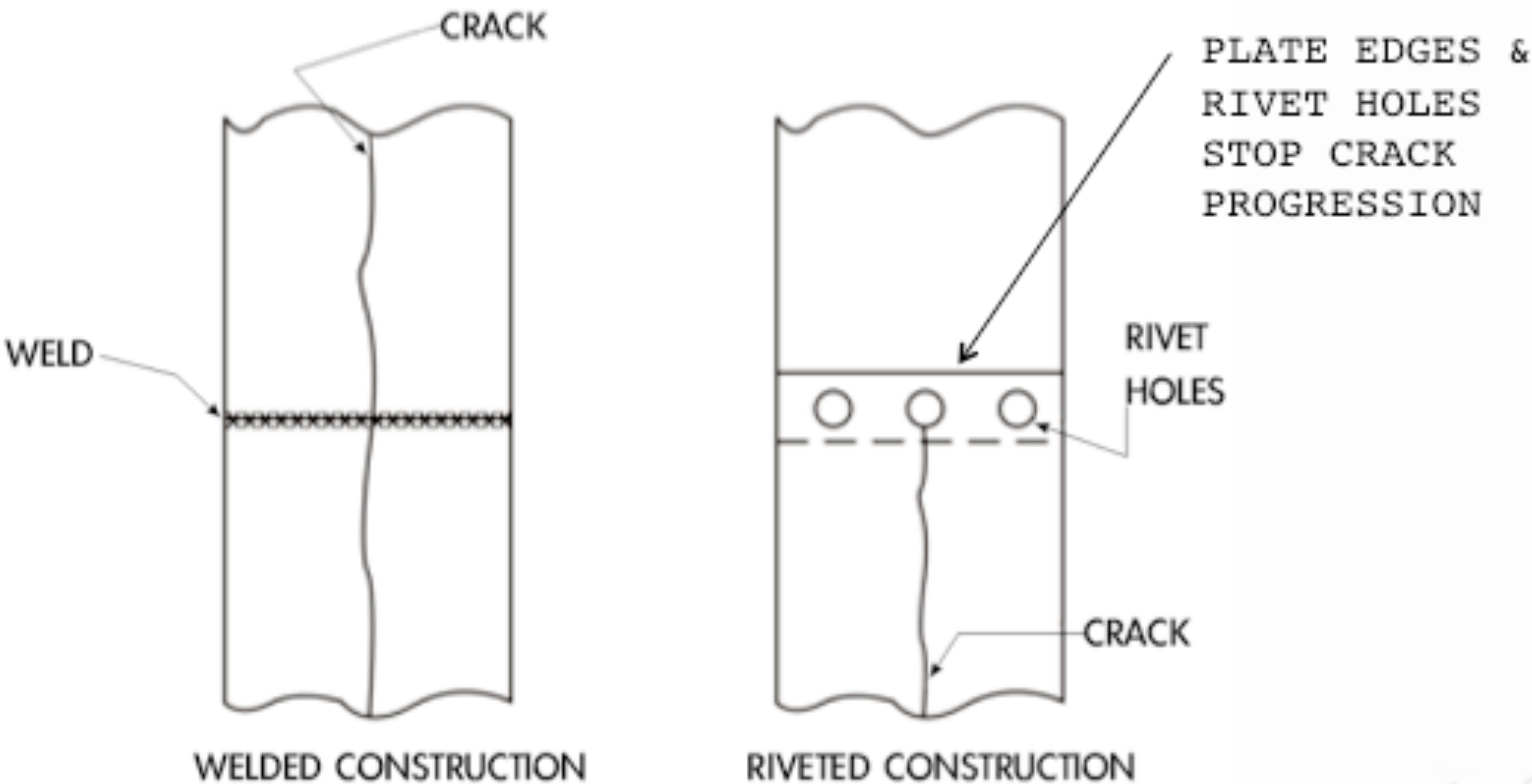
At low temperatures, steel becomes more brittle





Welds are not good crack arrestors

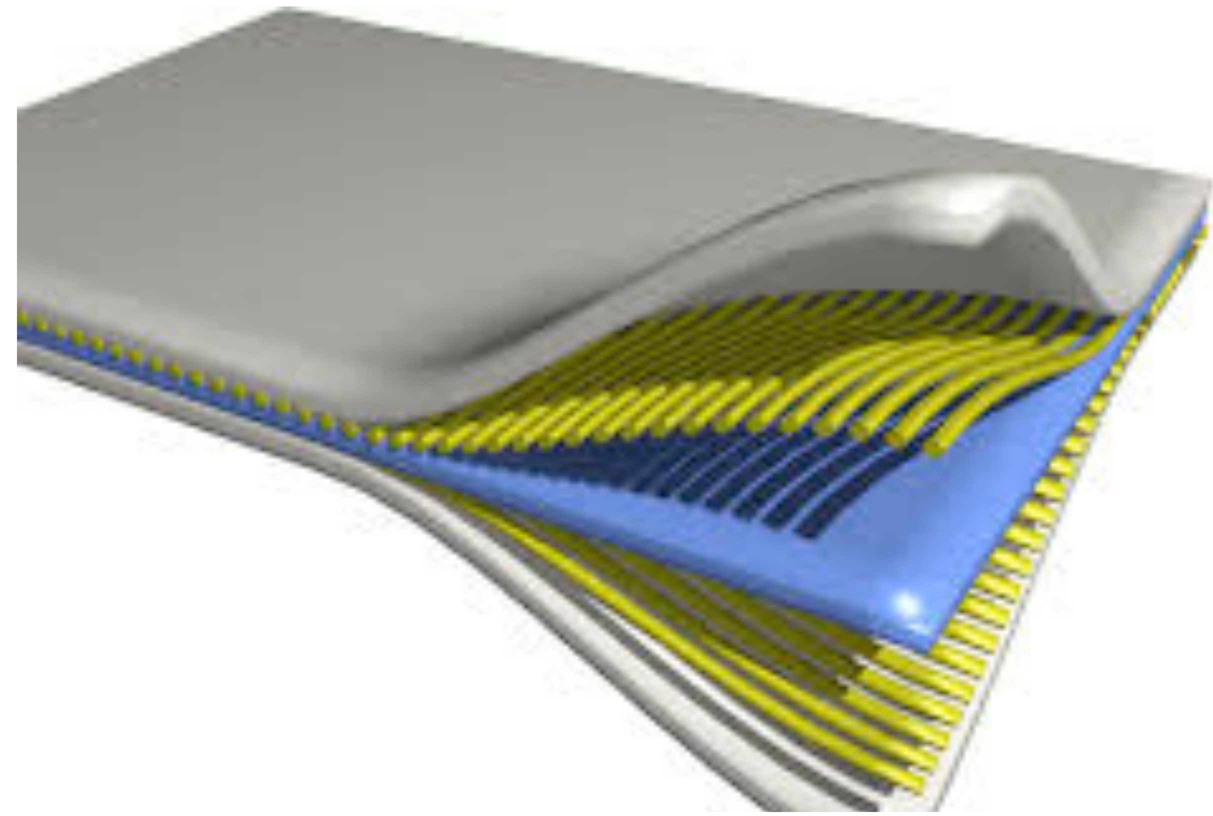

Utilization
Domain
Transition
Ductile
High scattering
base
Fragile
Toughness (joules)
Transition Temperature


CRACK
PLATE EDGES &
RIVET HOLES
STOP CRACK
PROGRESSION
WELD
RIVET
HOLES
CRACK
WELDED CONSTRUCTION
RIVETED CONSTRUCTION

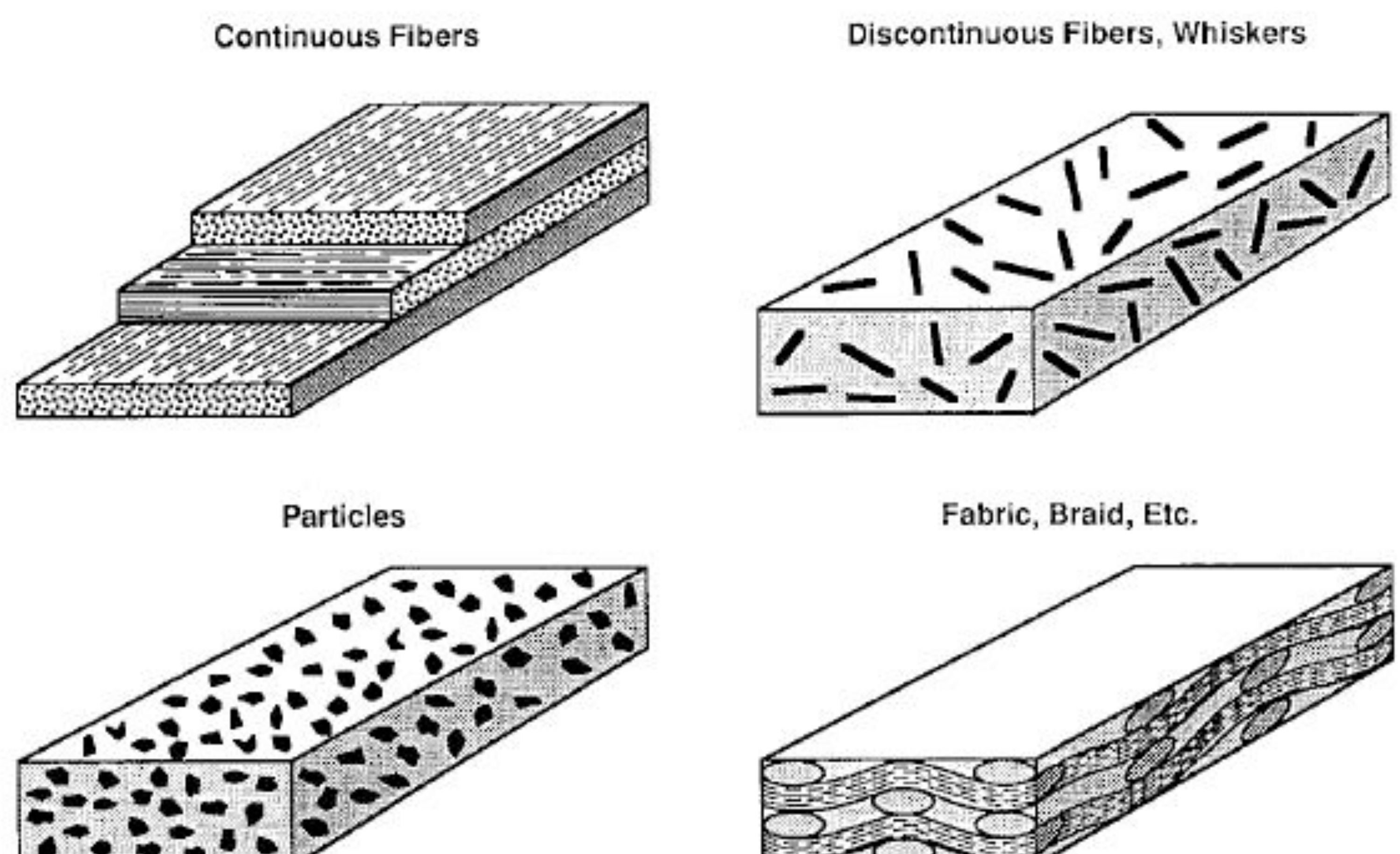
teaching...

New materials for more payload

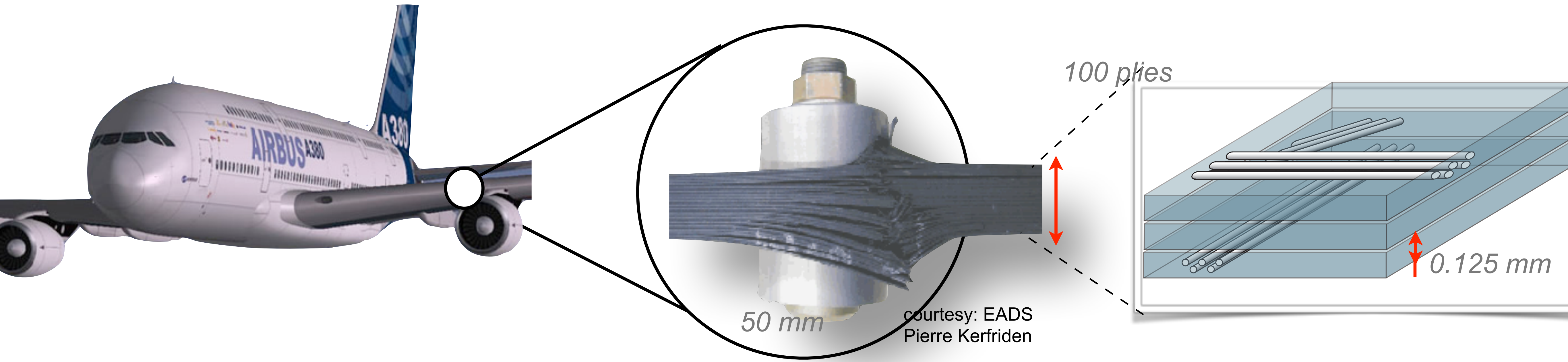
Introduction of composite materials have reduced the weight of structures by 20%



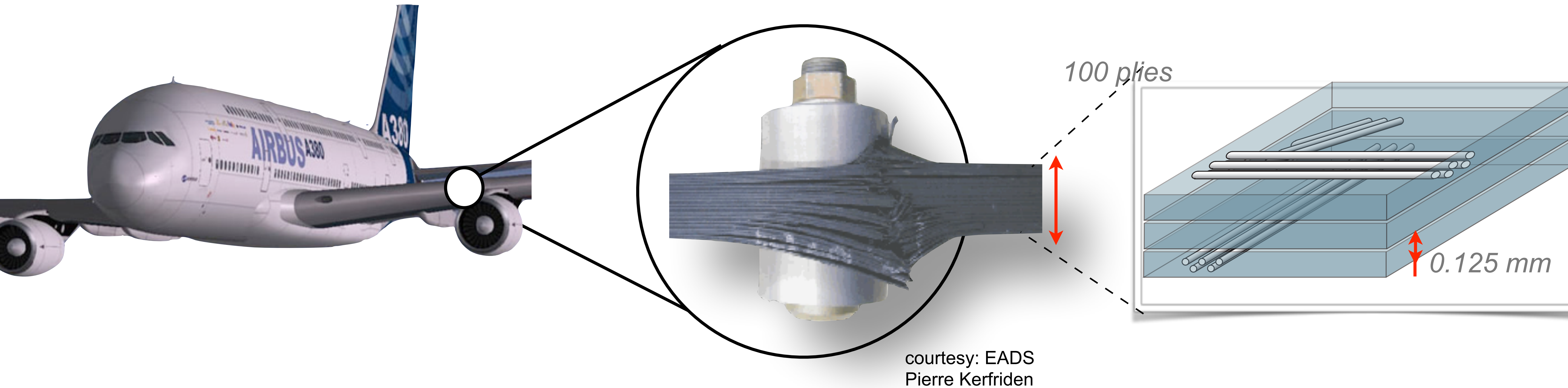
Over 1,000km savin
of 8,660kg of fuel
[A340-300]



Material complexity

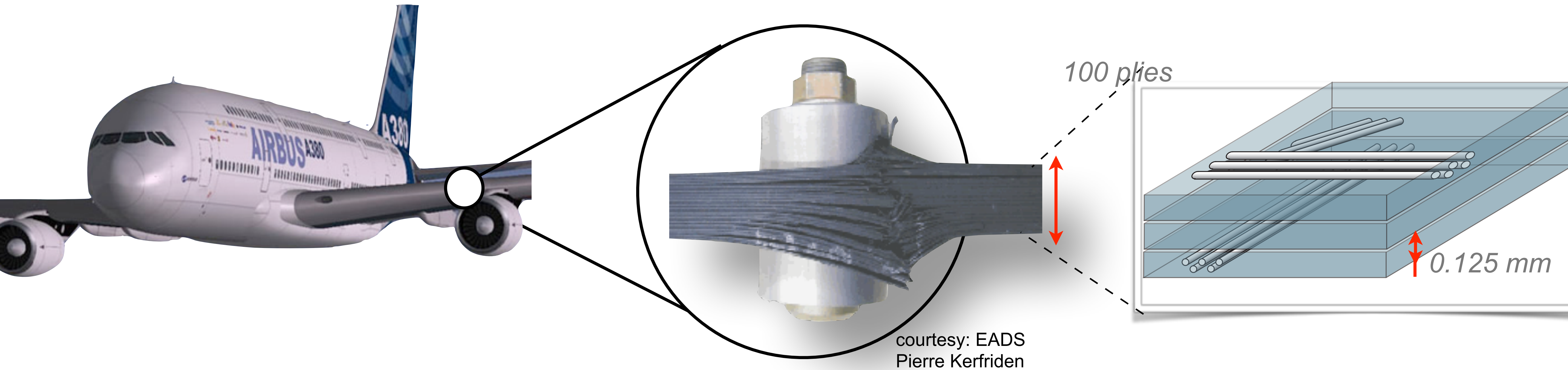


Material complexity



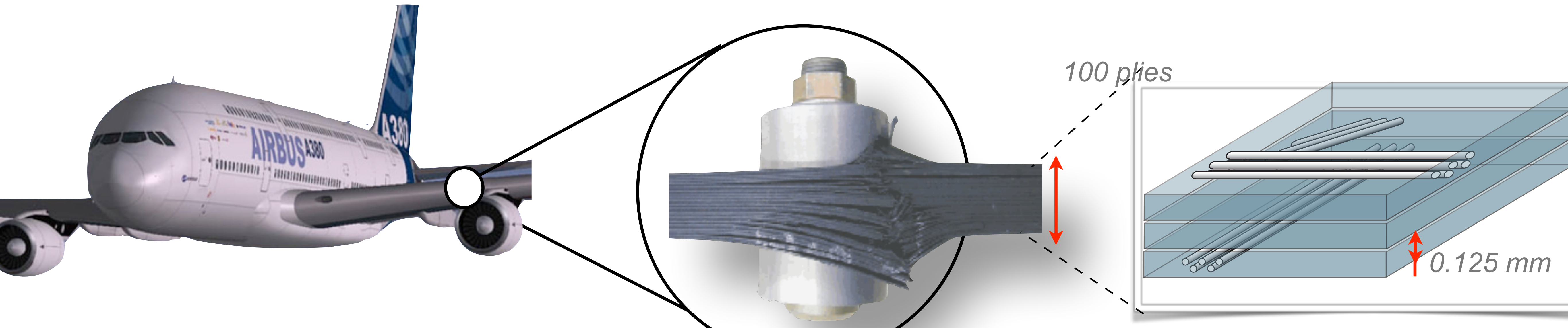
- Heterogeneous & multi-functional
- Experiments required to attain sufficient confidence in their behavior are increasingly costly

Material complexity



- Heterogeneous & multi-functional
- Experiments required to attain sufficient confidence in their behavior are increasingly costly
- Factor-of-Safety or probabilistic based methods cannot handle unknown unknowns
- Lack of similitude between testing (experimental) and operating conditions — also encountered in geophysics…

Material complexity

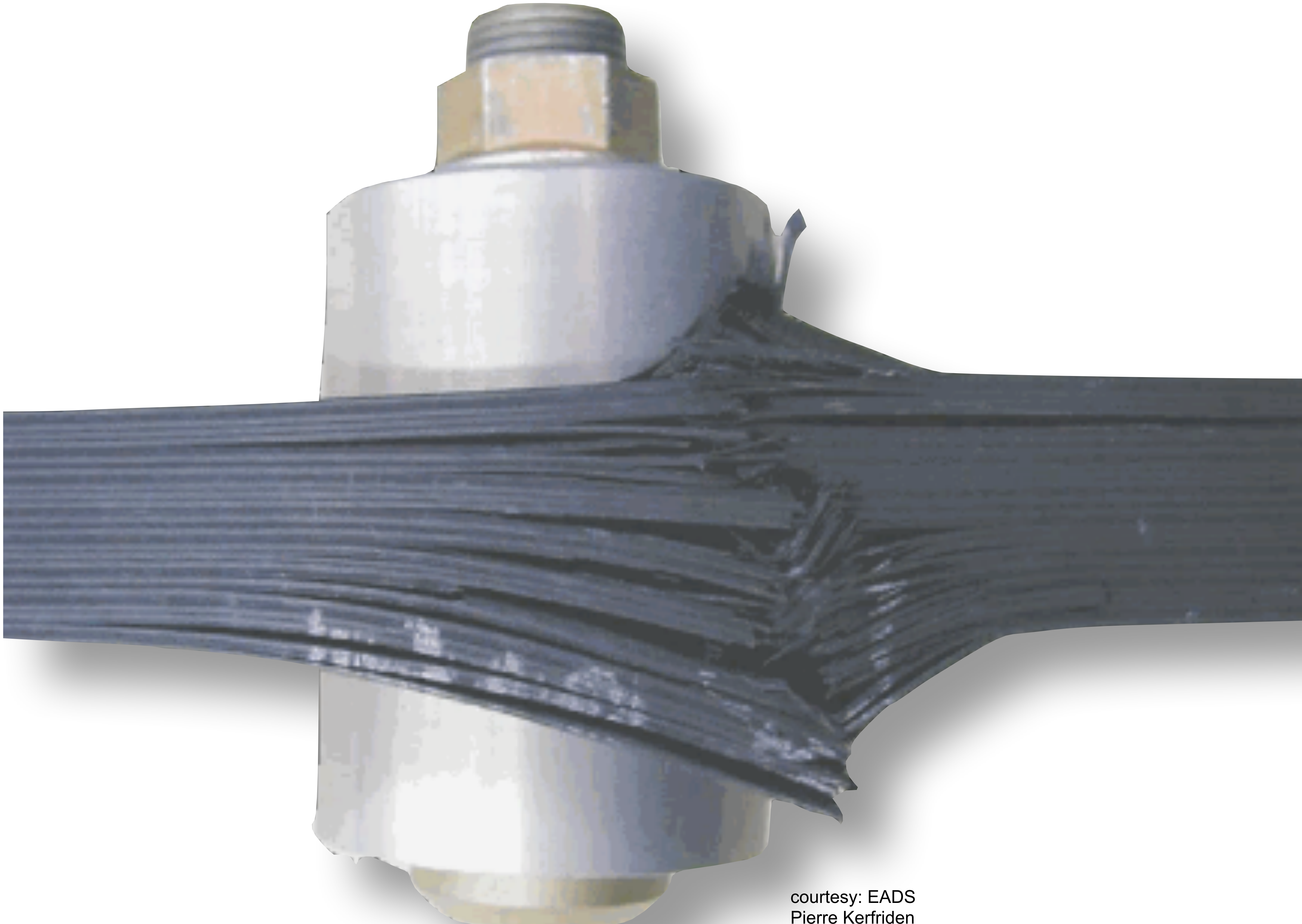


courtesy: EADS
Pierre Kerfriden

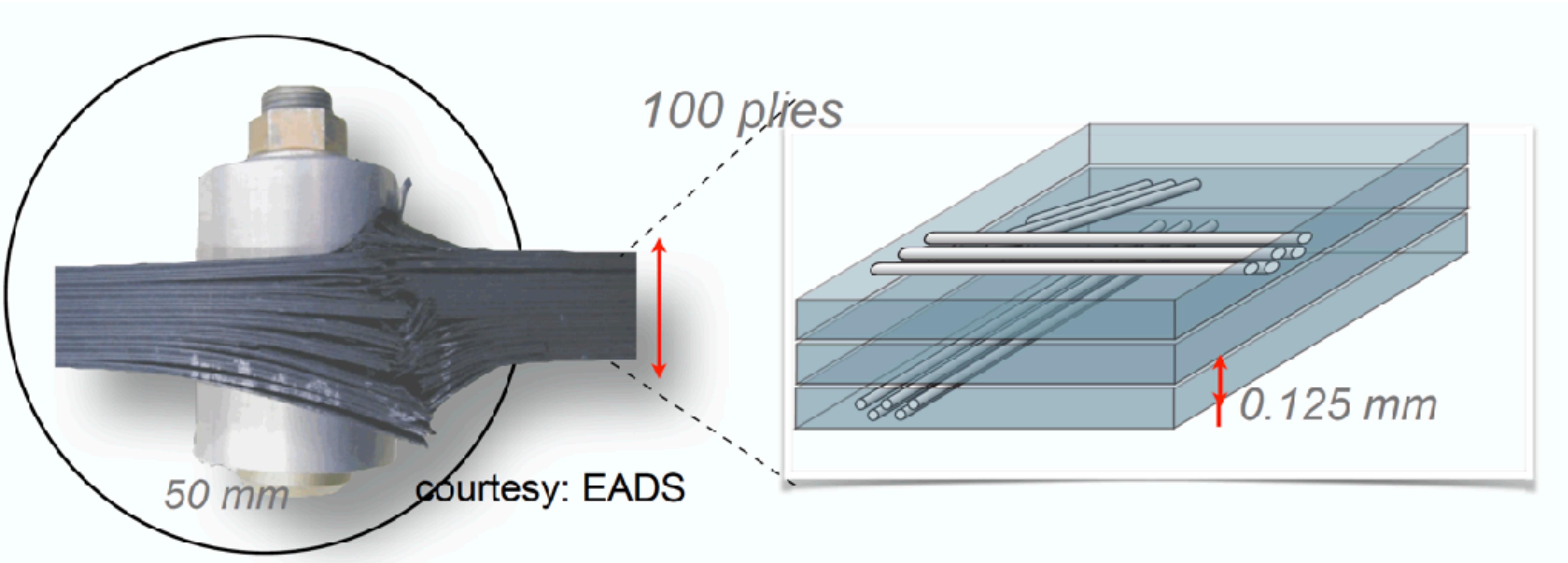
- Heterogeneous & multi-functional
- Experiments required to attain sufficient confidence in their behavior are increasingly costly
- Factor-of-Safety or probabilistic based methods cannot handle unknown unknowns - lack of similitude

- Move **away from heuristics** and experience-based engineering
- Develop **fundamental understanding** of physical processes (degradation, …)
- Reduce weight

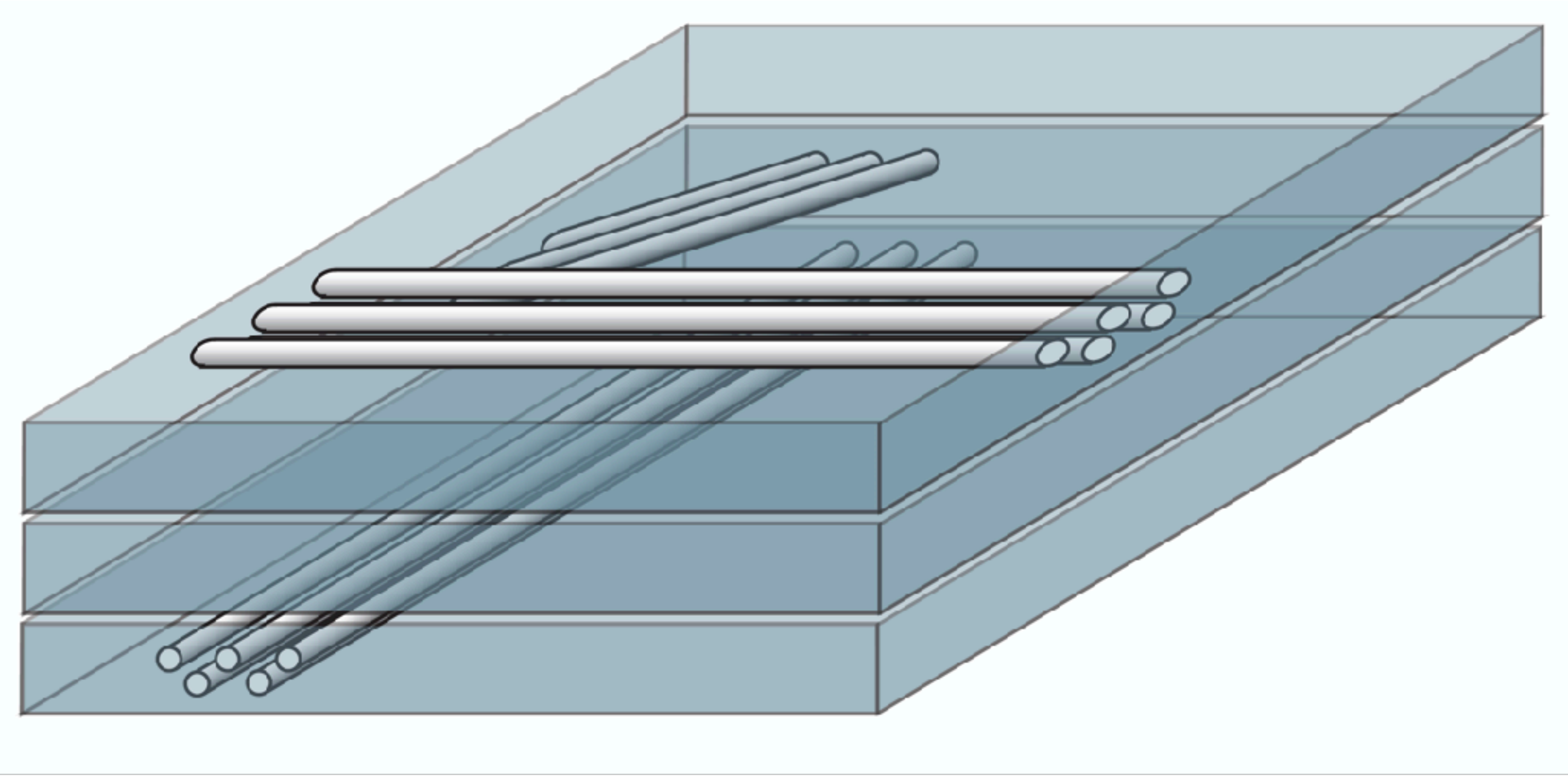
A bolted joint



courtesy: EADS
Pierre Kerfriden



100 plies
0.125 mm
50 mm
courtesy: EADS



courtesy: EADS
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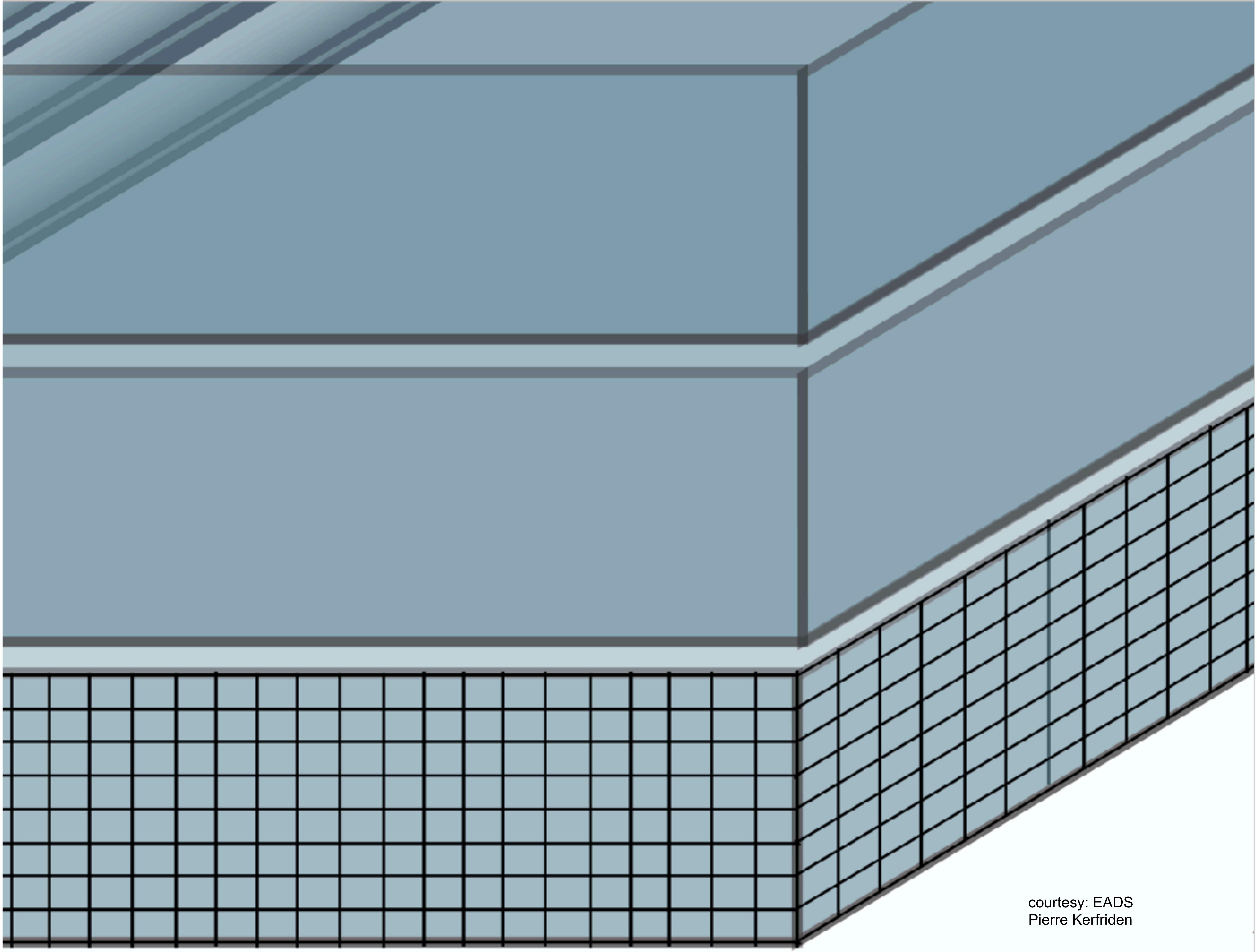
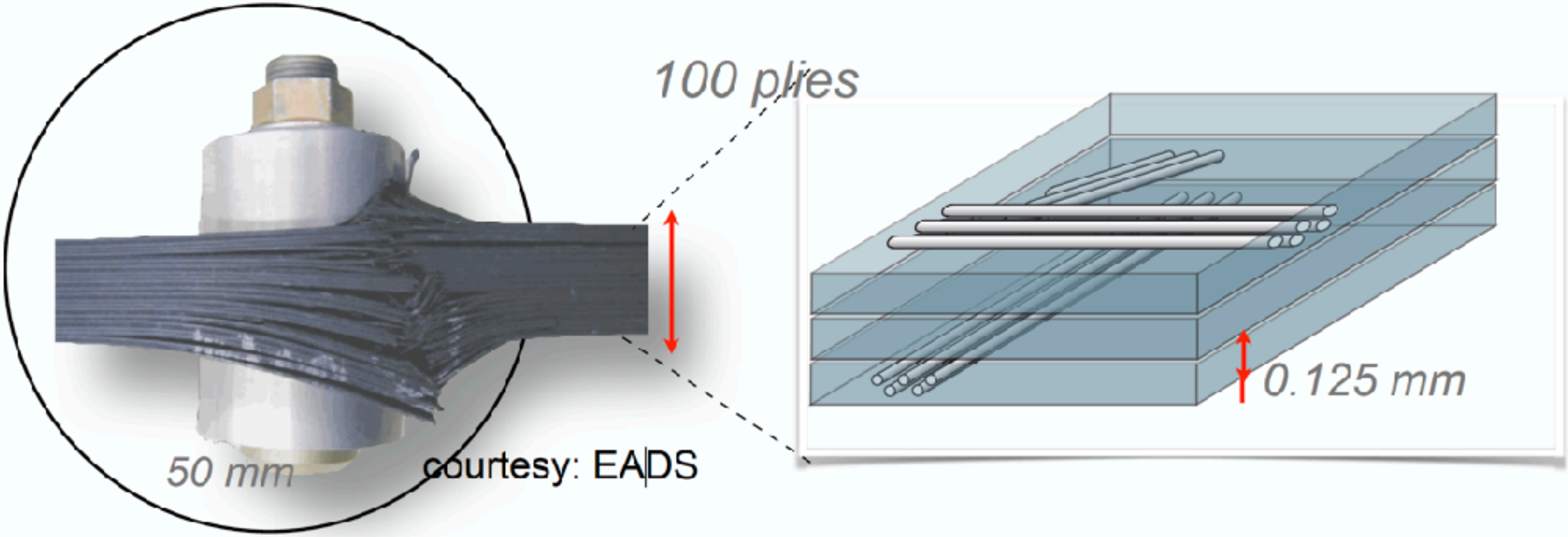
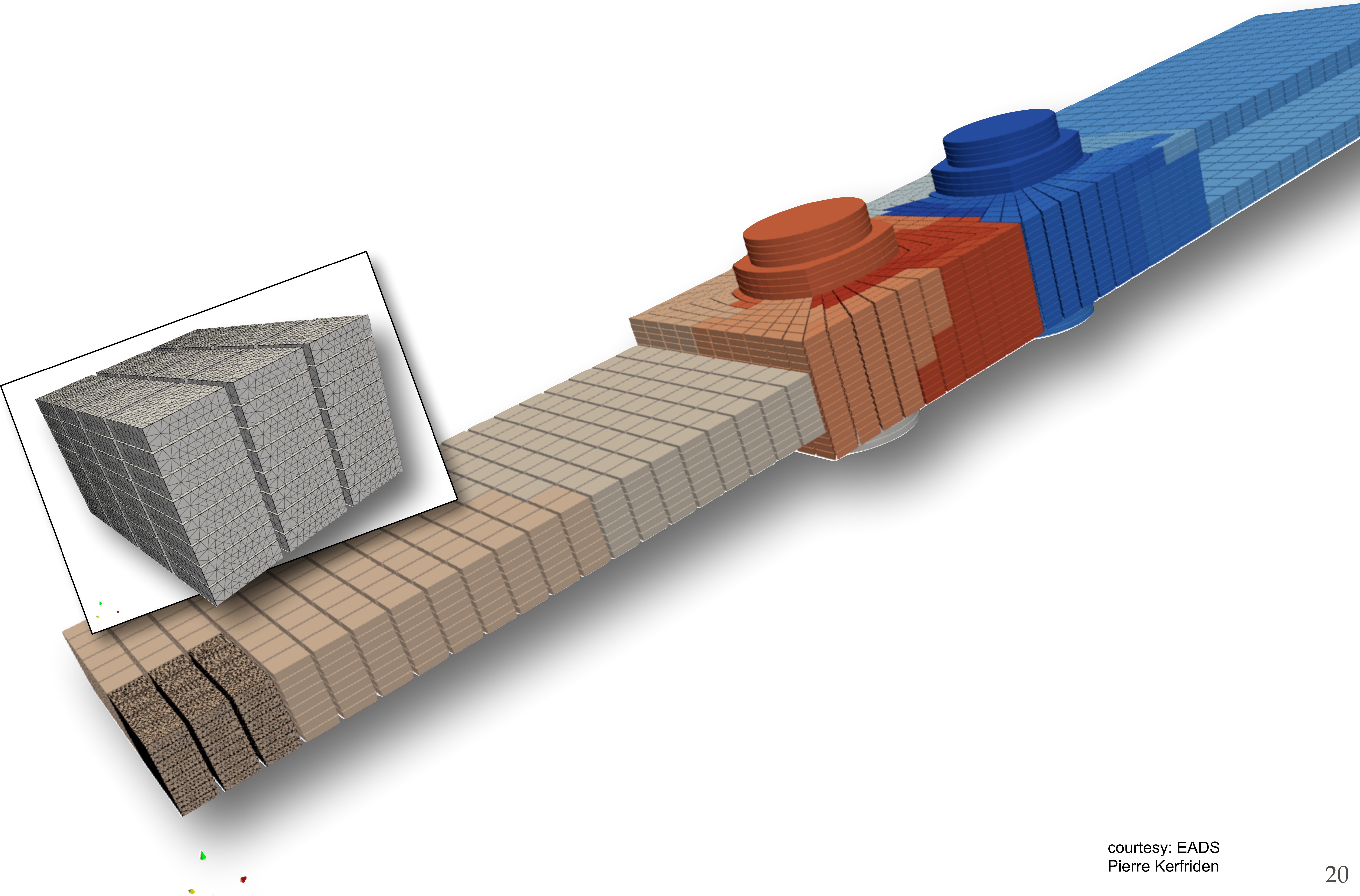

courtesy: EADS
Pierre Kerfriden

One single bolted joint

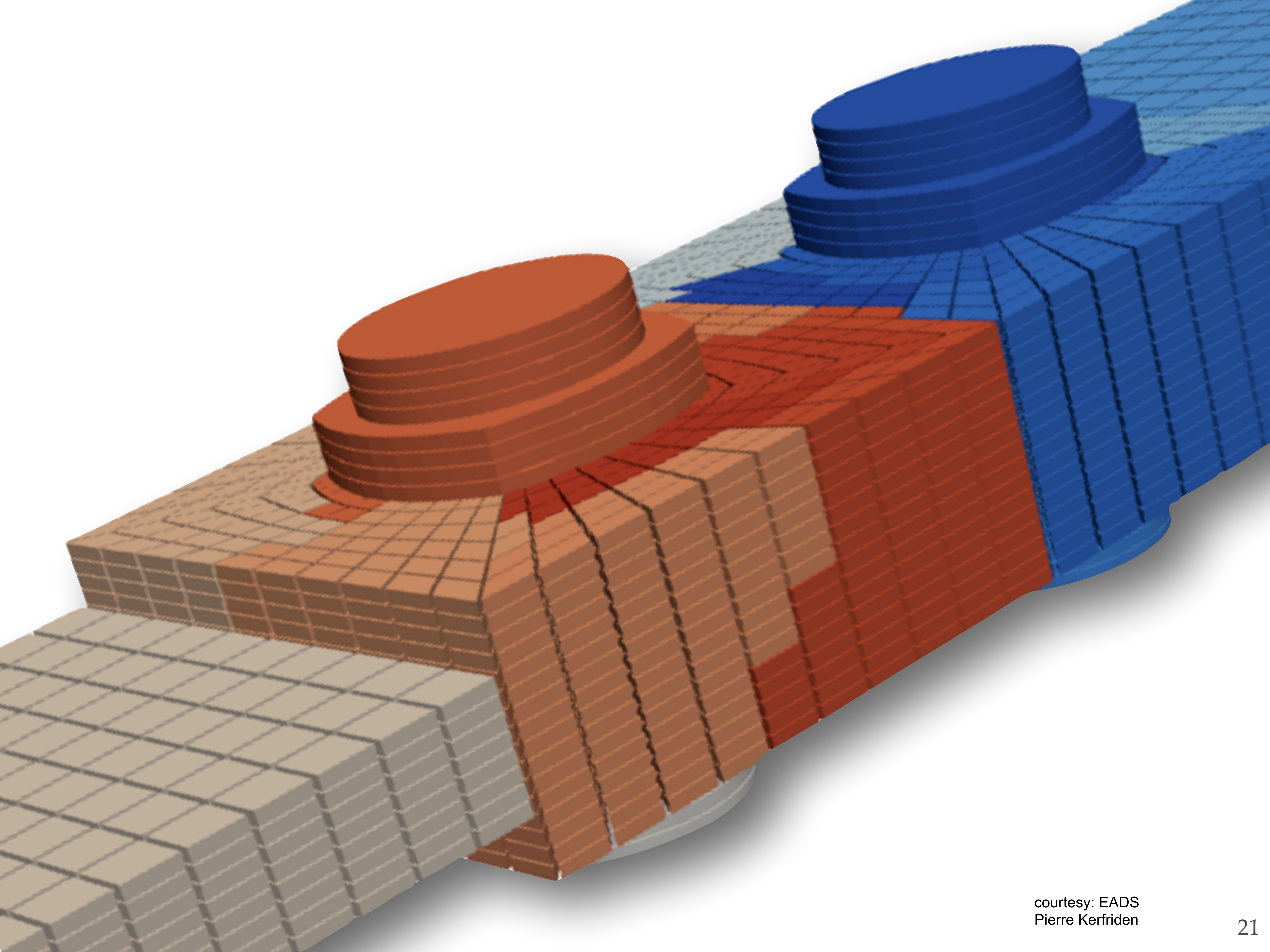


- 5 elements through the thickness of a ply => 0.025mm/element
- 50mm bolted joint area => 2,000 elements
- 50mm x 50mm x 100 plies => 2,000 x 2,000 x (100 x 5)

=> **2 billion elements**

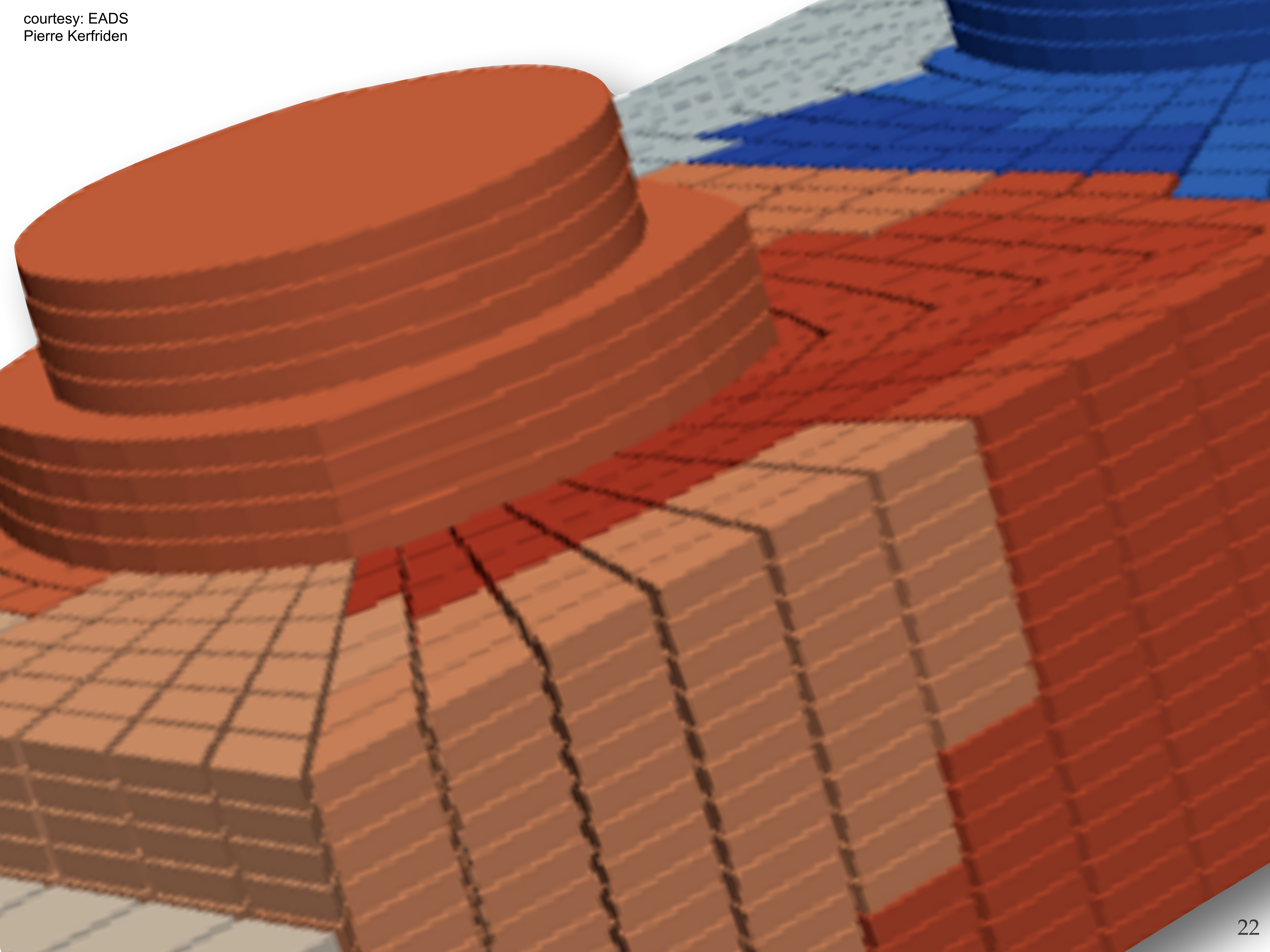


courtesy: EADS
Pierre Kerfriden



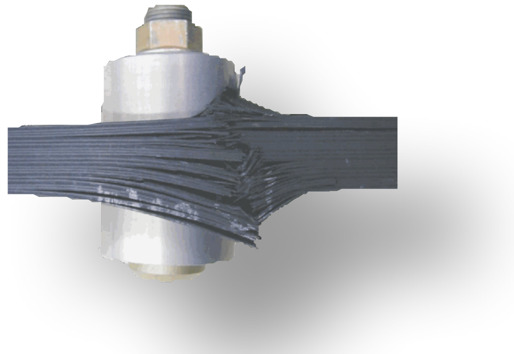
courtesy: EADS
Pierre Kerfriden

courtesy: EADS
Pierre Kerfriden

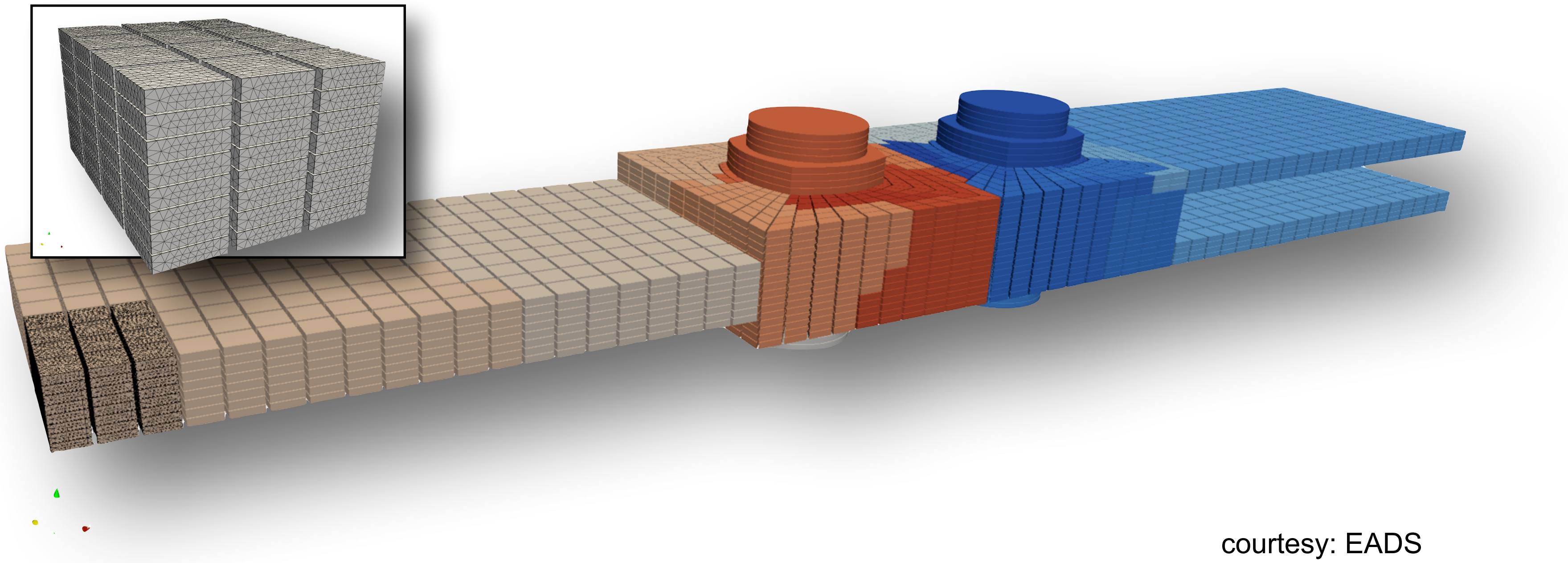


Large structures

whose behaviour is governed by small-scale effects



=> **intractable problem size**



courtesy: EADS
Pierre Kerfriden

How can the problem size be reduced but the accuracy controlled?

Challenge

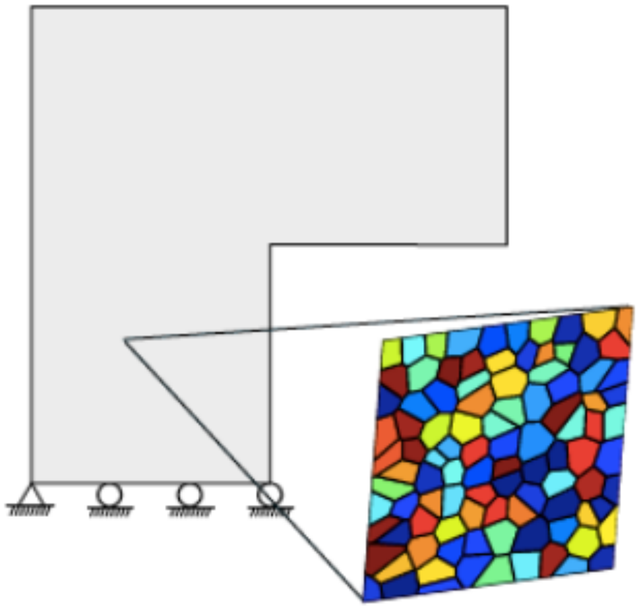
- Reduce the problem size
- Preserve essential features

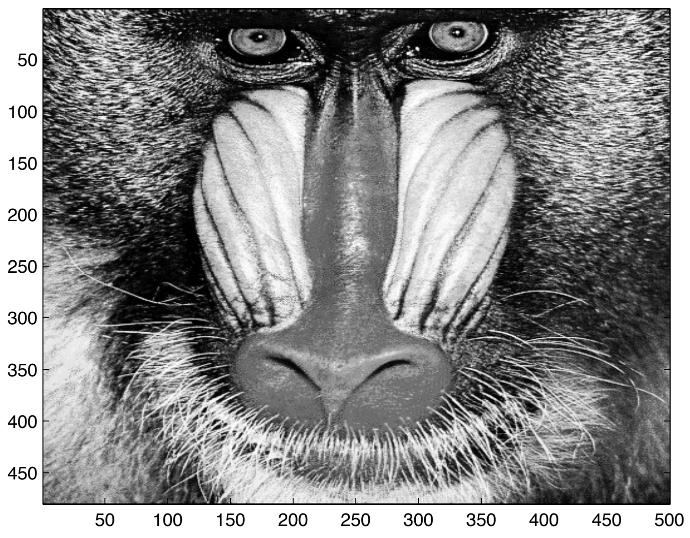
Reduce computational expense

Control the error

Physics based model reduction a.k.a**. Multiscale Methods**

Algebraic based model reduction a.k.a**. Machine Learning**





Challenge

- Reduce the problem size
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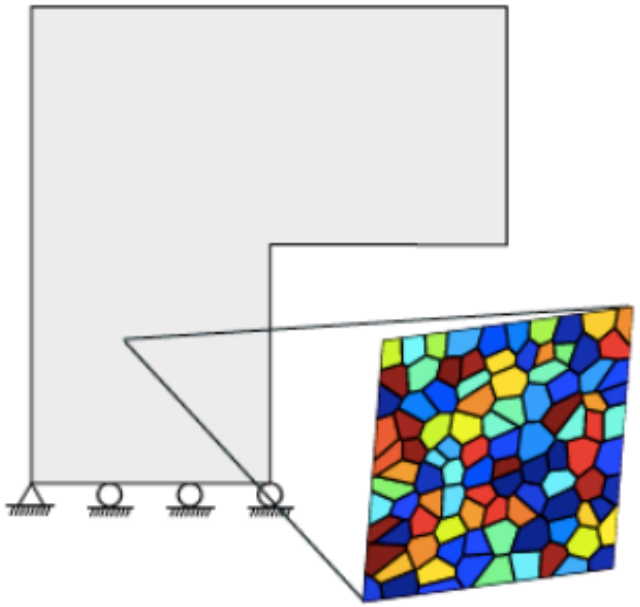
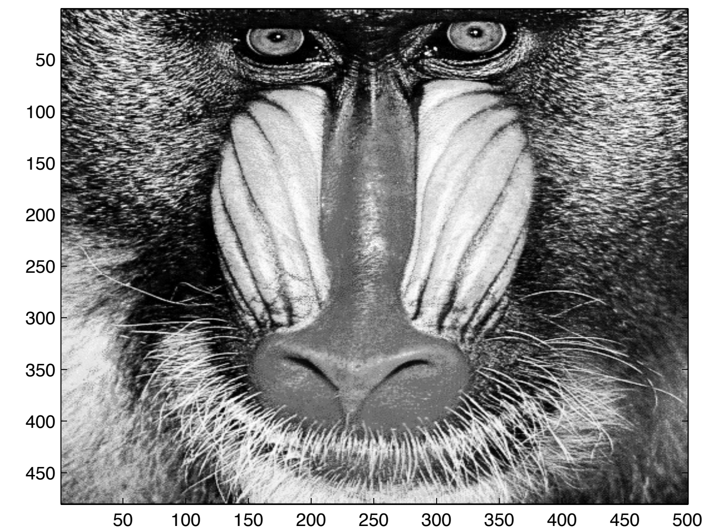
Reduce computational expense

Control the error

Physics based model reduction a.k.a. **Multiscale Methods**

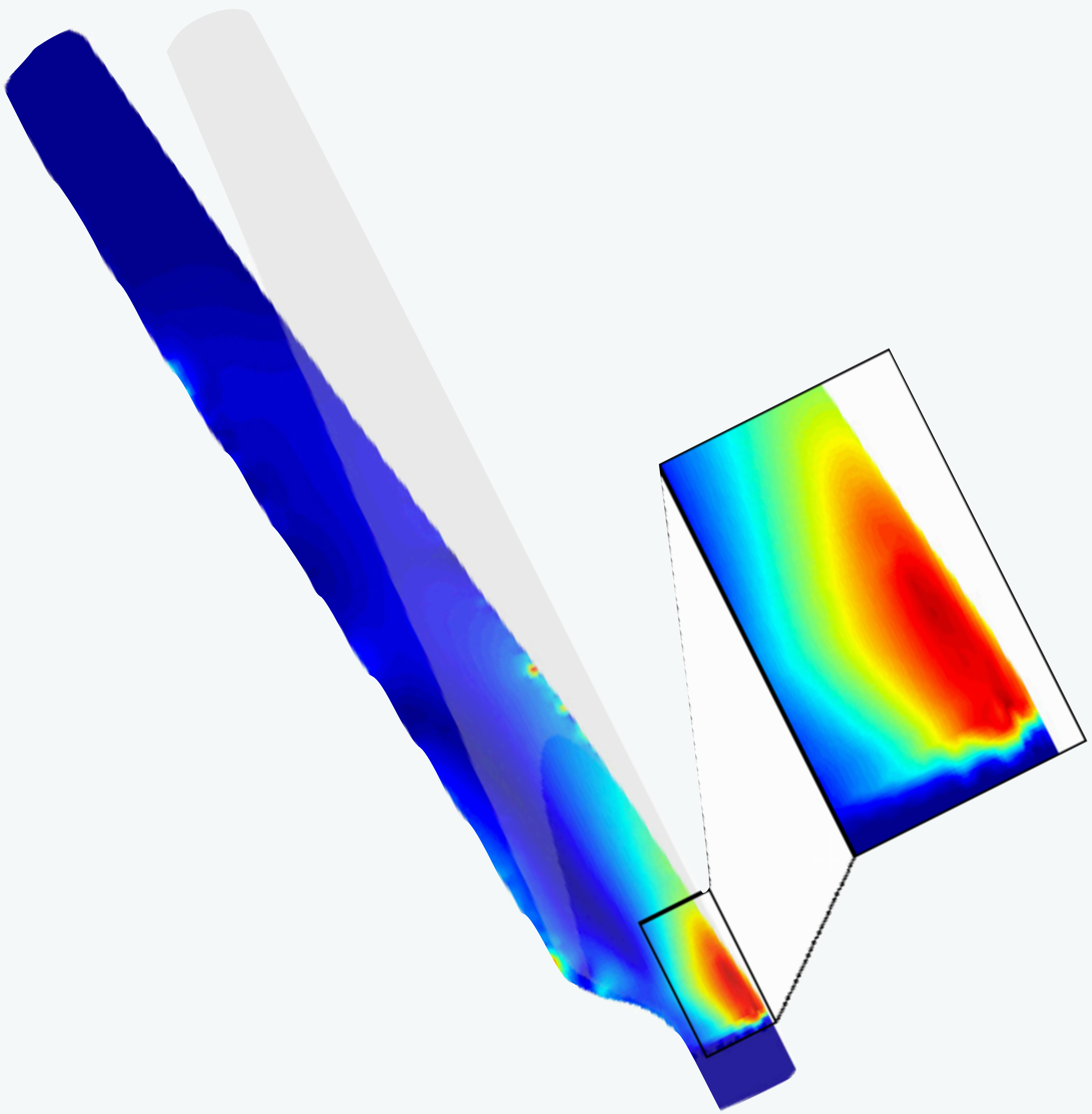
Algebraic based model reduction a.k.a. **Machine Learning**

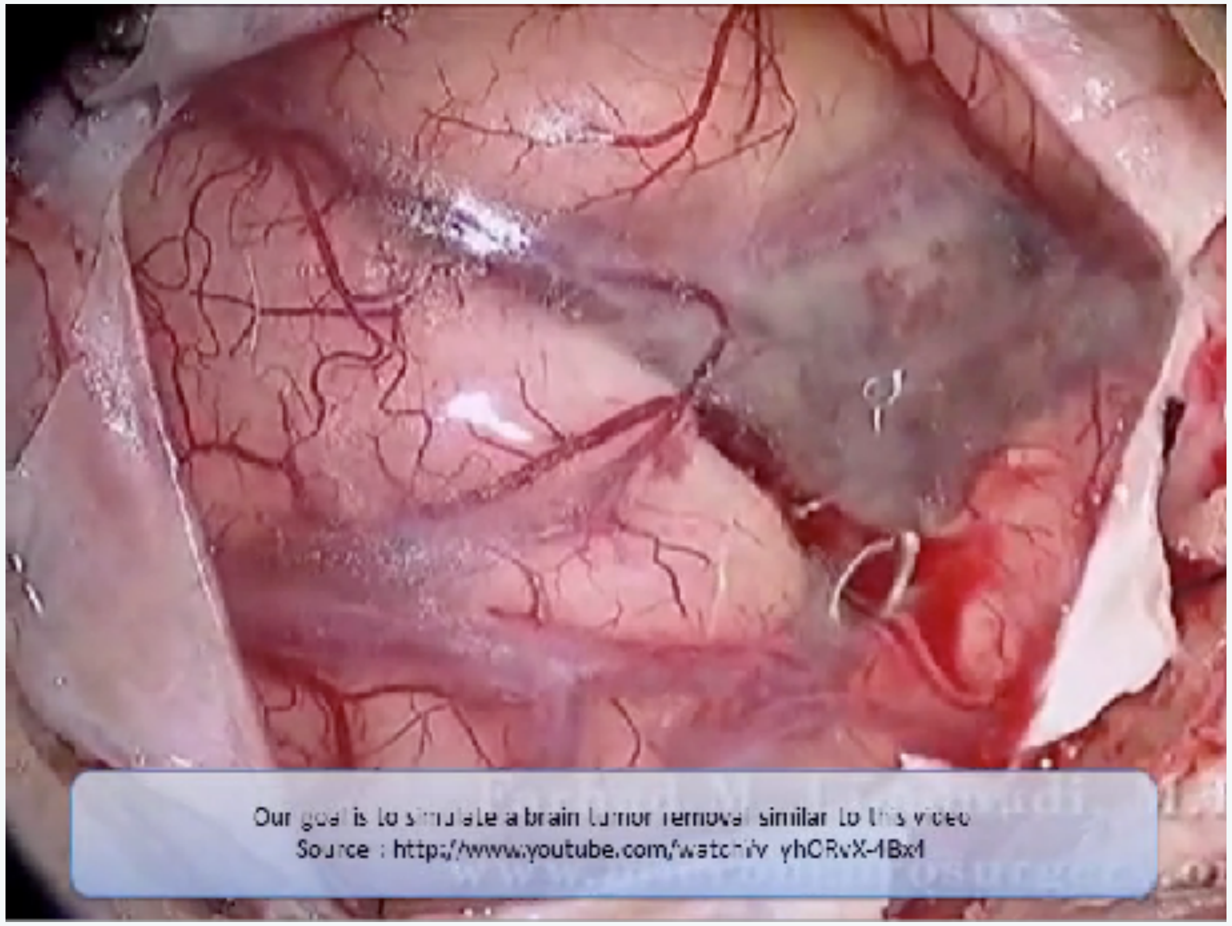




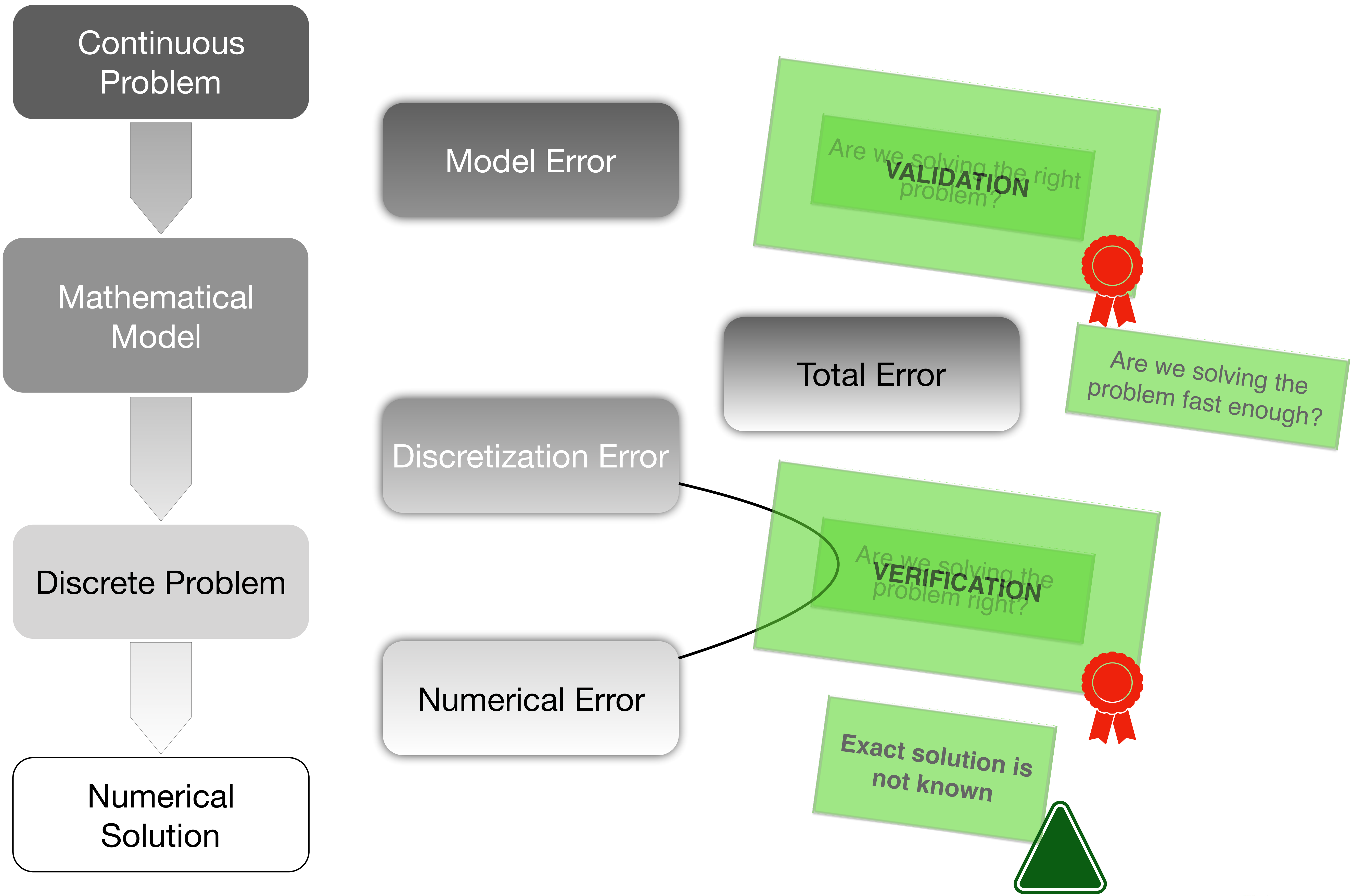
Lack of scale separation

A view from reduced order modelling and homogenisation





Mathematical Modelling

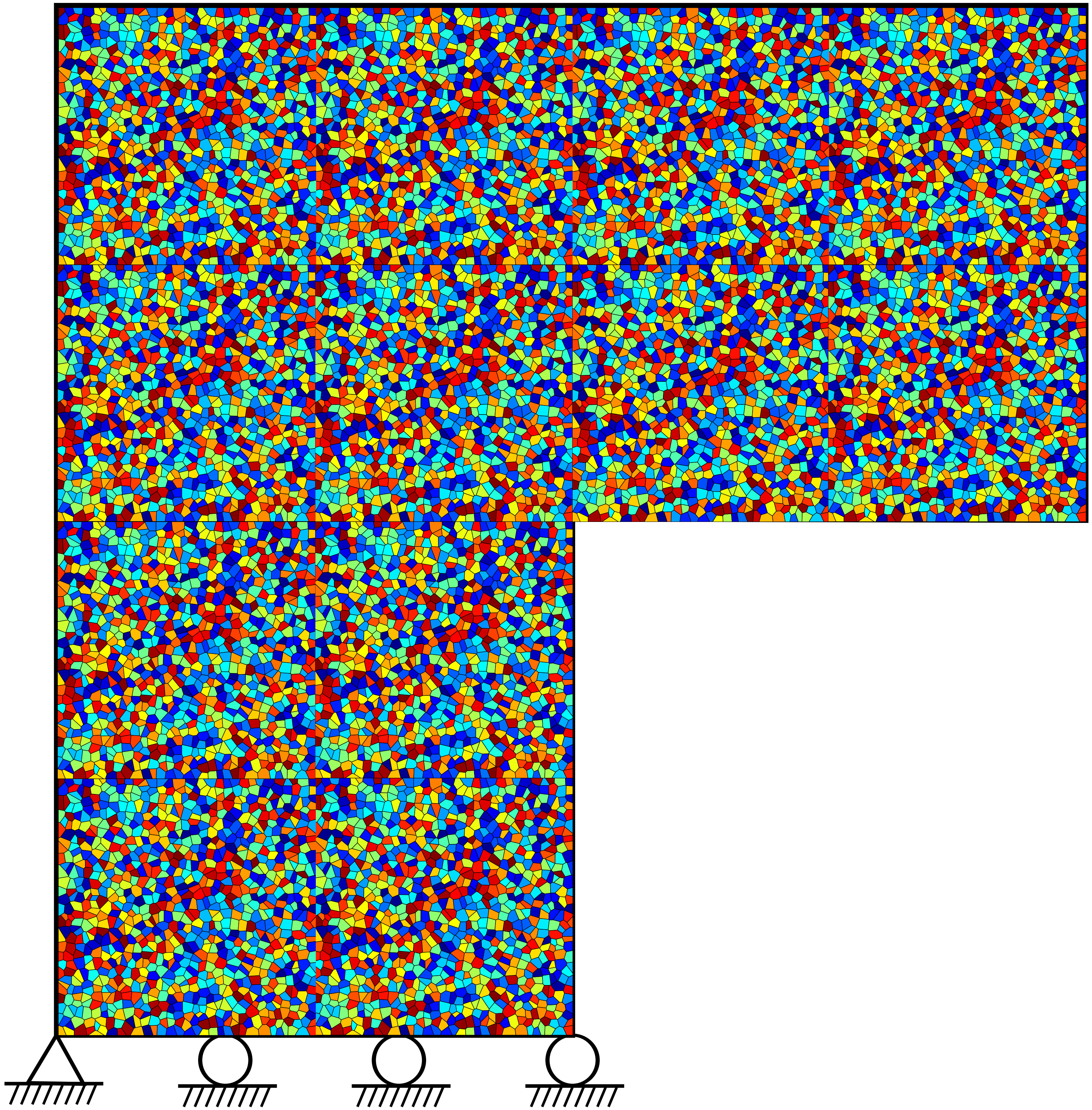


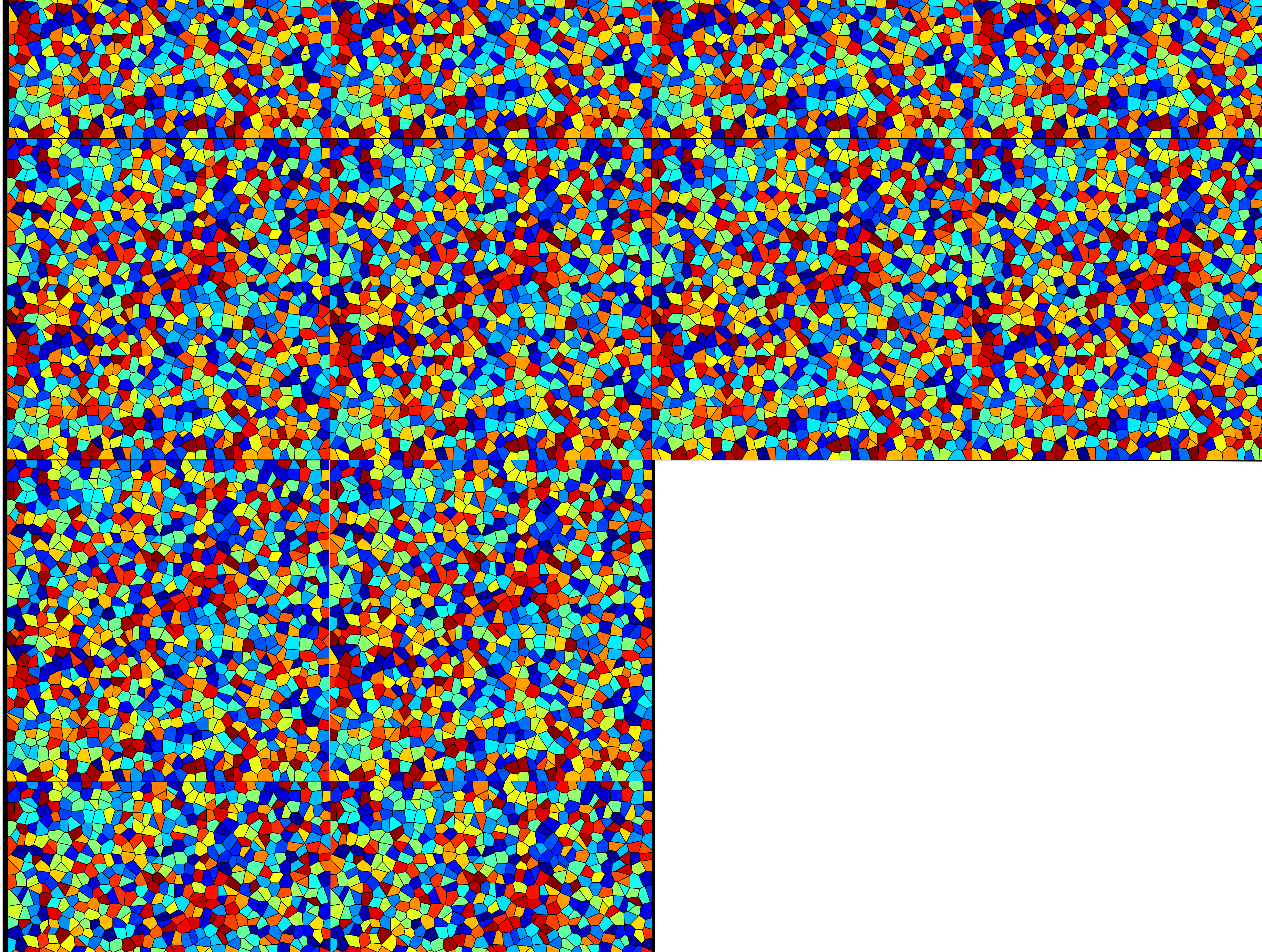
Stéphane Pierre Alain BORDAS, Department of Computational Engineering & Sciences University of Luxembourg and Cardiff University

Physics-based model reduction methods

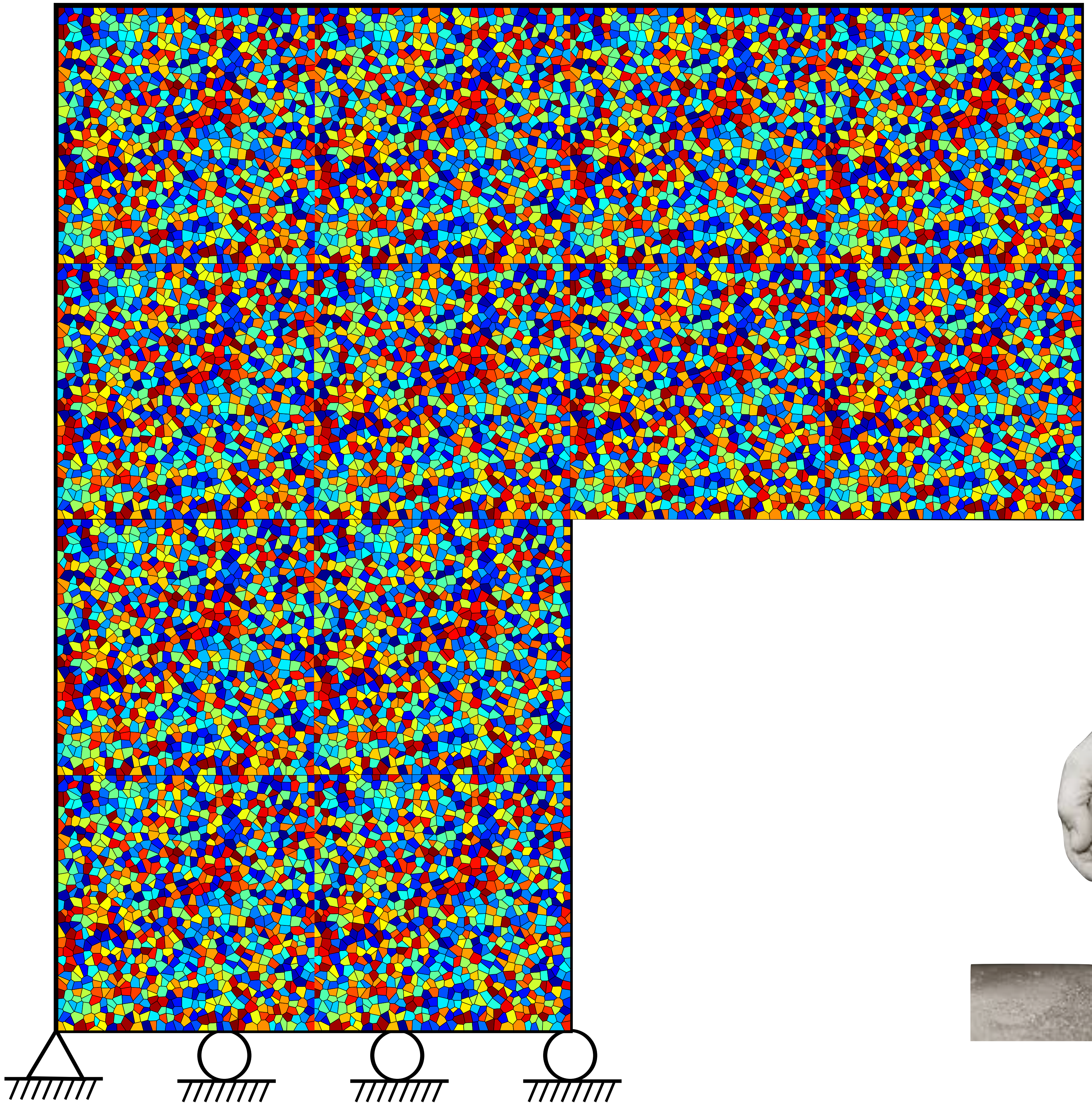
multi-scale methods

Full-scale



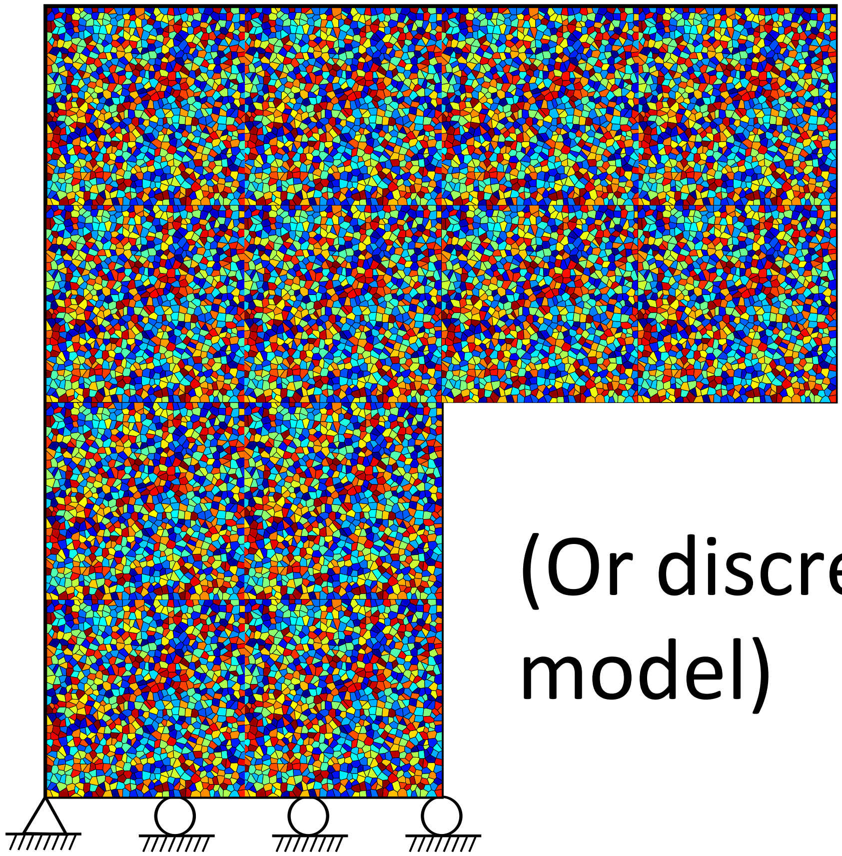


Full-scale



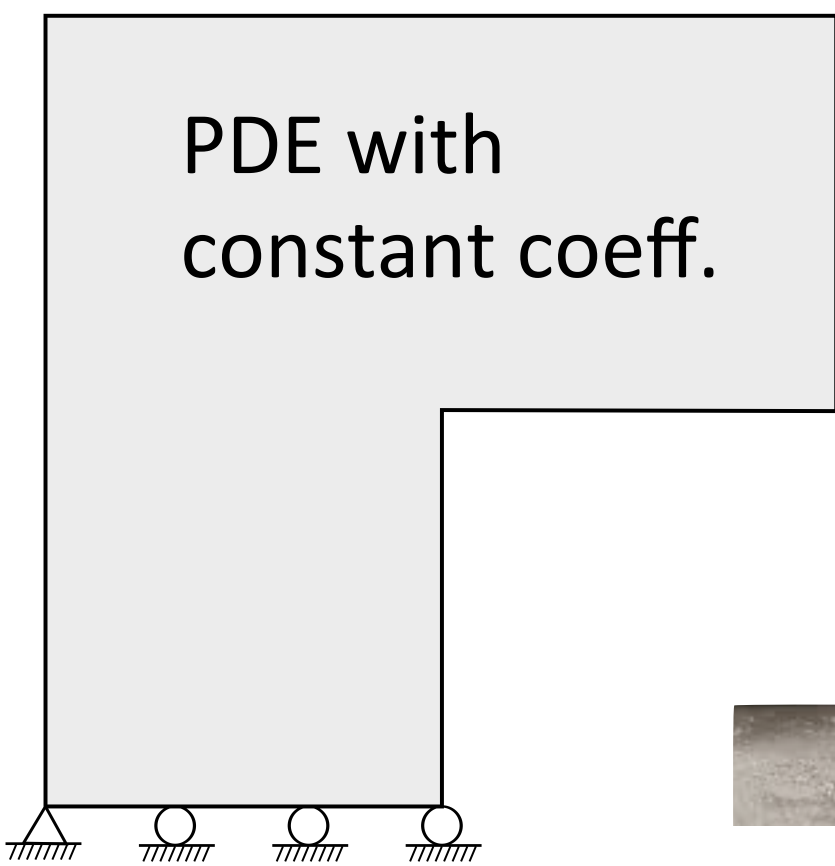
Multi-scale methods

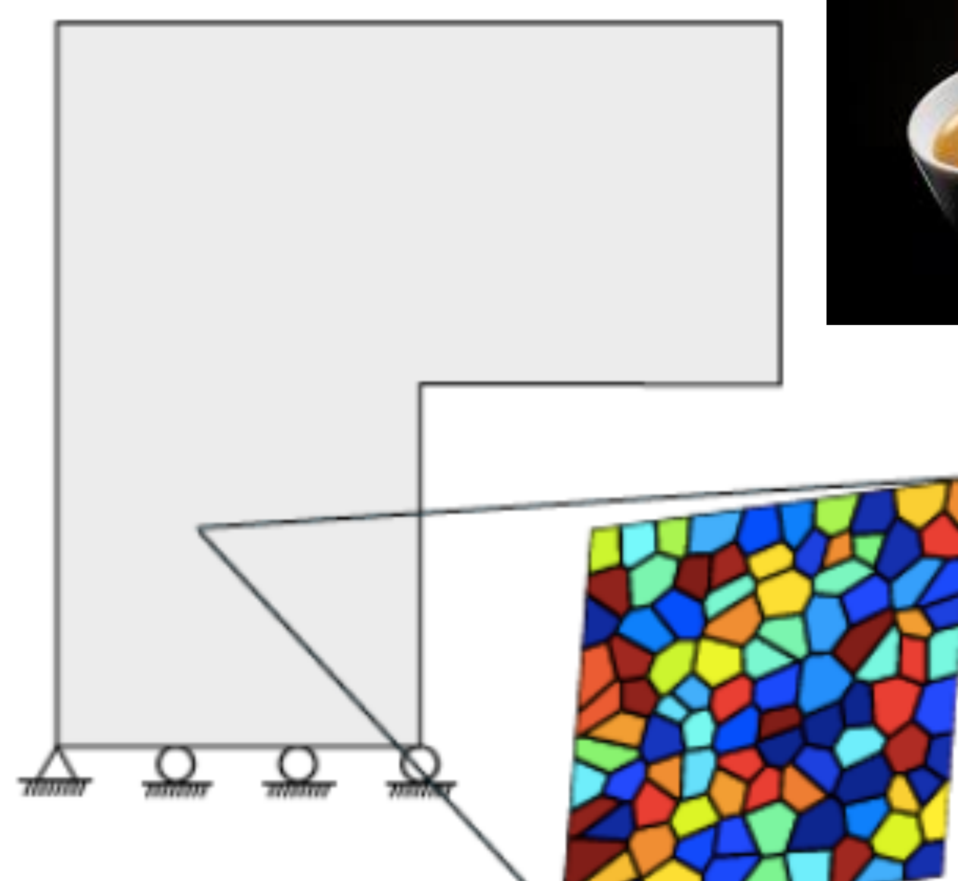
Replace the heterogeneous fine-scale model by an equivalent smoother model at the scale where the predictions are required



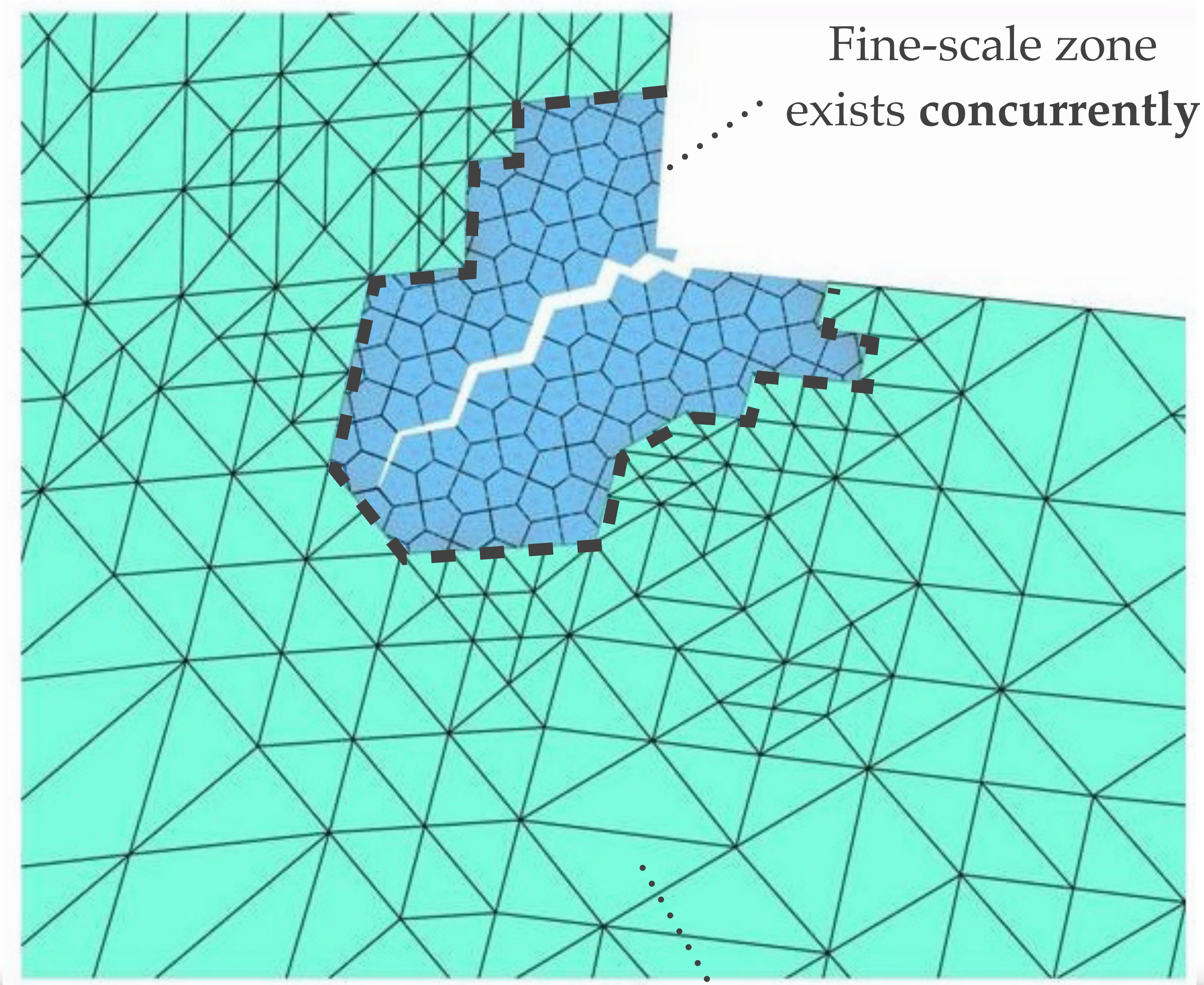
(Or discrete model)

↓ Homogenisation



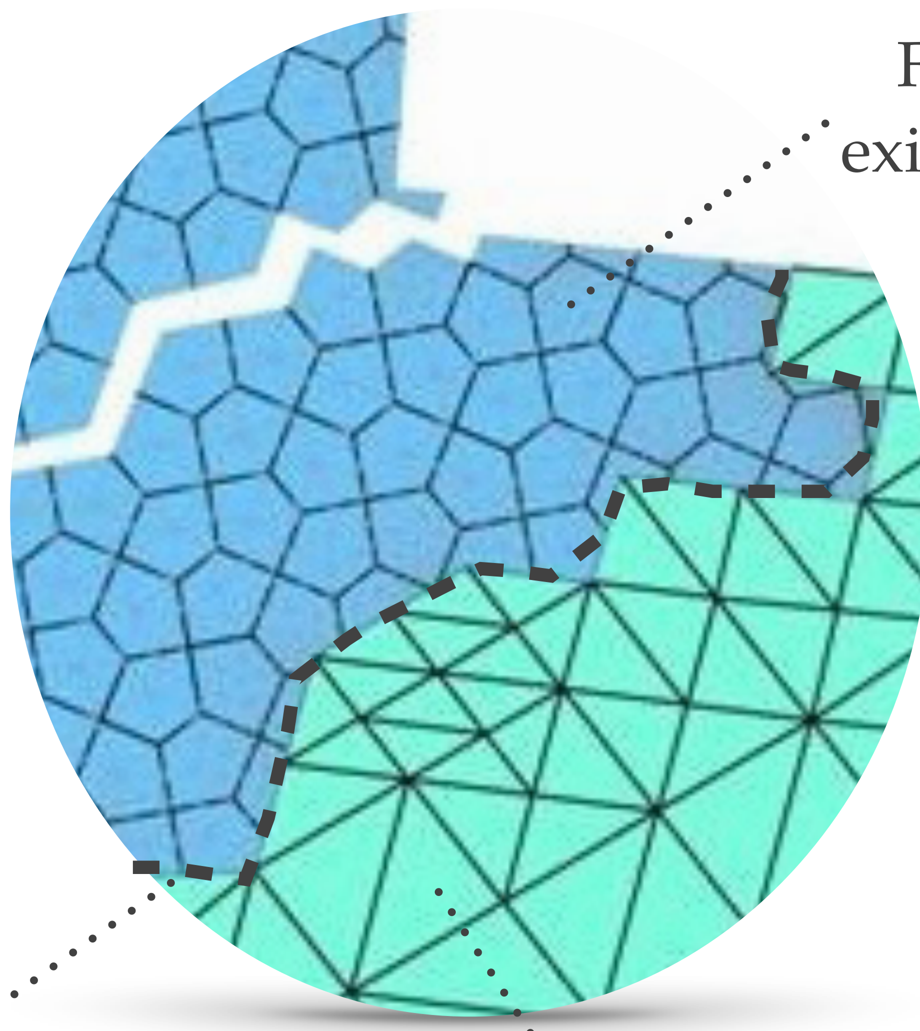


Concurrent methods



Akbari, Kerfriden, Bordas, 2014

Concurrent methods



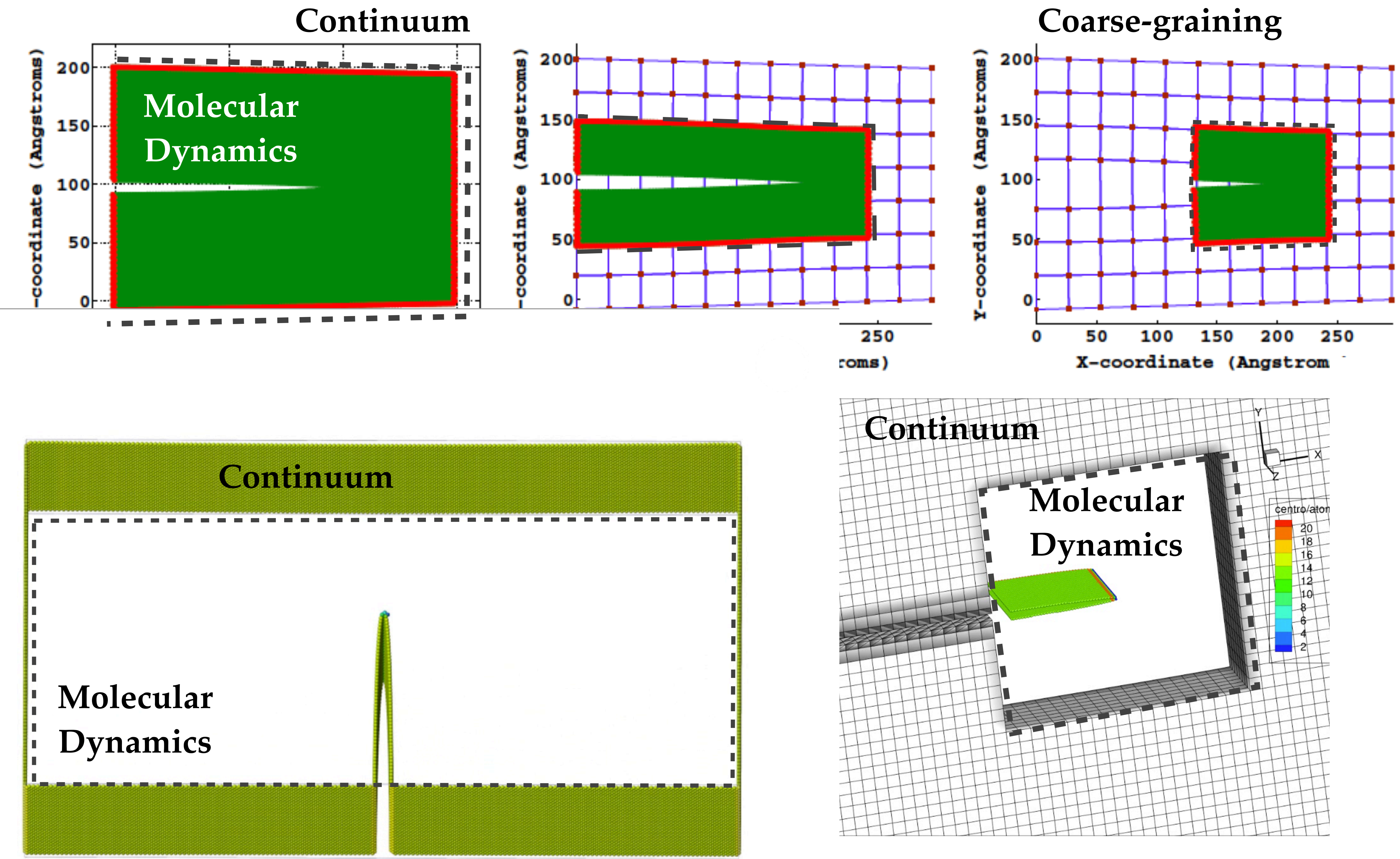
Fine-scale zone
exists **concurrently**

adaptive coarse-fine
interface

to the **coarse scale zone**

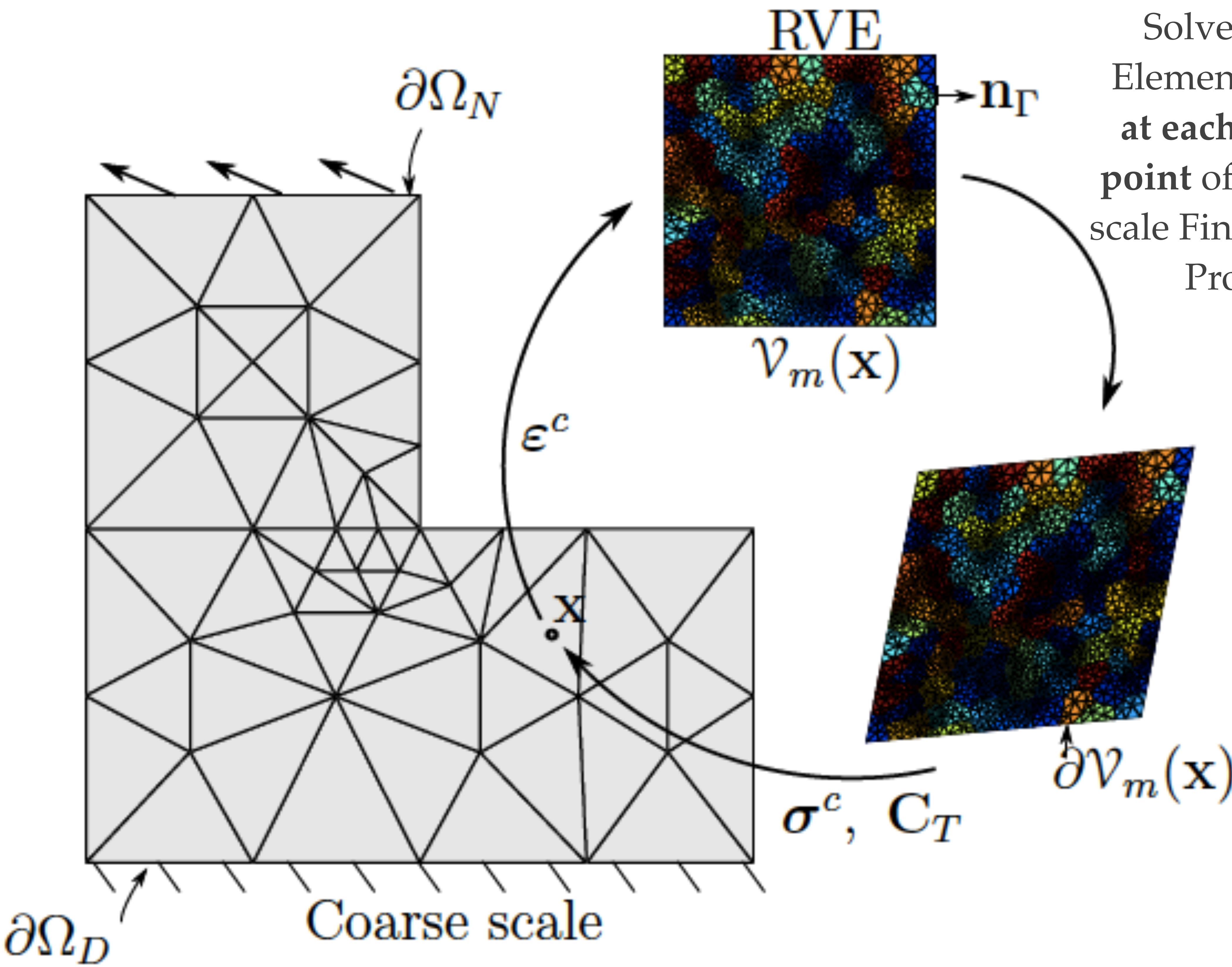
Akbari, Kerfriden, Bordas, 2014

Concurrent methods



Talebi, Ramaia, Rabczuk, Bordas, Kerfriden, 2014

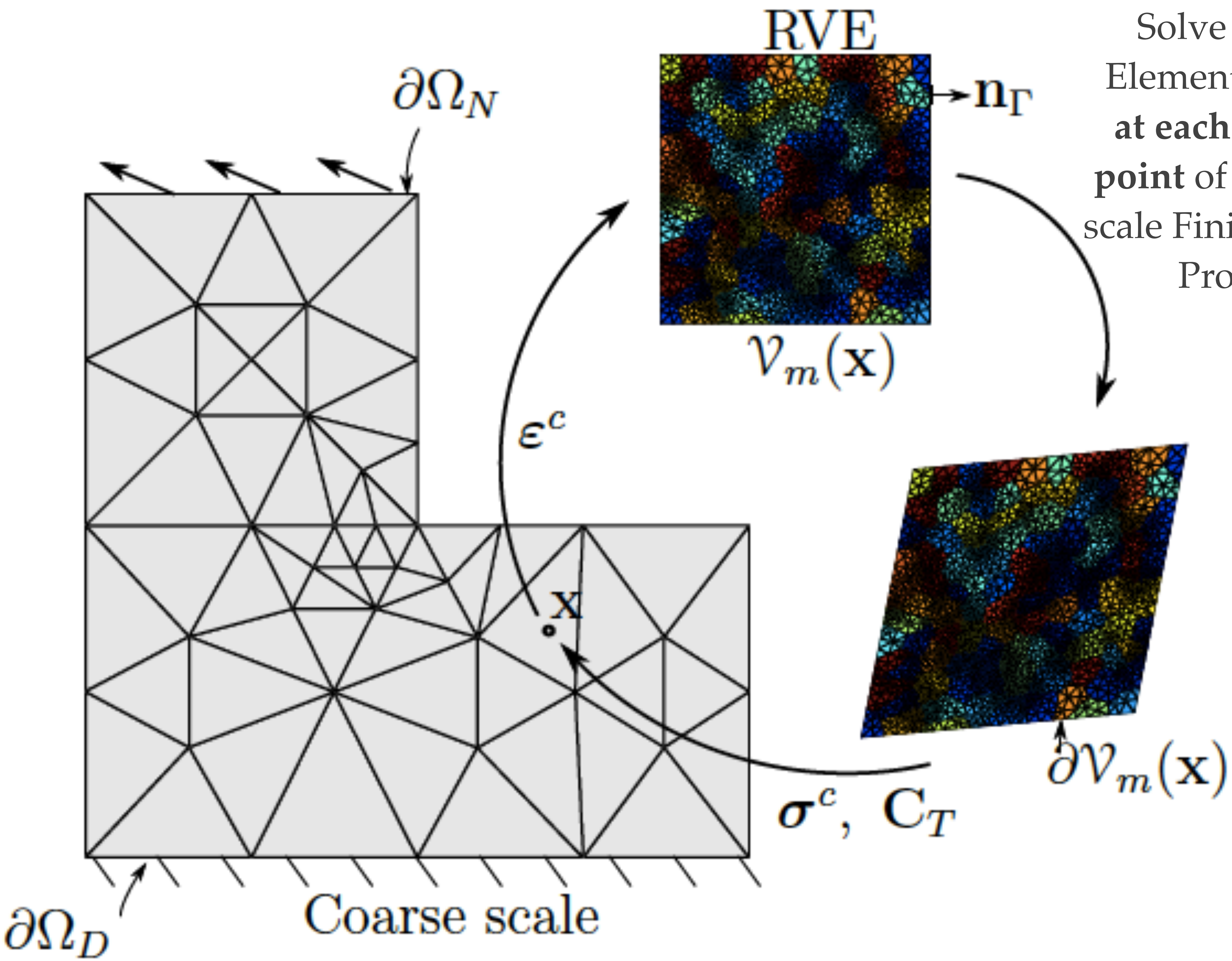
Hierarchical methods FE^2



Solve a Finite Element Problem **at each material point** of the coarse scale Finite Element Problem

Akbari, Kerfriden, Bordas, 2014

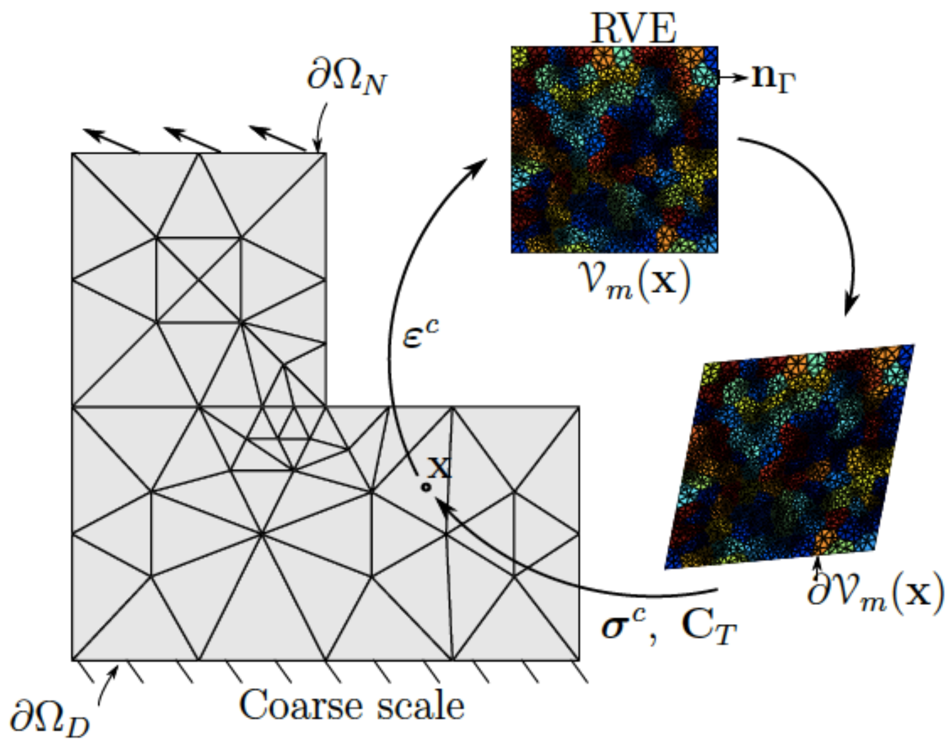
Hierarchical methods FE^2



Solve a Finite Element Problem **at each material point** of the coarse scale Finite Element Problem

Akbari, Kerfriden, Bordas, 2014

Hierarchical methods FE^2



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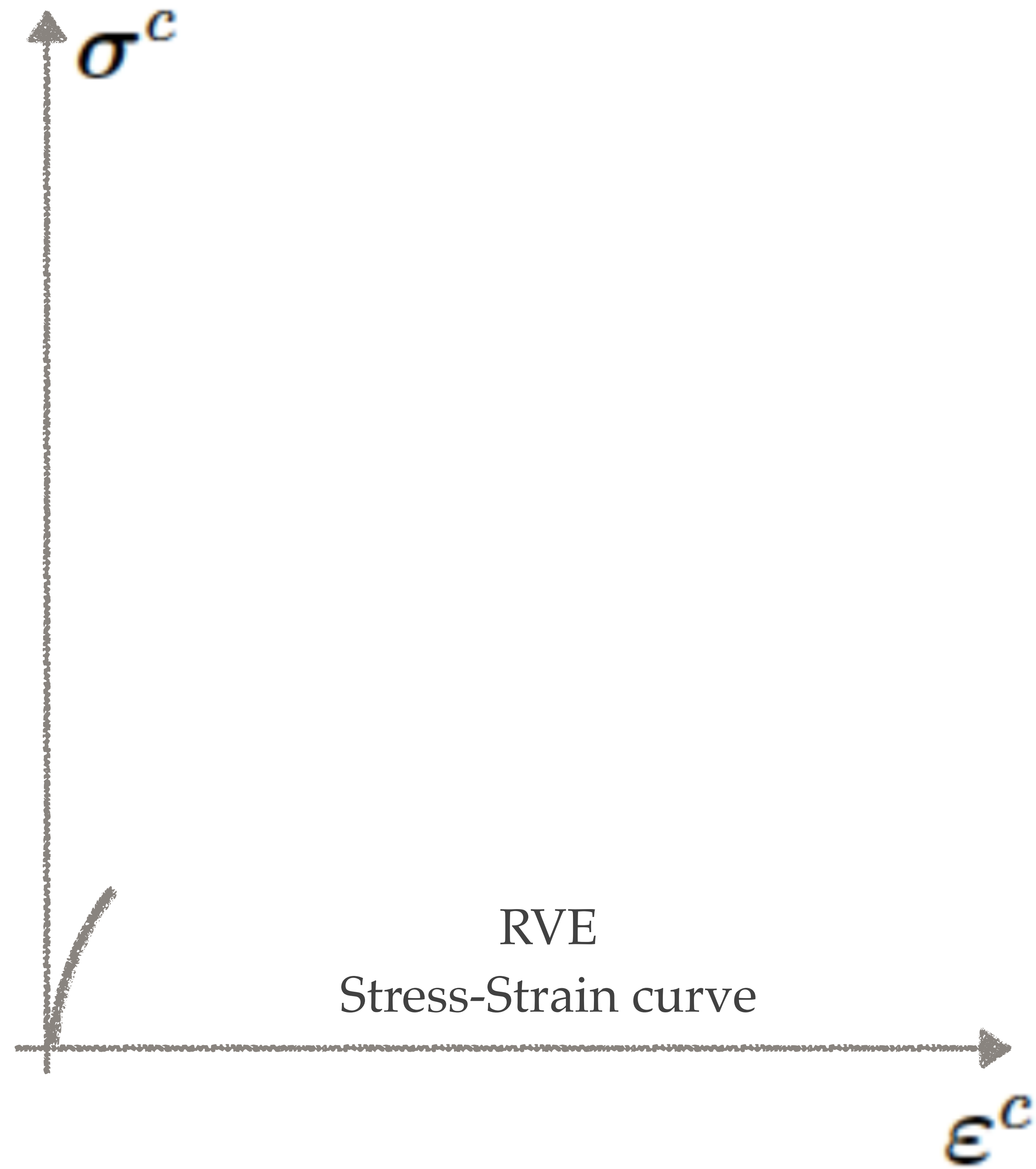
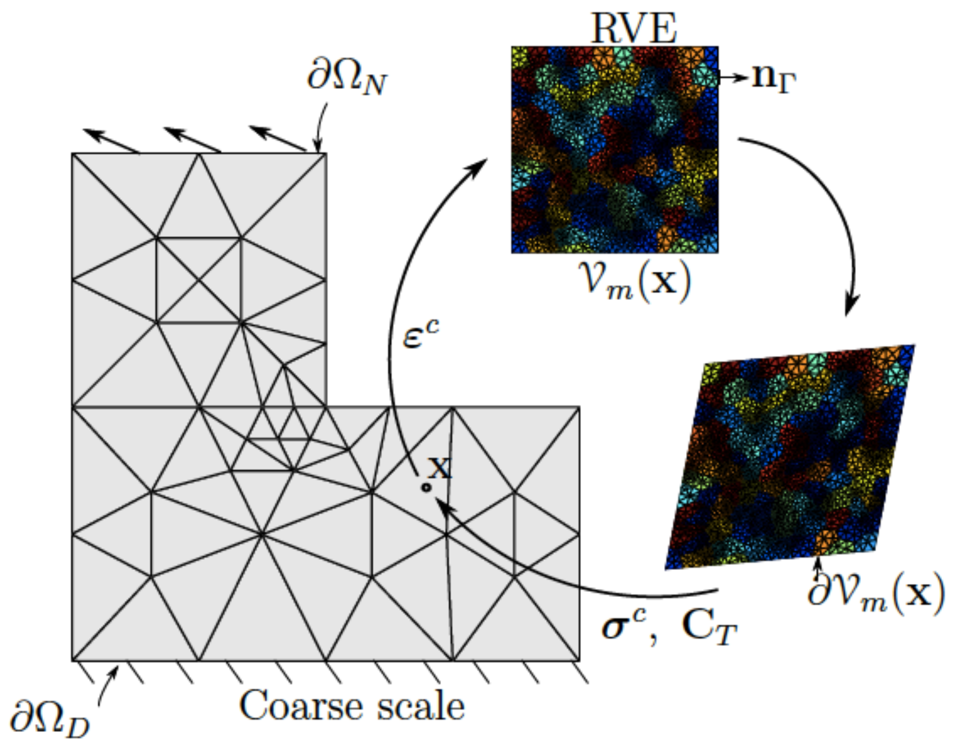


Feyel, Chaboche, 2000 - Akbari, Kerfriden, Bordas, 2014

Hierarchical methods FE^2



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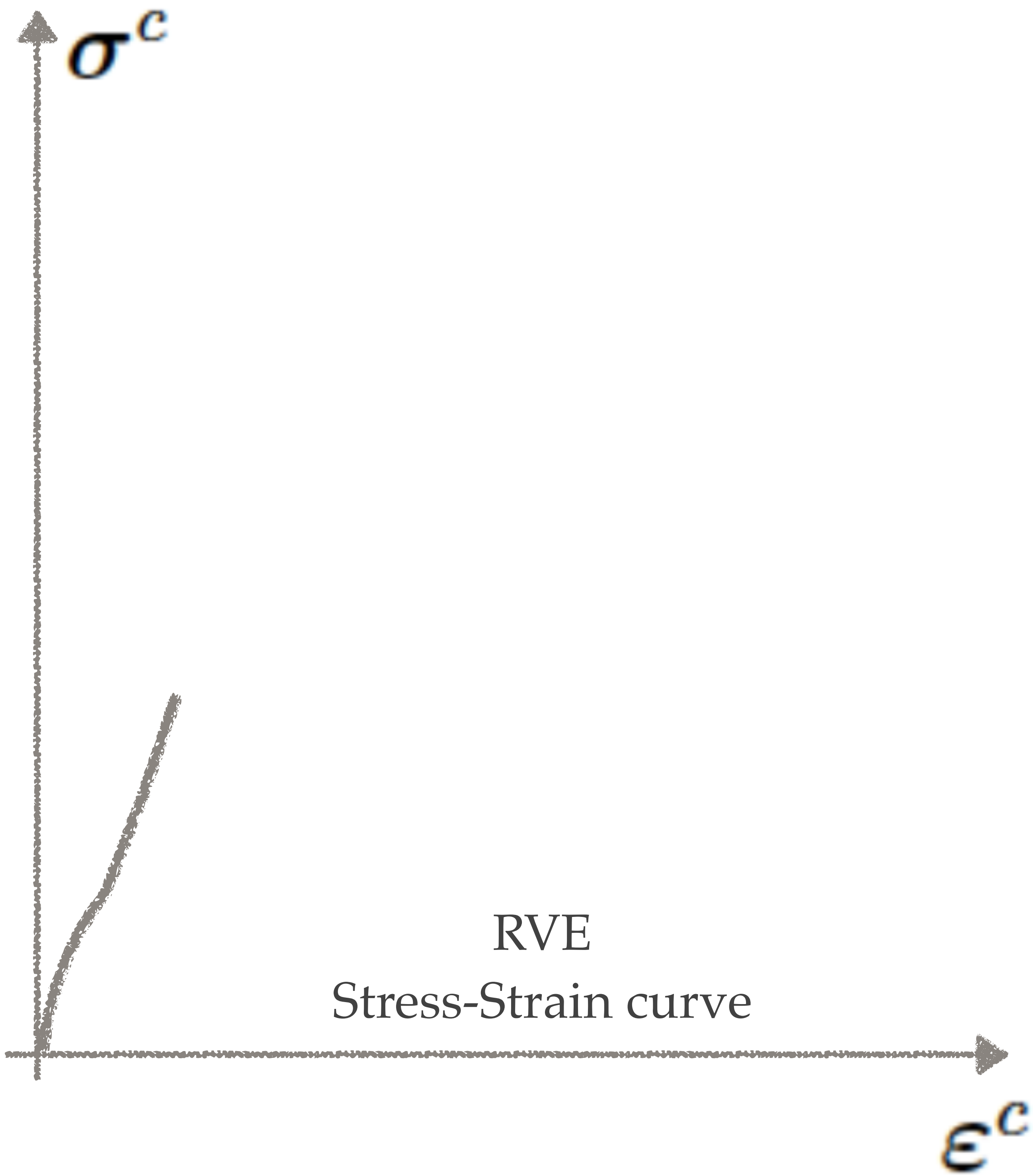
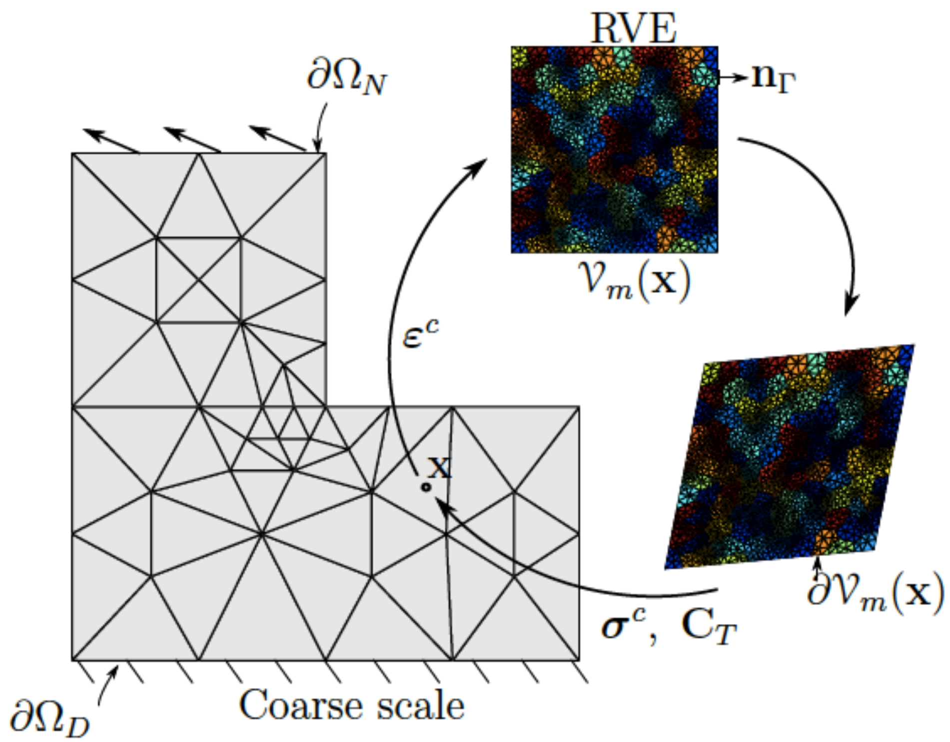
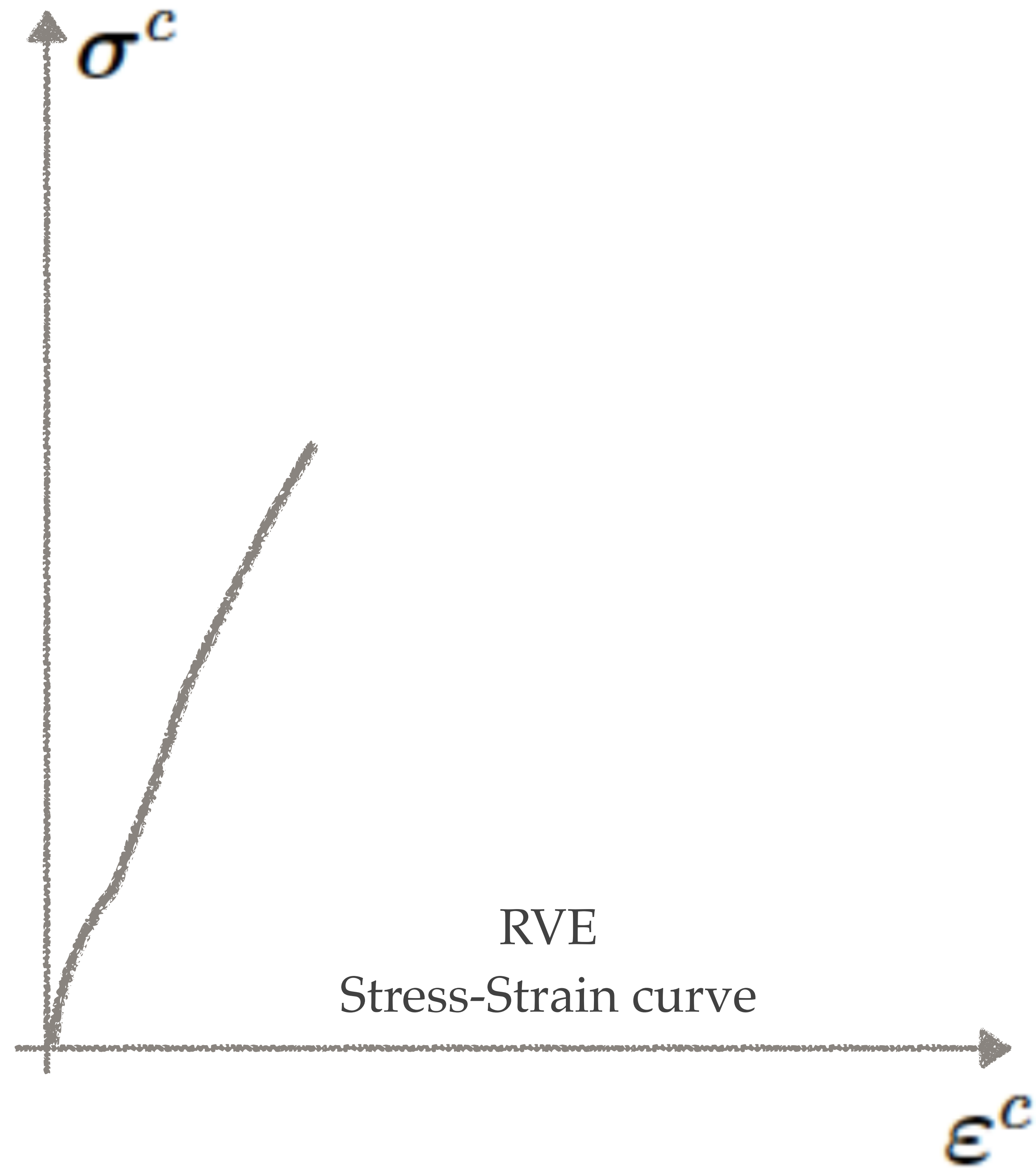


Feyel, Chaboche, 2000 - Akbari, Kerfriden, Bordas, 2014

Hierarchical methods FE^2





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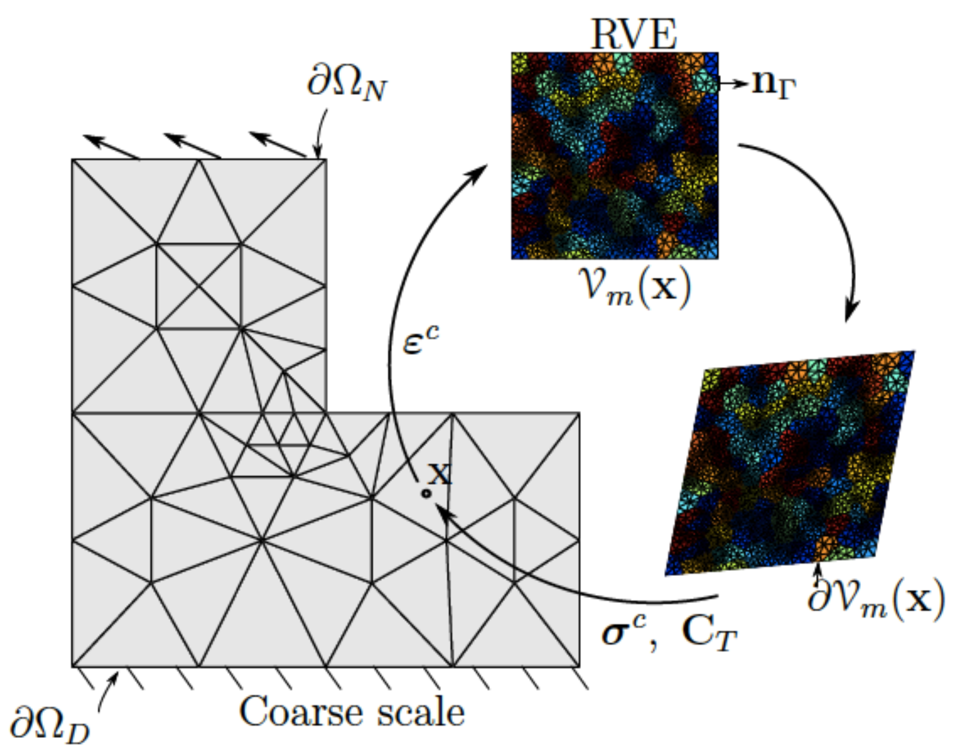
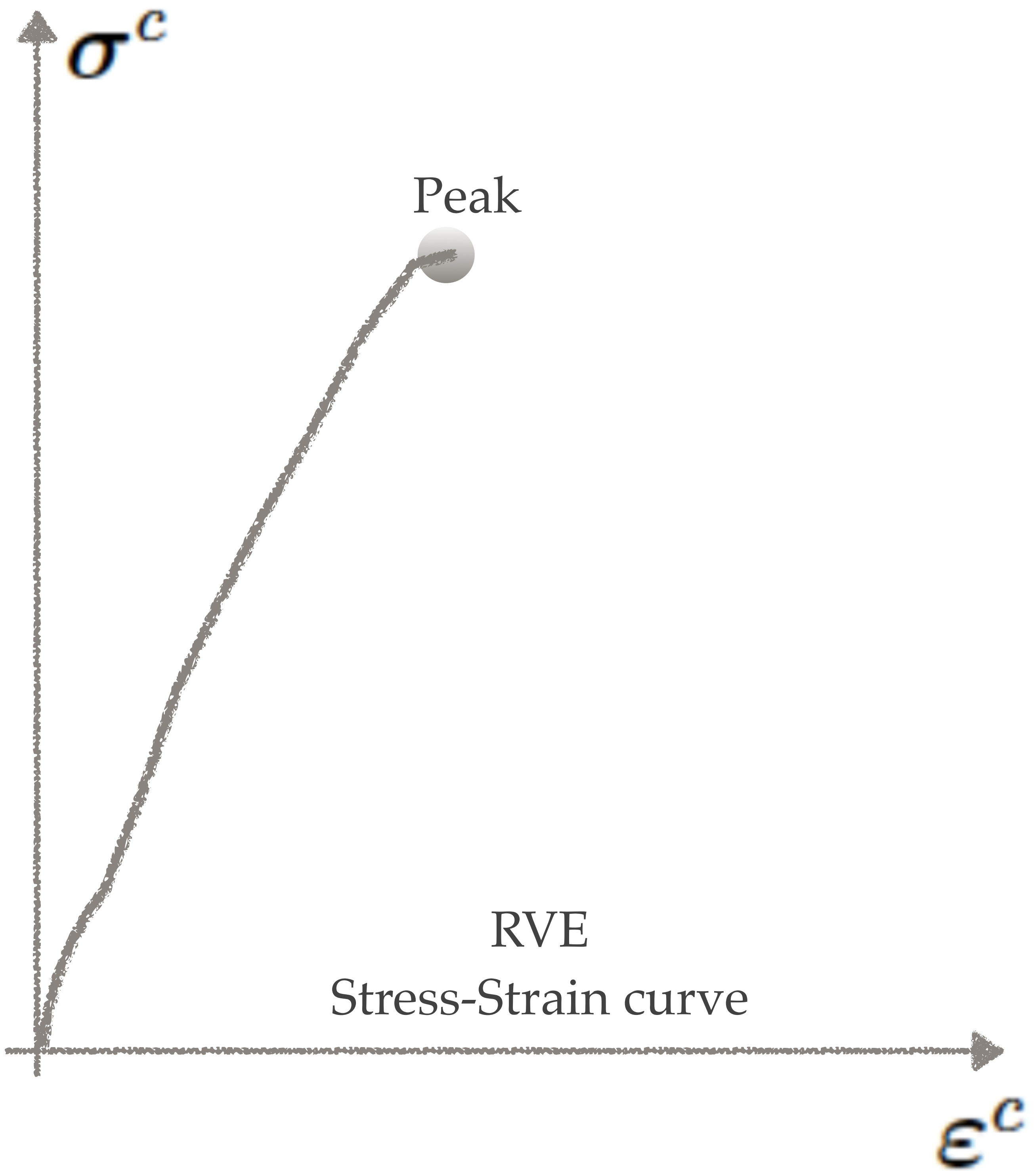
Feyel, Chaboche, 2000 - Akbari, Kerfriden, Bordas, 2014

Hierarchical methods FE^2

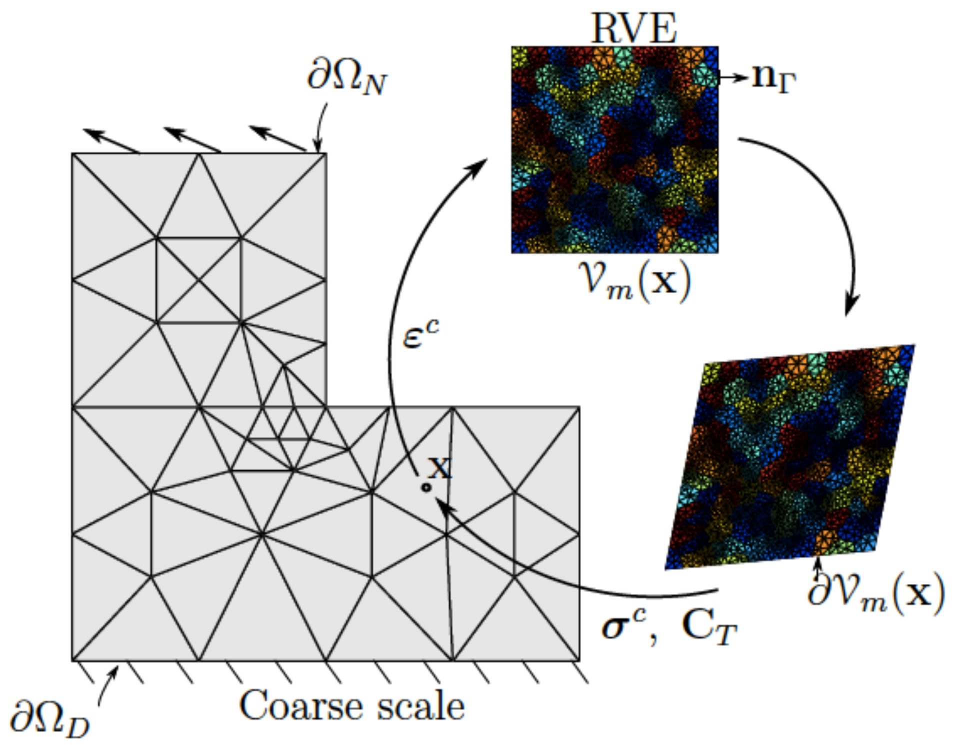




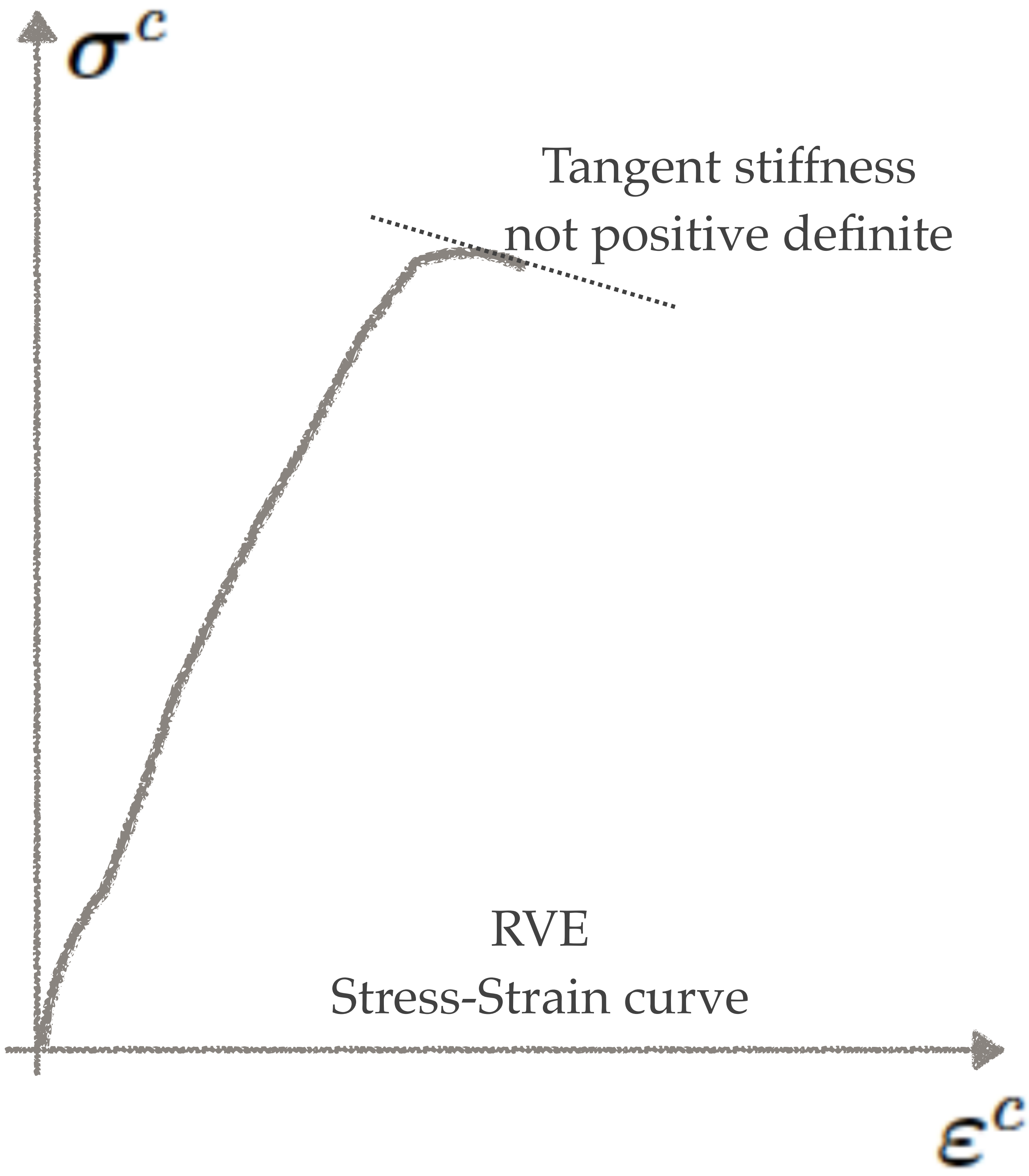
Peak

Feyel, Chaboche, 2000 - Akbari, Kerfriden, Bordas, 2014

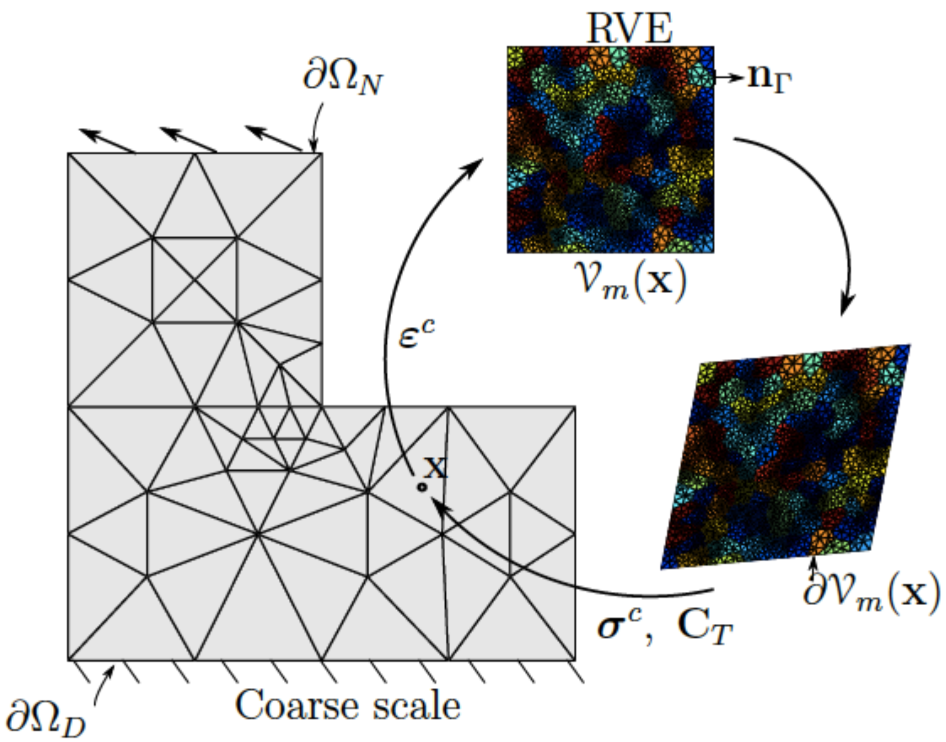
Hierarchical methods FE^2

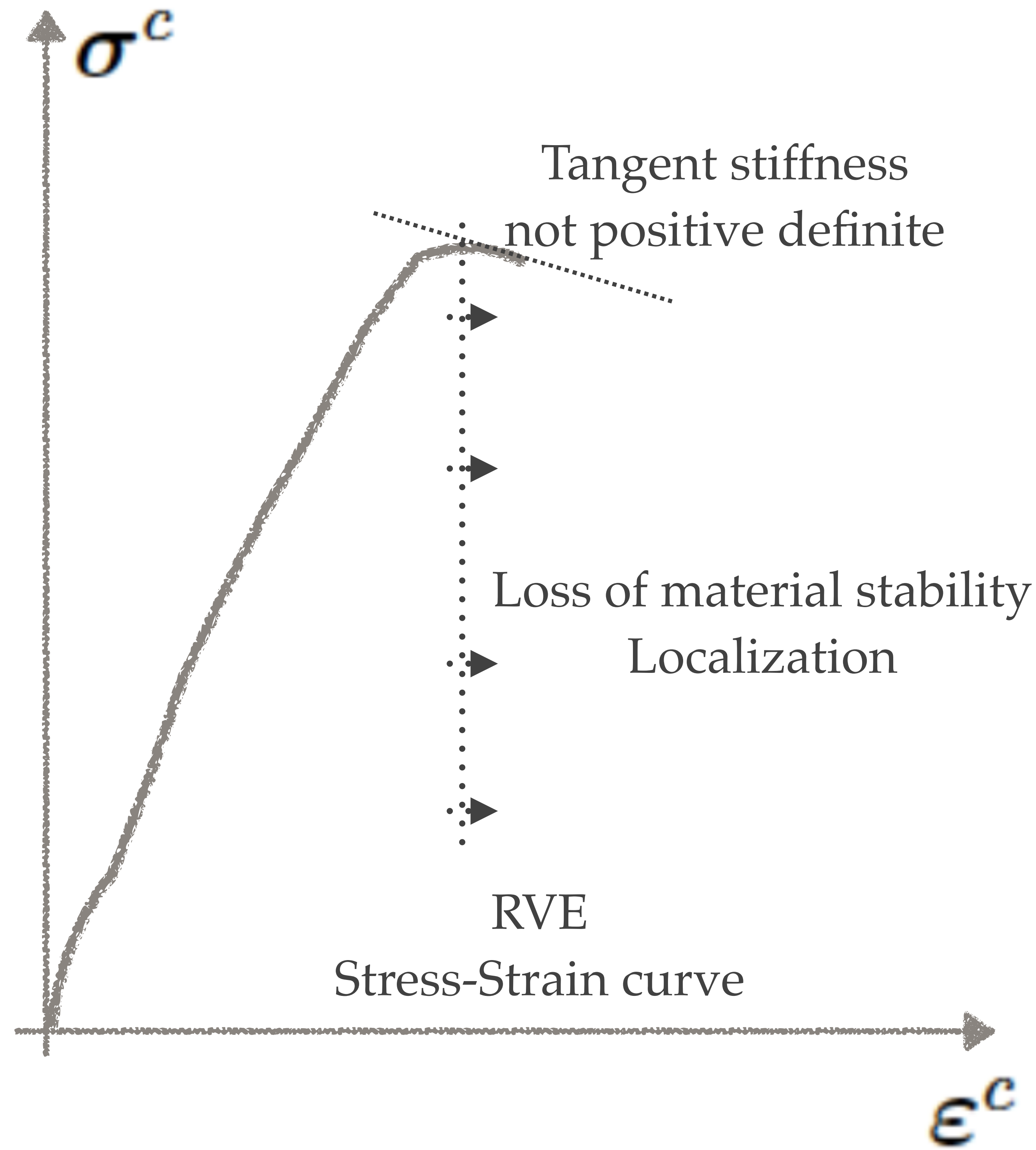


Unloading



Hierarchical methods FE^2



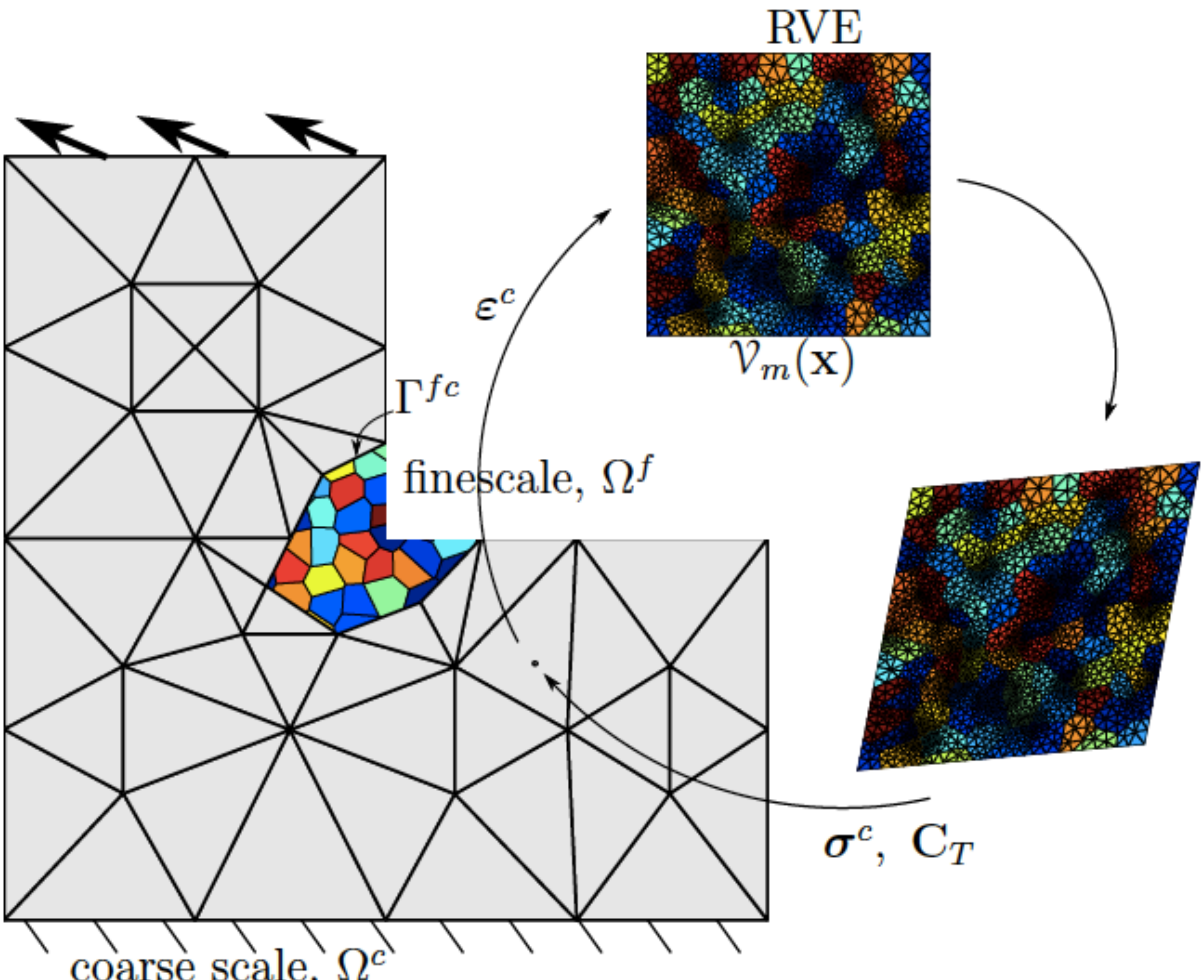


Unloading

RVE does not exist

Feyel, Chaboche, 2000 - Akbari, Kerfriden, Bordas, 2014

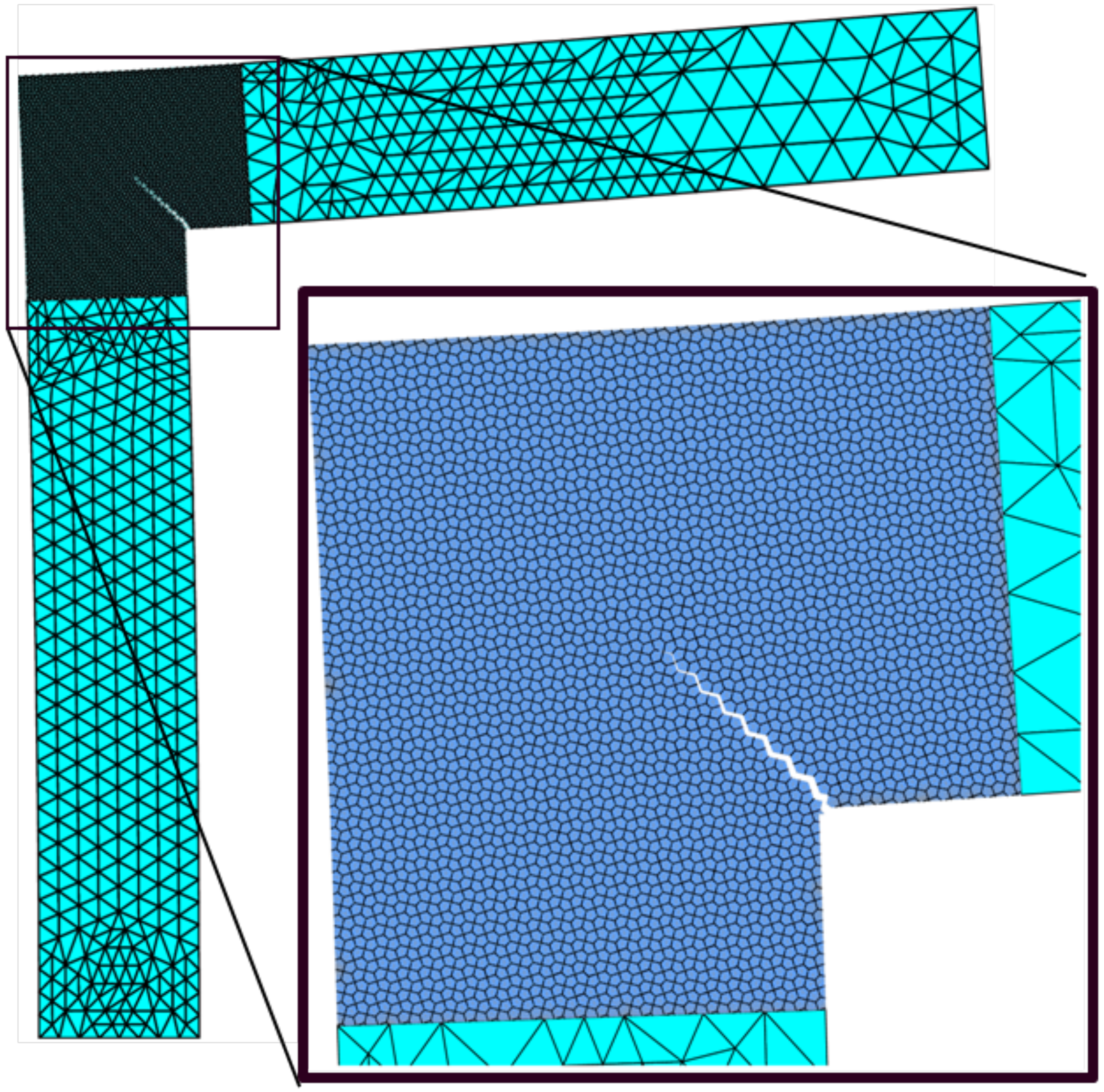
Hybrid methods



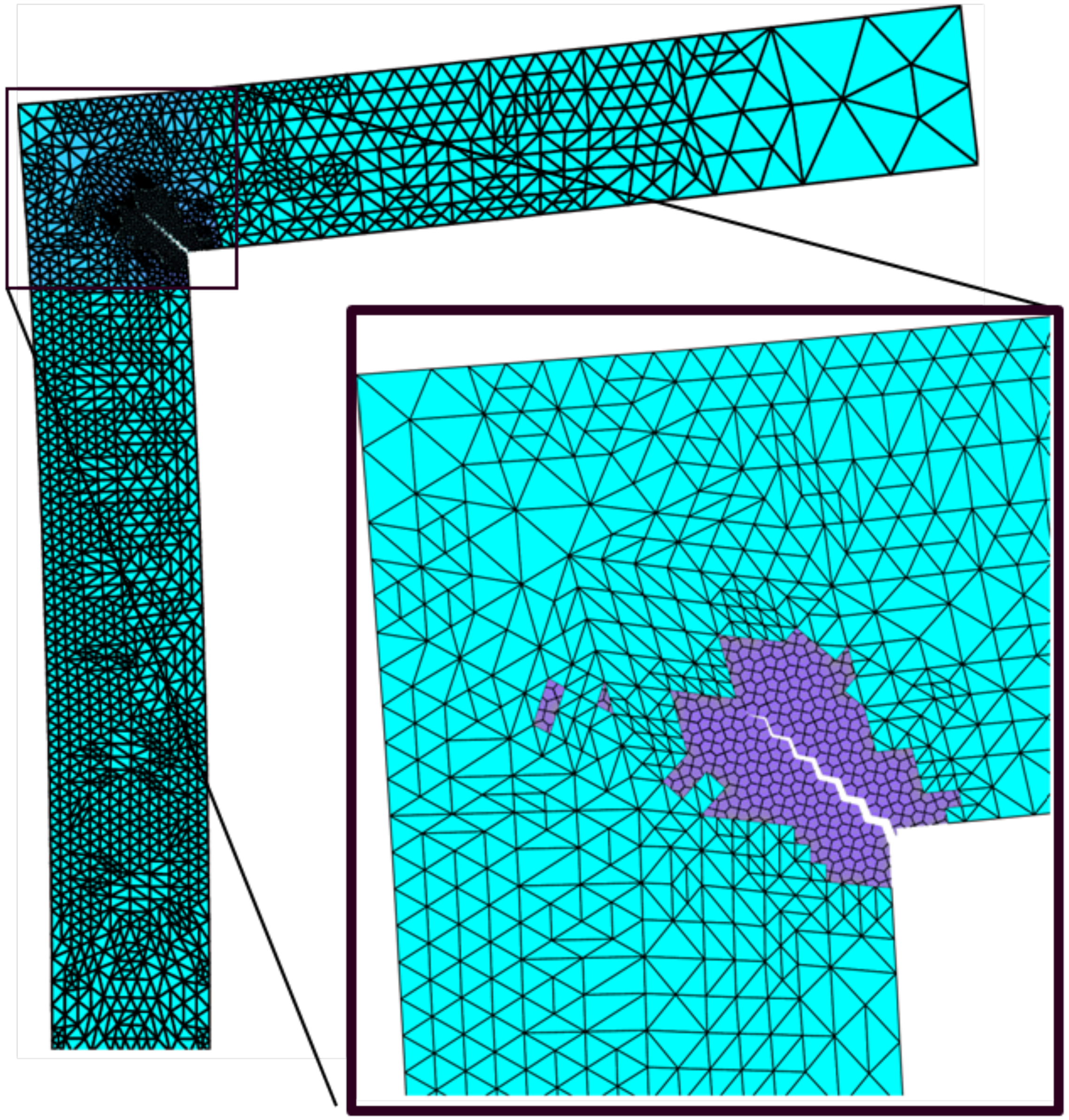
Akbari, Kerfriden, Bordas, 2014

Example

Direct Numerical Solution



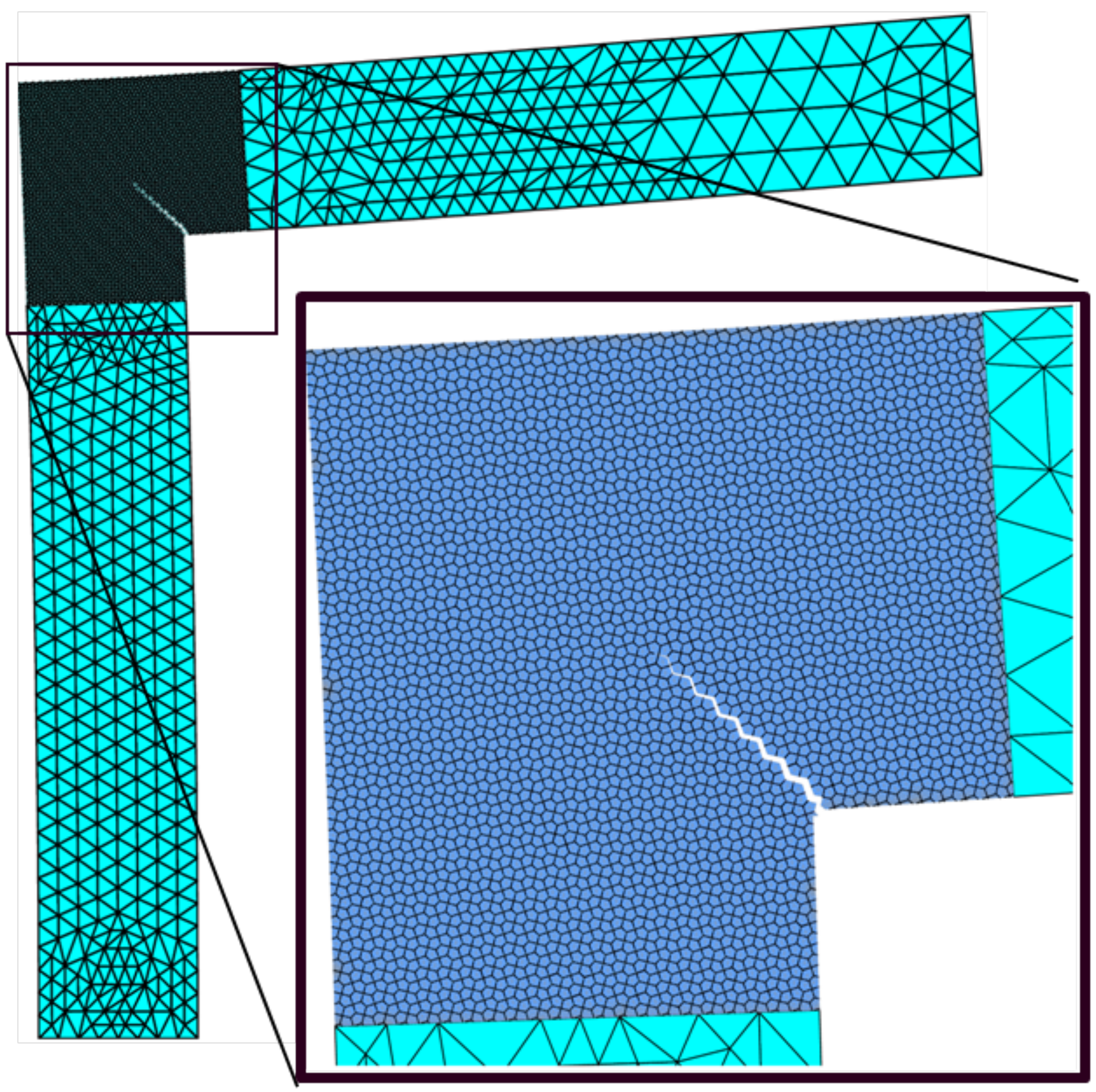
Adaptive Multiscale method



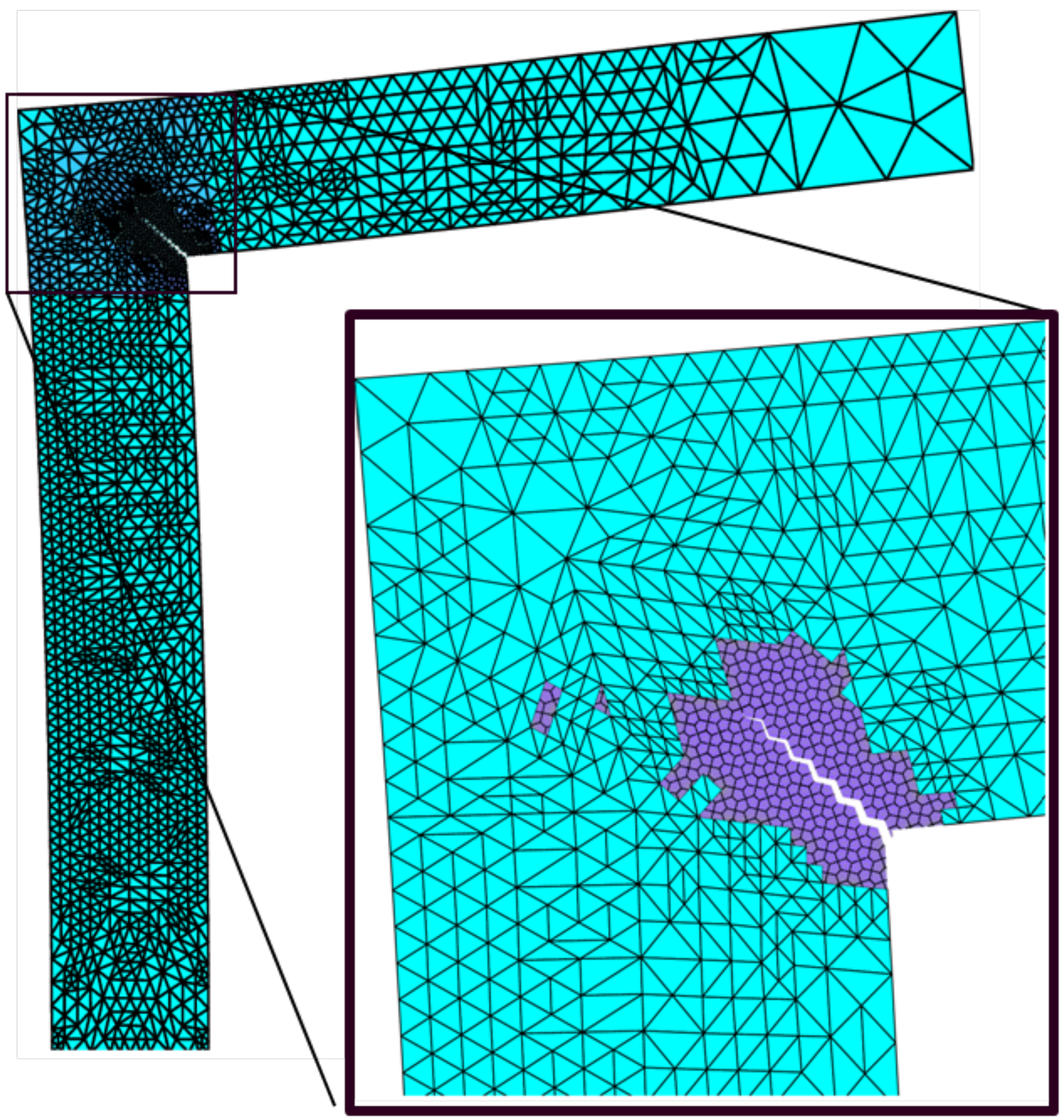
Example

20-100 times fewer unknowns in 2D ~ 1000 times fewer in 3D

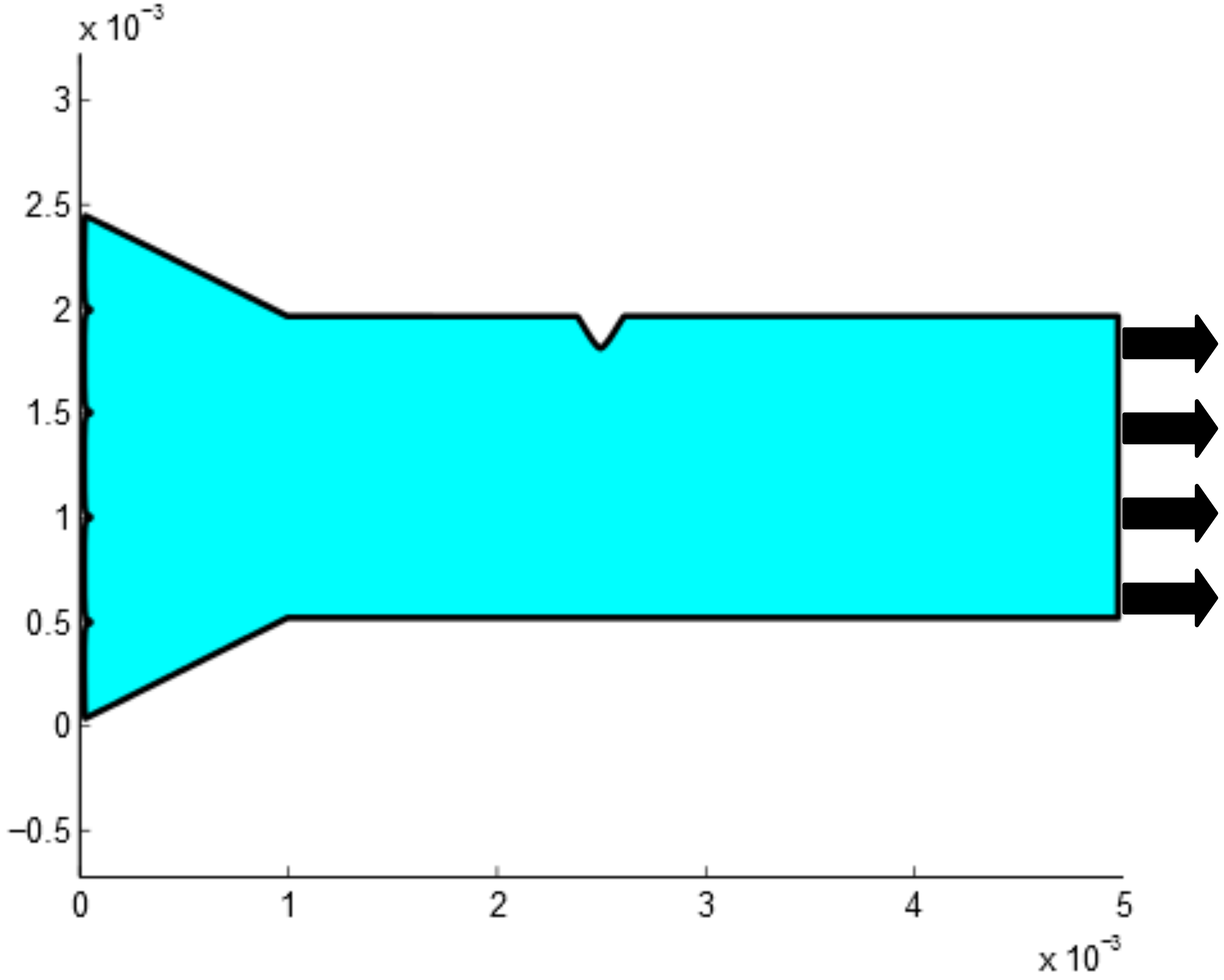
Direct Numerical Solution



Adaptive Multiscale method



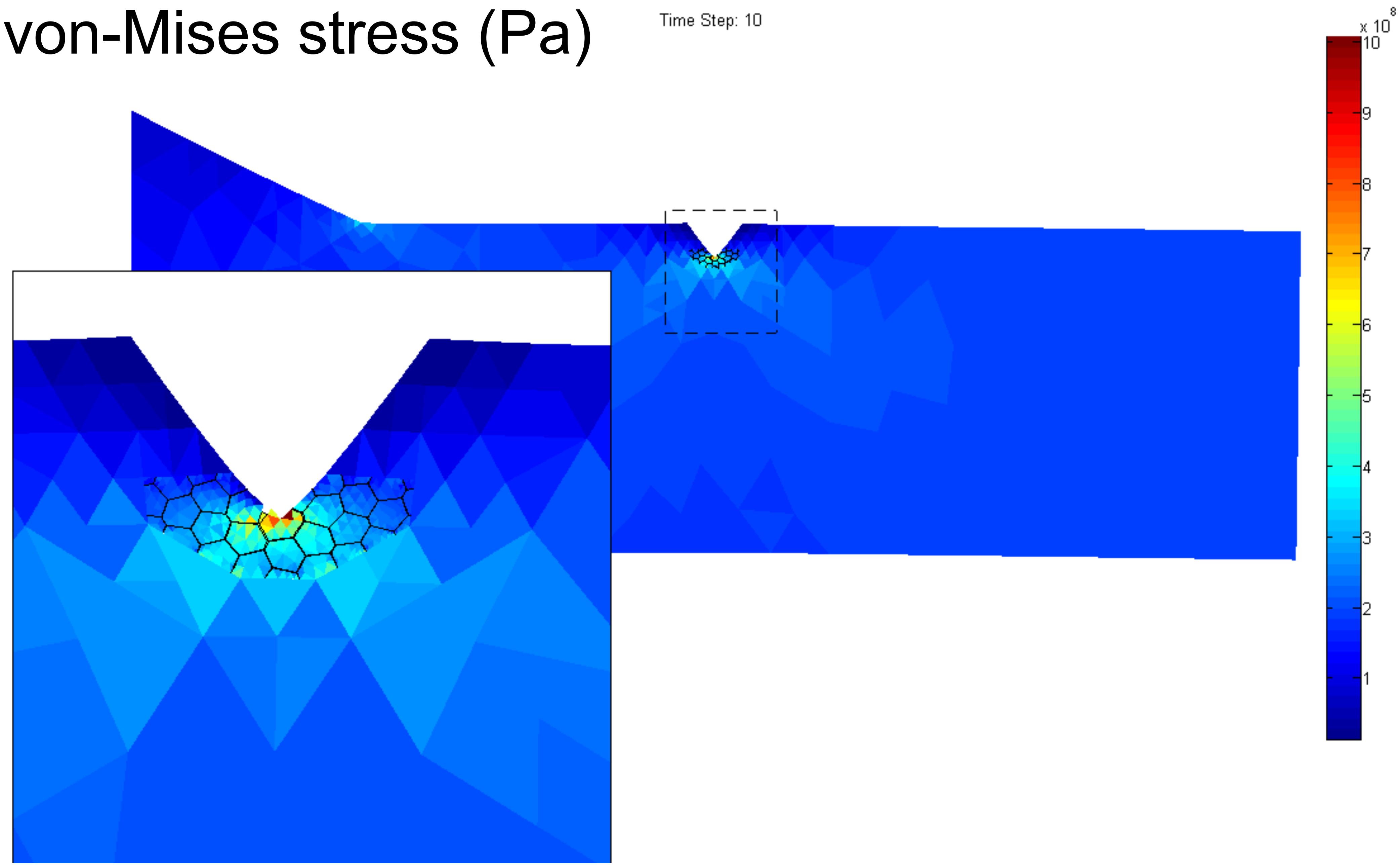
Results: uni-axial tension



❖ Sizes are in mm

Results: uni-axial tension

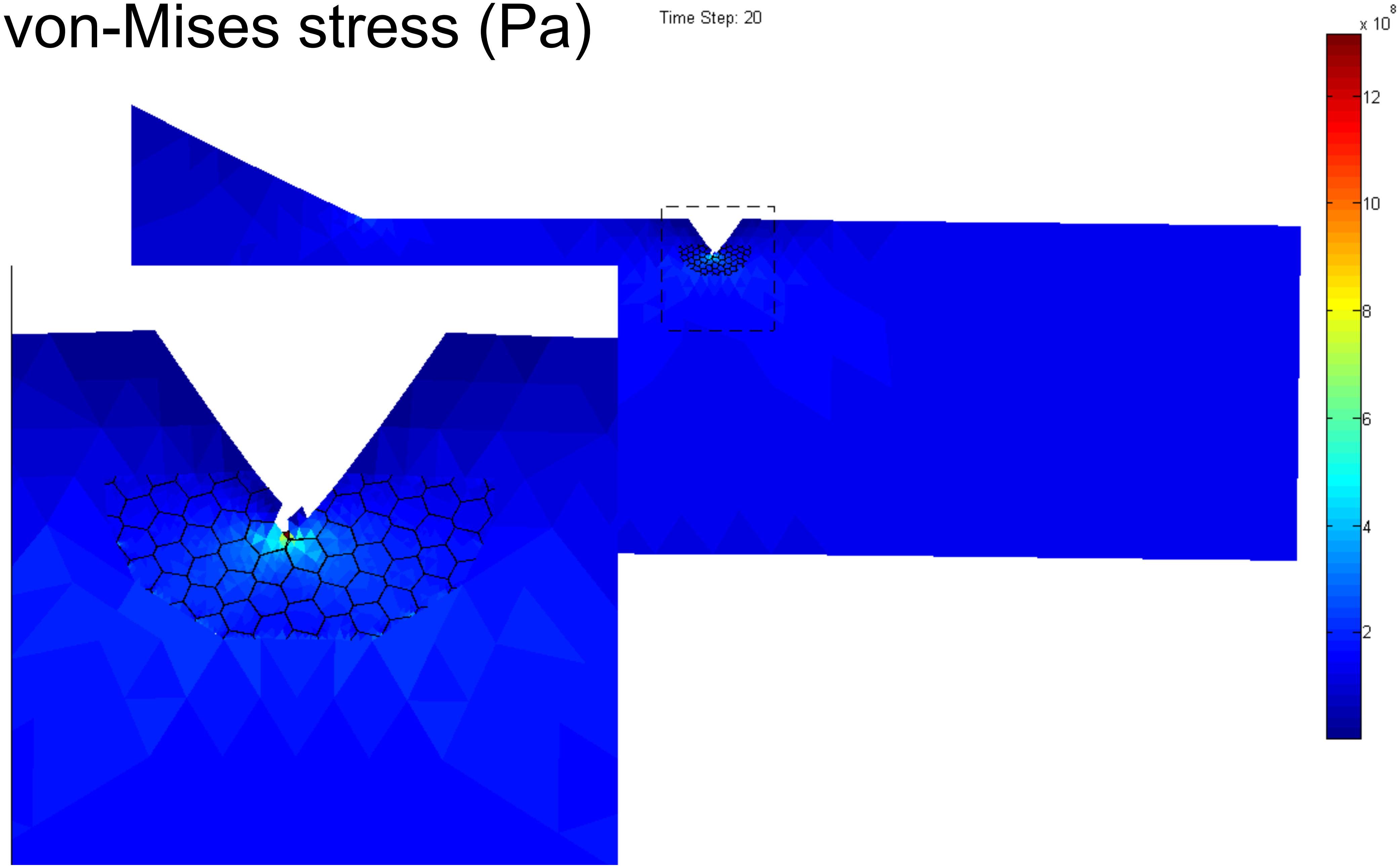
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

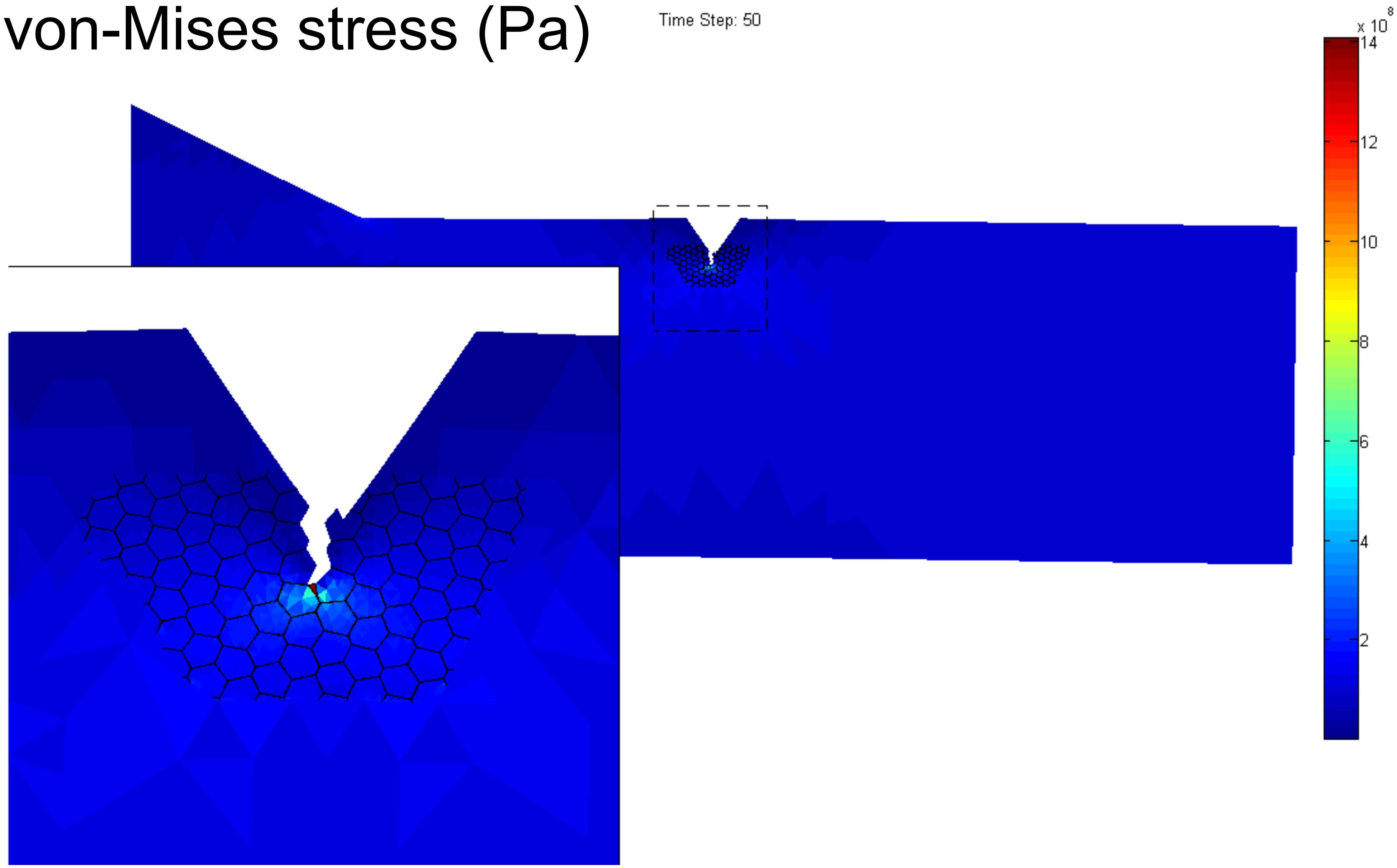
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

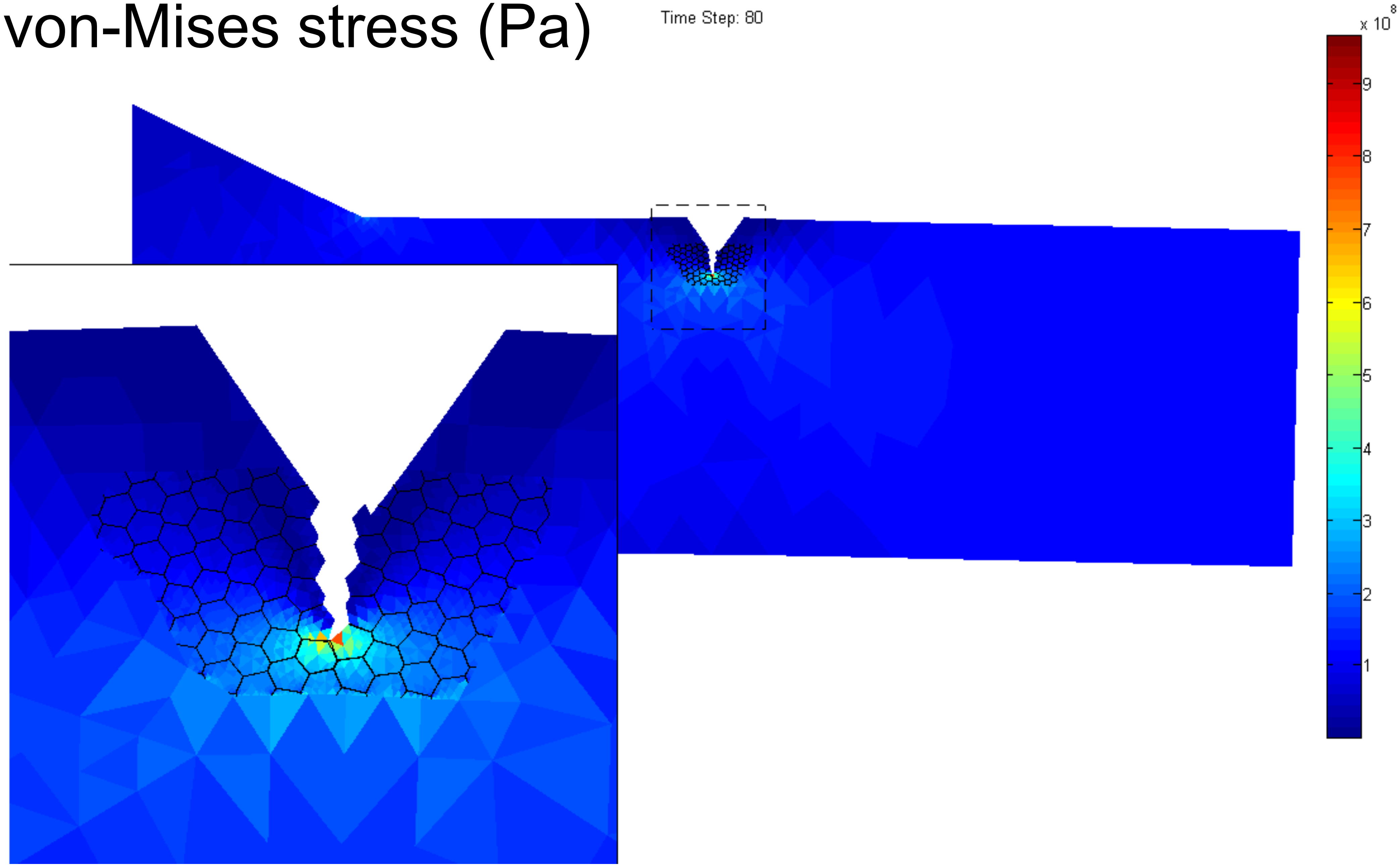
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

Results: uni-axial tension

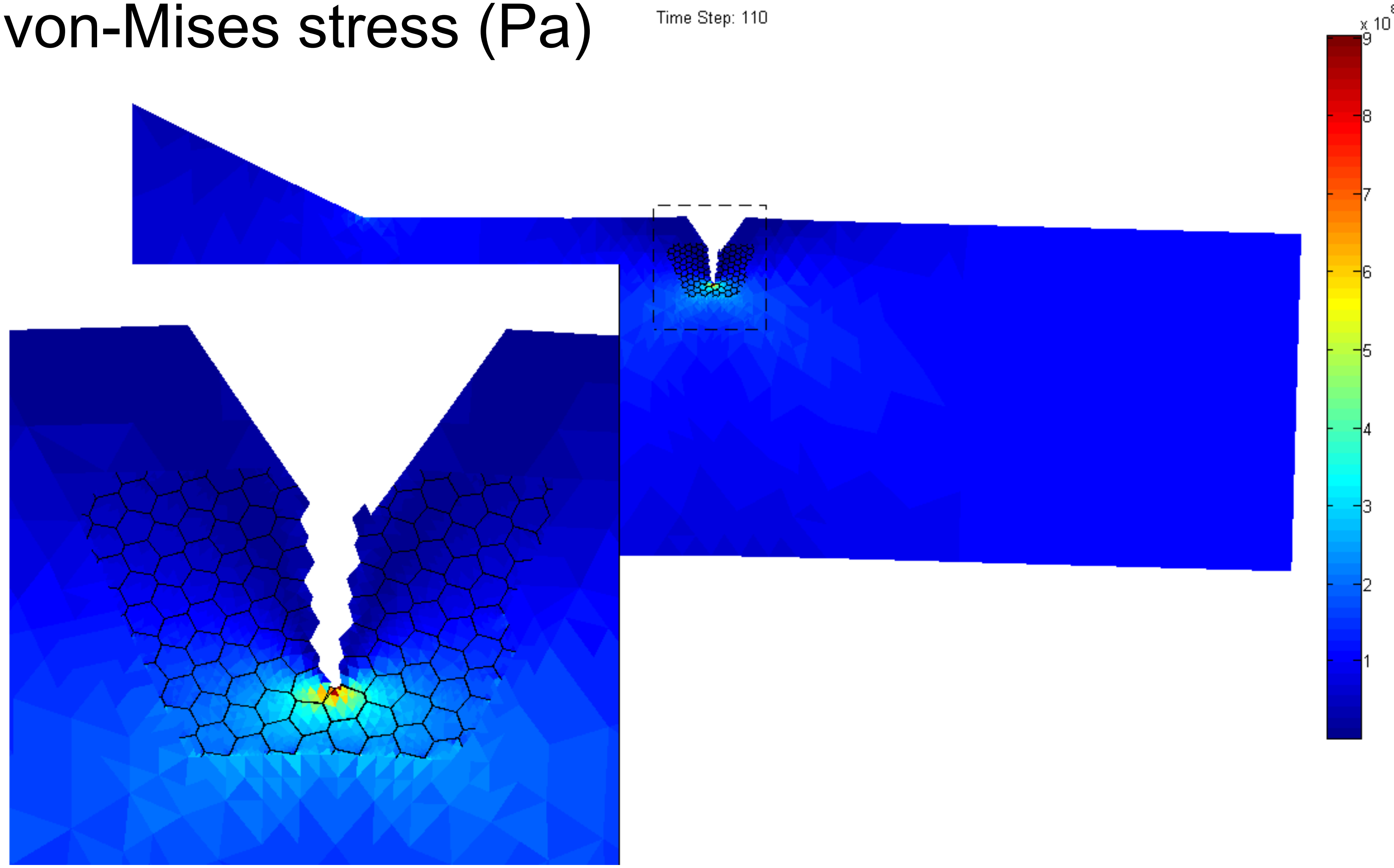
von-Mises stress (Pa)



❖ 100X (magnification of displacement)

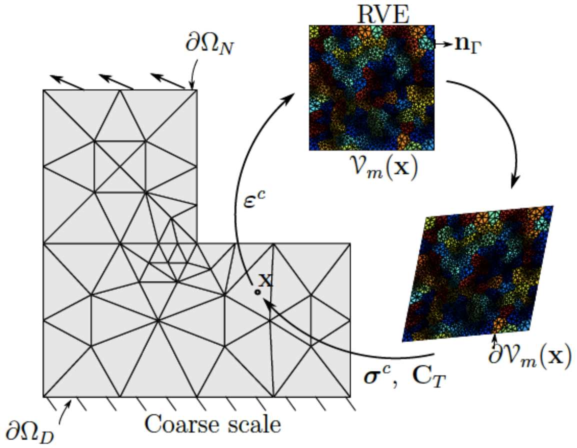
Results: uni-axial tension

von-Mises stress (Pa)

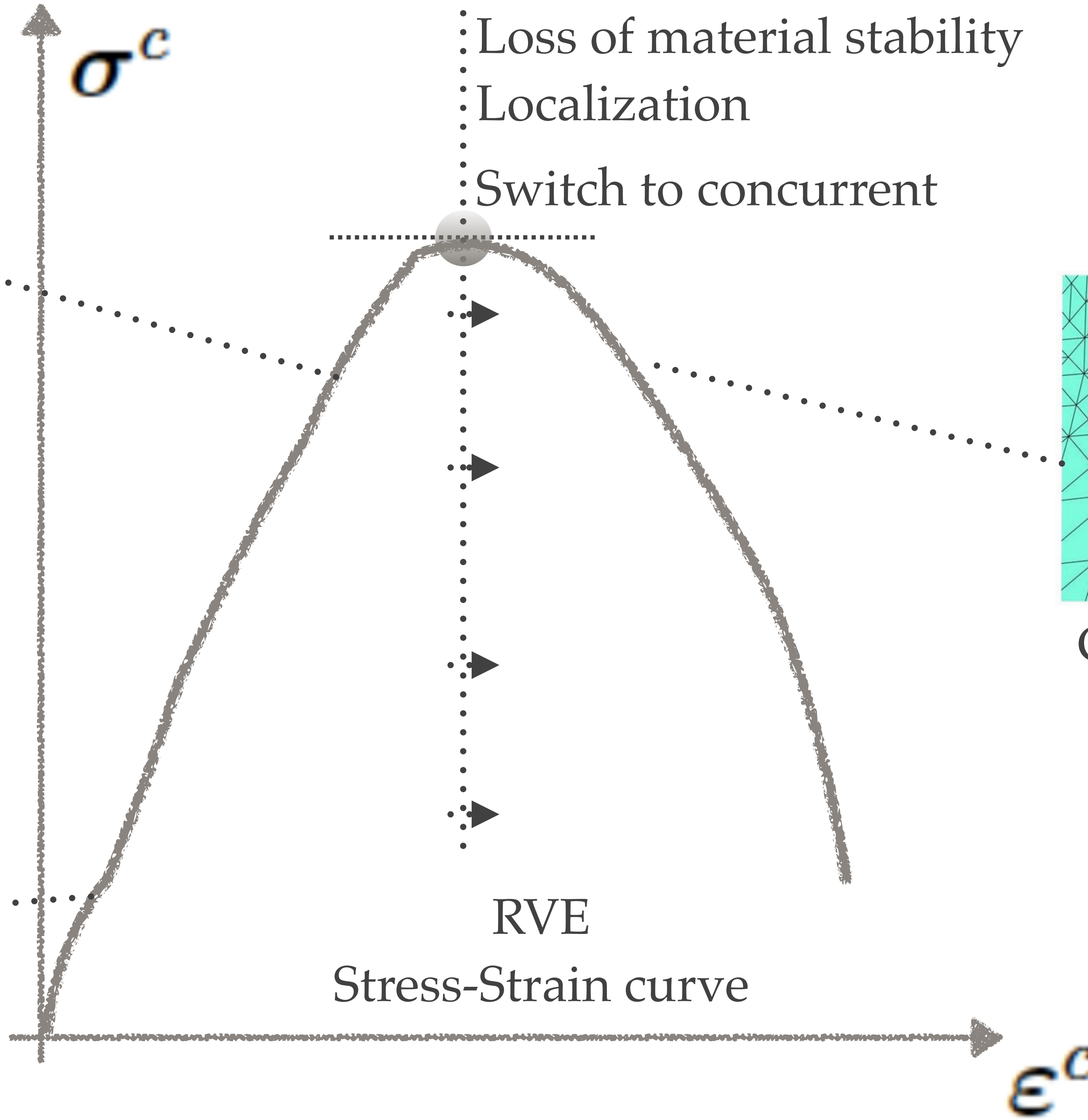


❖ 100X (magnification of displacement)

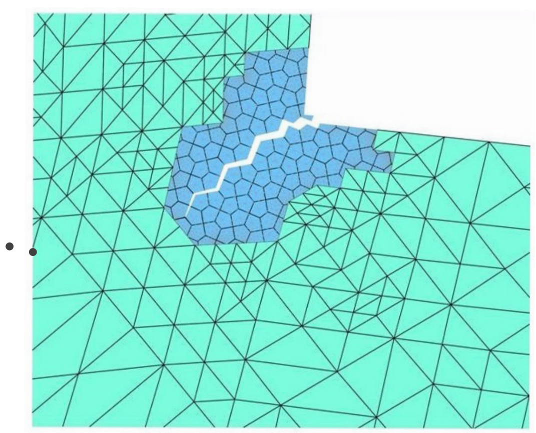
Adaptive multi-scale



Semi-concurrent
FE^2



Loss of material stability
Localization
Switch to concurrent



Concurrent

Analytical homogenisation

RVE
Stress-Strain curve

Feyel, Chaboche, 2000 - Akbari, Kerfriden, Bordas, 2014

Open problem

- model selection and error control

Possible approach

- machine learning and statistical inference, e.g. Bayesian statistics

Open problem

- statistical variability at the fine scale (geometry, material parameter)

Possible approach

- identification through small-scale experiments (costly, difficult to characterize interfaces)
- Monte Carlo

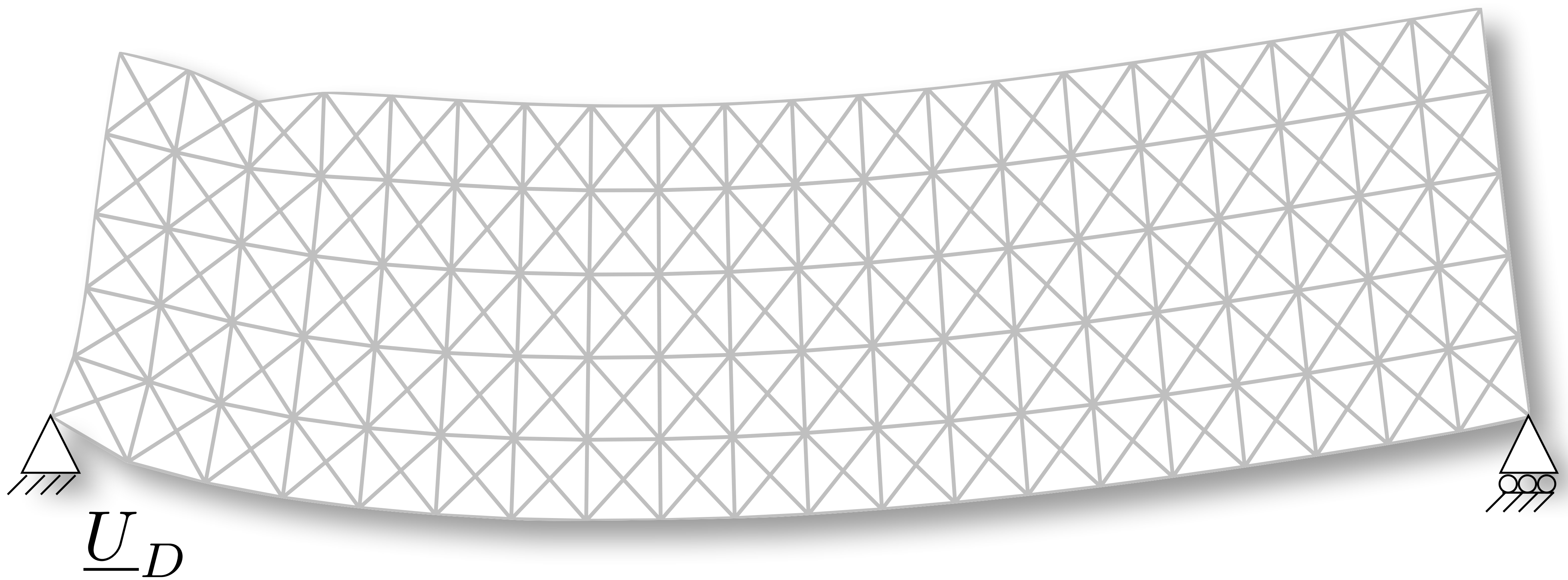
Algebraic model reduction methods

Use precomputed solutions to accelerate online simulations

Example - parametric problems

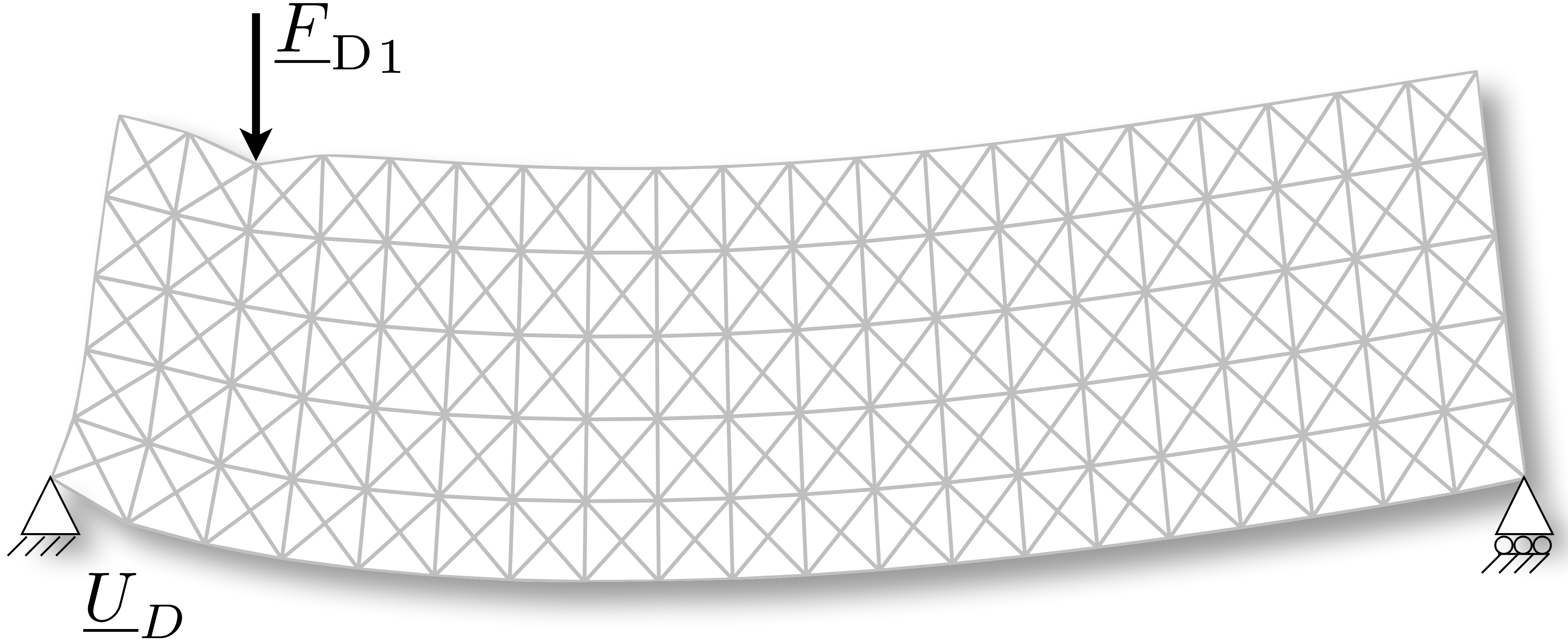
Method of separated representation

Lattice beam problem



Aim: accelerate the simulation using pre-computations

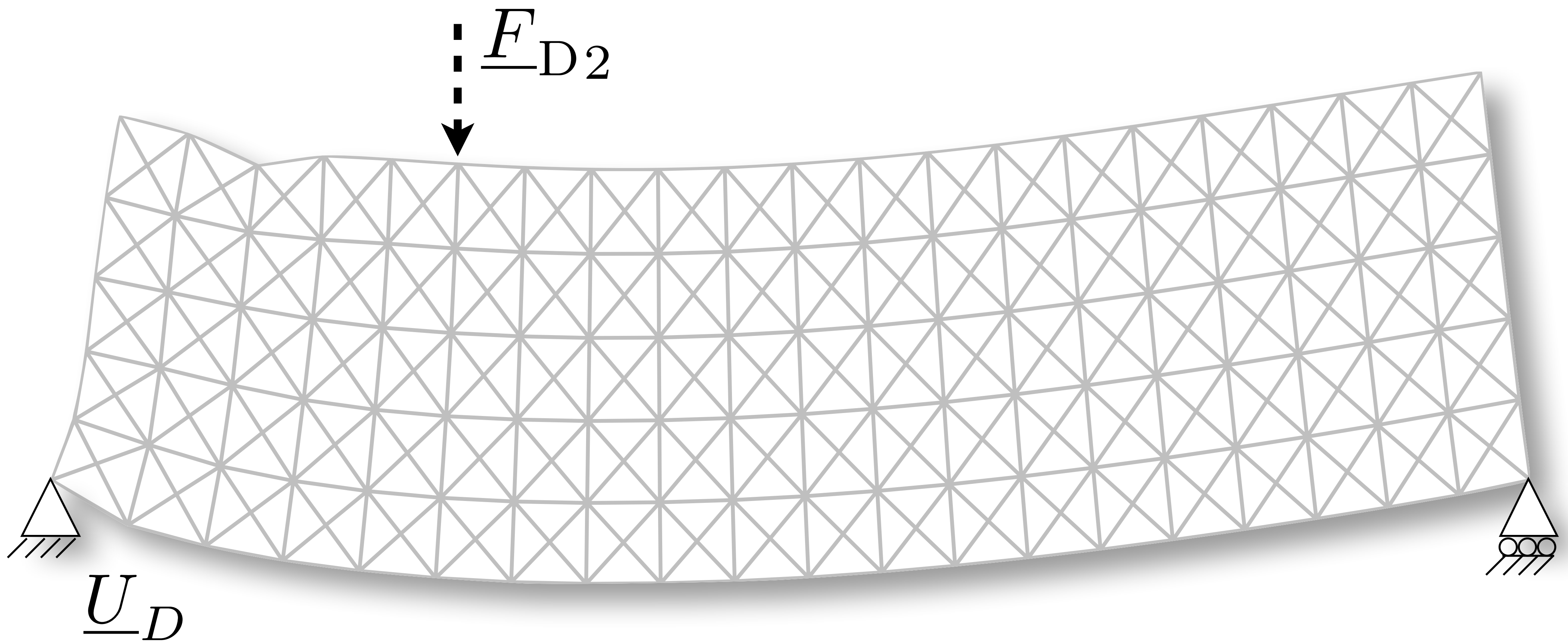
Lattice beam problem



$$\underline{\underline{\mathbf{S}}} = (\ \underline{\mathbf{S}}^1$$

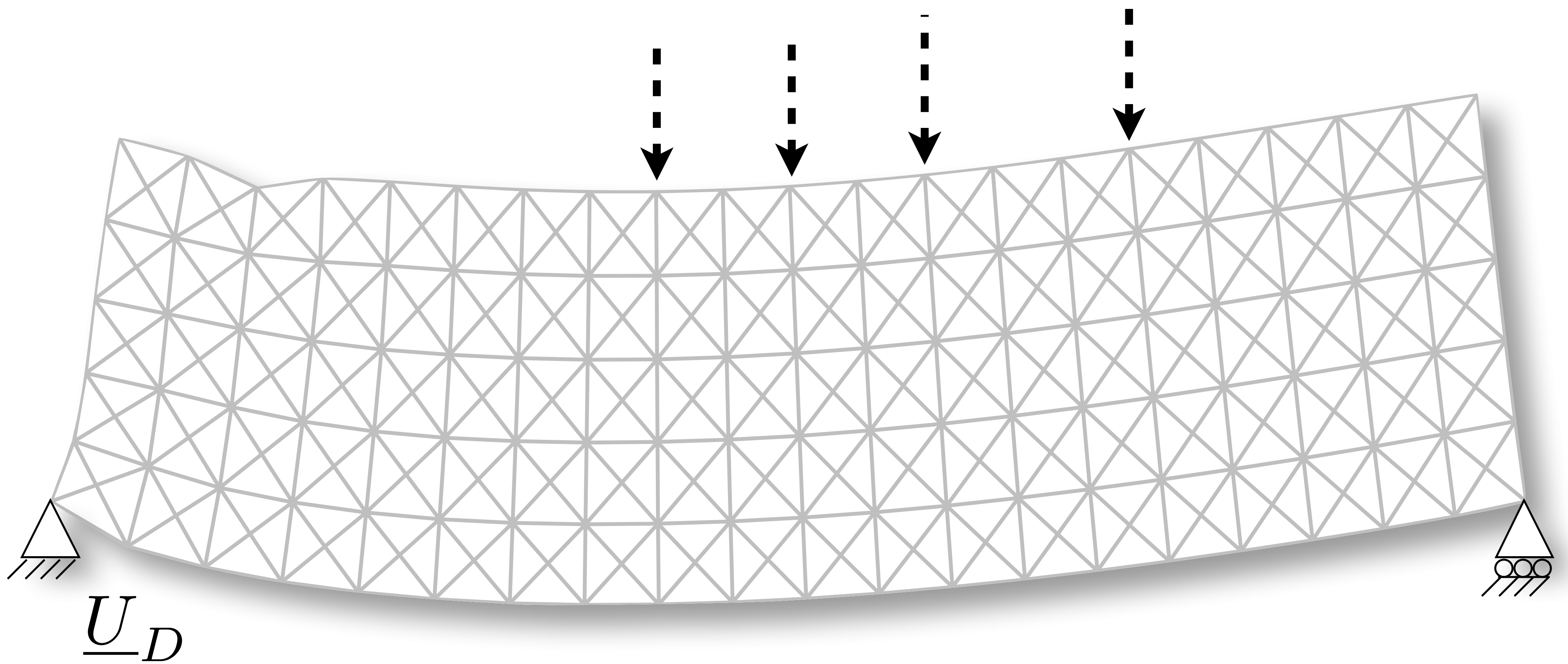
Compute solutions for several loading conditions

Lattice beam problem



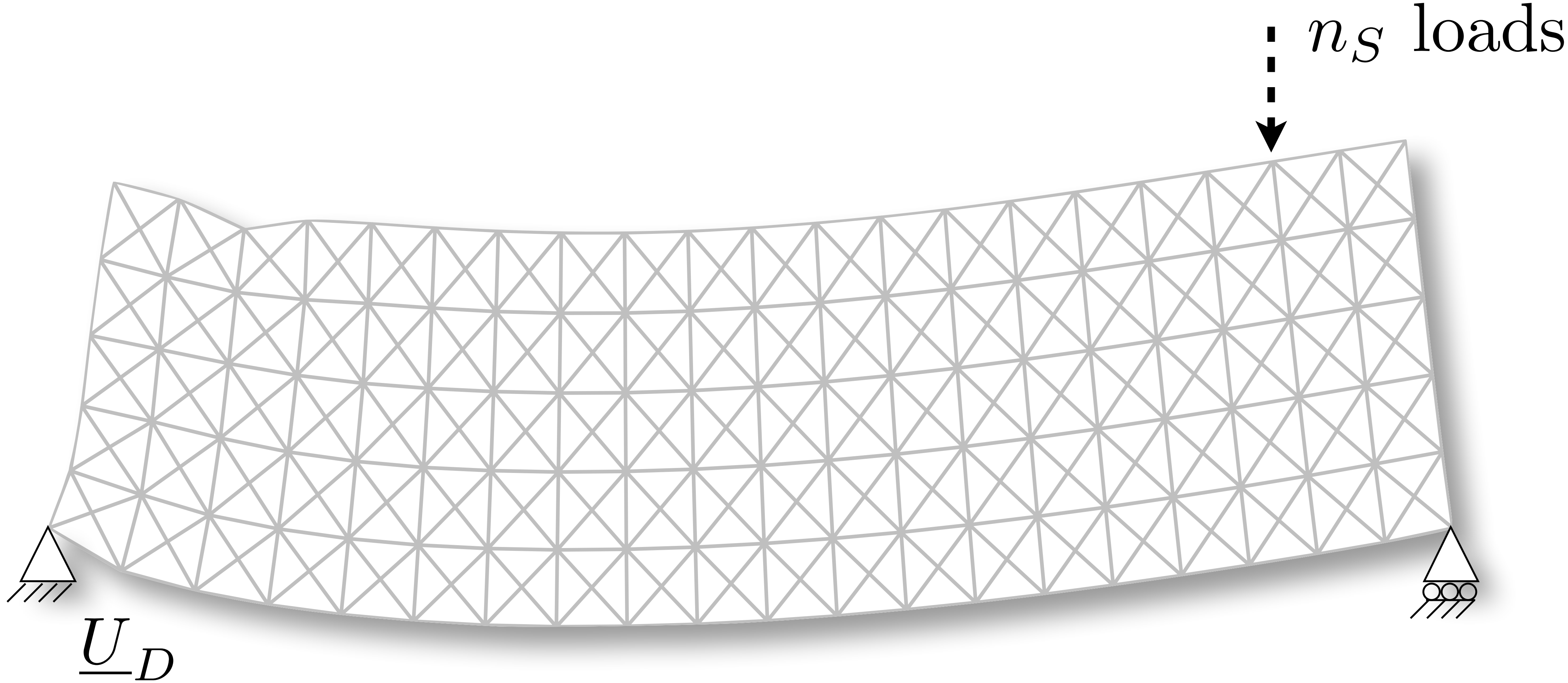
$$\underline{\underline{\mathbf{S}}} = (\ \underline{\mathbf{S}}^1 \quad \underline{\mathbf{S}}^2$$

Lattice beam problem



$$\underline{\underline{\mathbf{S}}} = \left(\underline{\mathbf{S}}^1 \quad \underline{\mathbf{S}}^2 \quad \dots \right.$$

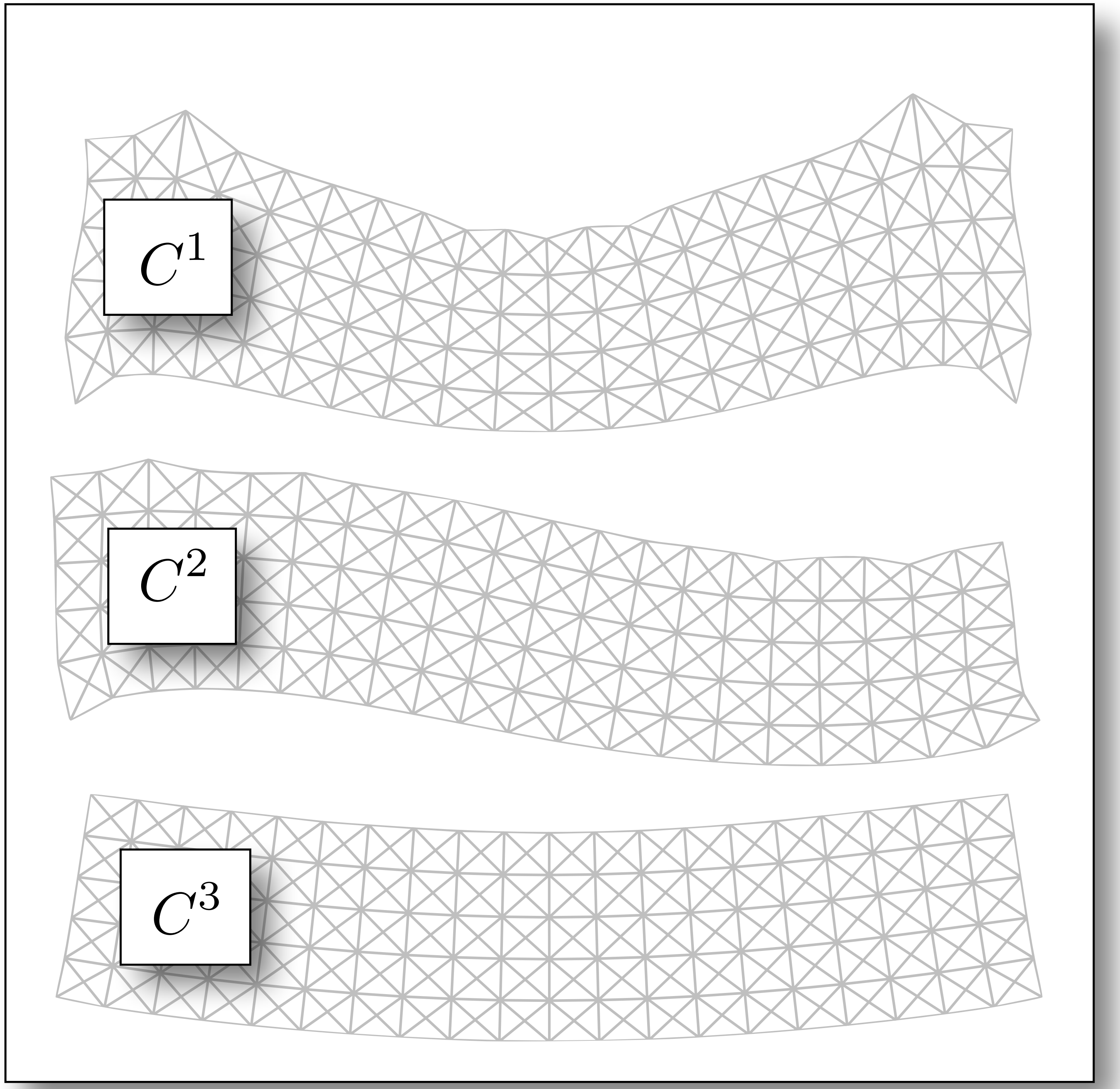
Lattice beam problem



$$\underline{\underline{\mathbf{S}}} = \begin{pmatrix} \underline{\mathbf{S}}^1 & \underline{\mathbf{S}}^2 & \dots & \underline{\mathbf{S}}^{n_S} \end{pmatrix}$$

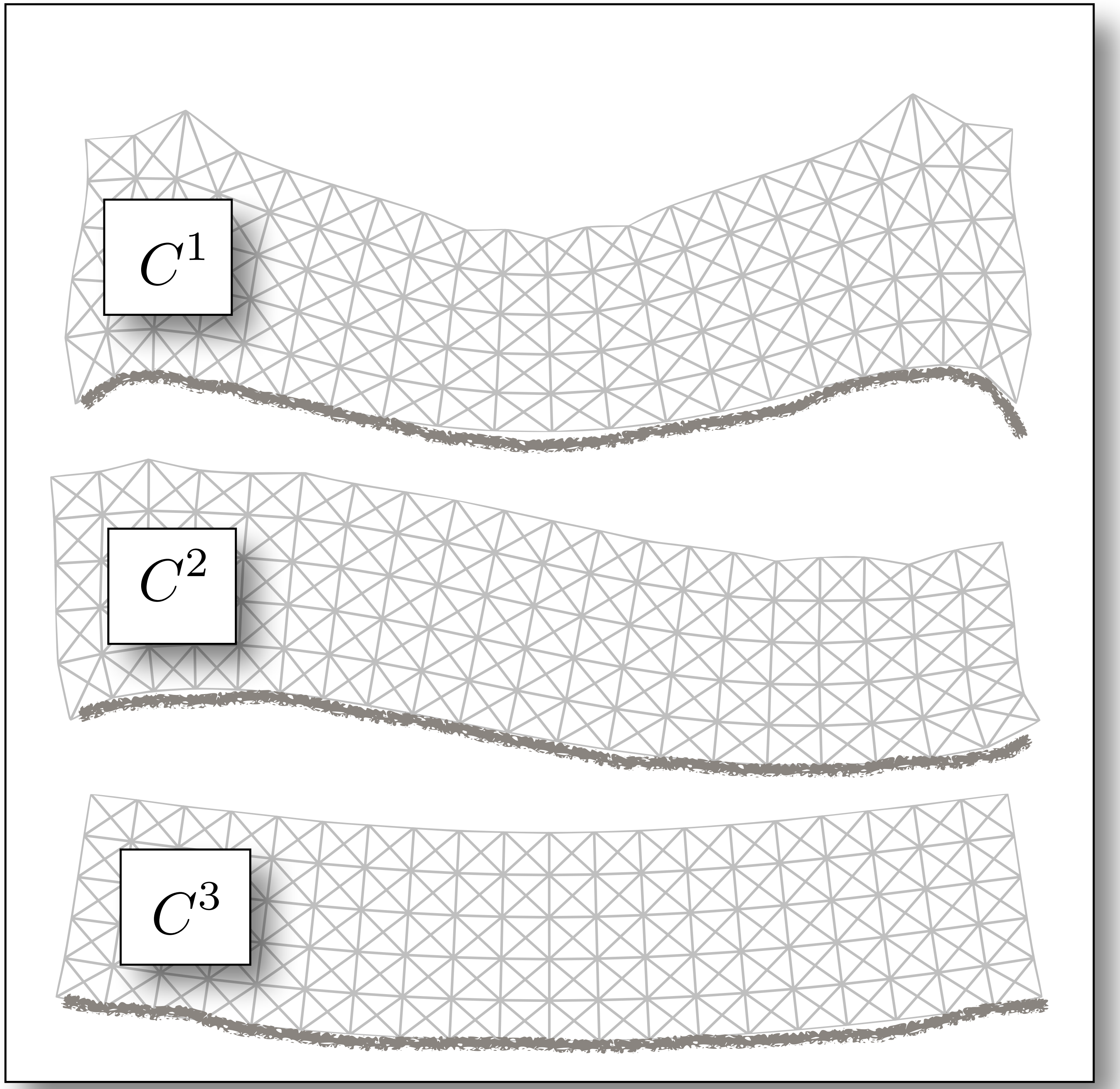
Perform singular value decomposition - POD to obtain "most energetic modes"

Reduced basis



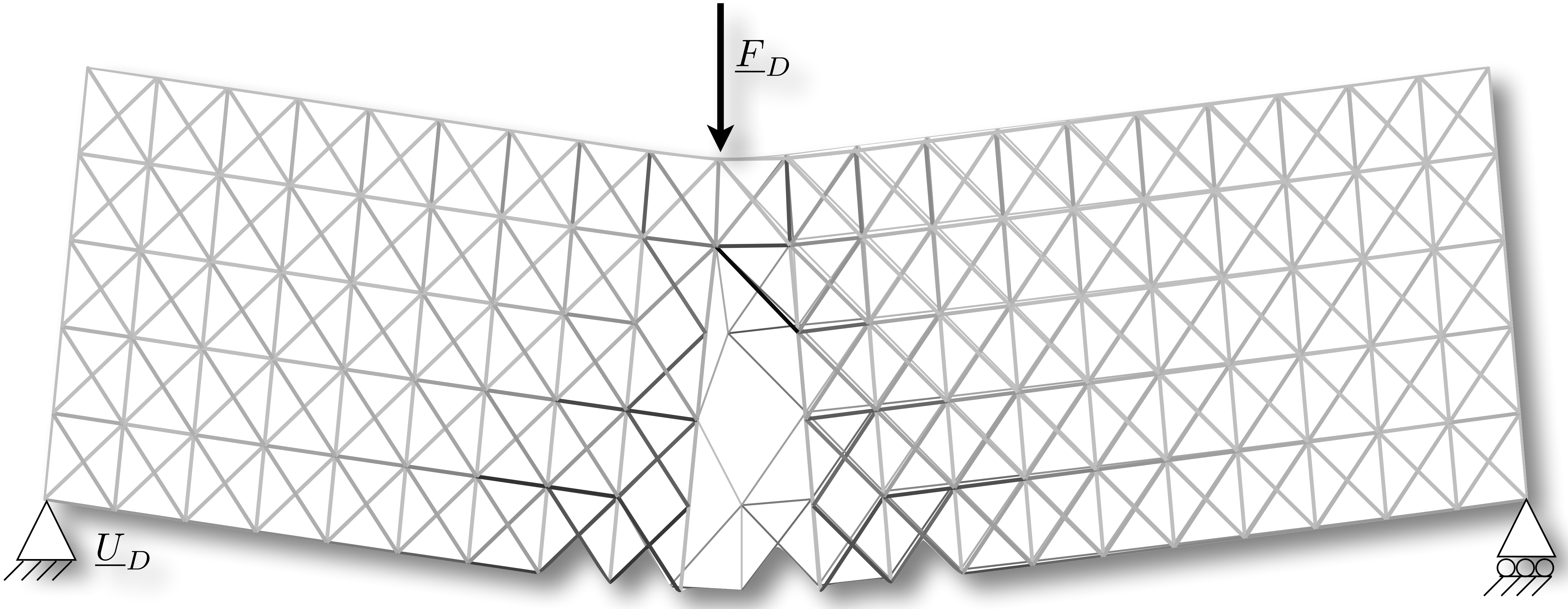
P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. *Bridging proper orthogonal decomposition methods and augmented Newton-Krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems*. Computer Methods in Applied Mechanics and Engineering, 200(5-8):850-866, 2011.

Reduced basis



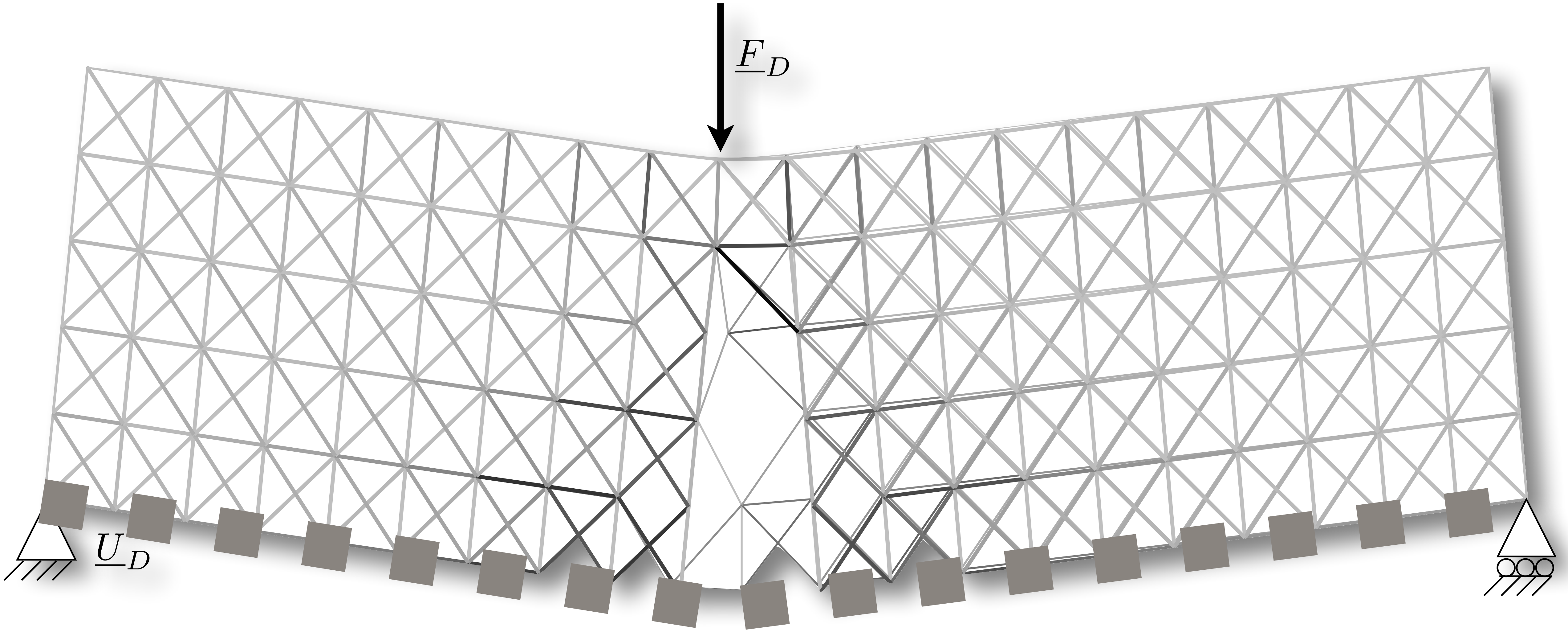
P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. *Bridging proper orthogonal decomposition methods and augmented Newton-Krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems*. Computer Methods in Applied Mechanics and Engineering, 200(5-8):850-866, 2011.

Beyond the elastic limit

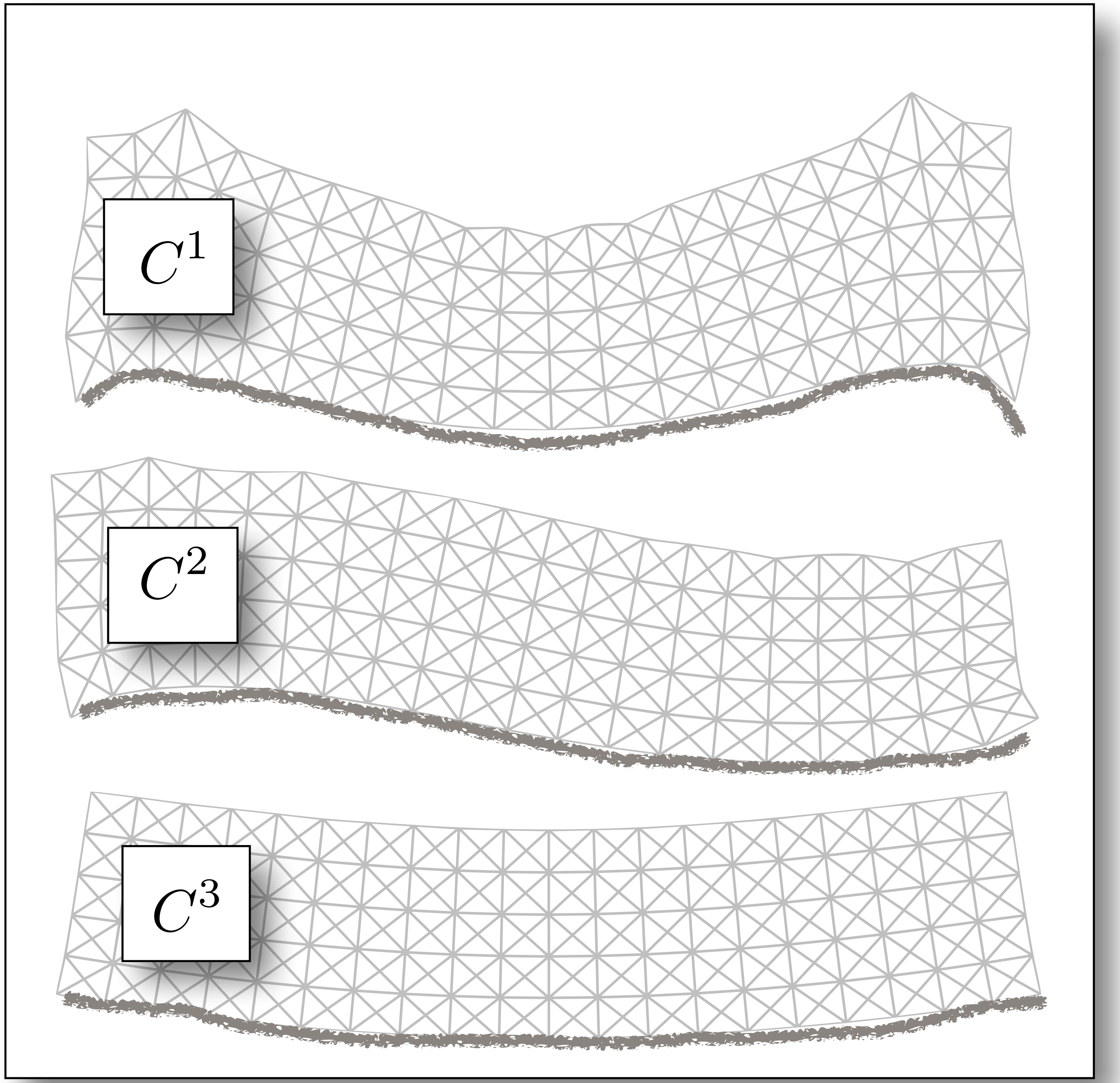


P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. *Bridging proper orthogonal decomposition methods and augmented Newton-Krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems*. Computer Methods in Applied Mechanics and Engineering, 200(5-8):850-866, 2011.

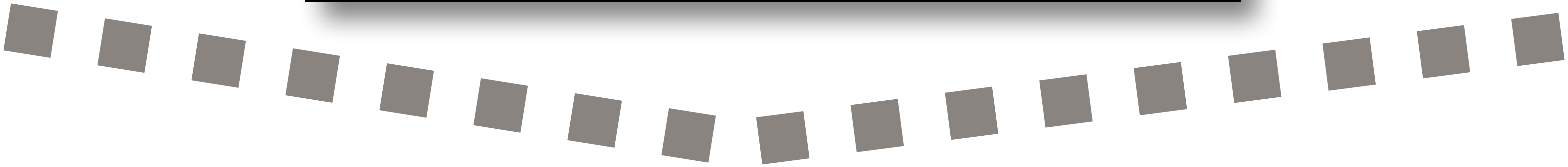
Beyond the elastic limit



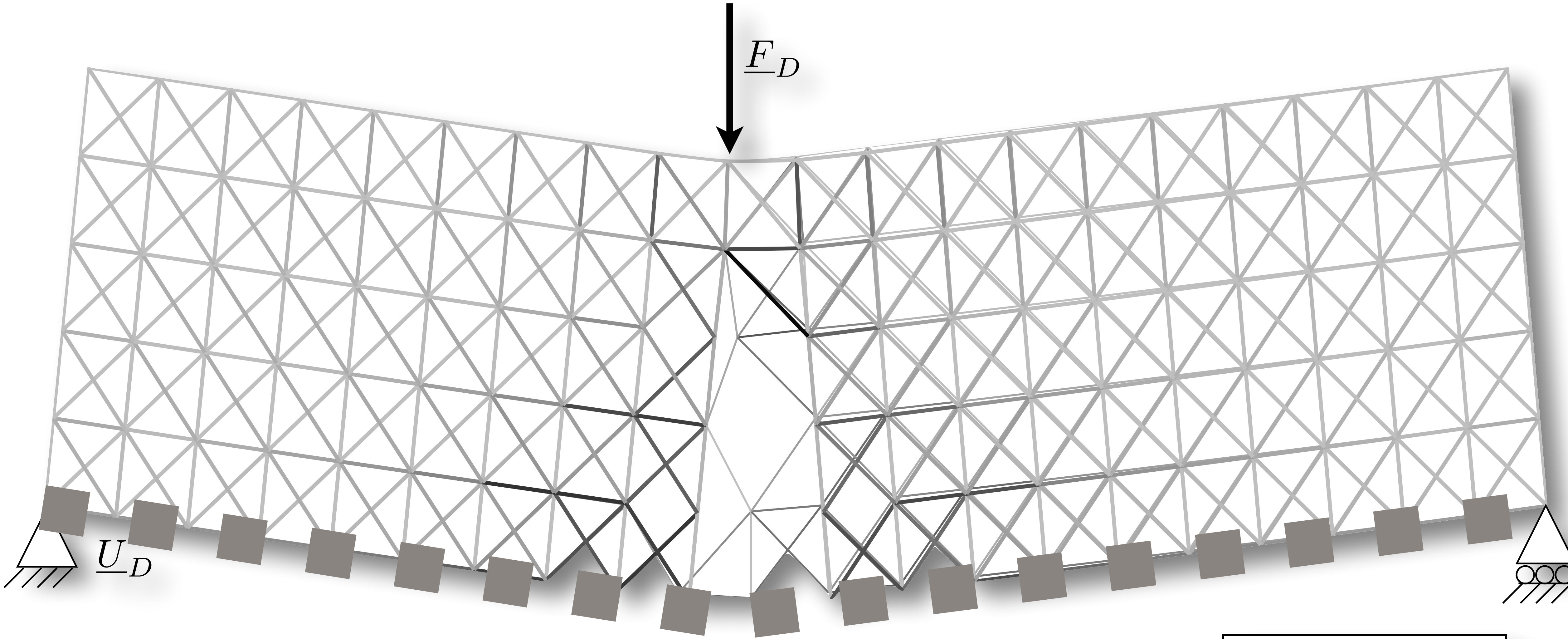
P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. *Bridging proper orthogonal decomposition methods and augmented Newton-Krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems*. Computer Methods in Applied Mechanics and Engineering, 200(5-8):850-866, 2011.



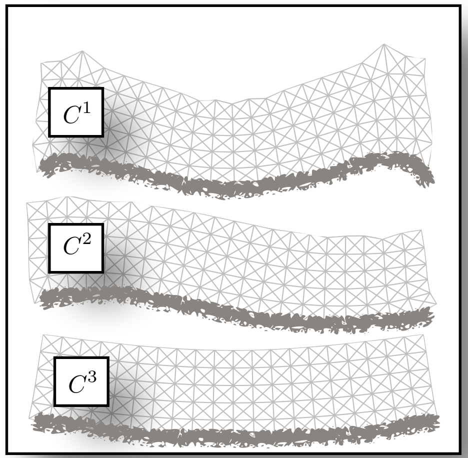
Smooth



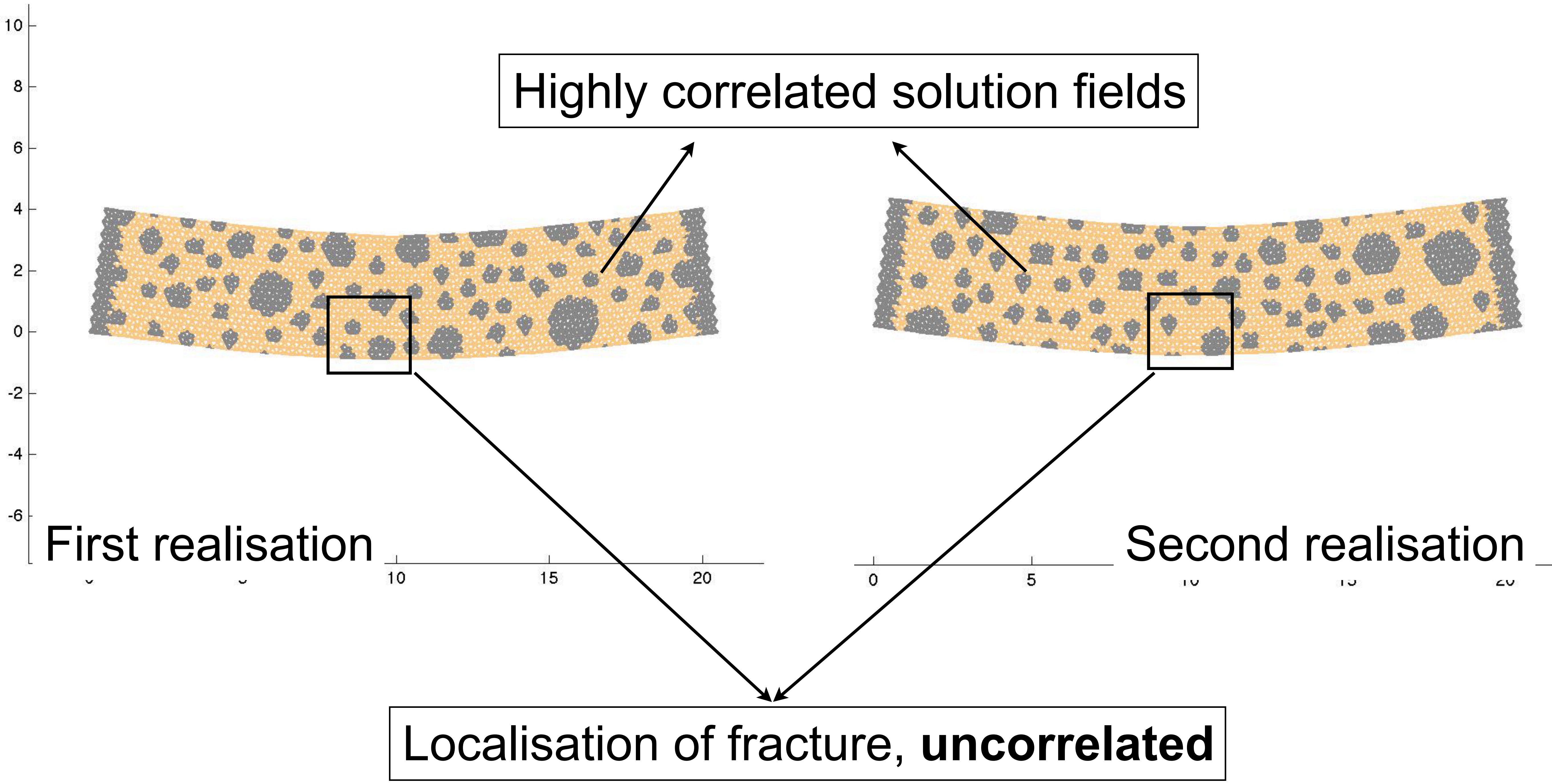
Kink



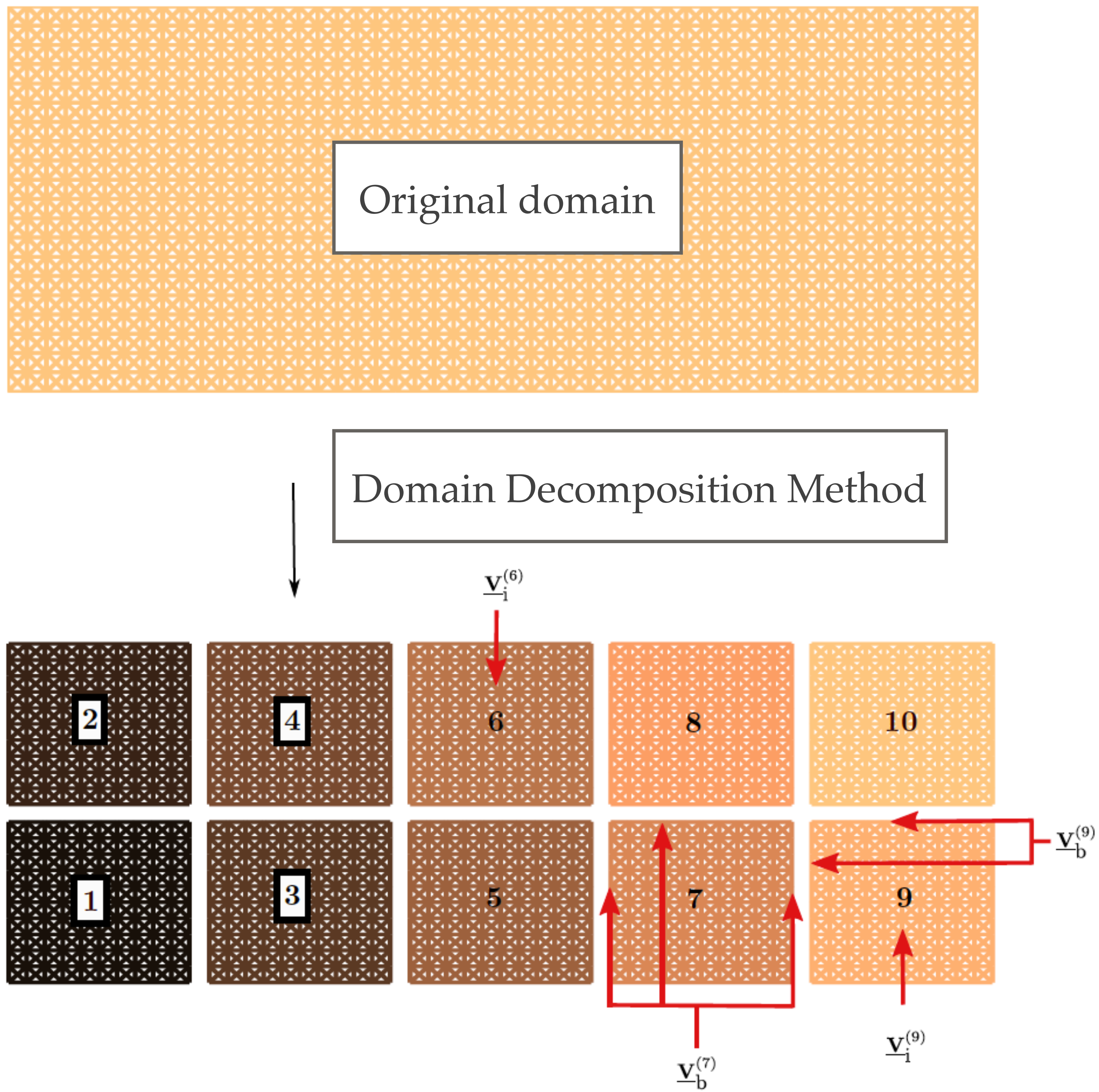
This solution is not in the snapshot !



Parametric / stochastic multiscale fracture mechanics



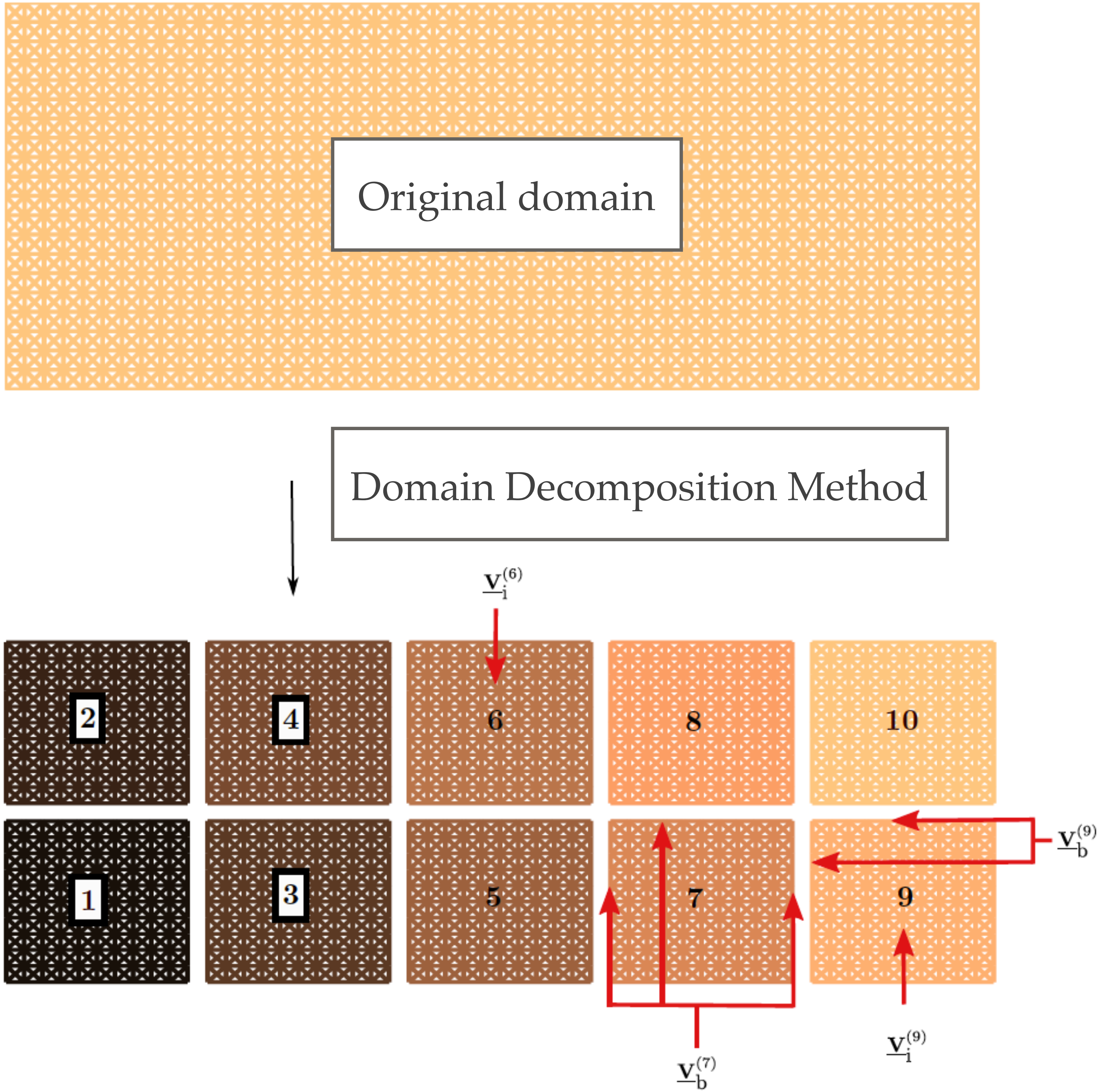
Partitioned POD/DDM

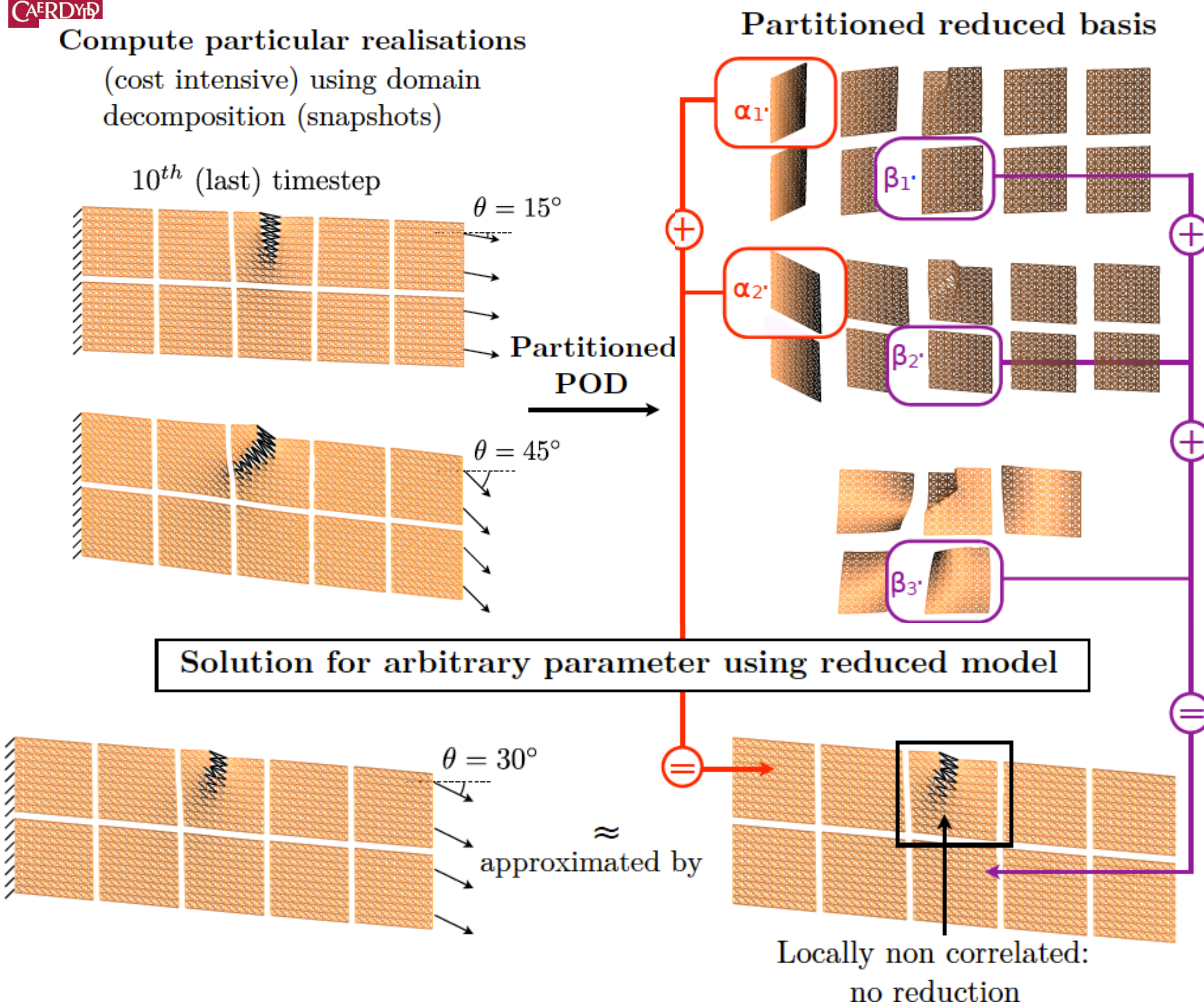


Partitioned POD/DDM

Domain Decomposition Method

Partitioned POD/DDM





- Decompose the structure into subdomains
- Perform a reduction in the highly correlated region
- Couple the reduced to the non-reduced region by a primal Schur complement

Adaptive equation-free multiscale modeling of metallic lattice with geometrical (and material) nonlinearity and variability

Li Chen [1,2]
Promoters: Thierry J. Massart, Lars Beex, Peter Berke, Stéphane Bordas

[1]Université libre de Bruxelles

[2]Université du Luxembourg

Conceptual idea of the work

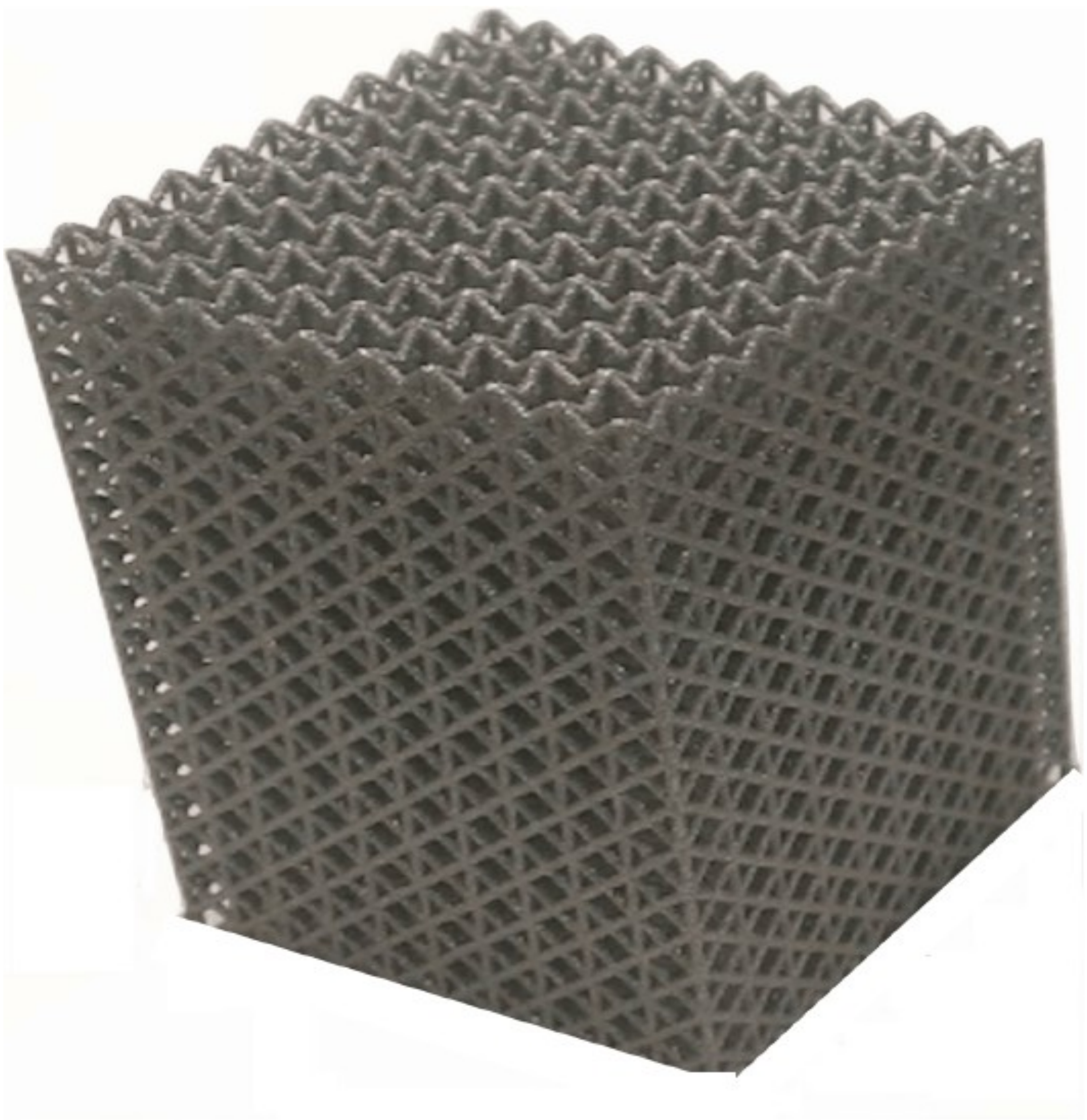
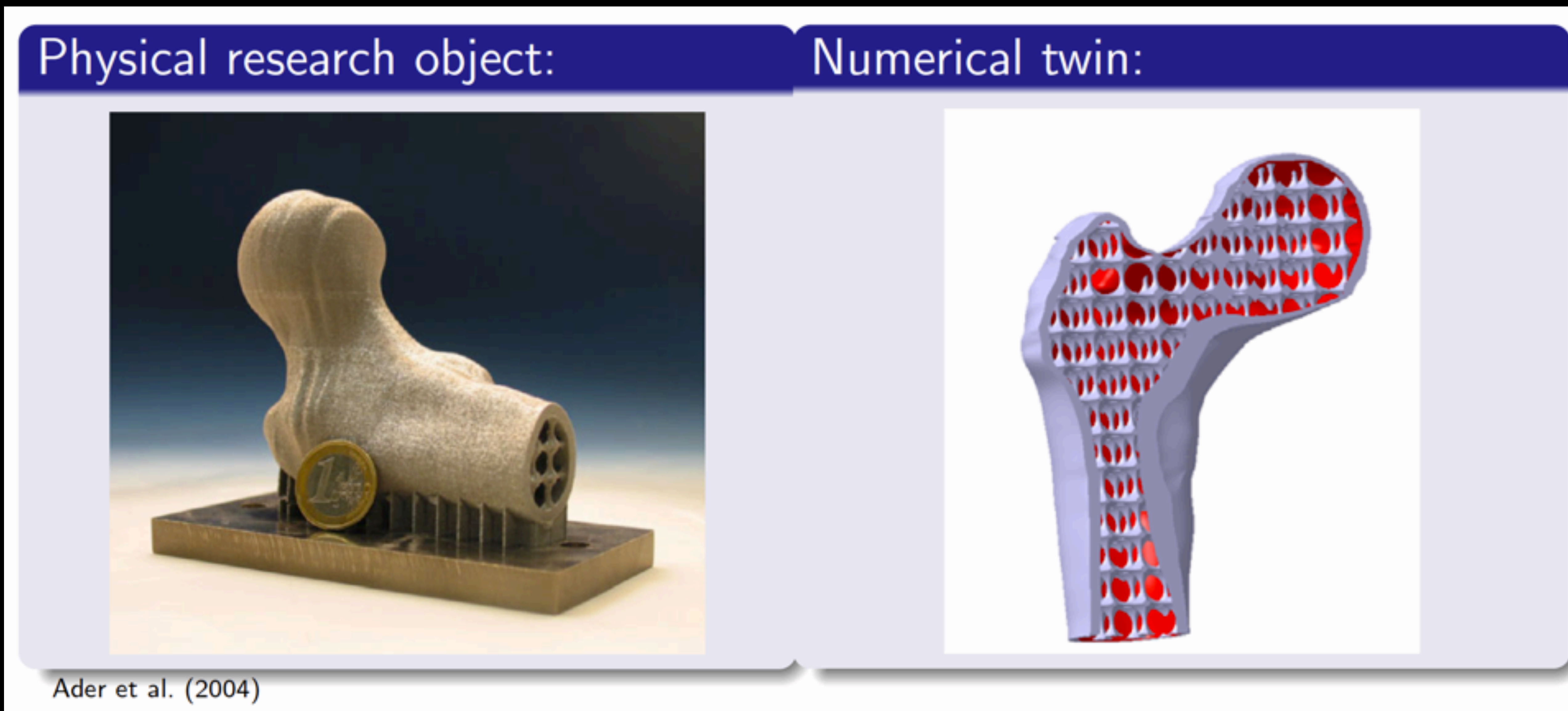


Figure: Metal lattice in the form of periodic mesostructural unit cells

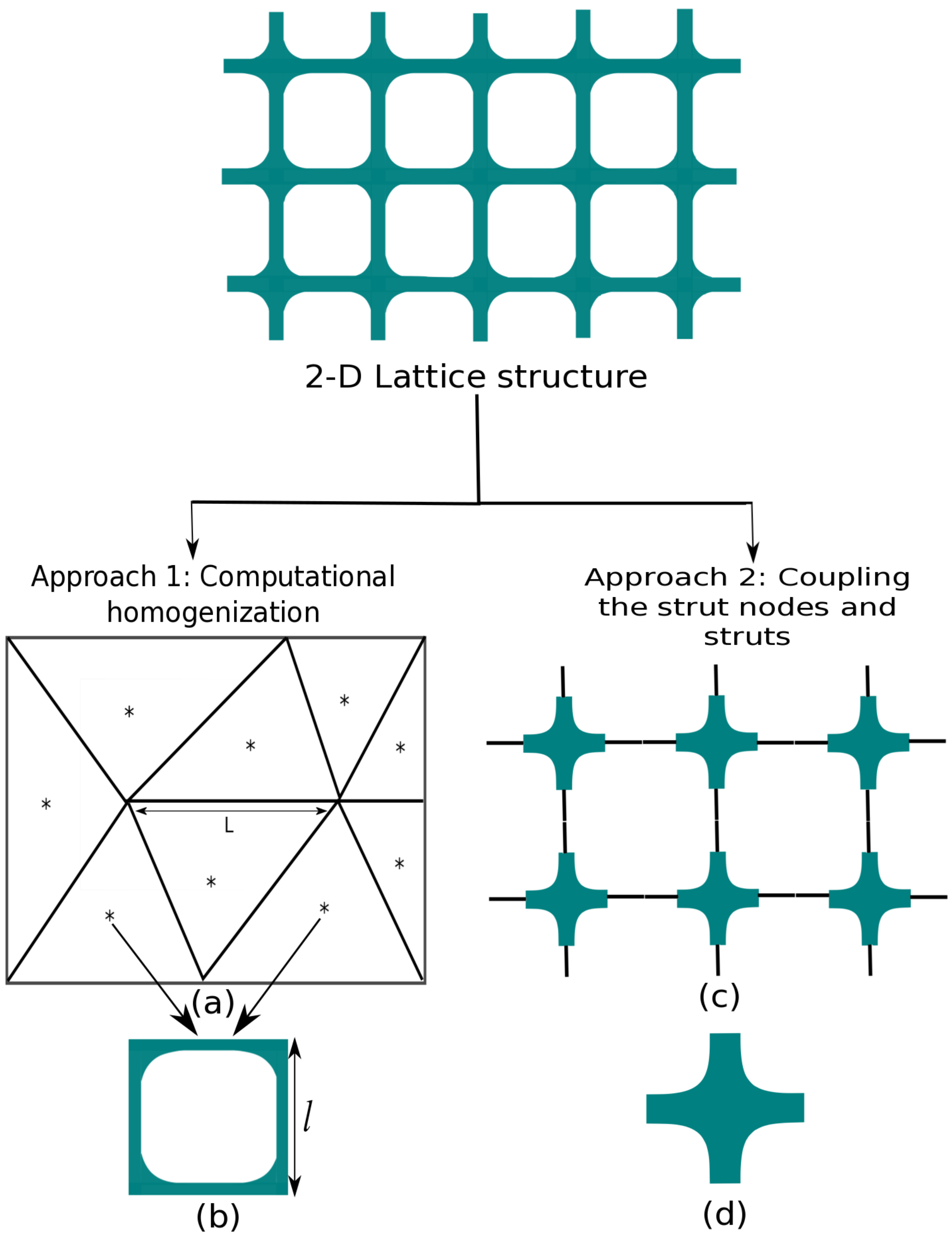
The problem

In case of certain metallic structures the strut nodes (i.e. the locations where several struts are connected) are relatively weak.

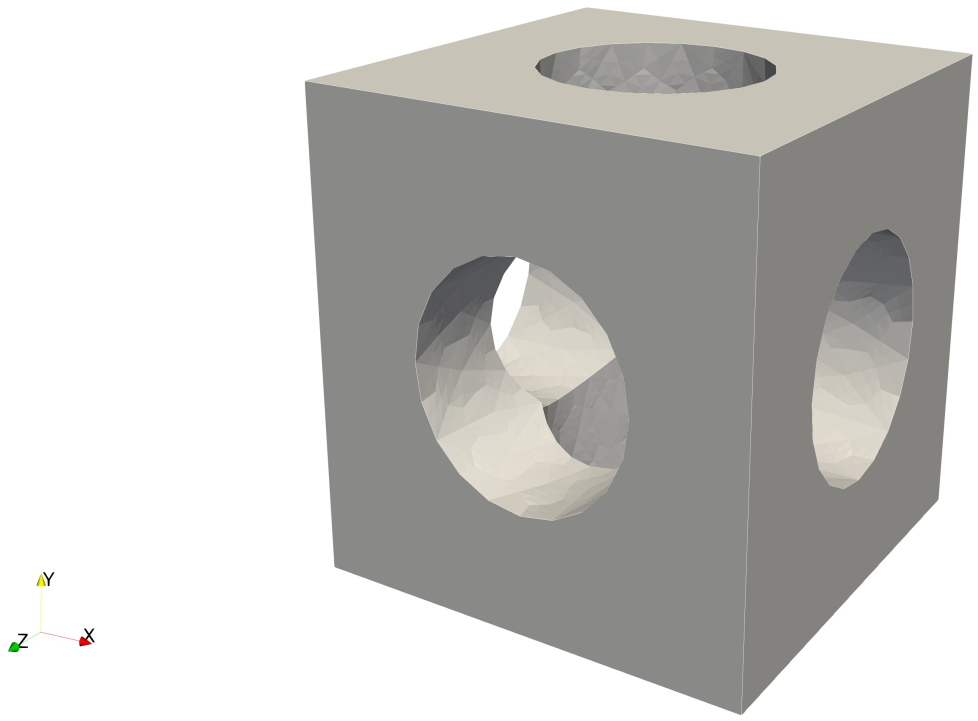
3d printed structures



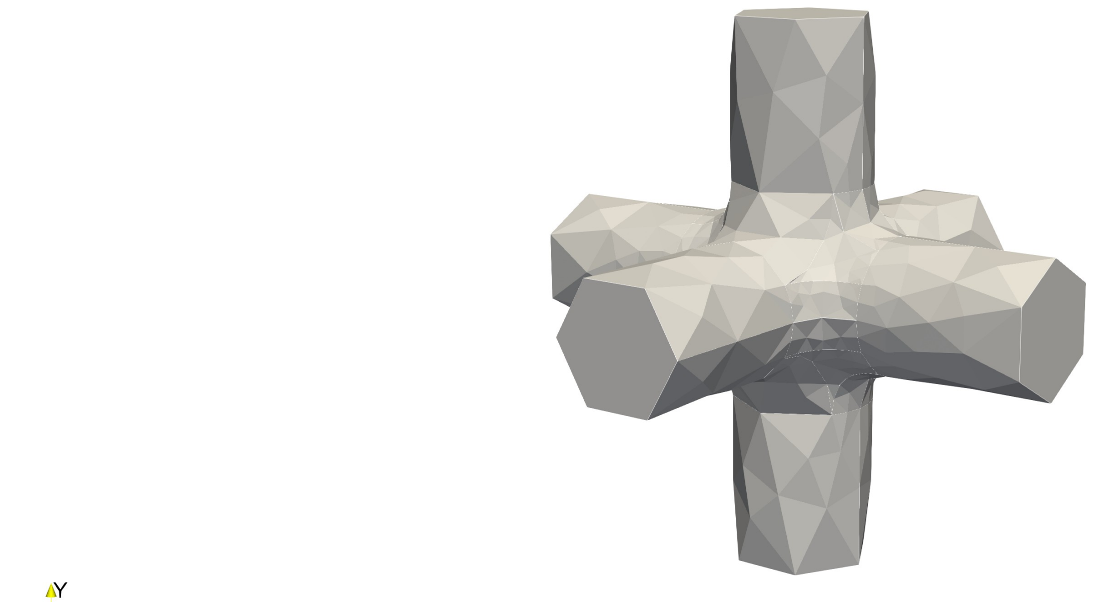
Conceptual idea of the work



Finite Element Model

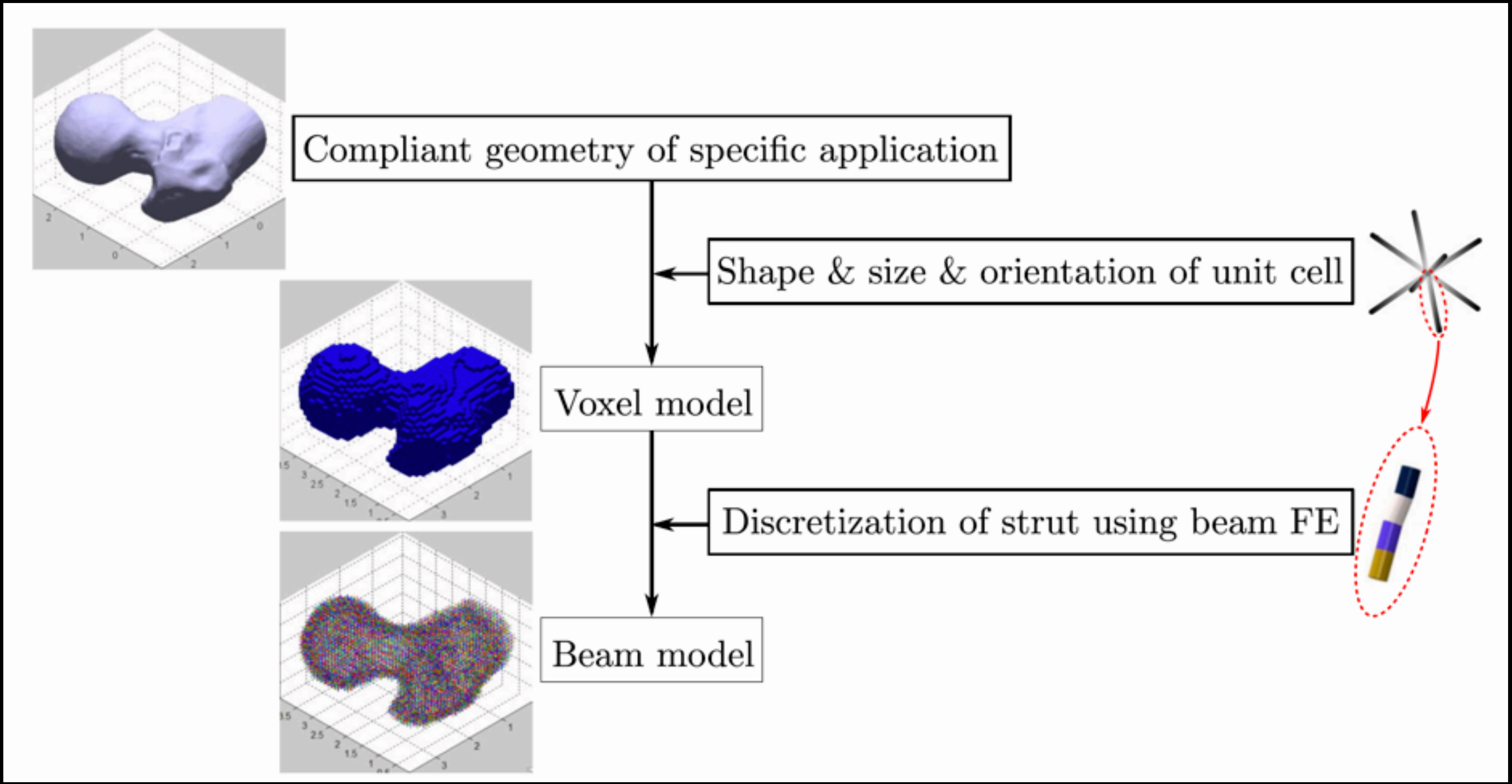


(a) 3D RVE

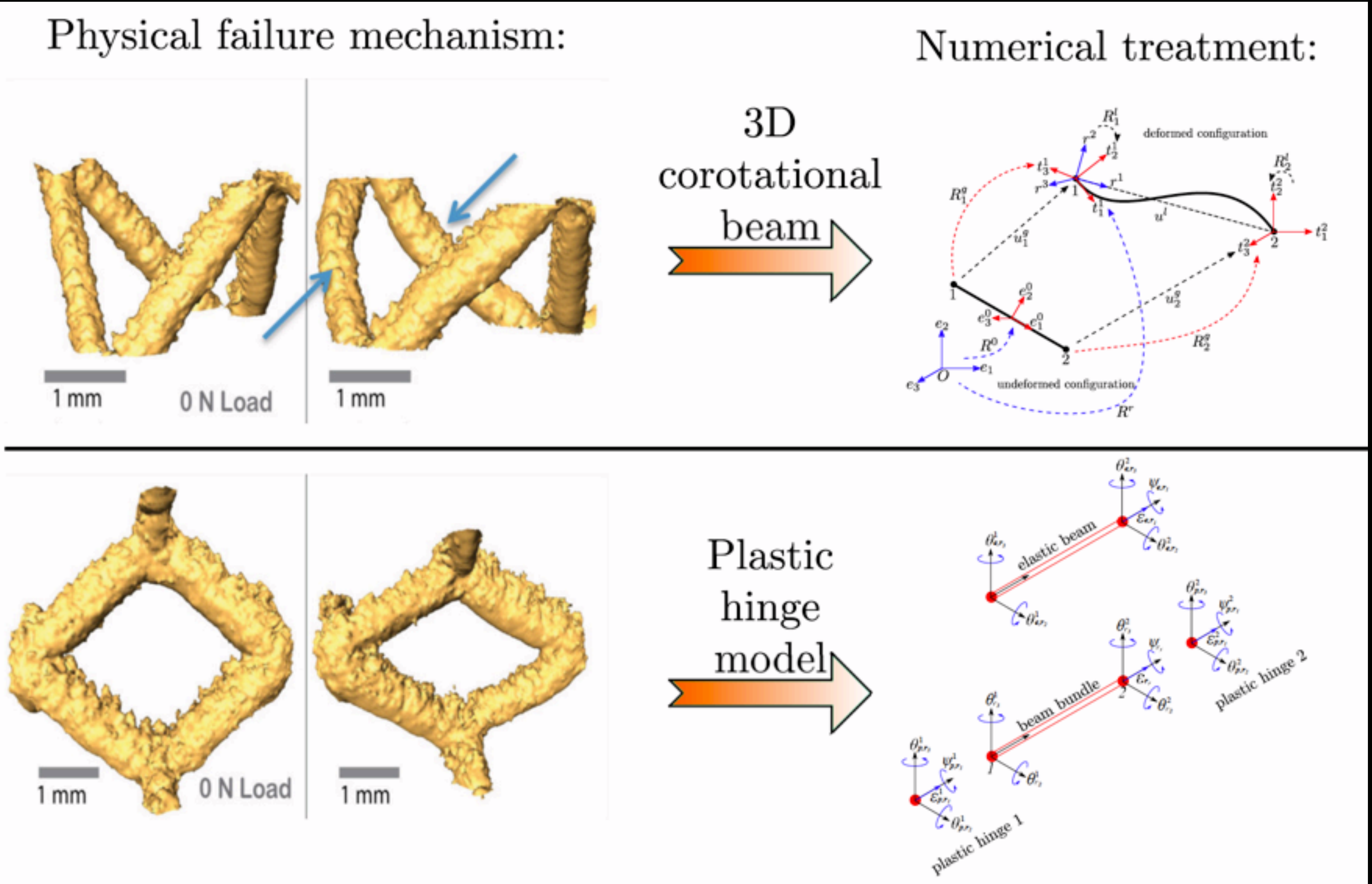


(b) 3D strut node

Beam model from image

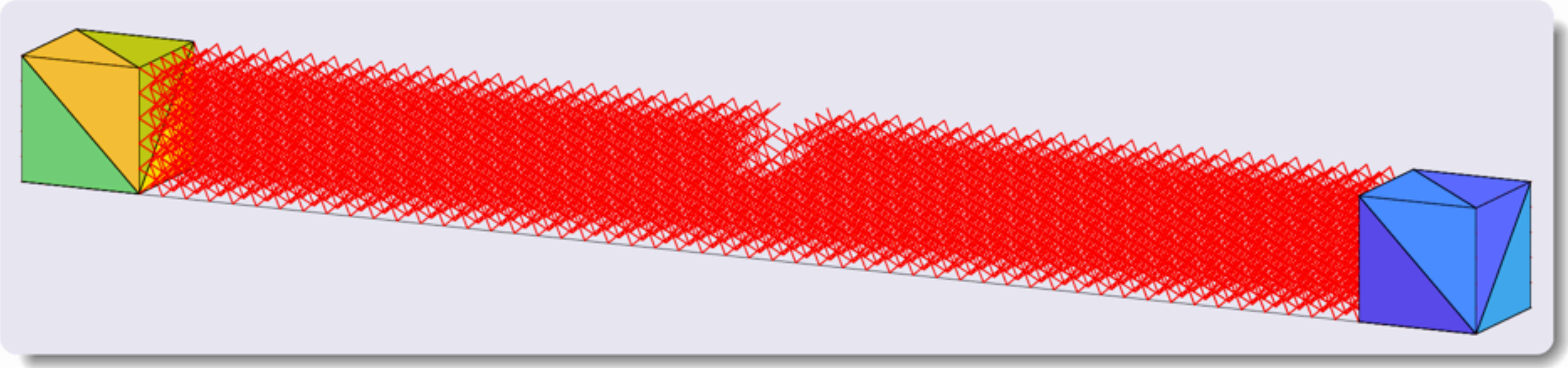


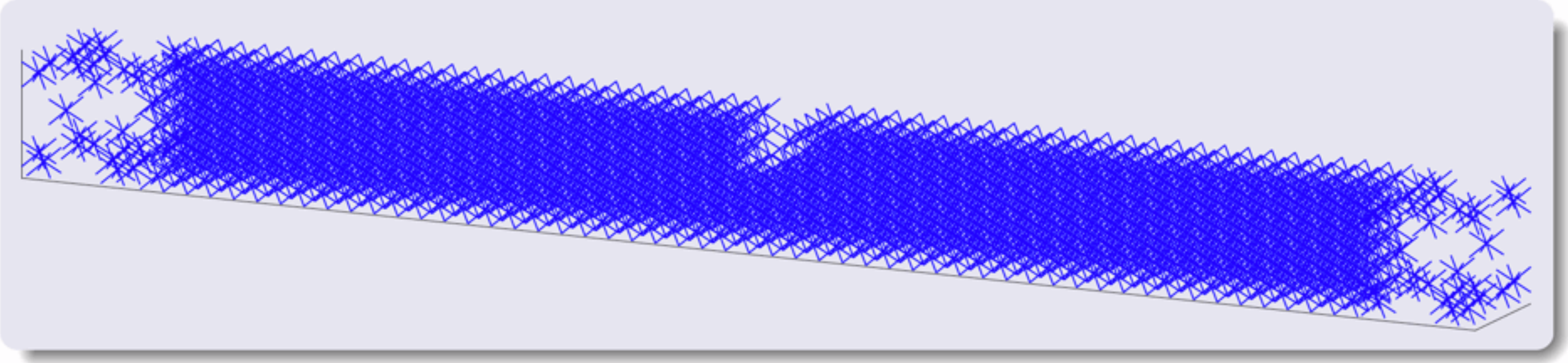
Beam model



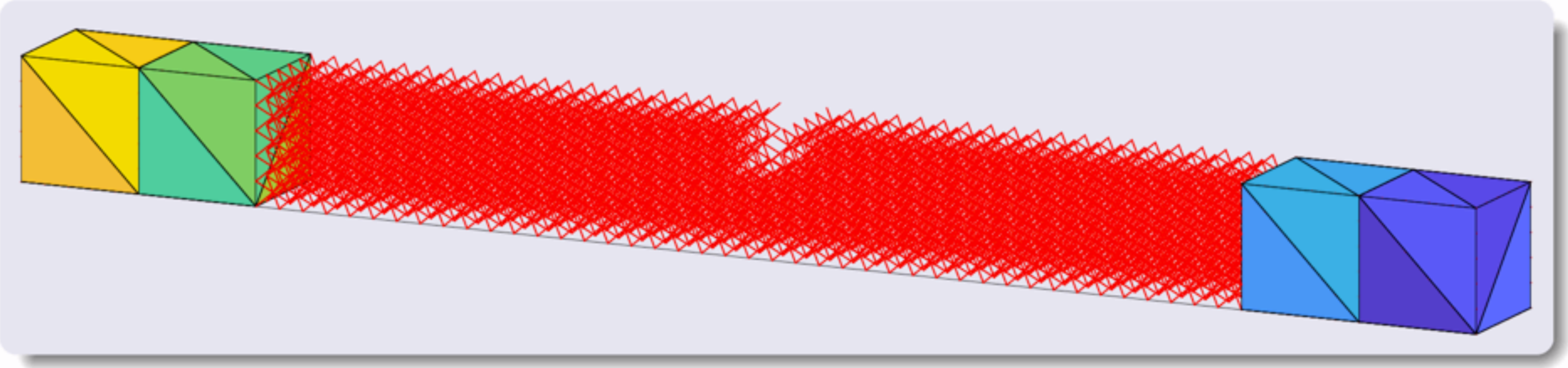
Carlton et al. (2017)

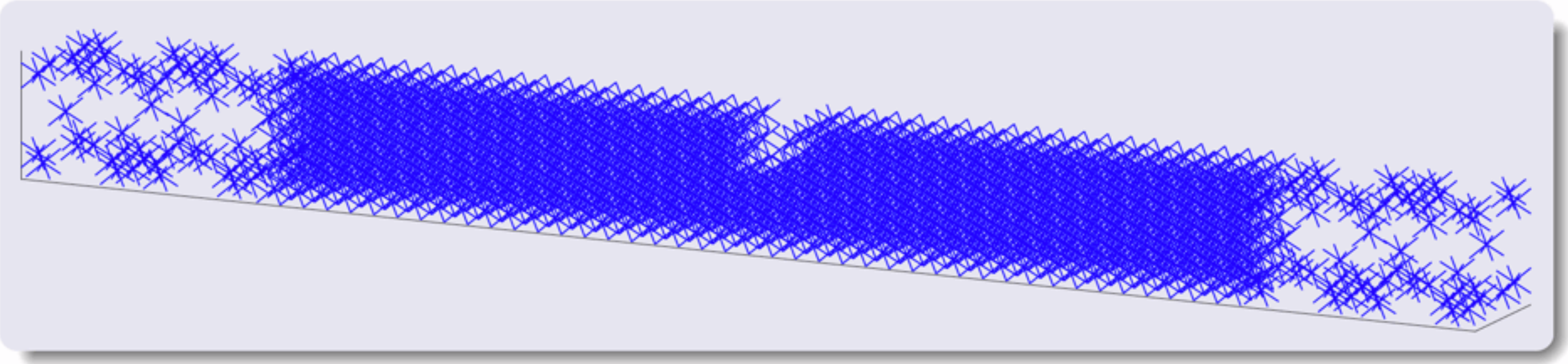
1st interpolation scheme and summation scheme:



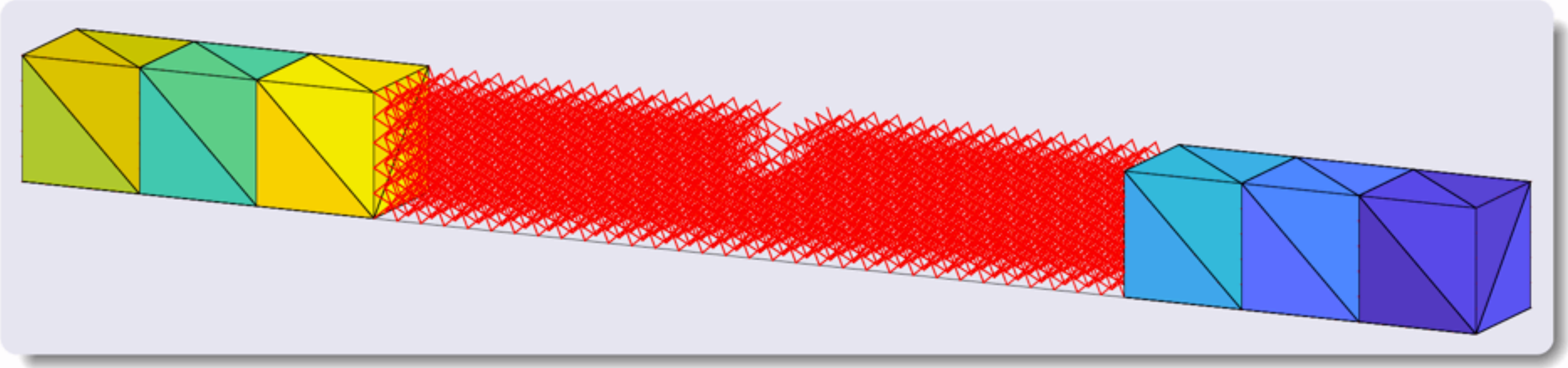


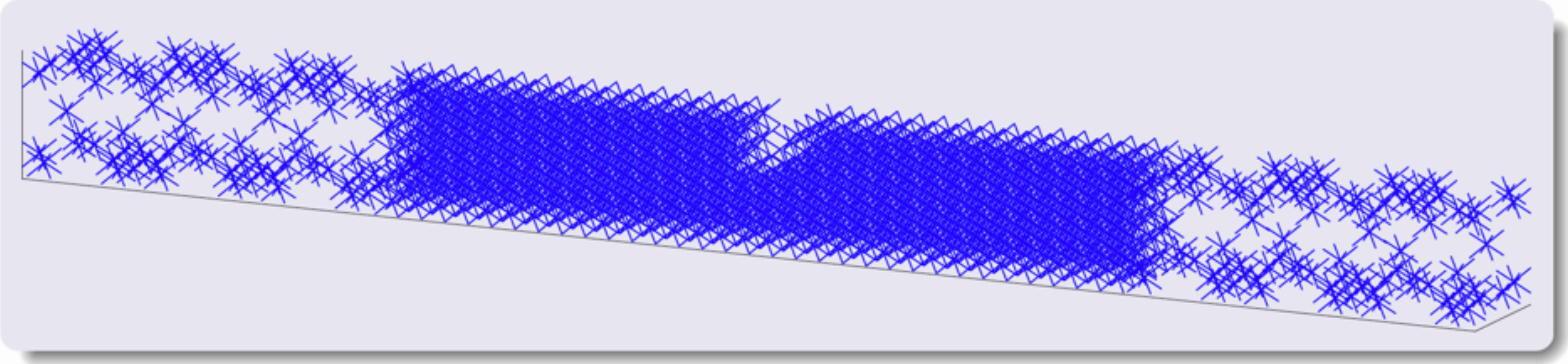
2nd interpolation scheme and summation scheme:



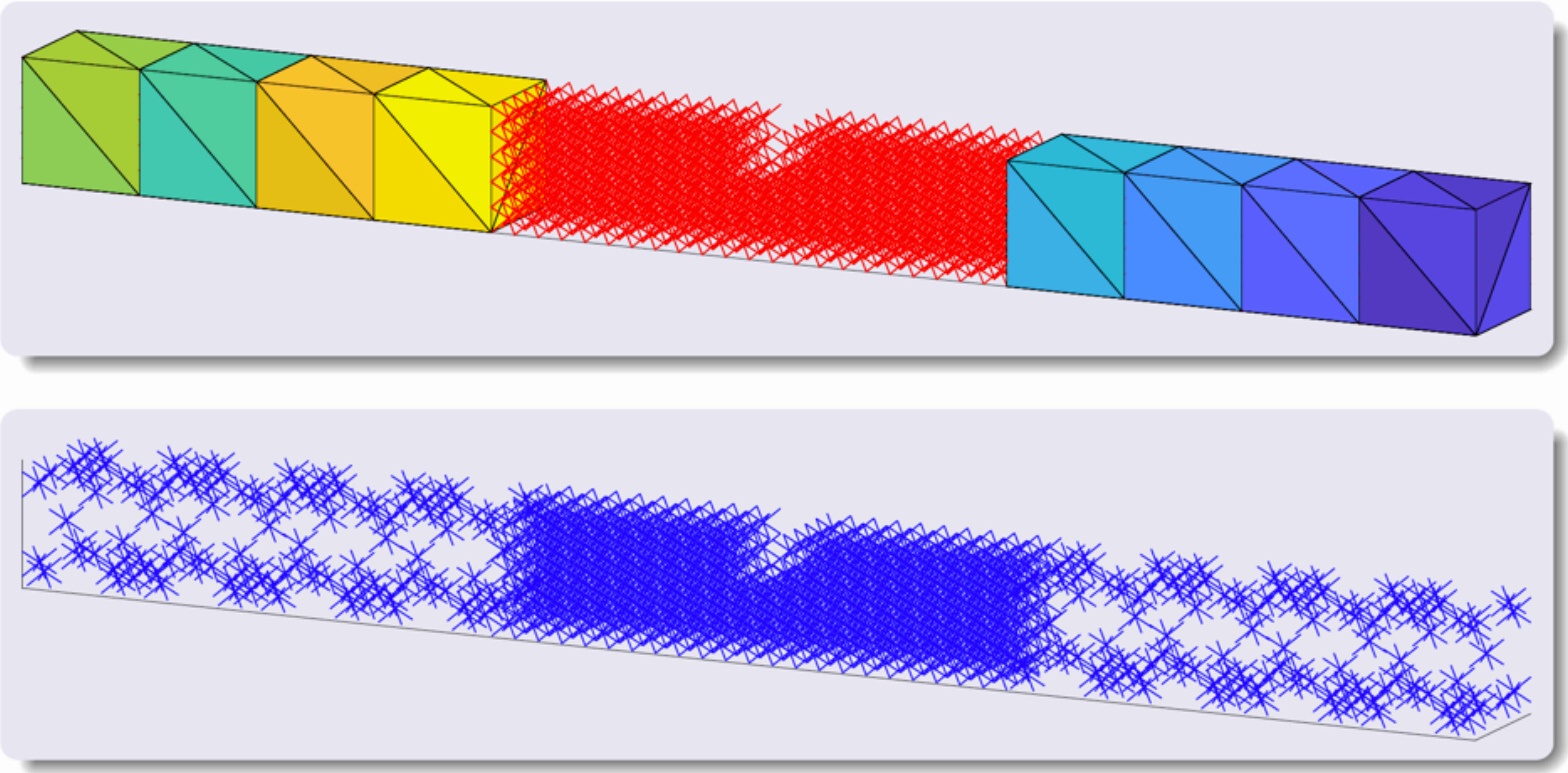


3rd interpolation scheme and summation scheme:

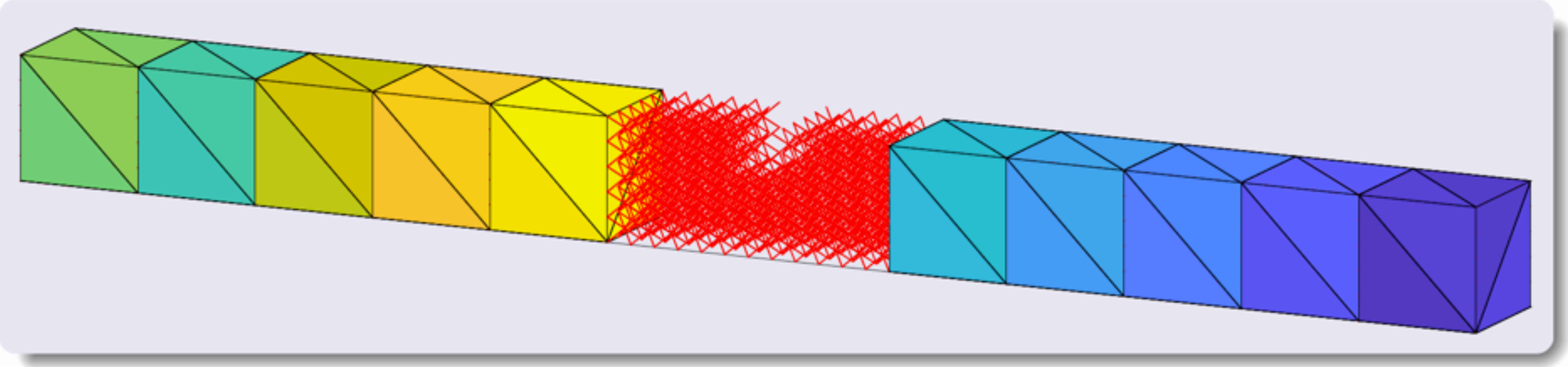


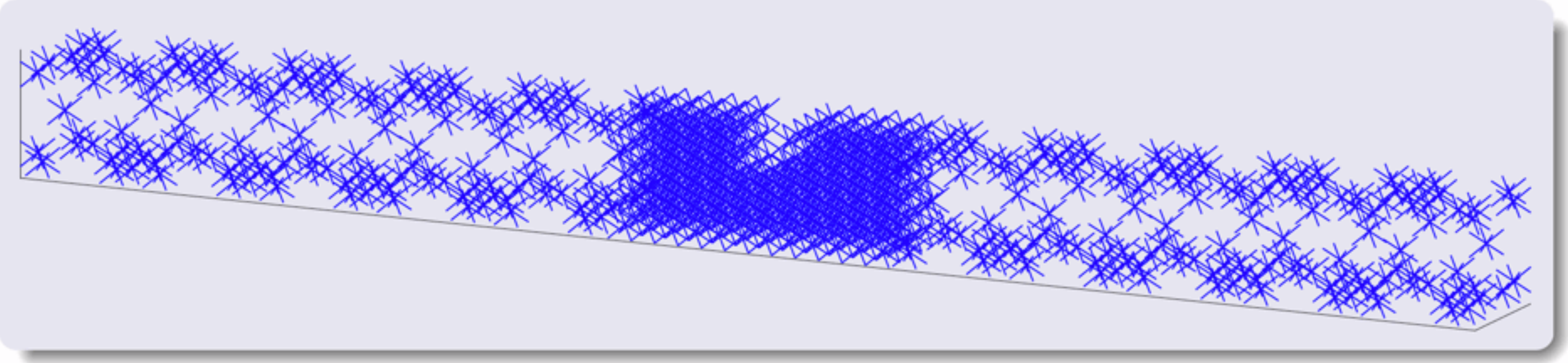


4th interpolation scheme and summation scheme:



5$^{\text{th}}$ interpolation scheme and summation scheme:





POD based hyper-reduction strategy

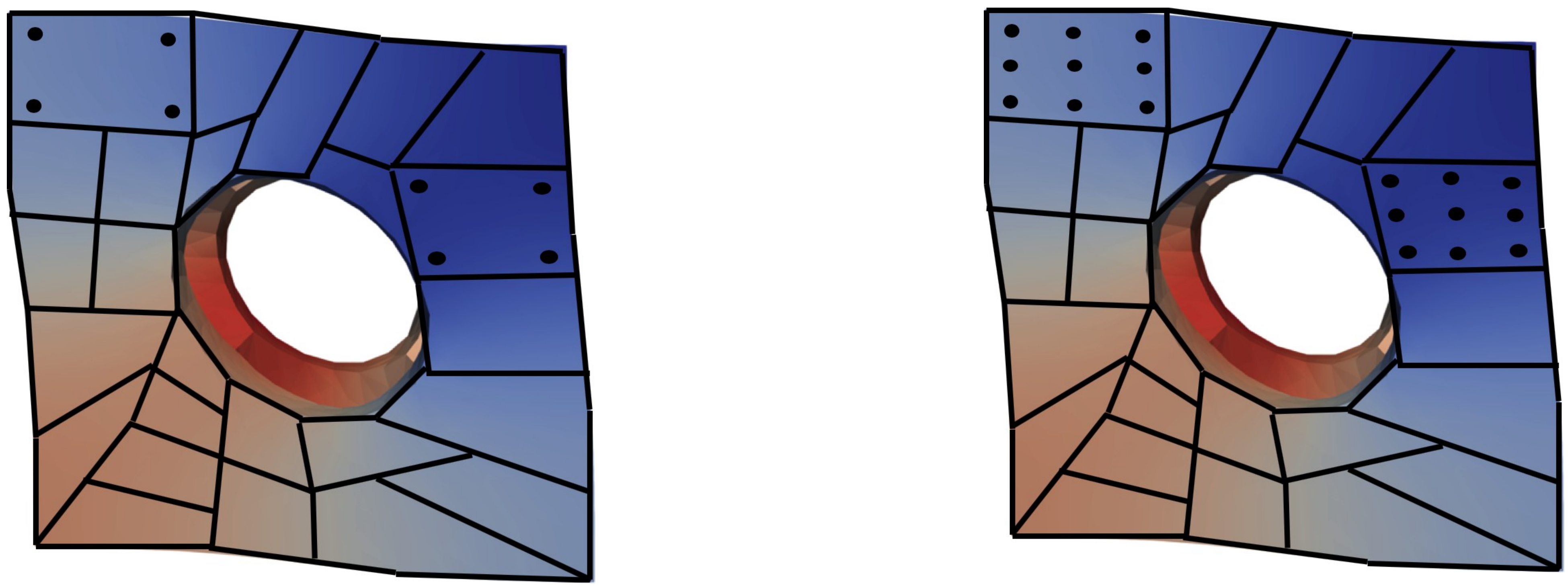
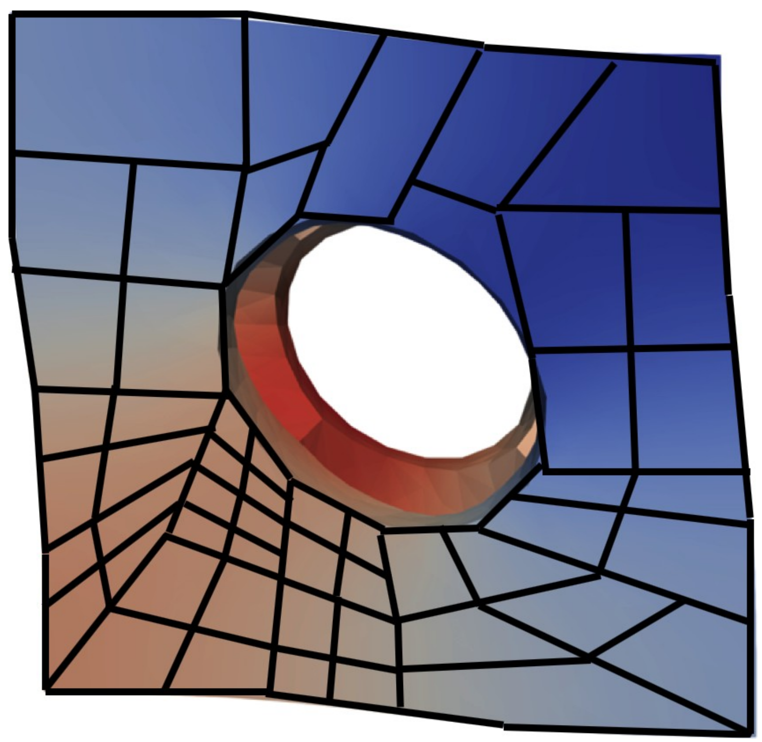
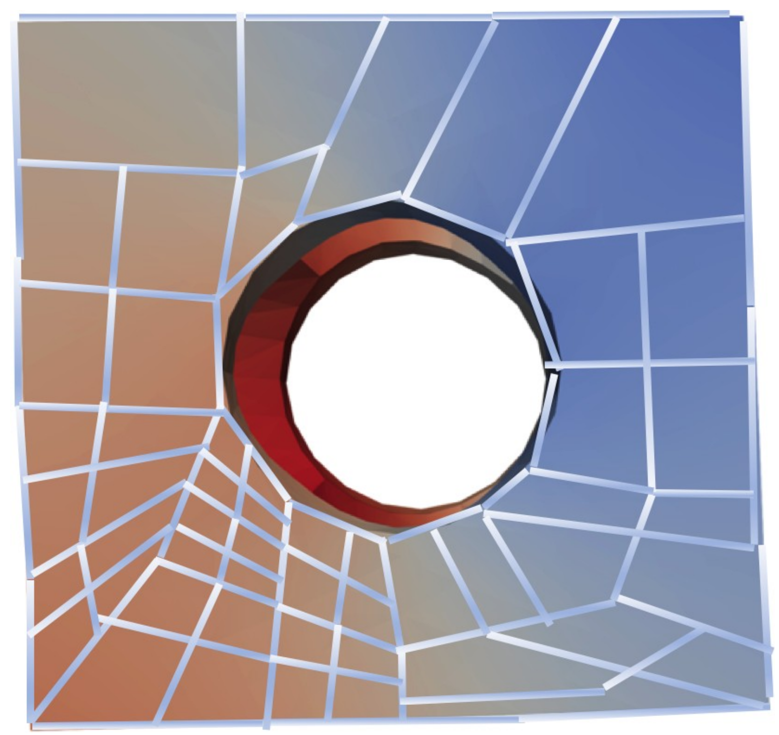


Figure: φ_1- Refined grid

POD based hyper-reduction strategy



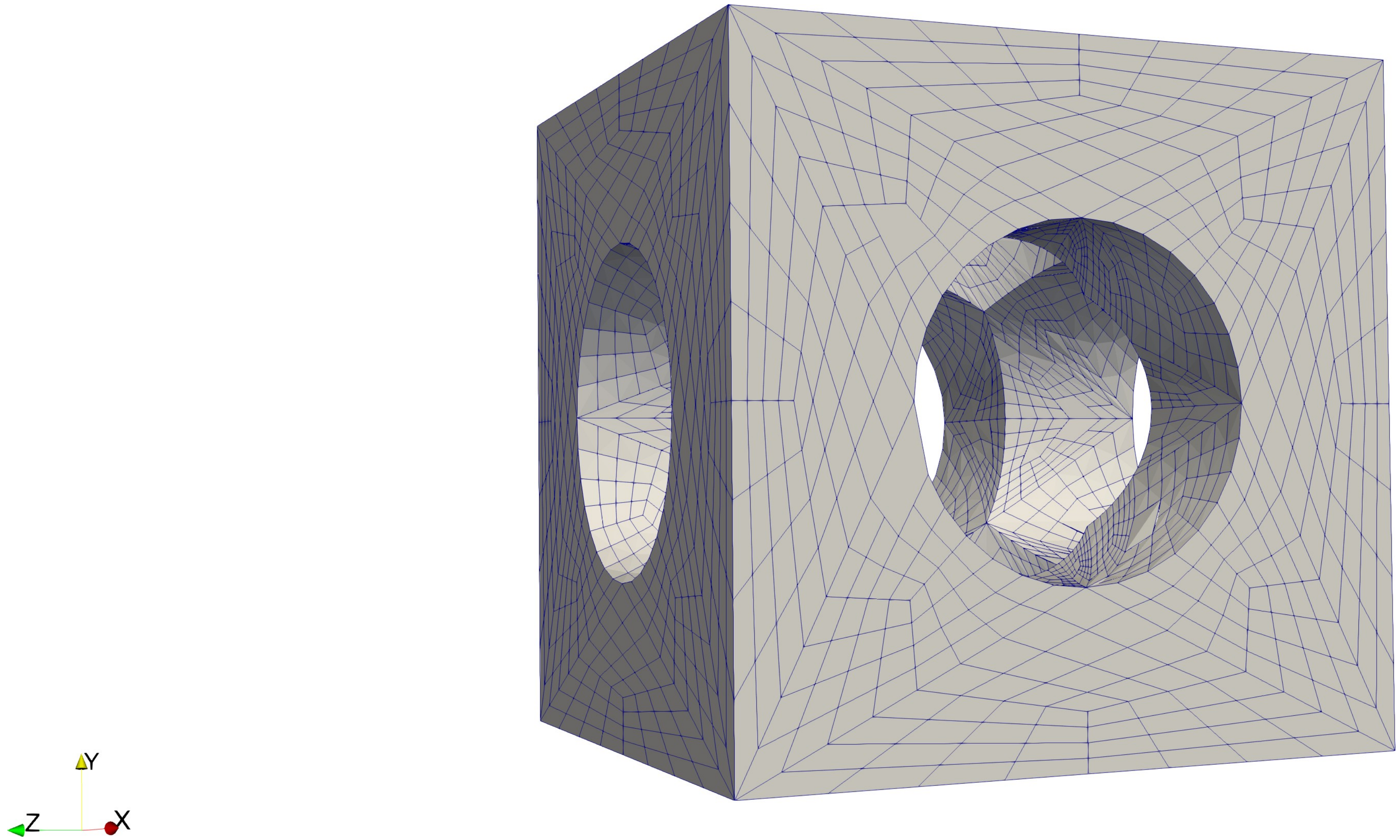
(a) φ_1- Final grid

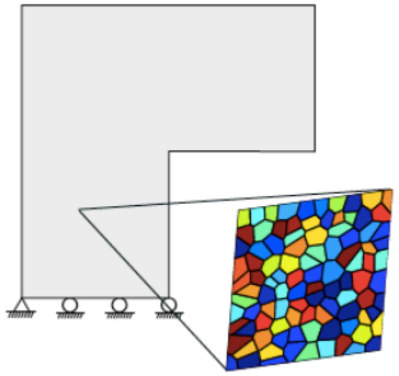


(b) φ_2- Initial grid

POD based hyper-reduction strategy

The final gird:

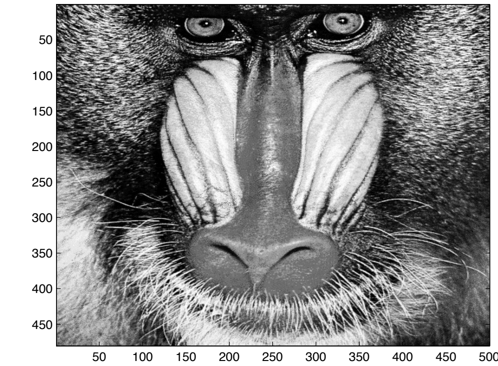




Challenges

Reduce the problem size

Preserve essential features



Reduce computational expense - Control the error

Physics based model reduction a.k.a. **Multiscale Methods**	**Algebraic based model reduction** a.k.a. **Machine Learning**
Representative volume elements do not exist after the onset of fracture	**The problem is not reducible in the fracture process zone**
Adaptive Multi-scale Methods: hierarchical - semi-concurrent - concurrent	**Adaptive Domain Decomposition Proper Orthogonal Decomposition**

WCCM2020 Symposium in Paris

14th World Congress on Computational Mechanics (WCCM XIV)
8th European Congress on Computational Methods in Applied Science and Engineering (ECCOMAS 2020)
July 19- 24, 2020, Paris, France

BENCHMARKING ADVANCED DISCRETISATION TECHNIQUES: PART I. MESH BURDEN ALLEVIATION WITH APPLICATIONS TO CAD-ANALYSIS TRANSITION, FRACTURE MECHANICS AND HIGHER-ORDER PDES

TRACK NUMBER 20

Elena Atroshchenko, Stéphane Bordas, Franz Chouly, Daniel Dias-Da-costa, Jakub Lengiewicz, Sundararajan Natarajan, Timon Rabczuk, Chongmin Song, Satyendra Tomar, Giulio Ventura, Eric Wyart

Key words: verification and validation, benchmarking, mesh-burden, IGA, XFEM, embedded discontinuities,

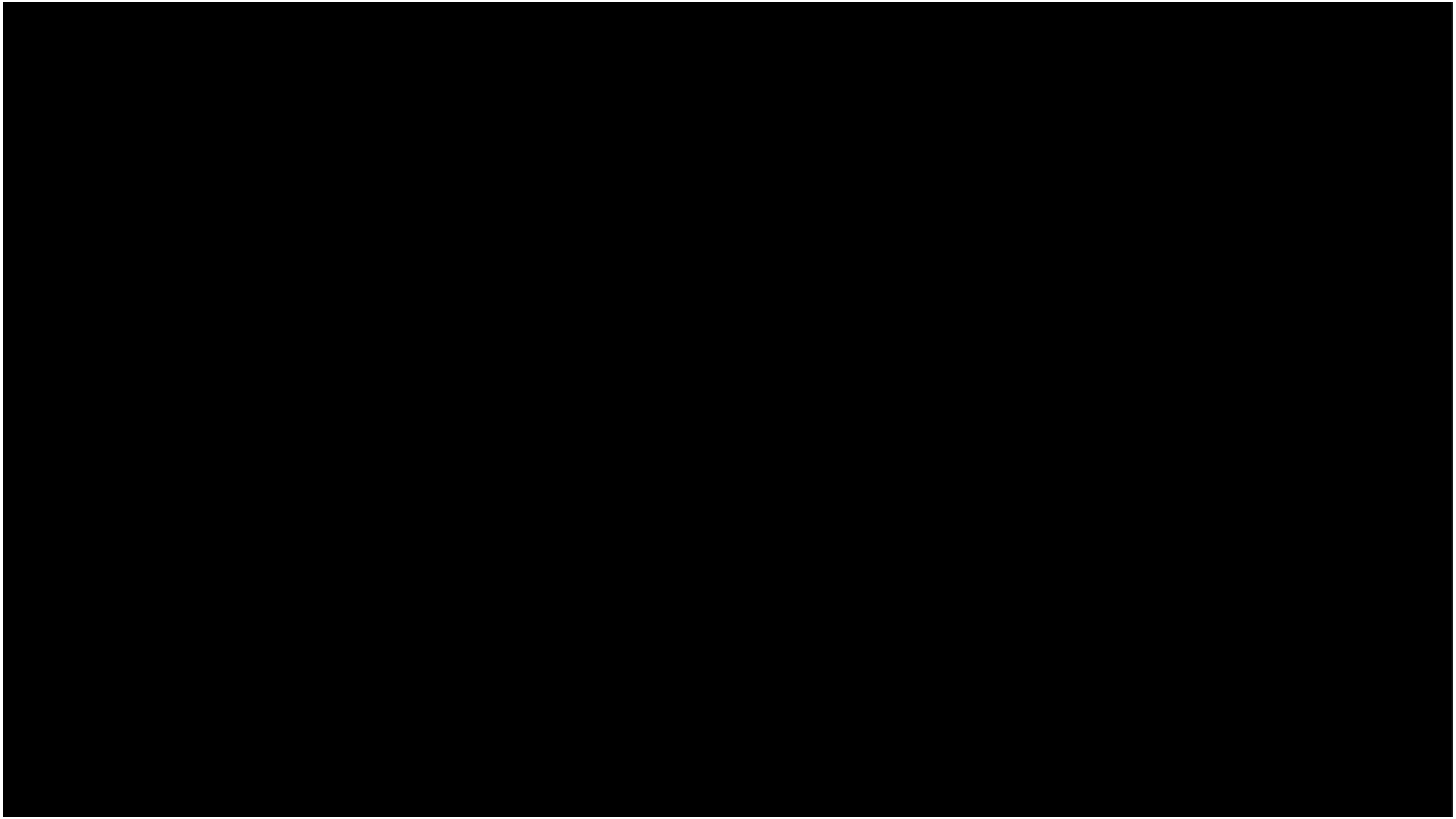
ABSTRACT

The last 50 years have seen the birth of a large number of "special" approximation methods aiming at complementing finite difference and finite element methods and alleviating their intrinsic difficulties. Major advances have been made, and yet, it is not always obvious to identify the most relevant advantages and drawbacks of a given approach.

This is the first of a series of symposia organised under the egis of ECCOMAS, IUTAM and EUROMECH. This series is organised by various groups involved in advanced discretisation techniques and aims at:

i) providing a set of benchmark problems and associated protocols for computational mechanics problems;
ii) providing a forum for long-term discussions around the theme of advanced discretisation methods; and
iii) unifying different groups of thought in the field of advanced discretisation methods.

In this first symposium, focus will be given to fracture approximation and mesh burden alleviation,



http://hdl.handle.net/10993/39442

Legato-team

University of Luxembourg

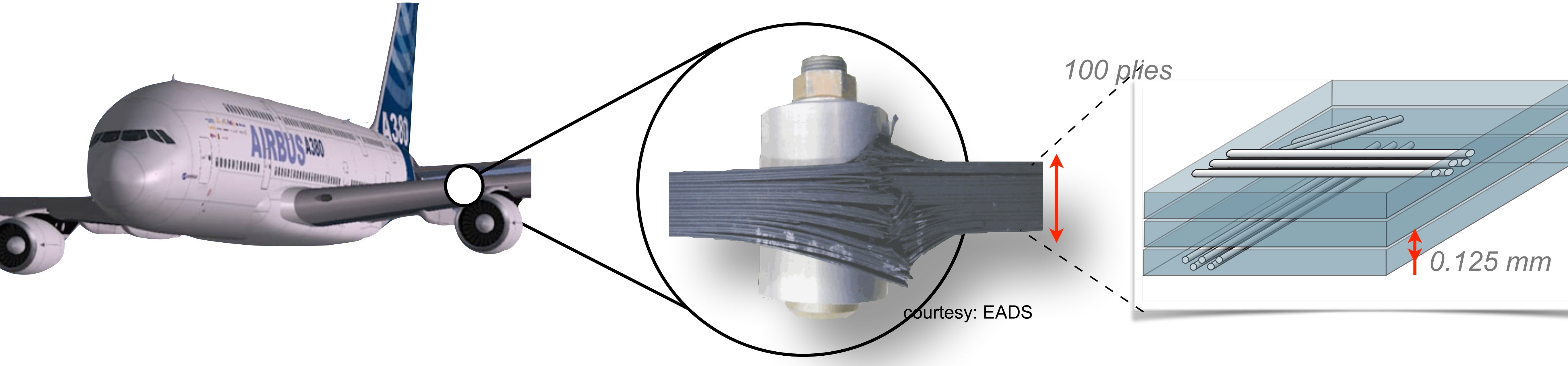
Department of Computational
Engineering Sciences
University of Luxembourg

Open problems

- how to define the reduced area?
- precomputation time (offline)

Future?

Material complexity

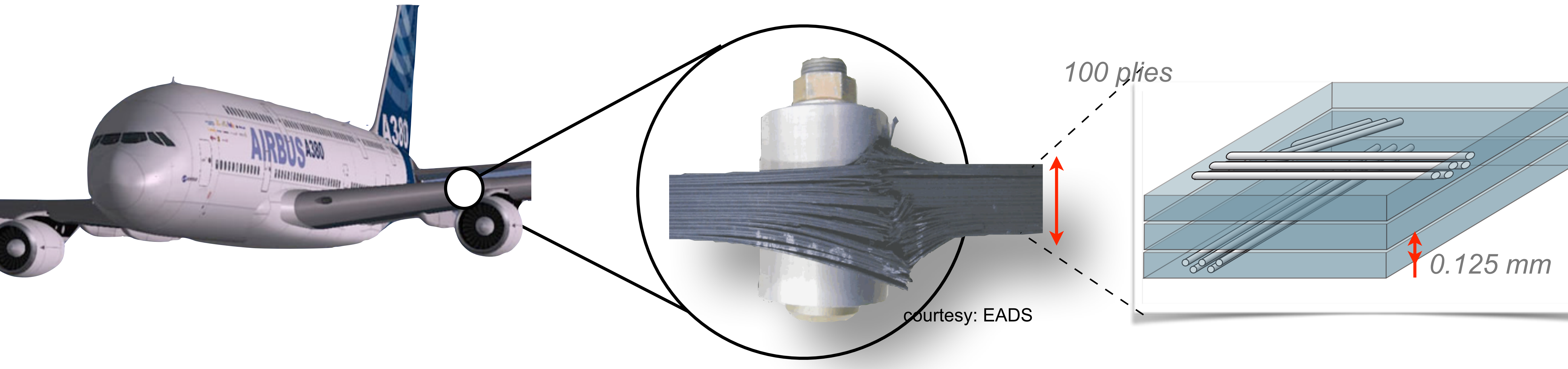


Heterogeneous & multi-functional materials

Can we optimise the material microstructure given macroscopic objective functions

Experiments required to attain sufficient confidence in their behavior are increasingly costly

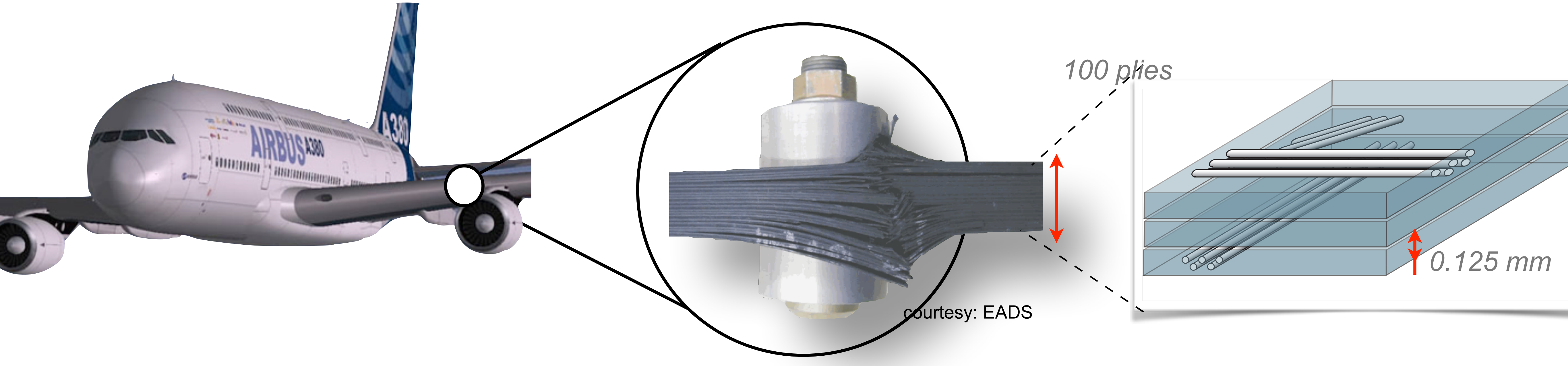
Material complexity



Factor-of-Safety or probabilistic based methods cannot handle unknown unknowns

Lack of similitude between testing (experimental) and operating conditions — also encountered in geophysics, medicine...

Challenges



- Move **away from heuristics** and experience-based engineering
- Develop **fundamental understanding** of physical processes (degradation, ...)

Digital twin concept

Actual aircraft

Digital aircraft model

Life prediction and extension

Situation awareness

High fidelity modeling and simulation

Certification and design methods

Requires real-time data assimilation, and model update…

Parallel with medicine

Mechanics

Macro (wing) - Micro (carbon fibres)

Environmental effects (Temperature, irradiation…)

Experimental condition dissimilarities

Medicine

Macro (Body, Physiology) to micro (microbes, needle/scalpel…)

Patient's environment, living conditions, habits…

Organ properties depend strongly on age, gender, …

Medicine

The average drug developed by a major pharmaceutical company costs at least <u>$4 billion</u>, and it can be as much as <u>$11 billion</u>.

Mechanics

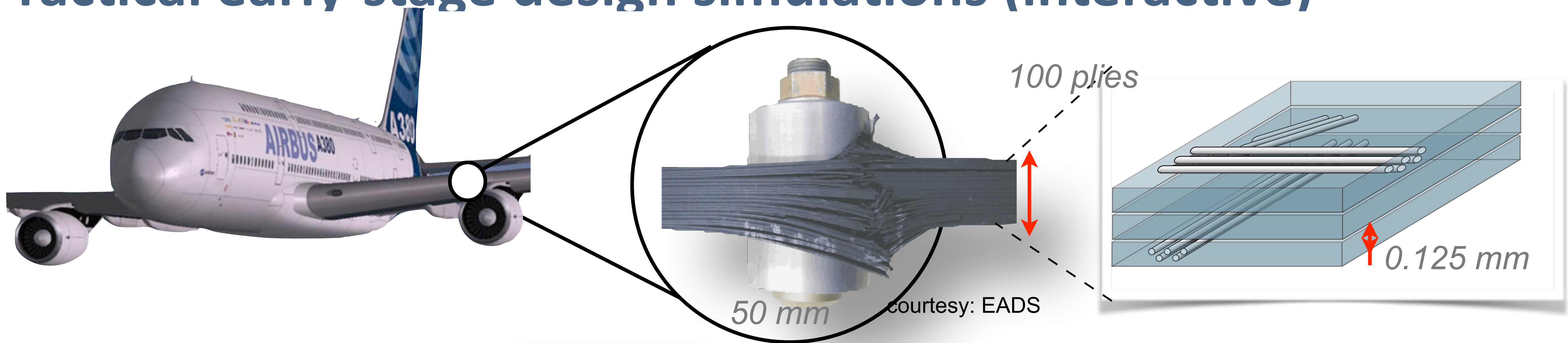
The development cost of the A380

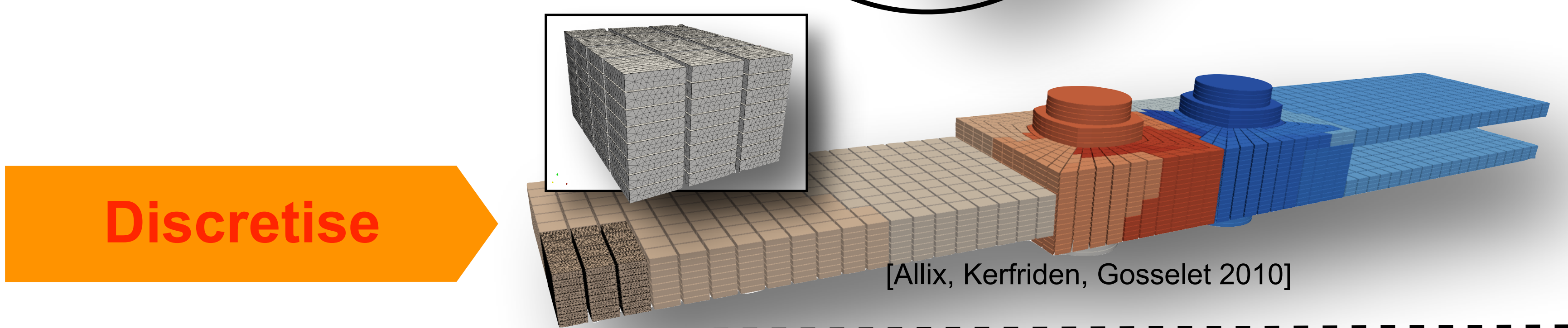
<u>11 billion</u> euros…

of the dreamliner…
<u>$32 billion</u>

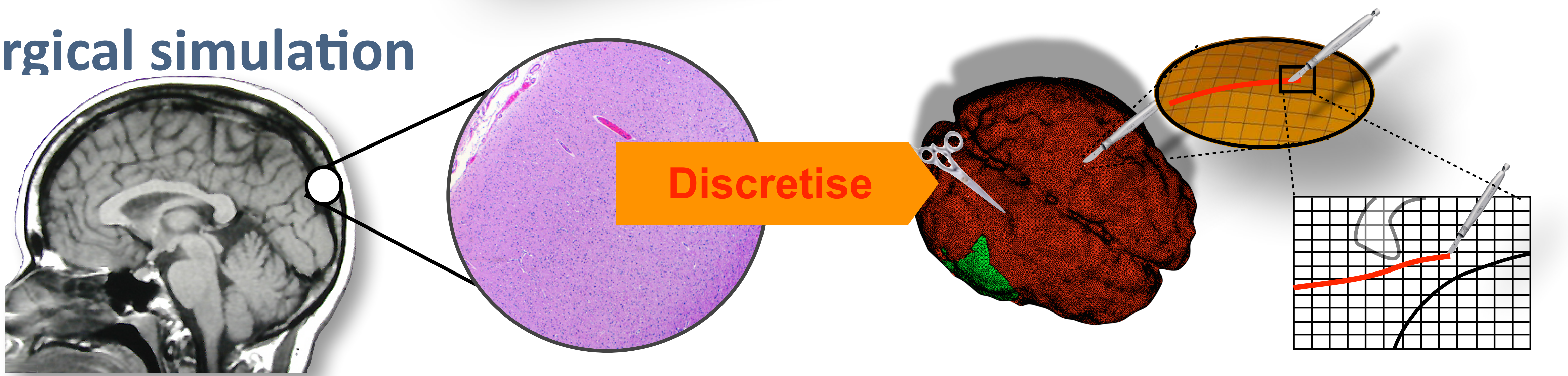
Patient/plane-specific simulation

Practical early-stage design simulations (interactive)





Surgical simulation

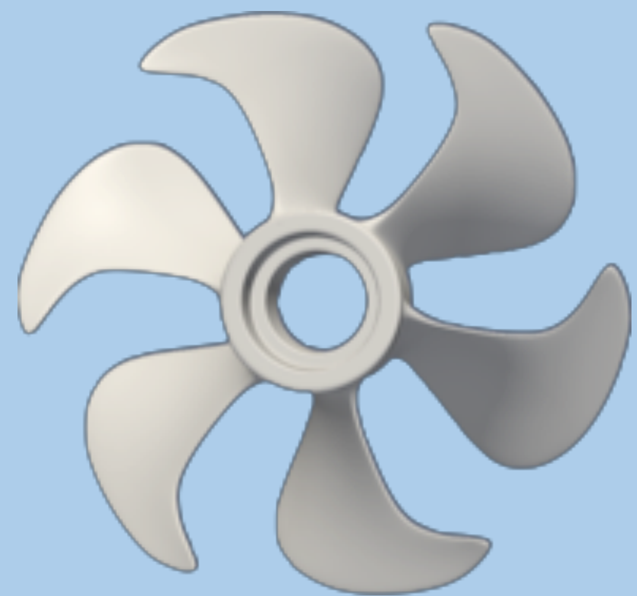


- Reduce the problem size while controlling the error (in QoI) when solving very large (multiscale) mechanics problems

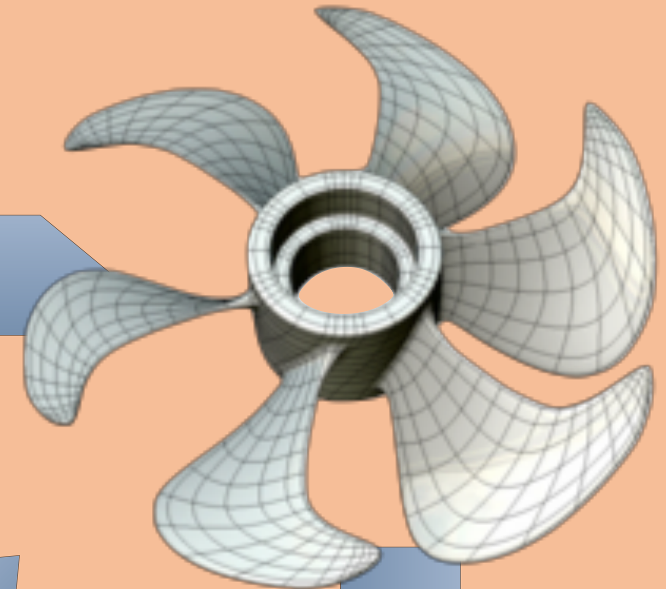
thanks for your attention

Partners and Funding

GEOMETRY



DISCRETISATION



Verification

A POSTERIORI ERROR

MATERIAL MODELS

Phenomenological

Elasticity/Plasticity

Crack growth law (Paris…)

Fracture energy

Maximum tensile strength

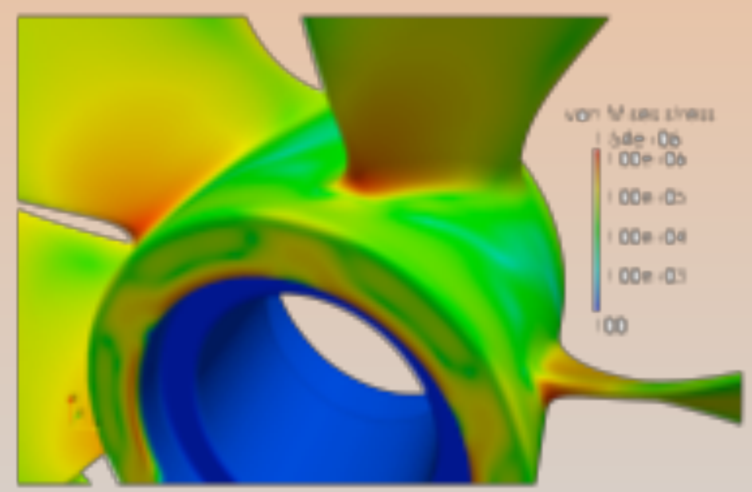
Multi-scale

Debonding, Fibre pull-out

Fibre breakage, interface

fracture, grains, dislocations,

NUMERICAL SOLUTION



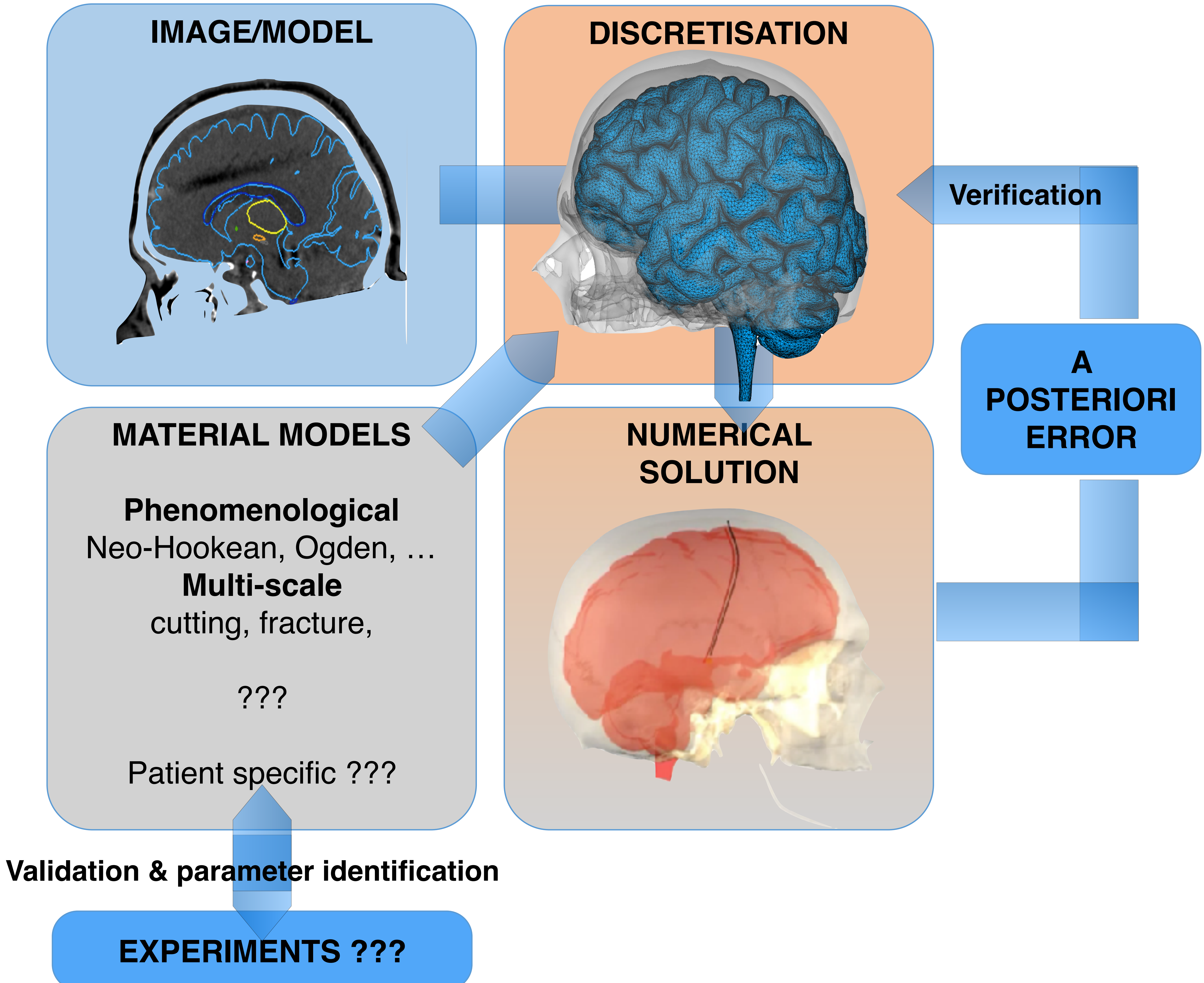
Validation & parameter identification

EXPERIMENTS

CONVENTIONAL APPROACH

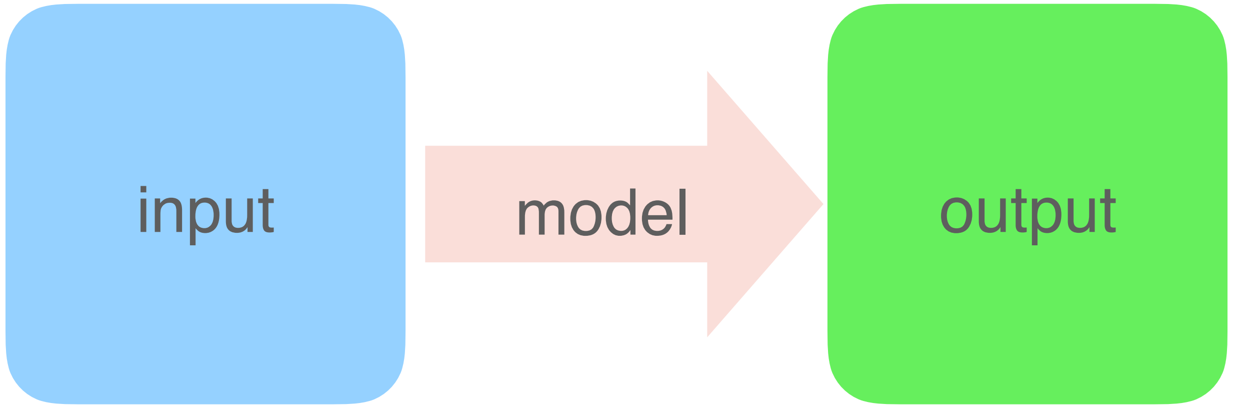
erc

RealTCut



IMAGE/MODEL
DISCRETISATION
Verification
A POSTERIORI ERROR
MATERIAL MODELS
Phenomenological
Neo-Hookean, Ogden, …
Multi-scale
cutting, fracture,
???
Patient specific ???
NUMERICAL SOLUTION
Validation & parameter identification
EXPERIMENTS ???

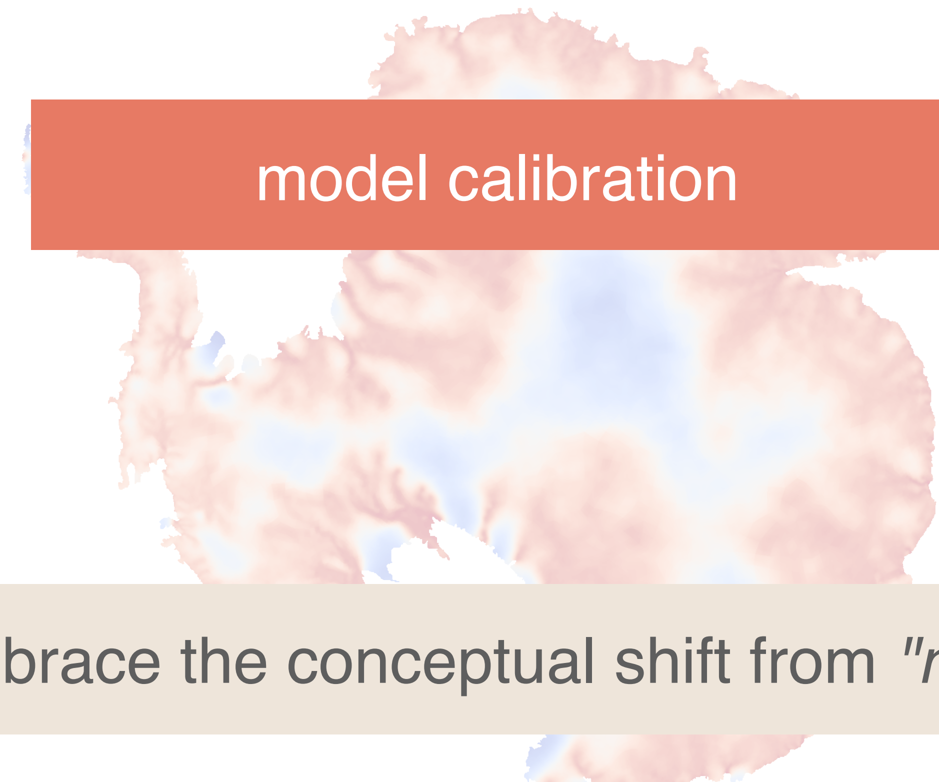
Data-driven Modelling



$$f : \mathbf{x} \rightarrow \mathbf{y}$$

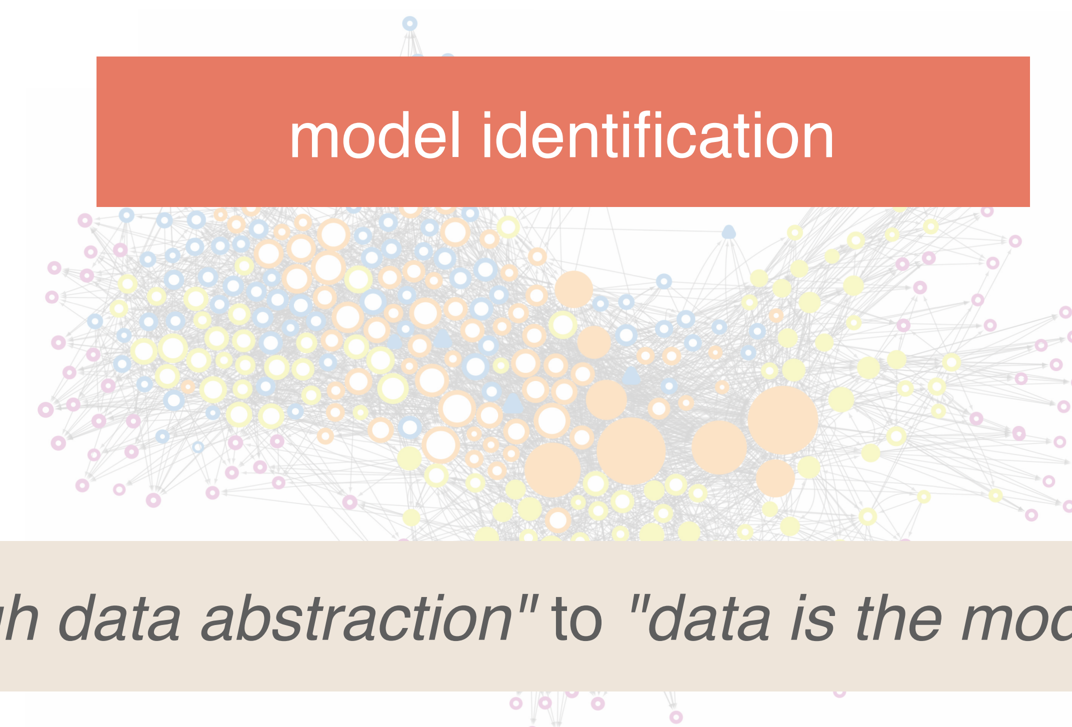
The structure of *f* is known but its parameters are not.

model calibration



There is no a priori knowledge about the function *f* available.

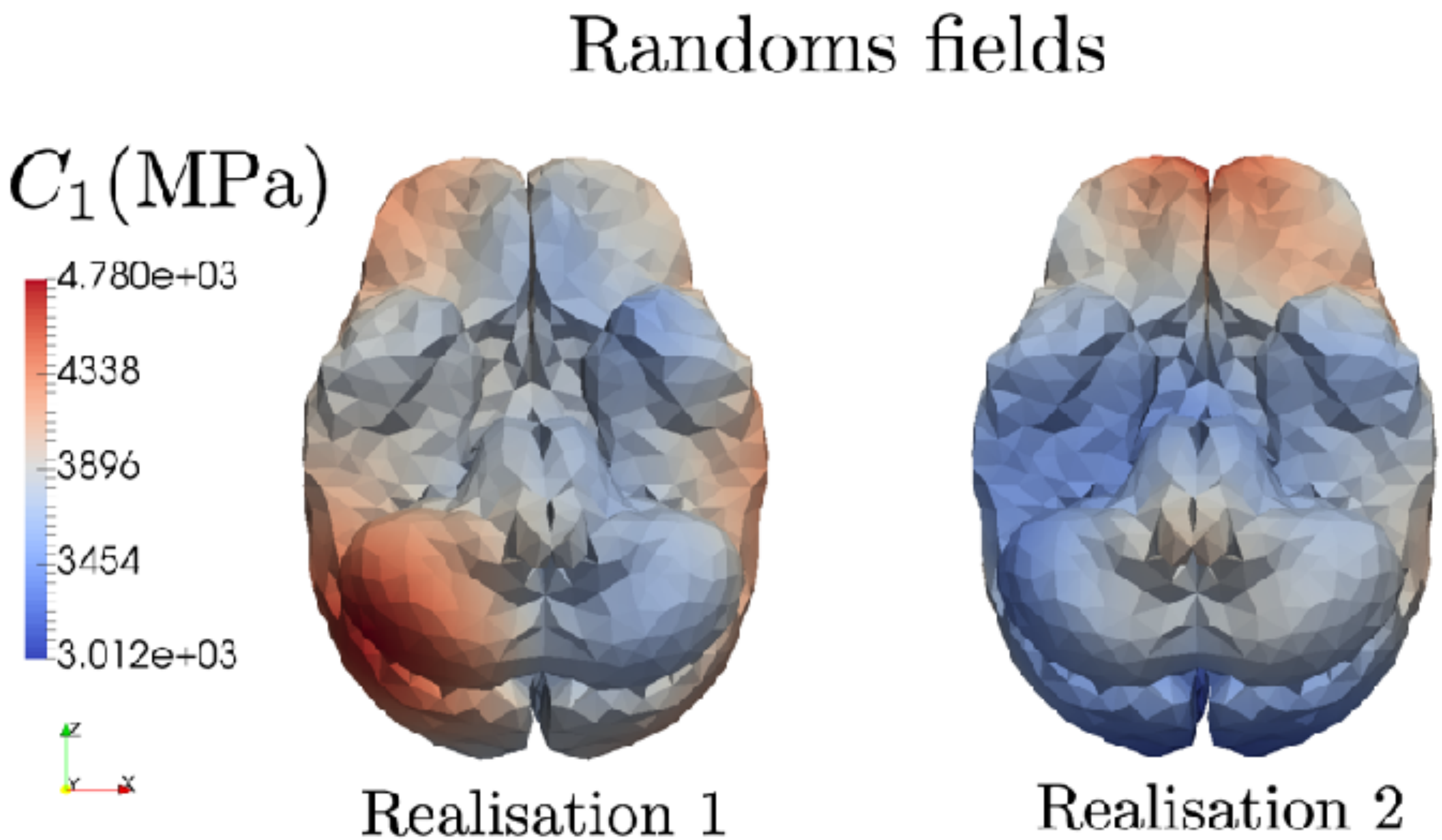
model identification



Embrace the conceptual shift from *"model through data abstraction"* to *"data is the model"*.

Assuming the material model is representative, what is the influence of each parameter in the model?

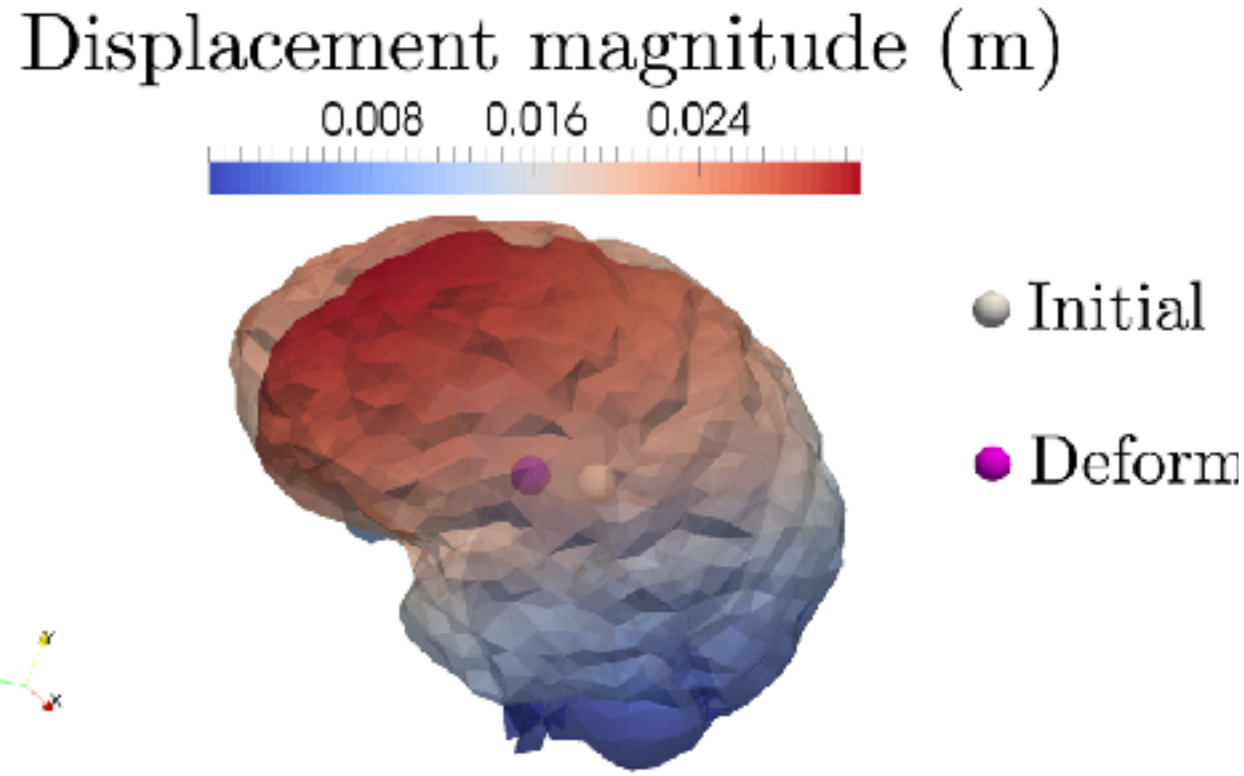
- Different methods: Karhunen–Loève expansion [Adler 2007], Fast Fourier transform [Nowak 2004].

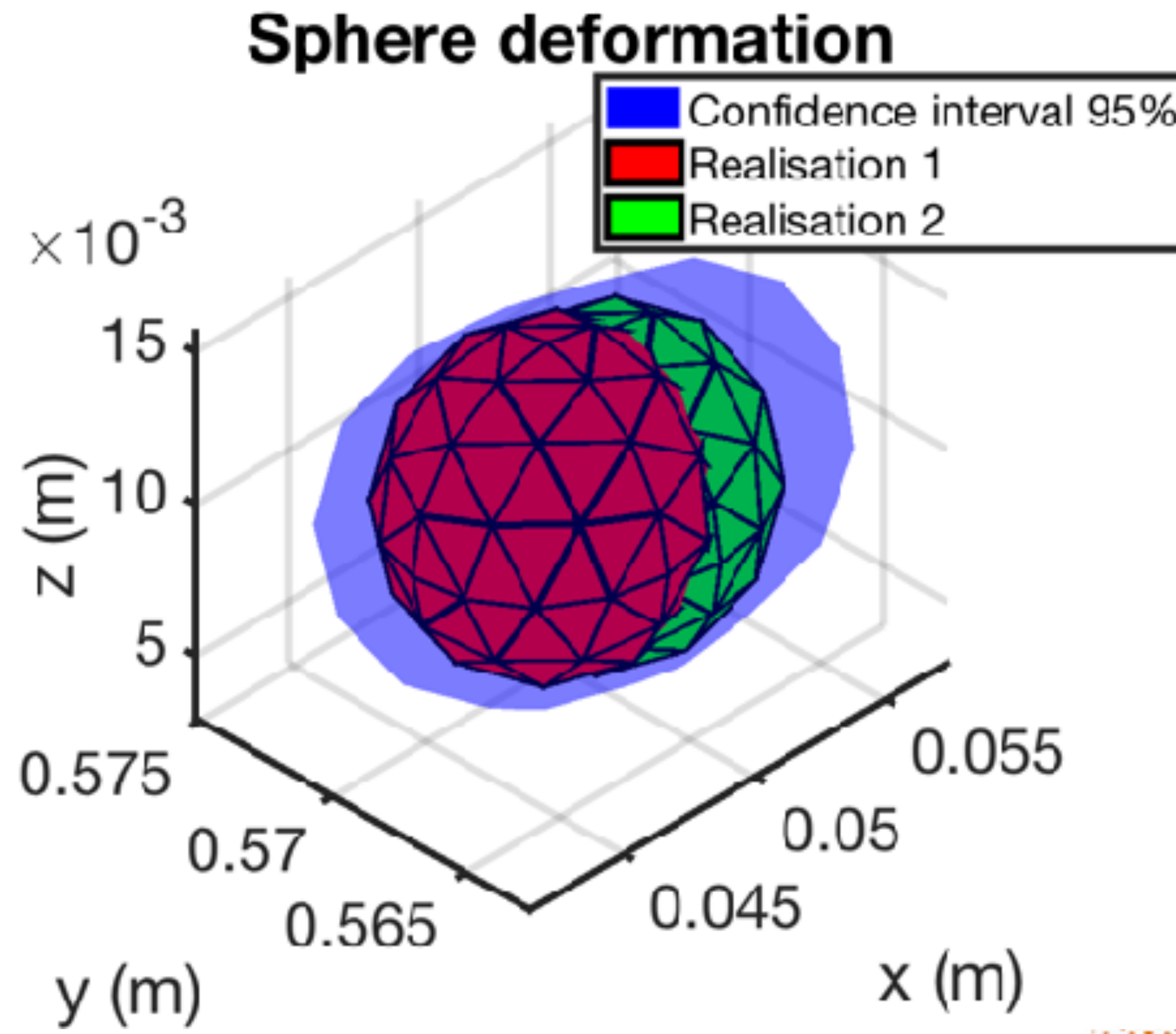


Two realisations of RF, with a log-normal distribution, for the parameter C_1 (in MPa).

Stéphane Pierre Alain BORDAS, Department of Computational Engineering & Sciences University of Luxembourg
RealTCut

Confidence level in predicting the target location

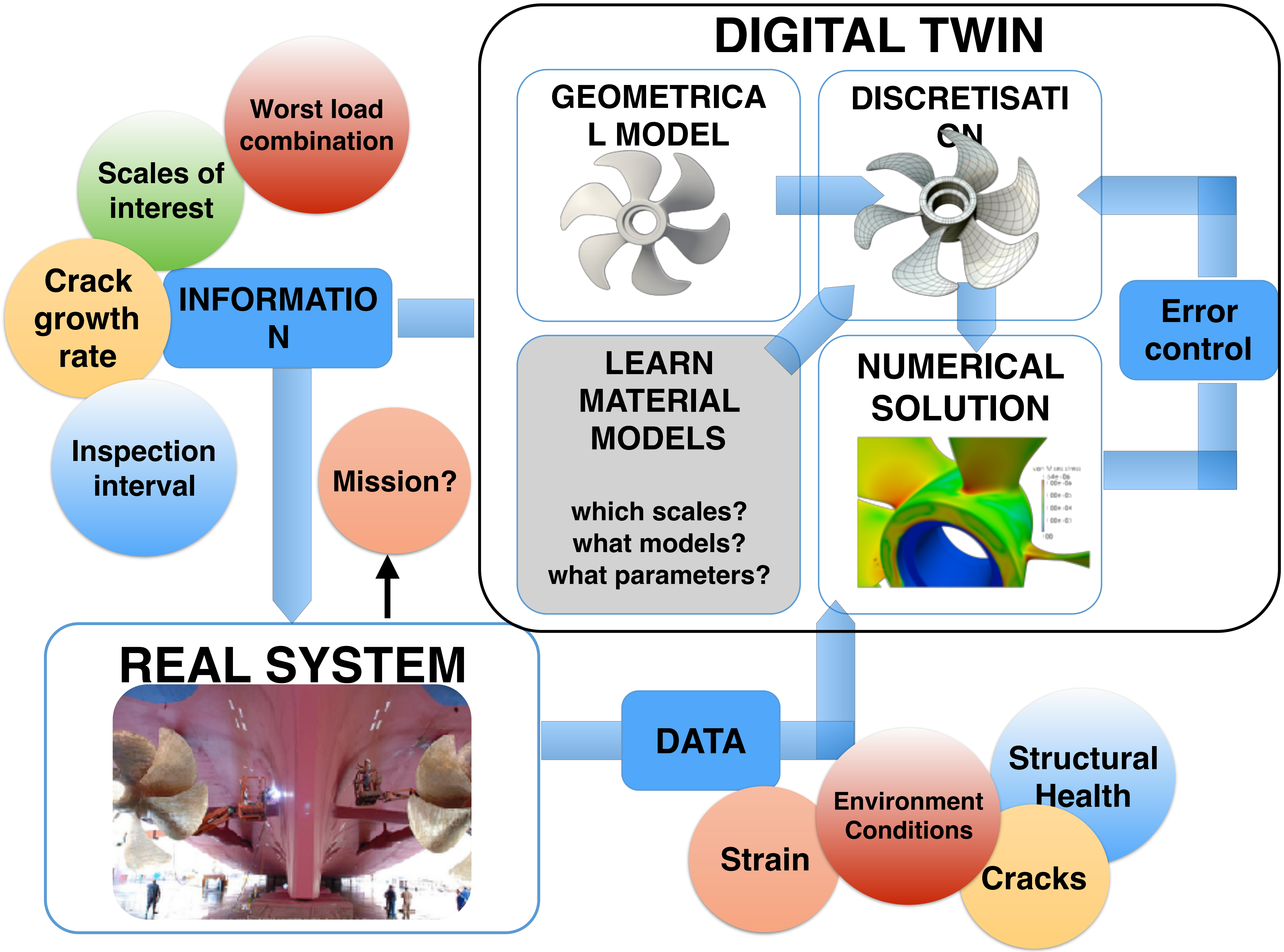


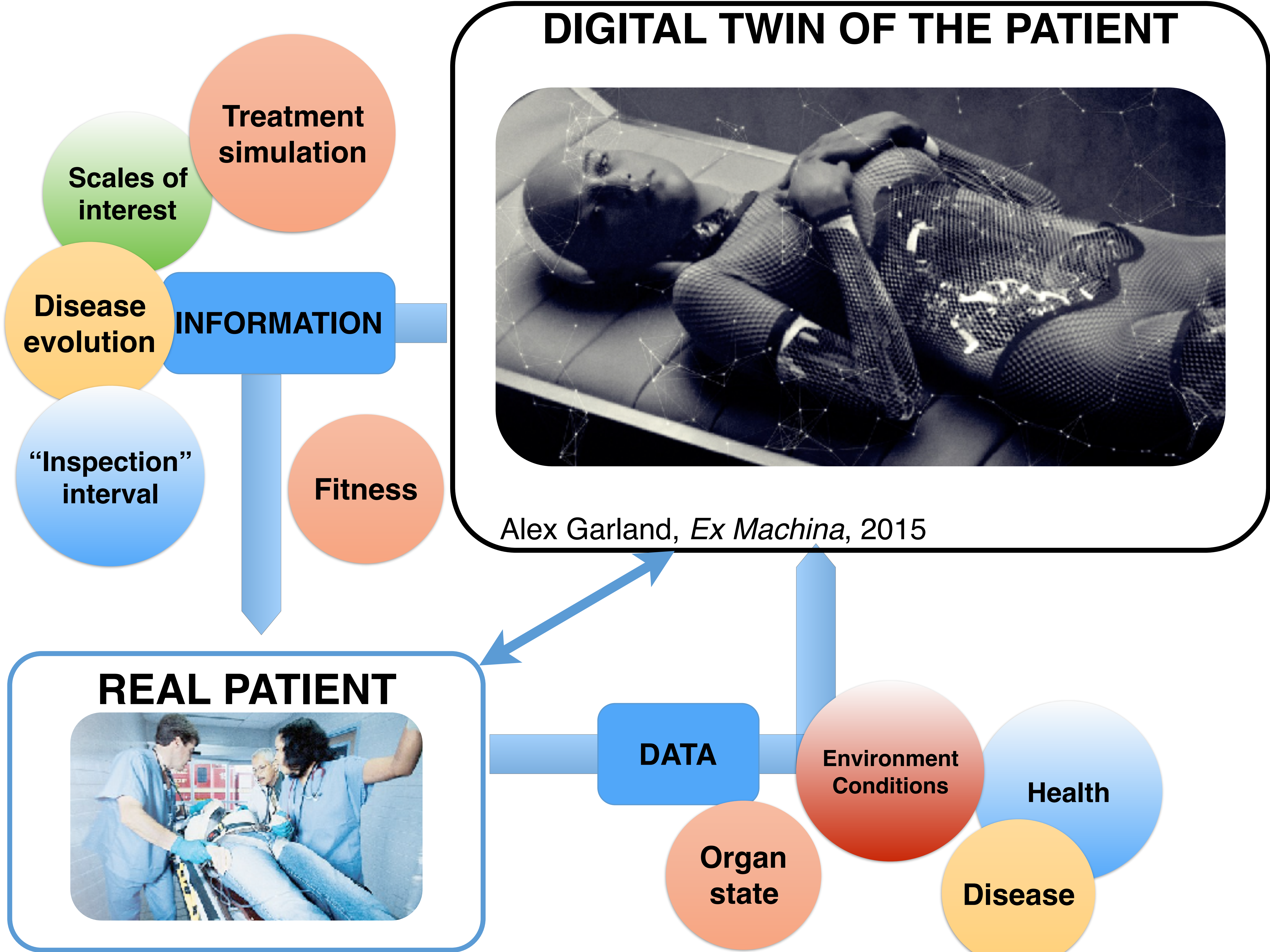


What is the influence of material parameters on computed quantities of interest?

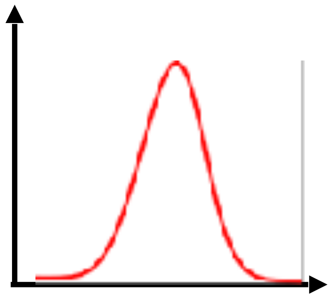
RealTCut

Possible approach

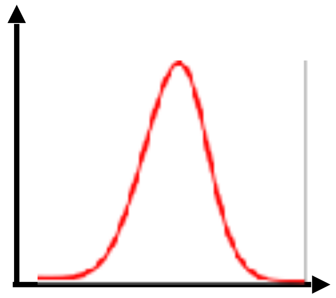



DIGITAL TWIN OF THE PATIENT
Alex Garland, Ex Machina, 2015
Treatment simulation
Scales of interest
Disease evolution
INFORMATION
"Inspection" interval
Fitness
REAL PATIENT
DATA
Environment Conditions
Health
Organ state
Disease

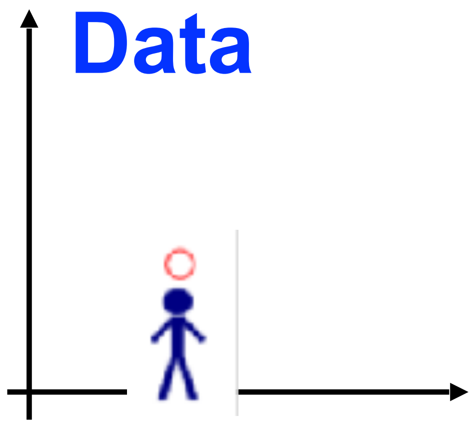
Prior
Knowledge



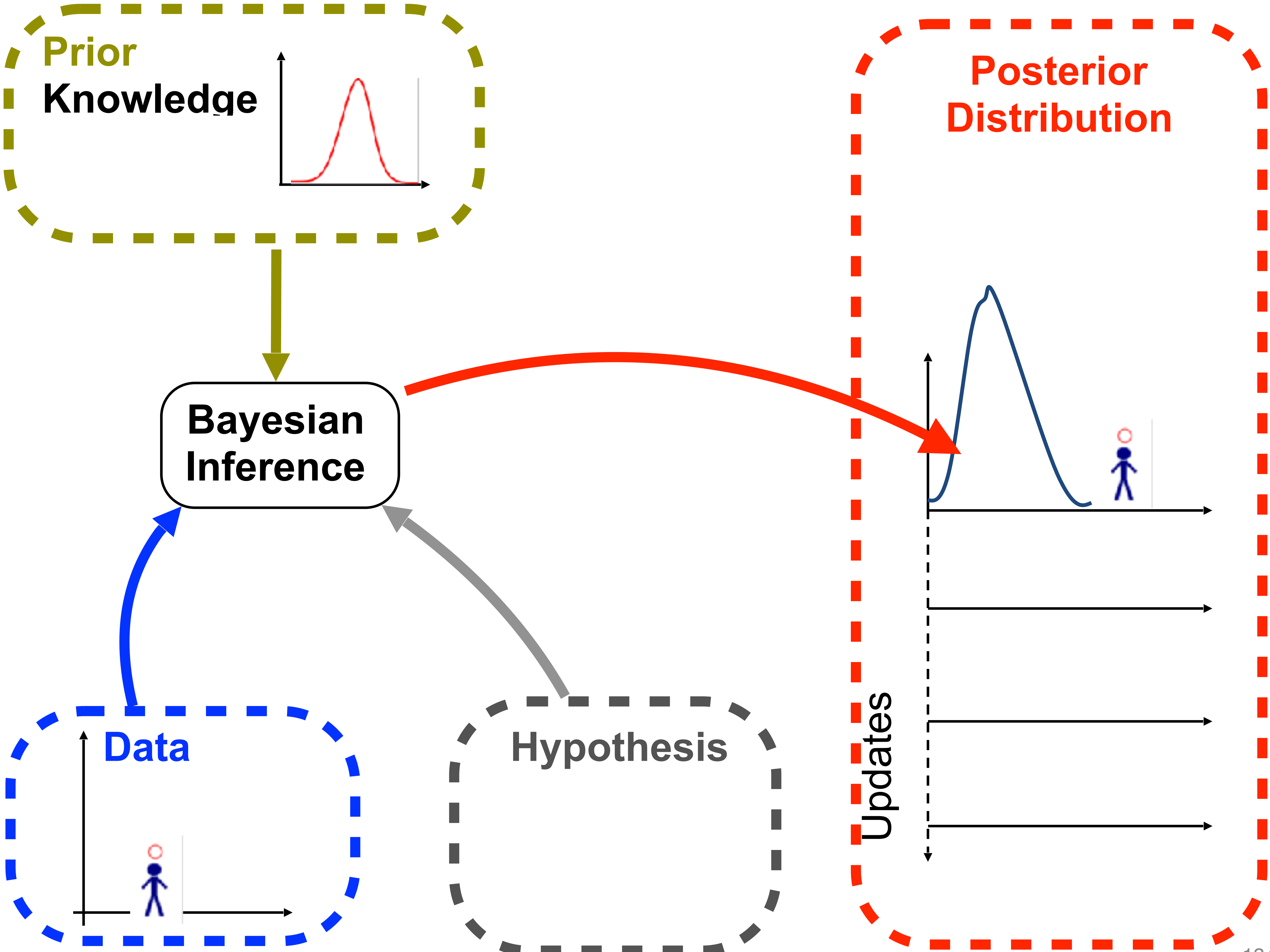
Prior Knowledge



Data



Hypothesis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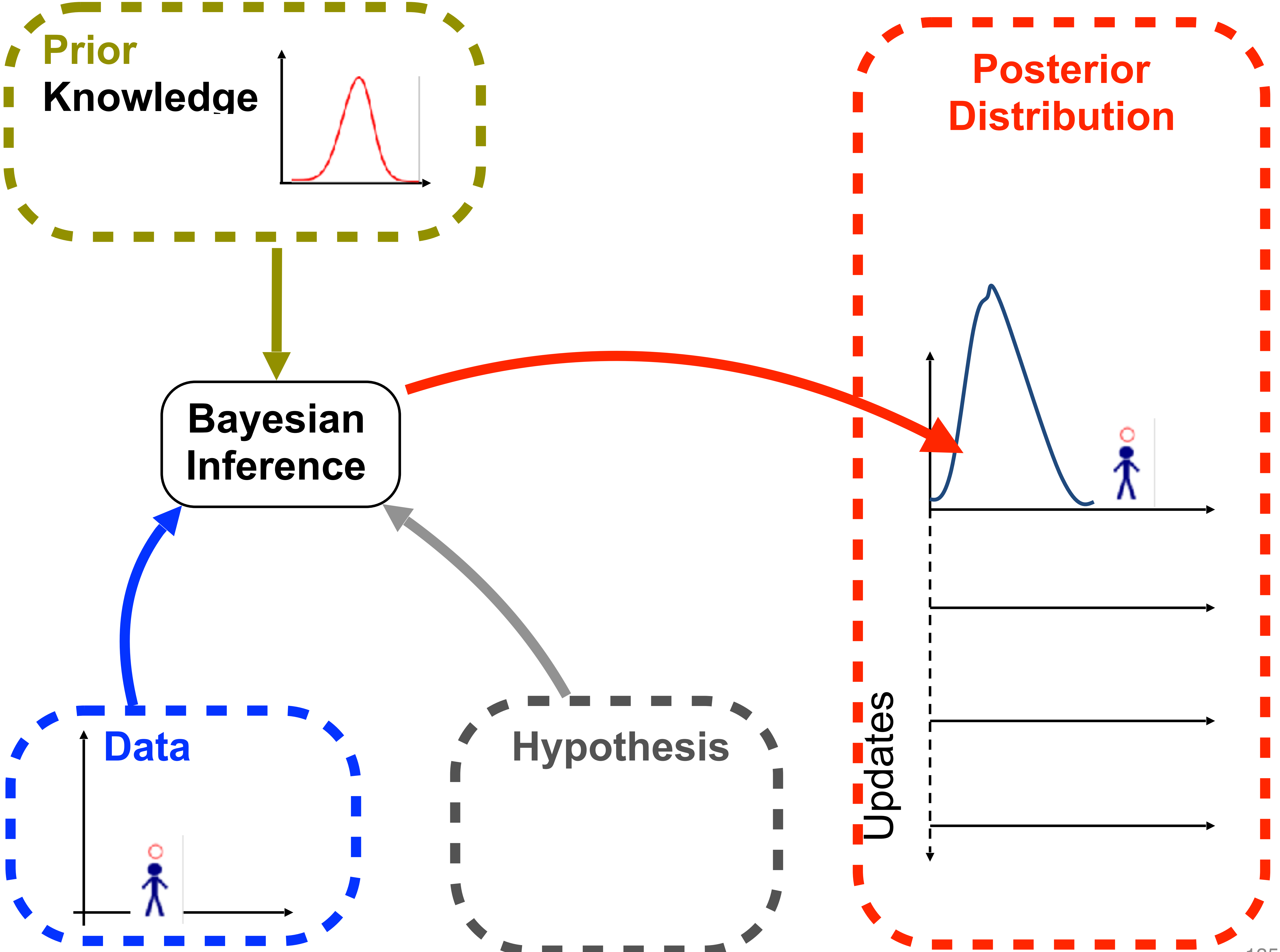
Knowledge
Bayesian
Inference
Posterior
Distribution
Updates
Data
Hypothesis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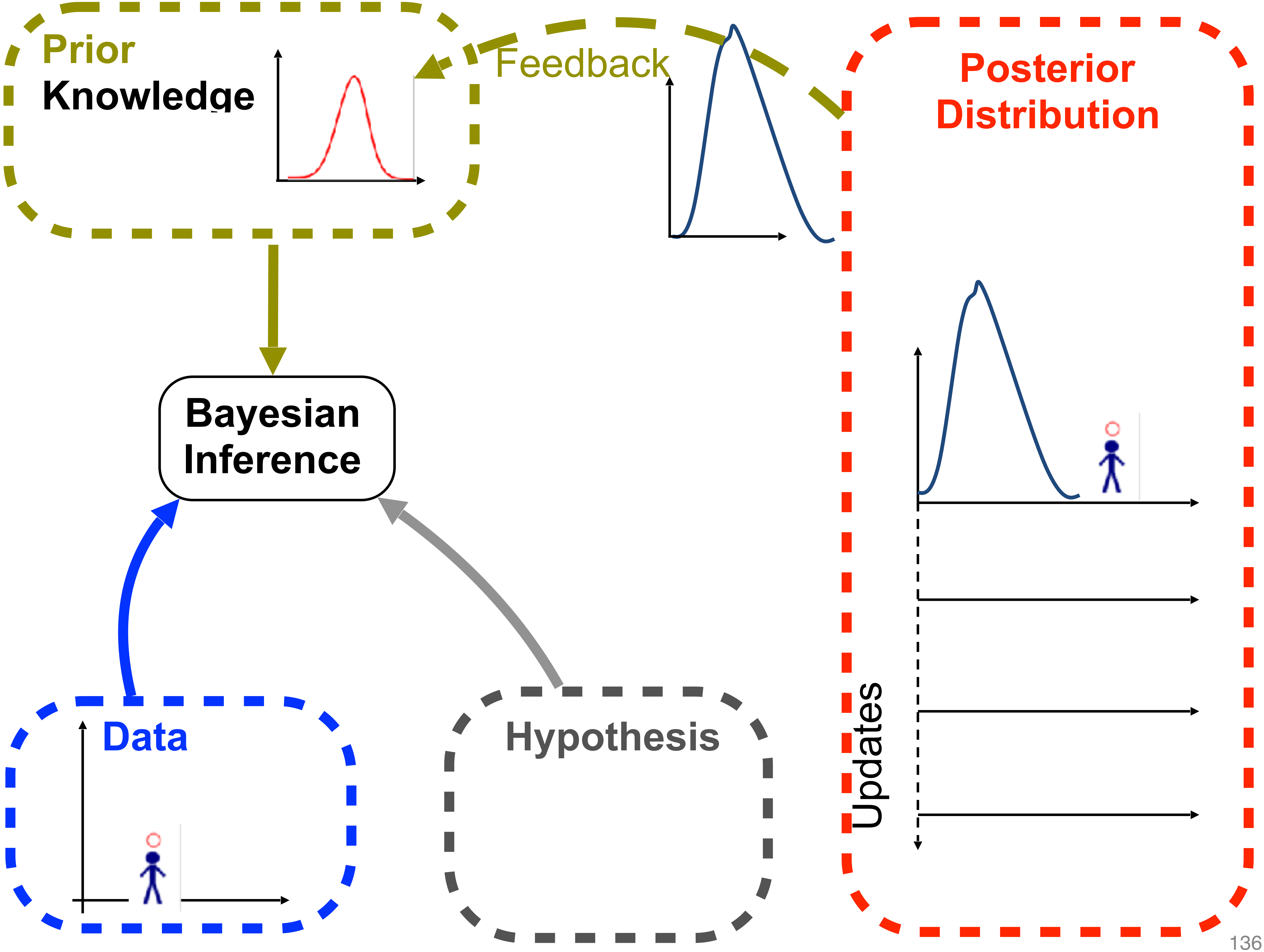
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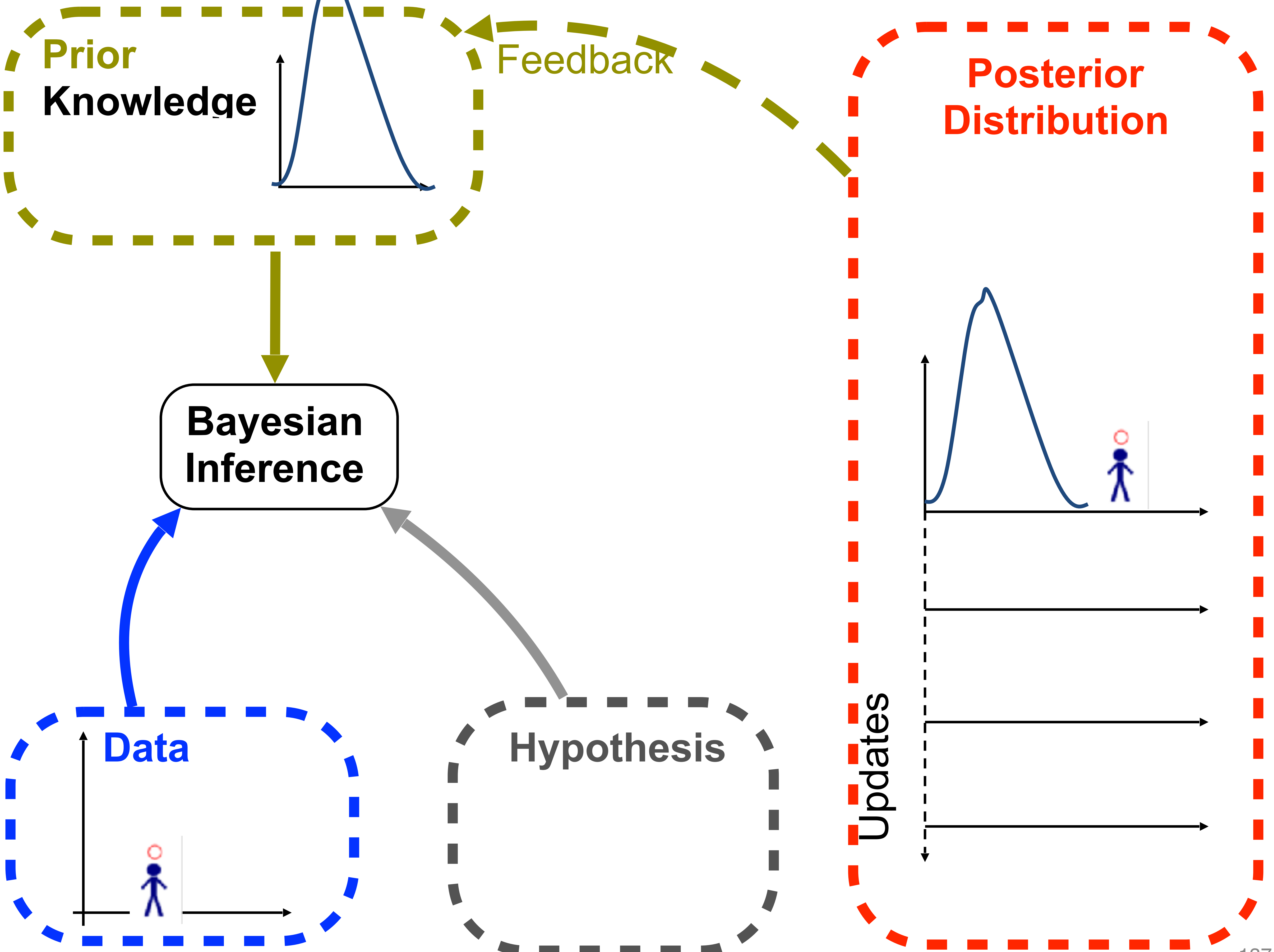
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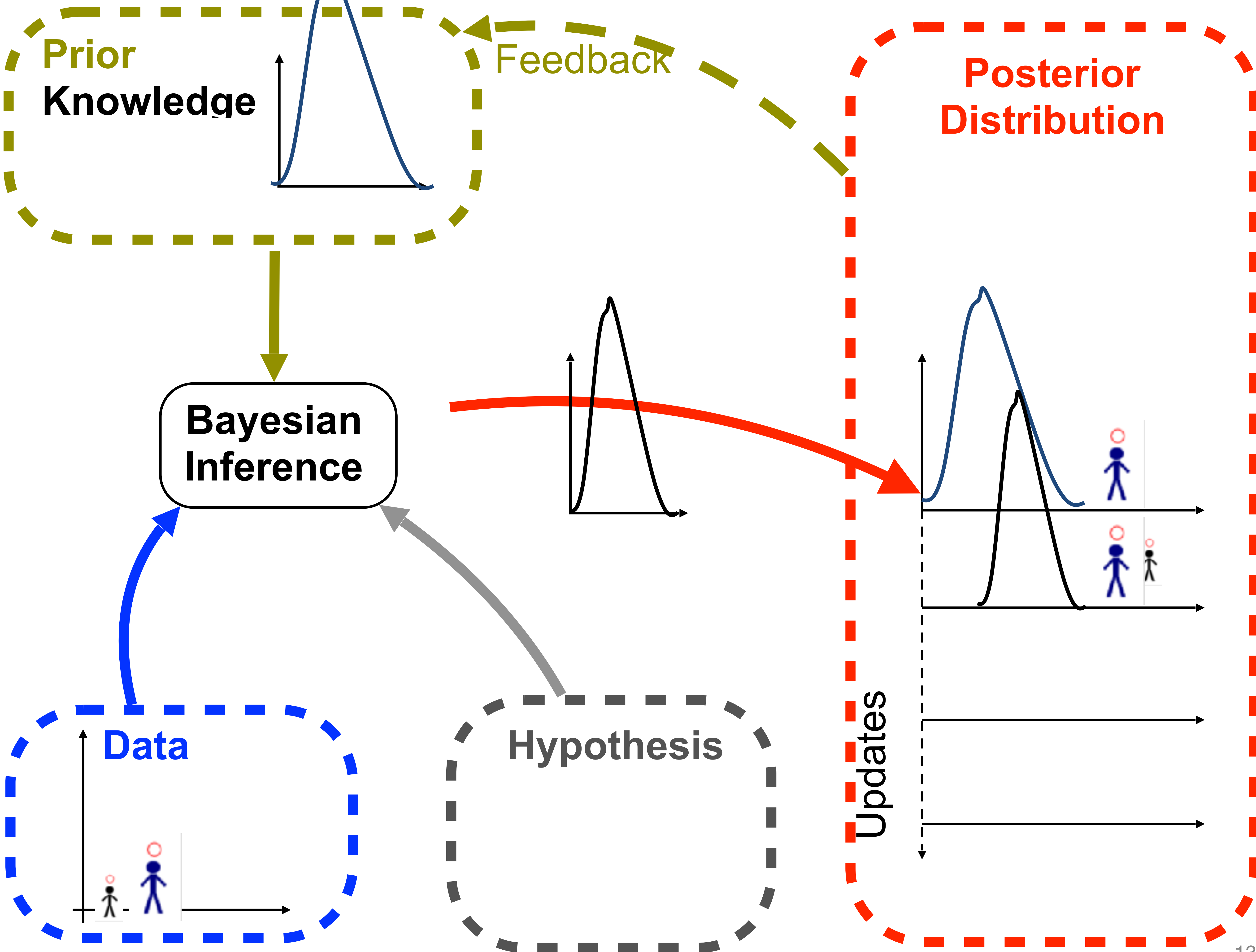
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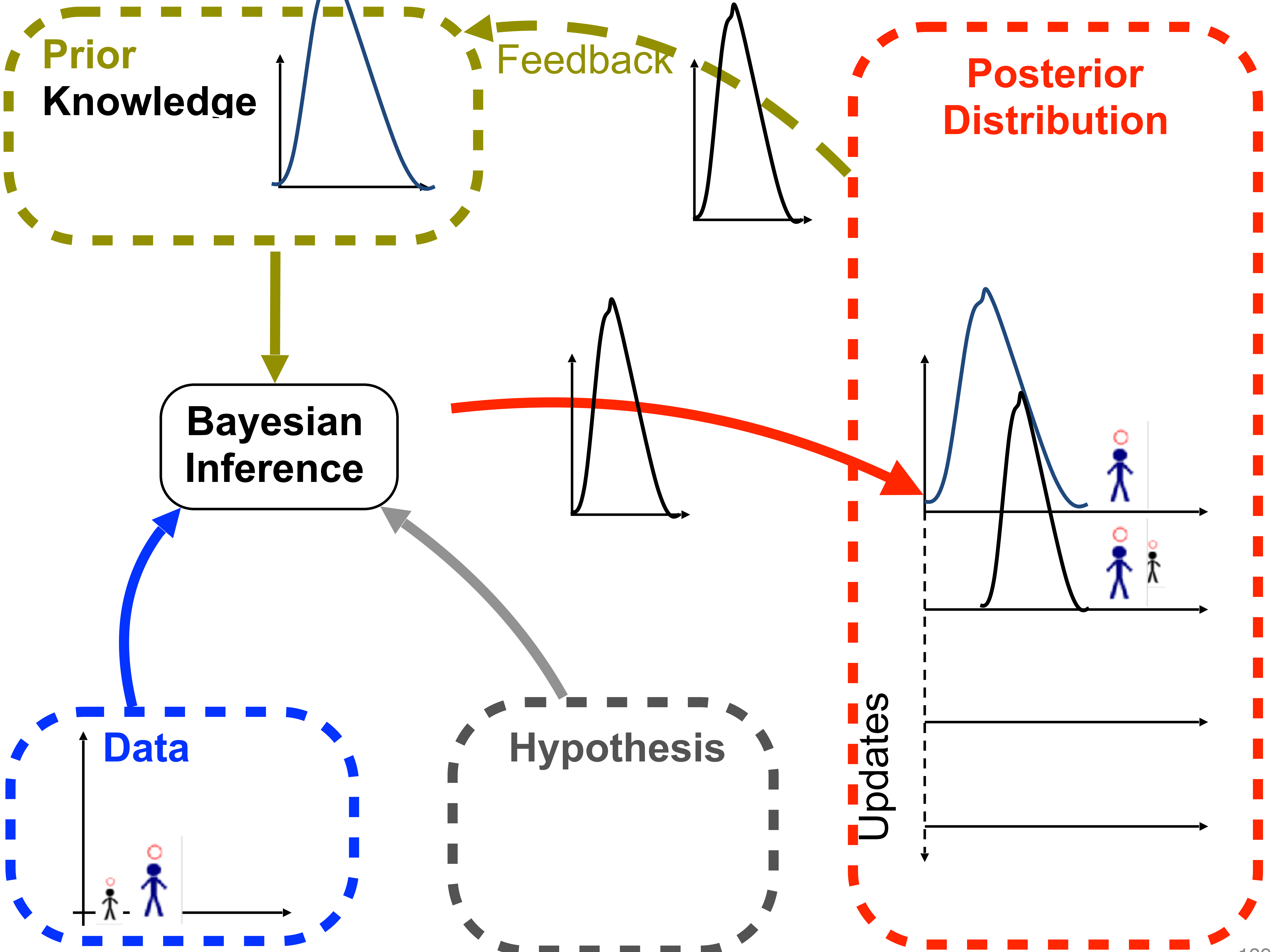

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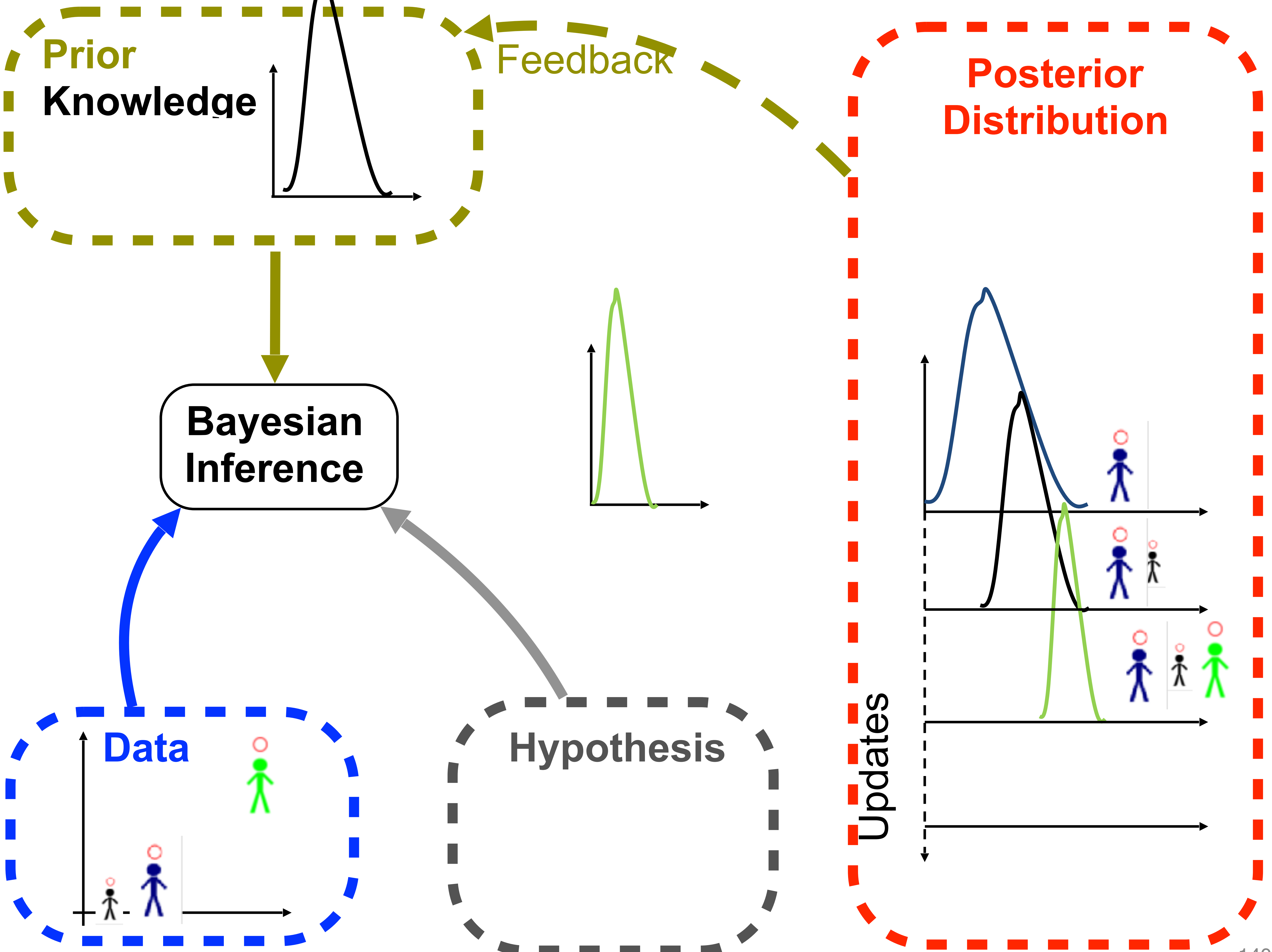
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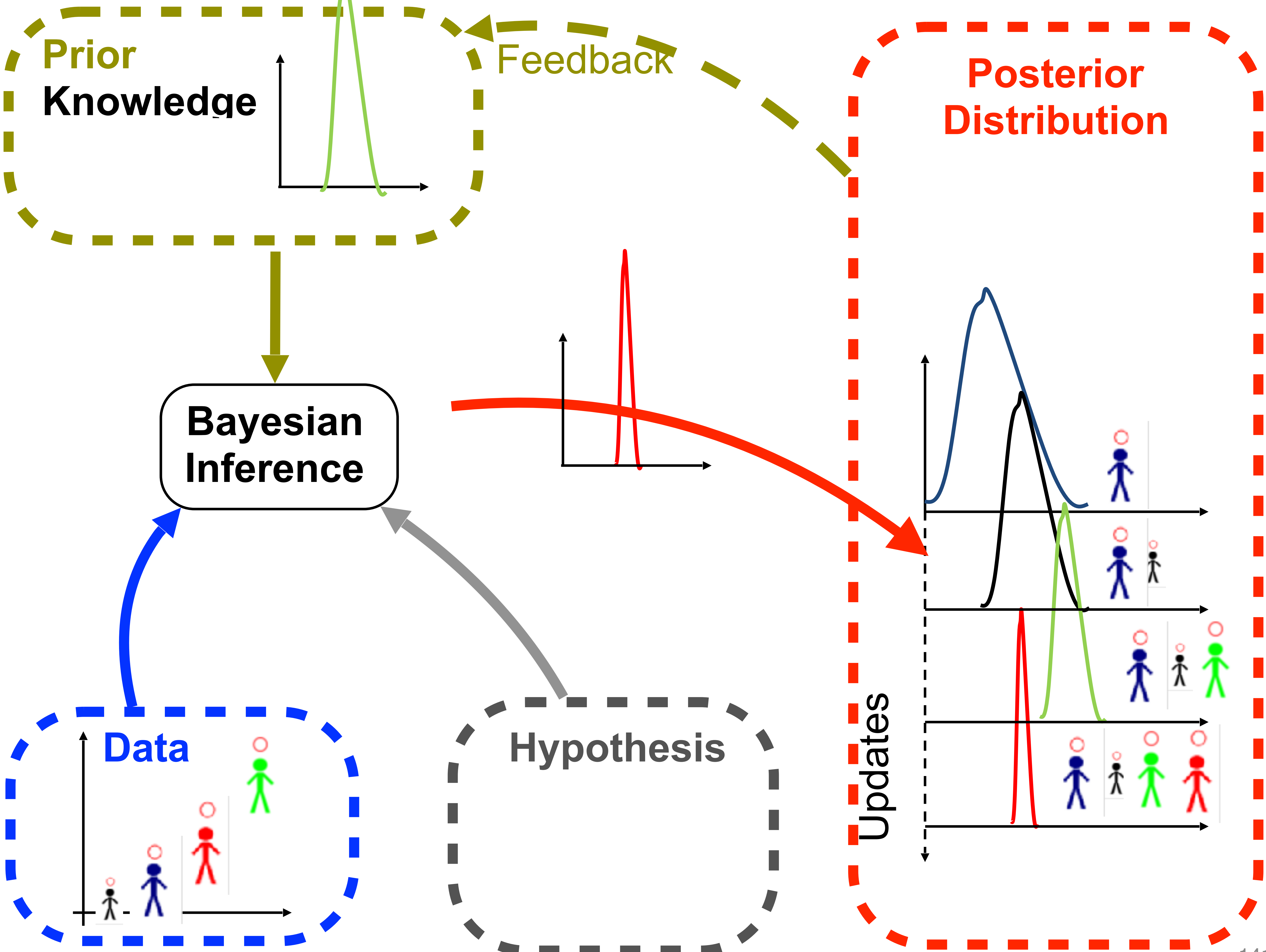
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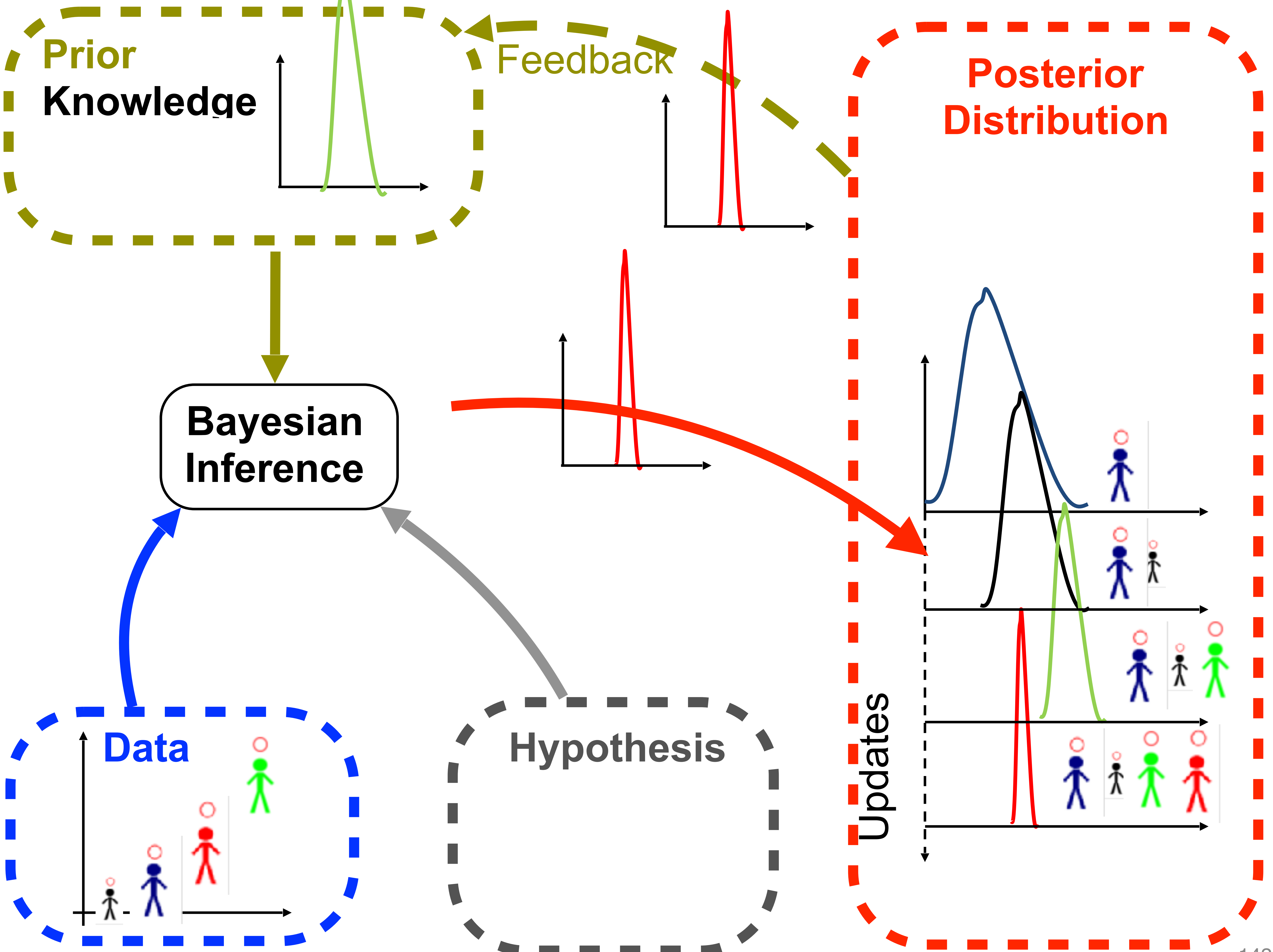
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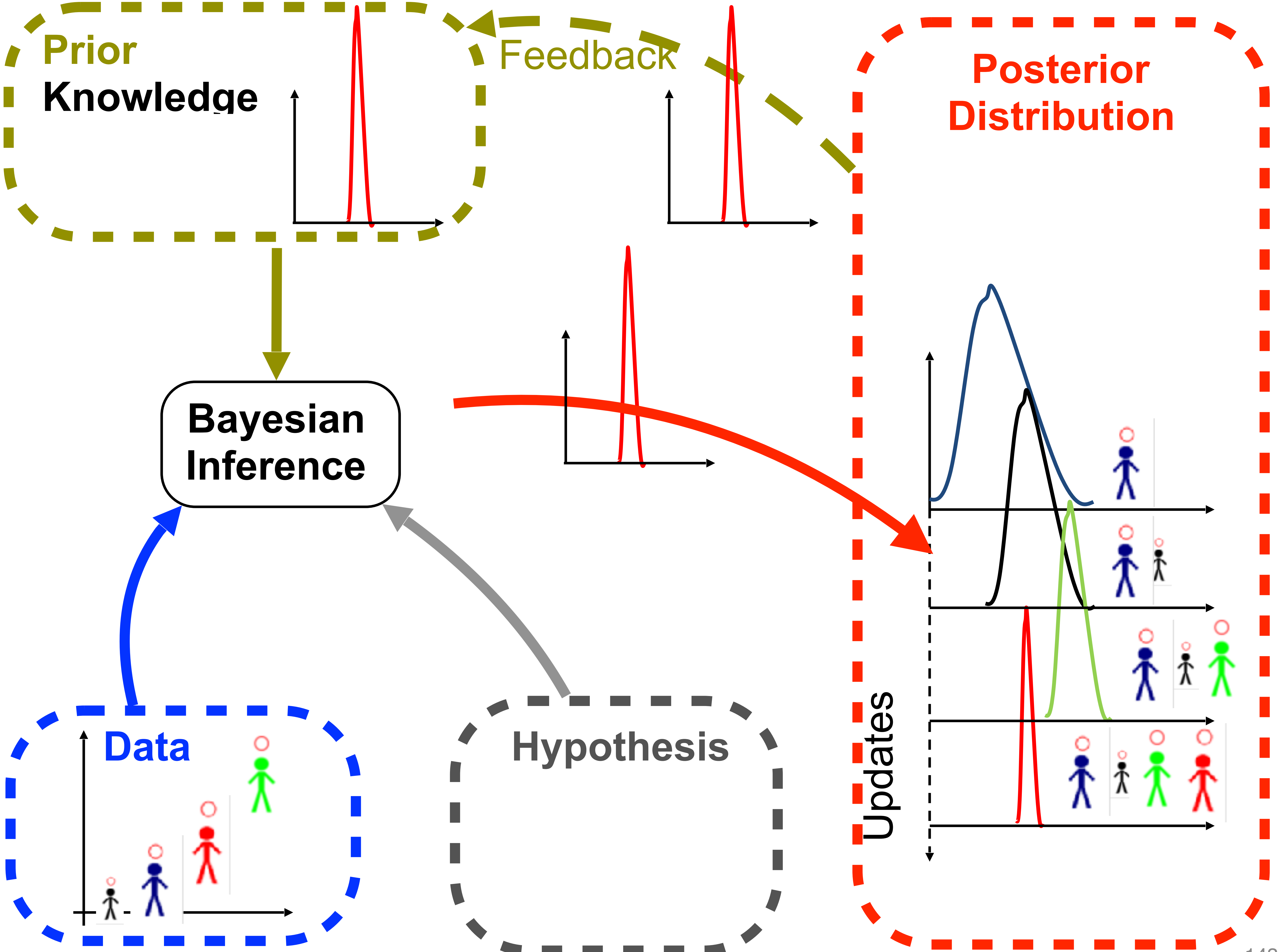
Data
Hypothesis


Prior
Knowledge
Feedback
Posterior
Distribution
Bayesian
Inference
Data
Hypothesis
Updates