

White-Box and Asymmetrically Hard Crypto Design

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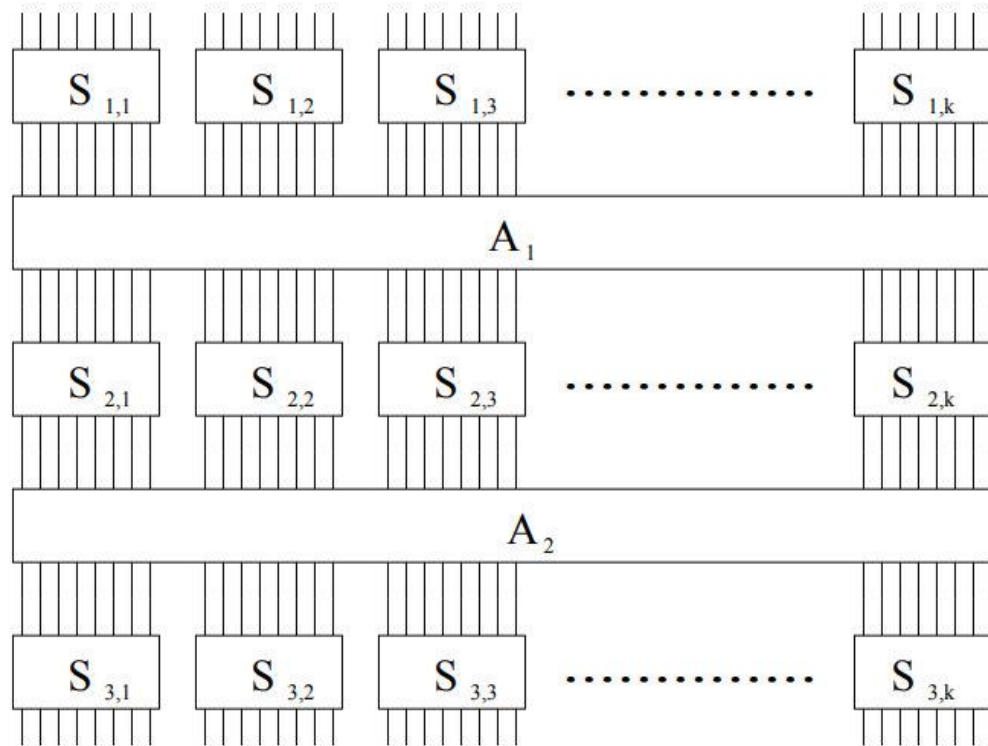
University of Luxembourg

18-May-2019

Plan of the talk

- The ASASA story
- Resource Hardness Framework
- Other ideas

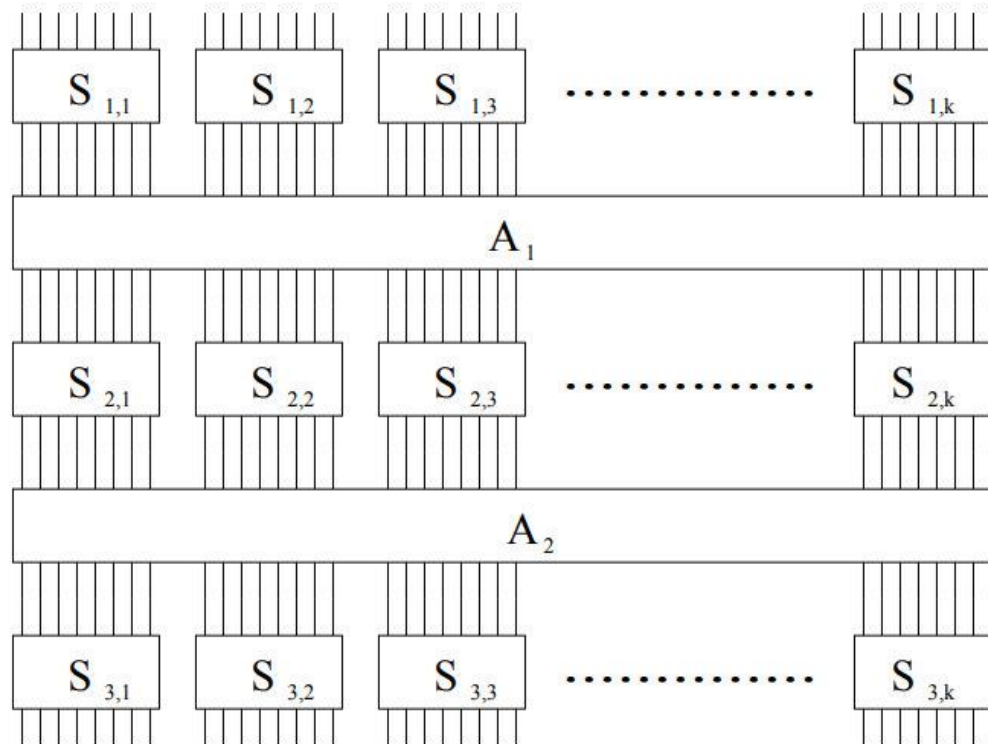
Structural cryptanalysis of SASAS*



- Scheme with unknown keyed S-boxes and Affine mappings
- For 128-bit block, 8-bit S-boxes, secret key-size is 2^{17} bits

*Biryukov, Shamir, Structural Cryptanalysis of SASAS, Eurocrypt'2001

Structural cryptanalysis of SASAS*



- For 128-bit block, 8-bit S-boxes, secret key-size is 2^{17} bits
- **Multiset attack** complexity is 2^{16} chosen texts and 2^{28} time

*Biryukov, Shamir, Structural Cryptanalysis of SASAS, Eurocrypt'2001

Structural cryptanalysis of SASAS

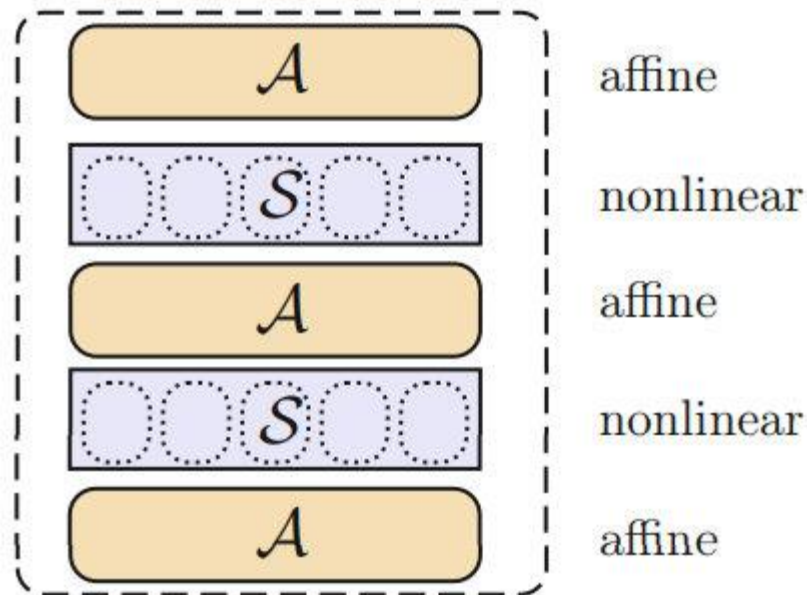
- What this has to do with WBC?

Structural cryptanalysis of SASAS

- Many early obfuscations were broken because **SASAS** and shorter ciphers are structurally very weak (and simple **ASA** was used in many WBC schemes)
- Strong diffusion in ciphers prevents from building tables with more rounds since lookup tables explode

The ASASA attempt*

- One scheme we couldn't break in 2001 was ASASA (with bijective S-boxes)
- (ASASA with non-bij. S-boxes was proposed as PK scheme by PatarinGoubin'97 and broken by Ding-Feng'99, Biham'00)



*Biryukov, Bouillaguet, Khovratovich, Cryptographic Schemes based on ASASA., AC'2014

The ASASA attempt*

- Defined strong and weak white box crypto in [BBK'14] a la [Wyseur'09] (Strong WBC=PK, i.e. no ability to decrypt, was the main goal of the paper, also now called one-wayness (OW))
- Built strong and weak WBC from ASASA
- Strong WBC was based on multivariate crypto, expanding S-boxes+noise

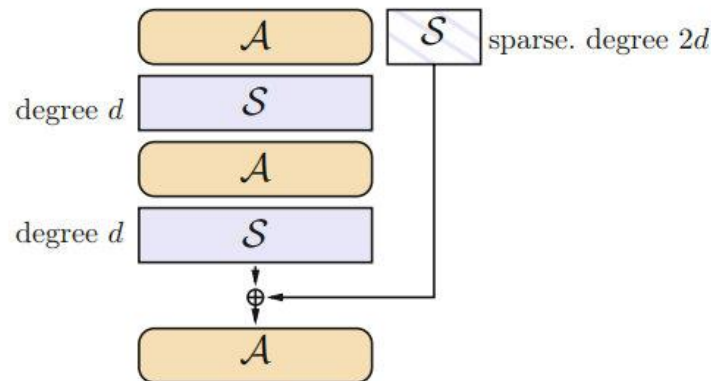


Fig. 2. Small perturbations to defeat decomposition attacks as injection of sparse high-degree polynomials

The ASASA attempt*

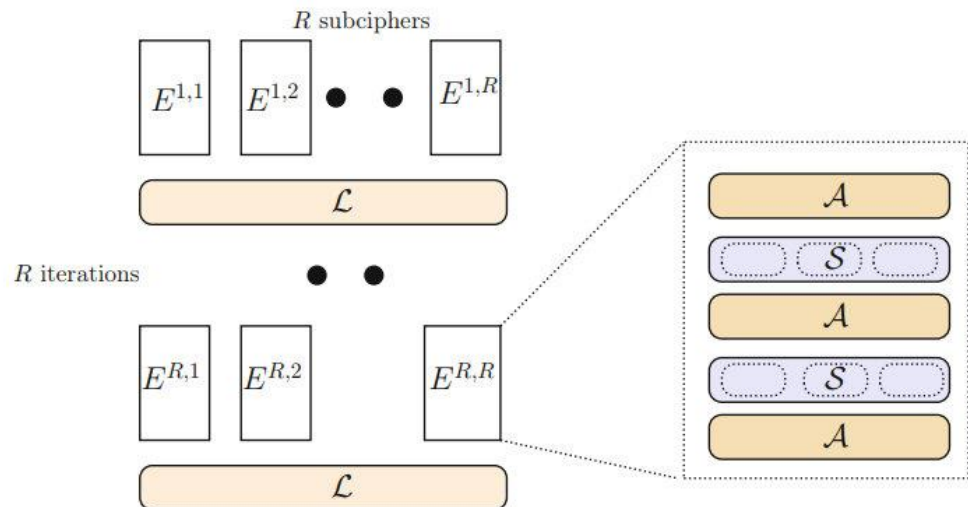
- Built strong and weak WBC from ASASA
- Strong WBC was based on multivariate crypto, expanding S-boxes+noise
- Strong and some weak WBC broken in 3 nice cryptanalytic papers [GPT'15,DDKL'15,MDFK'15]

*Biryukov, Bouillaguet, Khovratovich, Cryptographic Schemes based on ASASA., AC'2014

The ASASA attempt

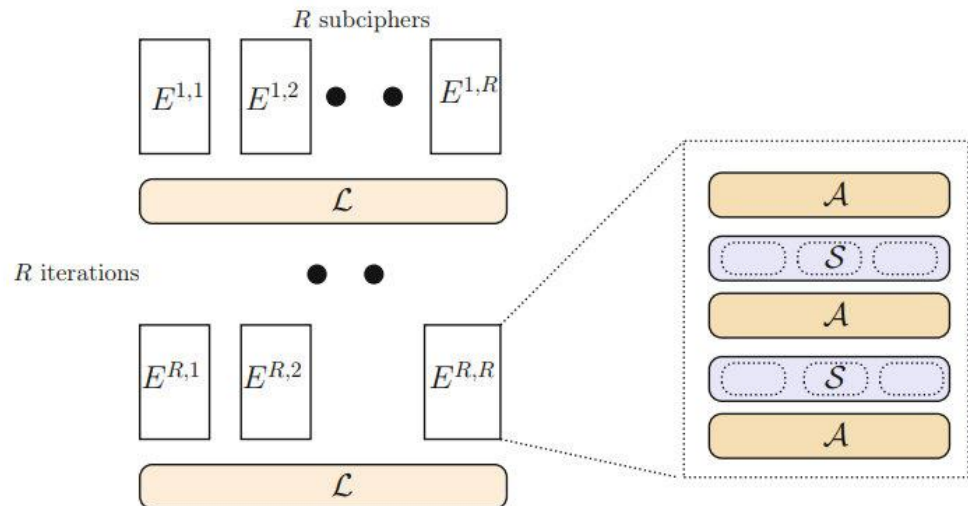
A few more details on our weak WBC scheme

- SPN, recursive approach, assuming ASASA or ASASASA mini-ciphers are secure against decomposition



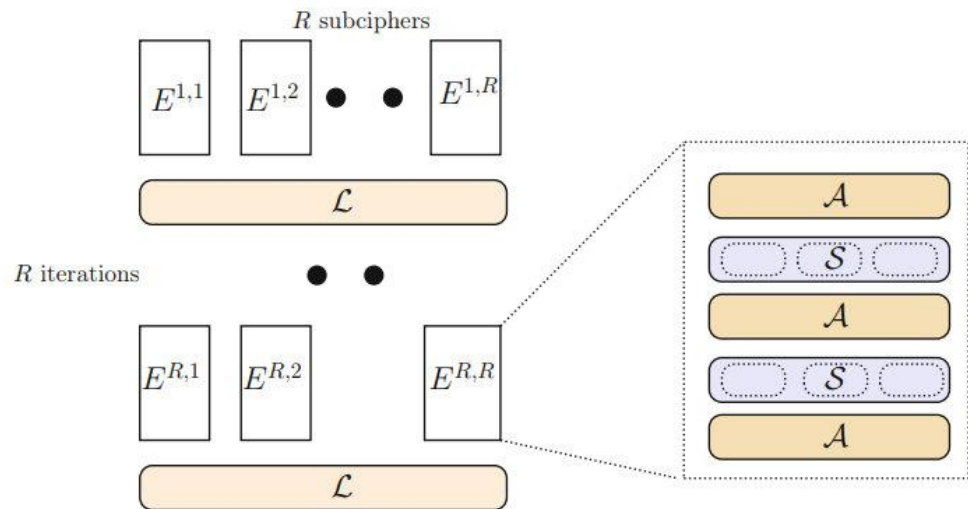
The ASASA attempt

- ASASASA instances still unbroken
- Overall approach is valid, just needs more rounds r , description size grows linearly with r .



The ASASA attempt

- ASASASA instances still unbroken
- Overall approach is valid, just needs more rounds.
- Motivated more research on weak-WBC and nice constructions
SPACE [BI15], PuppyCipher [FKKM16], SPNBox [BIT16]



Weak white-box

- "We note that a white-box implementation can be useful as it forces the user to use the software at hand", -Marc Joye'08

Weak white-box

- Incompressibility \approx Space-hardness \approx Code-hardness
- Generalize: Resource R -hardness

Force to use implementation with special properties:

- *Inefficient* in resource R
- Password-protected (access control)
- Tagged/watermarked (tracing)

Resource Hardness Framework*

Efficiency metrics for crypto algorithms:

- Speed (Time complexity, parallel or sequential)
- Code-size (ROM)
- Memory complexity (RAM)

Sometimes *inefficiency* of algorithms in these metrics is required

*Biryukov, Perrin, “Symmetrically and Asymmetrically Hard Cryptography, Asiacrypt’17

Resource Hardness Framework

Sometimes inefficiency of crypto algorithms in these metrics is required (*several research areas that do not always talk to each other*)

- Weak whitebox-crypto (code size hardness)
- Password hashing (memory hardness)
- Key derivation functions (KDF) (time hardness)
- Big key encryption (code size hardness)
- Time-lock puzzles, PoSW, VDFs (sequential time hardness)
- Proof-of-X (all kinds of hardness)

Resource Hardness Framework

Symmetric vs Asymmetric Resource hardness:

- Symmetric – computation is R hard for all the users
- Asymmetric – computation is easy for “privileged” users knowing the secret K

Resource Hardness Framework

	Time	Memory	Code size
Applications	KDF, time-lock	Password hashing, egalitarian computing	White-box crypto, big-key encryption
Symmetrically hard functions	PBKDF2 [Ka100]	Argon2 [BDK16], Balloon [BCGS16]	XKEY2 [BKR16], WHALE (Sec. 5.2)
Asymmetrically hard functions	RSA-lock [RSW96], SKIPPER (Sec. 5.1)	DIODON (Sec. 2.4.3)	White-box block ciphers [BBK14, BI15, FKKM16] [BIT16]

Table 1: Six types of hardness and their applications.

Resource Hardness Framework

Definition 2 (R-hardness). *We say that a function $f : \mathcal{X} \rightarrow \mathcal{Y}$ is R-hard against 2^P -adversaries for some tuple $R = (\rho, u, \epsilon(p))$ with $\rho \in \{\text{Time, Code, RAM}\}$ if evaluating the function f using less than u units of the resource ρ and at most 2^P units of storage is possible only with probability $\epsilon(p)$. More formally, the probability for a 2^P -adversary to win the efficient approximation game, which is described below, must be upper-bounded by $\epsilon(p)$.*

- 1. The challenger chooses a function f from a predefined set of functions requiring more than u units of ρ to be evaluated.*
- 2. The challenger sends f to the adversary.*
- 3. The adversary computes an approximation f' of f which, unlike f , can be computed using less than u units of the resource ρ .*
- 4. The challenger picks an input x of \mathcal{X} uniformly at random and sends it to the adversary.*
- 5. The adversary wins if $f'(x) = f(x)$.*

Resource Hardness Framework

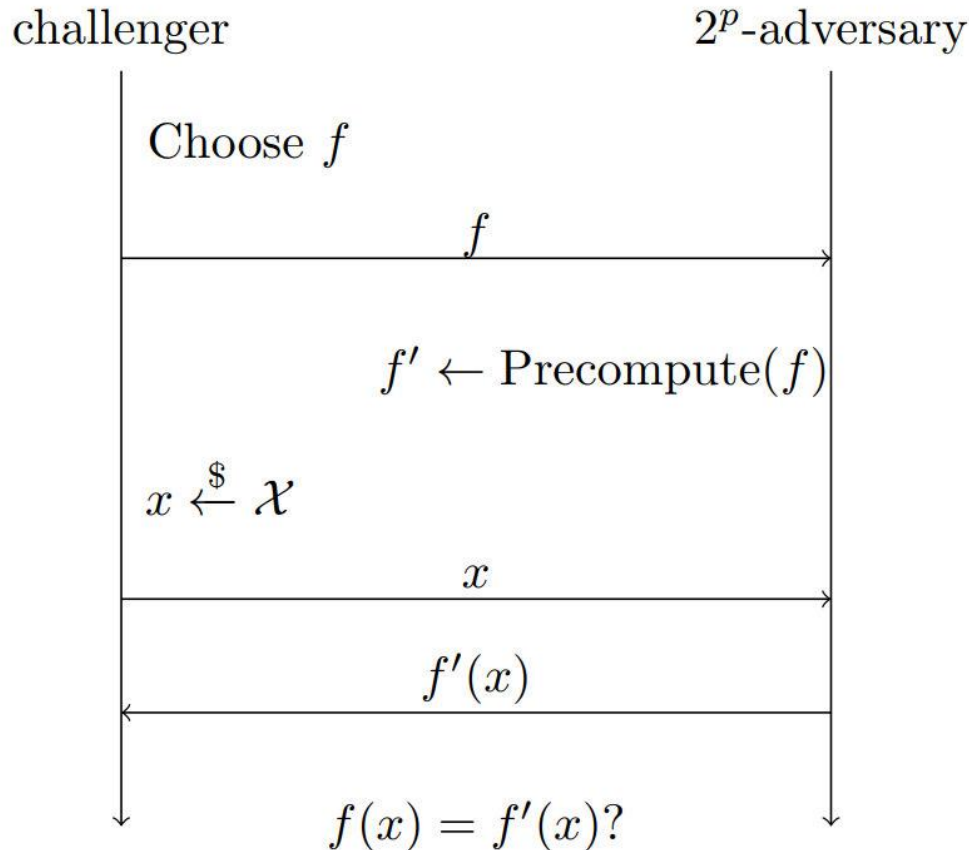


Figure 2: The game corresponding to the definition of $(\rho, u, \epsilon(p))$ -hardness against 2^p -adversaries.

Resource Hardness Framework

- How to achieve required R -hardness?
- The framework allows us to construct primitives with any hardness type:
the idea of *plugs* with specific hardness type

Plugs: Time-Hardness

Symmetric:

- **IterHash (t,n)** – iterates t-bit hash n times ($n < 2^{t/2}$ to avoid cycles)

Asymmetric

- **RSALock(t,n)** (time-lock) n squarings mod N , $N=pq \approx 2^t$

$$\text{RSALock}_n^t(x) = x^{2^n} \pmod{N}$$

Secret owner first computes $e=2^n \pmod{(p-1)(q-1)}$

Then he computes $x^e \pmod{N}$ (or CRT)

Plugs: Code-Hardness

Symmetric:

- **BigLUT** (t, v) – a table with 2^t random v -bit entries

Asymmetric

- **BcCounter**(t, v) = $E_k(0^{v-t} || x)$, E_k is a v -bit block cipher with secret key k , $|k| \geq v$

Secret owner knows k

Hardness for the common user:

$(\text{Code}, 2^p, 2^{p-t})$ -hard

Plugs: Code-Hardness

Symmetric:

- BigLUT (t,v) – a table with 2^t random v-bit entries

Asymmetric

- BcCounter(t,v) = $E_k(0^{v-t} || x)$, E_k is a v-bit block cipher with secret key k ,
 $|k| \geq v$, $|x|=t$, $t < v$

Secret owner knows k

Improvement for small t : (parallel application of ℓ tables $|x| = v$)

$$f(x_0 || \dots || x_{\ell-1}) = \bigoplus_{i=0}^{\ell-1} E_k(\text{byte}(i) || 0^{n-t-8} || x_i)$$

Hardness for the common user:

$$(\text{Code}, 2^p, \max(2^{p-v}, (2^{p-t}/\ell)^\ell))\text{-hard.}$$

Plugs: Memory-Hardness

Symmetric:

- [Argon2\(t,M\)](#) with input size t and memory size M
(memory hard password hashing function)

Asymmetric

- [Diodon](#) (more details later)

Our collection of R -hard plugs

Hardness	Symmetric	Asymmetric
Time	IterHash $_{\eta}^t$ (Time, η , 2^{p-t})	RSALock $_{\eta}^t$ (Time, η , 2^{p-t})
Memory	Argon2 (RAM, $M/5$, 2^{p-t})	DIODON (RAM, $M/10$, 2^{p-t})
Code	BigLUT $_v^t$ (Code, 2^p , 2^{p-t})	BcCounter $_v^t$ (Code, 2^p , 2^{p-t})

Table 2: Possible plugs, i.e. sub-components for our constructions which we assume to be R -hard against 2^p -adversaries.

Modes of Plug Usage

The plugs can be used in different modes

- Plug-then-randomize (PTR)
- Hard block cipher mode (HBC)
- Hard sponge mode (HSp)

Mode: Plug-then-Randomize

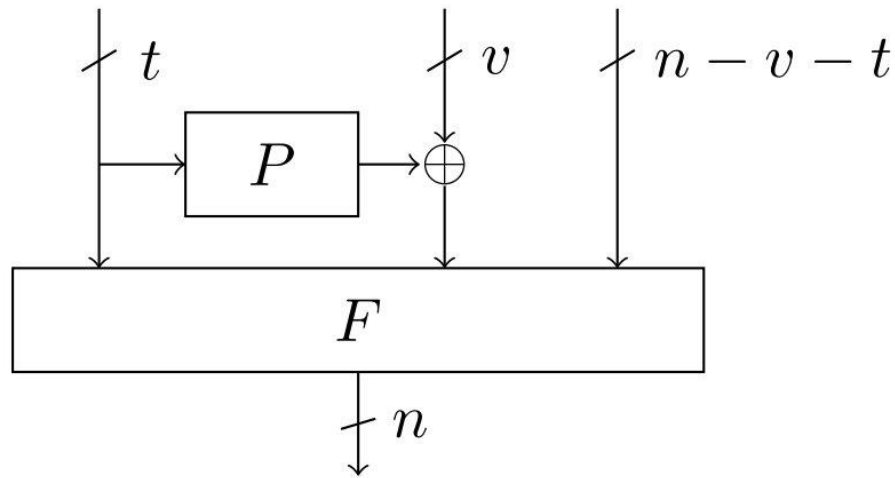


Figure 3: Evaluating the plugged function ($F \cdot P$)

Here F is a random (permutation) oracle

Iterate to increase hardness:

$(\rho, u, \max(\epsilon(p)^r, 2^{p-n}))$ -hard against 2^p -adversaries

Mode: Hard block cipher

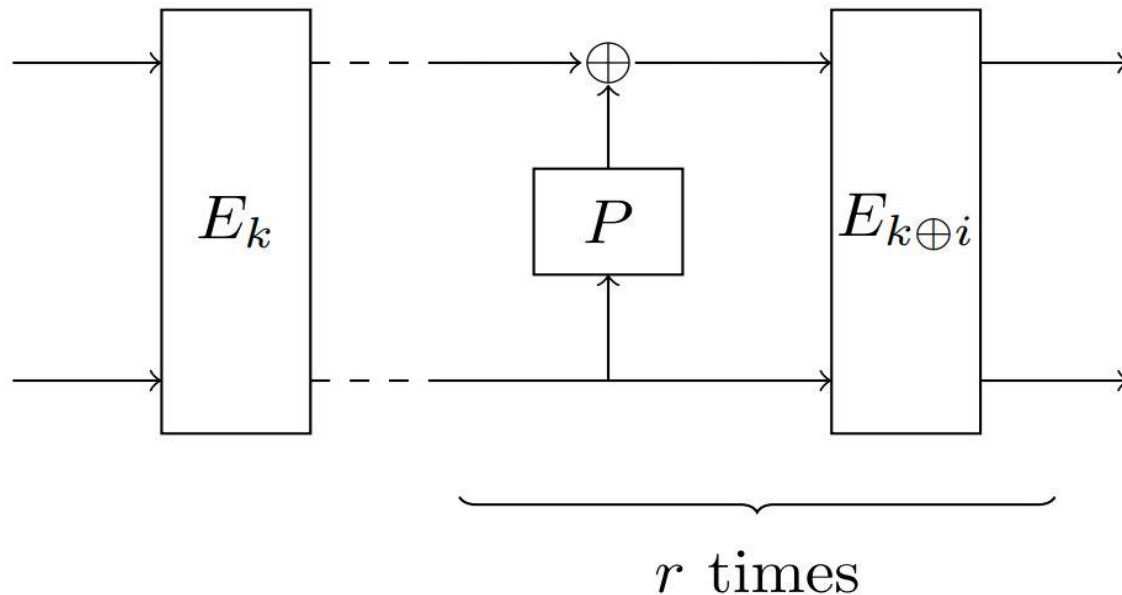


Figure 4: The HBC block cipher mode.

- Given related-key-secure n -bit block cipher E_k , $k \geq n$
 $(\rho, u, \max(\epsilon(p)^r, 2^{p-n}))$ -hard against 2^p -adversaries

Example: Time-hard block cipher *Skipper*

Algorithm 5 SKIPPER encryption

Inputs: n -bit plaintext x ; k -bit key k ; RSA modulus N

Output: n -bit ciphertext y

$y \leftarrow \text{AES}_k(x)$

for all $i \in \{1, 2\}$ **do**

$y_1 \parallel y_2 \leftarrow y$, where $|y_1| = 88$ and $|y_2| = 40$

$y_2 \leftarrow y_2 \oplus T_{40}(y_1^{2^n} \bmod N)$

$y \leftarrow \text{AES}_{k \oplus i}(y_1 \parallel y_2)$

end for

return y

- The plug is: $(\text{Time}, \eta, 2^{-40})$ -hard. *Skipper* is:
 $(\text{Time}, \eta, \max(2^{48-128}, (2^{-40})^2))$ -hard

Hard Sponge Mode (HSp)

- Sponges can be used to construct hash functions, stream ciphers, MACs and AE

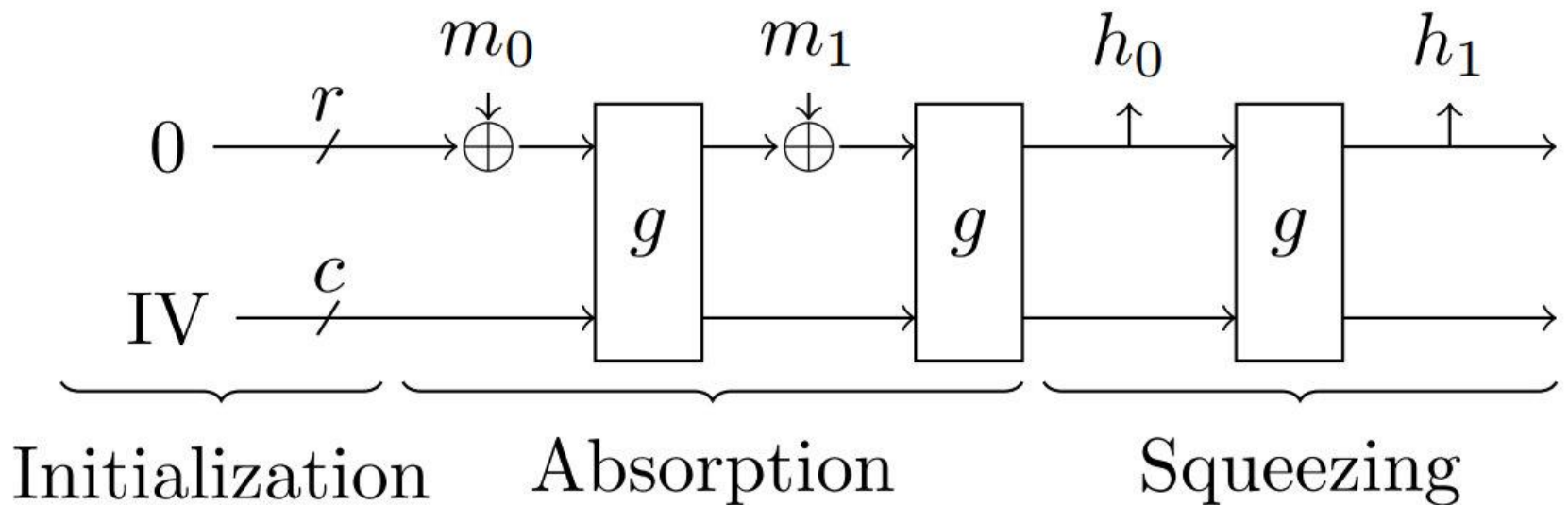


Figure 5: A sponge-based hash function.

Hard Sponge Mode (HSp)

- Iteratively use Plug-then-Randomize mode

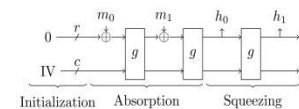
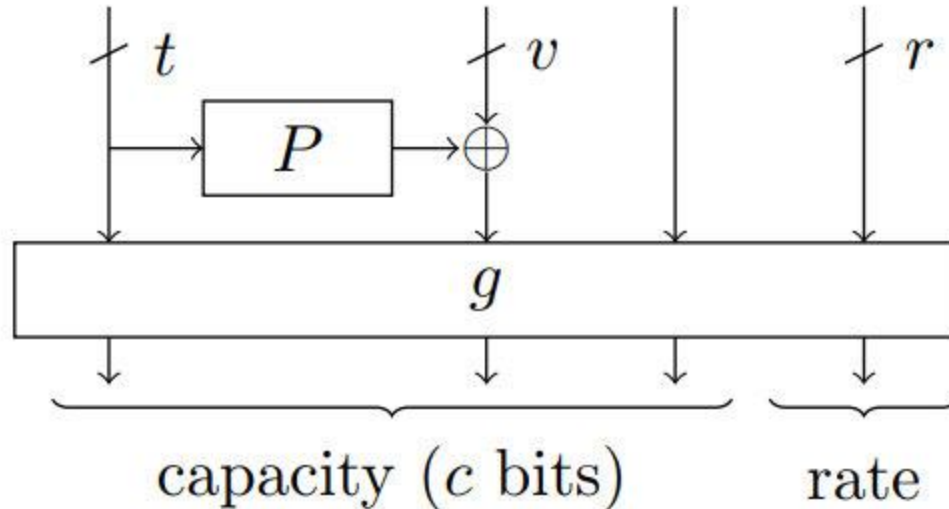


Figure 5: A sponge-based hash function.

- In the paper: Code-hard hash function based on Keccak which we called Whale.

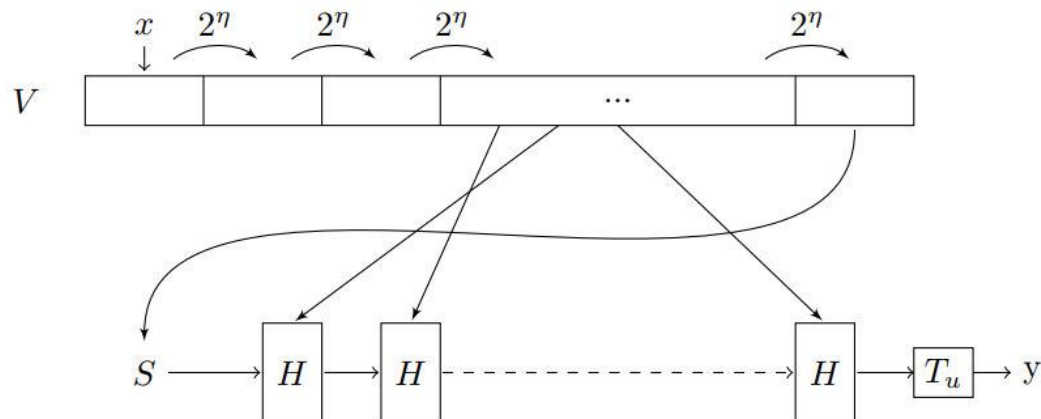
Example: Memory-Hard function *Diodon*

Algorithm 1 DIODON Asymmetrically memory-hard function

Inputs: t -bit block x ; RSA modulus N of n_p bits; M, L ;

Output: u -bit output y

```
 $V_0 = x$   
for all  $i \in \{1, \dots, M - 1\}$  do  
     $V_i = V_{i-1}^{2^\eta} \bmod N$   
end for  
 $S = V_{M-1}$   
for all  $i \in \{0, \dots, L - 1\}$  do  
     $j = S \bmod M$   
     $S = H(S, V_j)$   
end for  
return  $T_u(S)$ 
```



Example: Memory-Hard function *Diodon*

Algorithm 2 DIODON for privileged users

Inputs: t -bit block x ; RSA factors q, q' ; η ; M, T ;

Output: u -bit output y

$$e = 2^{(M-1) \times \eta} \bmod (q-1)(q'-1)$$

$$S = x^e \bmod (qq')$$

for all $i \in \{0, \dots, L-1\}$ **do**

$$j = S \bmod M$$

$$e_j = 2^{j \times \eta} \bmod (q-1)(q'-1)$$

$$S = H(S, (x^{e_j} \bmod (qq'))))$$

end for

return $T_u(S)$



Resource hardness Framework

Parameters	Conservative	Fast
t	128	128
u	128	128
n_p	2048	1024
η	2048	1
M	4,000	8,000,000
L	4,000	20,000
RAM (basic user)	1 Mb	1 Gb
Time (basic user)	10.00 s	9.87 s
Time (privileged)	13.49 s	10.65 s

n_p – bits in RSA modulus; t, u – input/output sizes; M, L – upper/lower chain length

Resource hardness Framework

Parameters	Conservative	Fast
t	128	128
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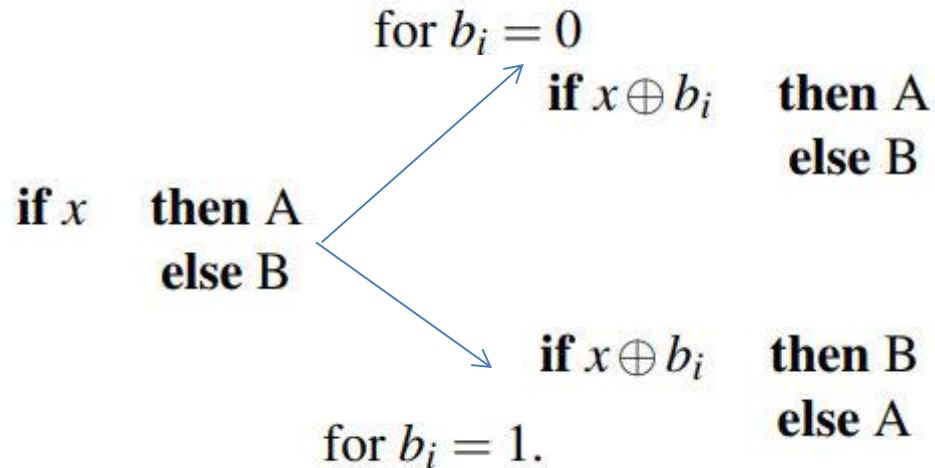
Open problem: Diodon is based on scrypt which has lousy linear TM-tradeoff. Also slow due to RSA. Improve?

Few other things

R -hardness and code obfuscation

Using obfuscation idea from [BK'16*]:

- Compiler that runs some resource hard function $F(pwd, x)$
- Computes R -hard bits $F(pwd, x) = b_i$ and then makes code transformations:

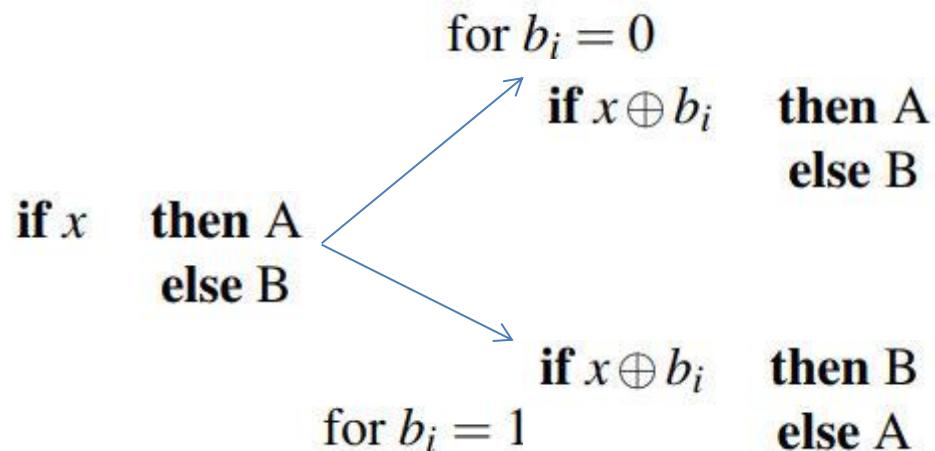


*Biryukov, Khovratovich, Egalitarian Computing, Usenix'16

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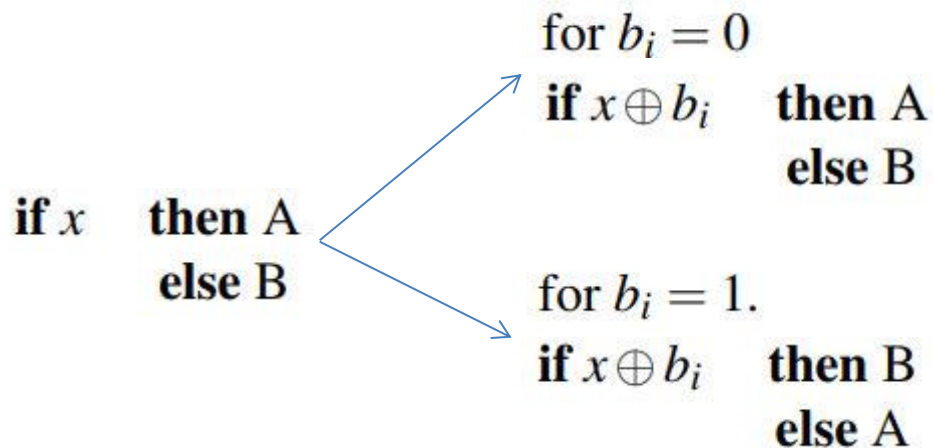


- The user will have to run R -hard function $F(pwd, x)$ at least once

R -hardness and code obfuscation

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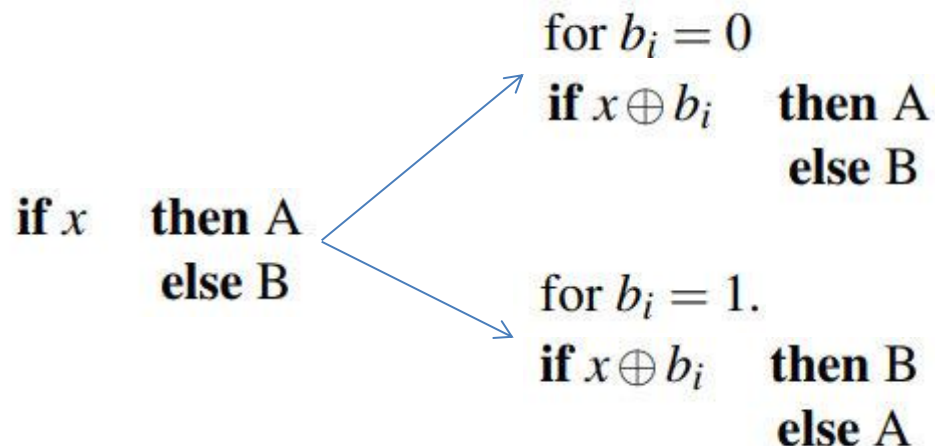


- This could work well for previously unseen code.

R-hardness and code obfuscation

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Would this approach work to make
Incompressible, password protected INC-AES ?

R-hardness and code obfuscation

- Not really. Unless we already have K -unextractable/unbreakable UBK-AES.
- However it shows hope that at least in some cases $UBK \Rightarrow INC$

Related topics

Related research topics

- Code Obfuscation (for structure hiding)
- Cross-pollination with GreyBox crypto (for value hiding)
- IO
- Malicious crypto – adversarial crypto design
- PK crypto based on new ideas

Open problems

- Can we design a WBC-friendly cipher?
- Would Even-Mansour cipher be a good candidate?
- Design Diodon-like asymmetric memory hard functions with non-linear TM tradeoffs and faster operations
- INC-PWD-AES?

End

(and we are hiring postdocs on
WBC and other topics)

cryptolux.org