Trade and Vertical Differentiation*

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Abstract

This paper discusses a trade model with many countries, many goods produced in multiple quality versions, and non-homothetic preferences. It embeds in the same model a series of results that have been empirically confirmed: high-income countries specialize in the production of high-quality goods and trade more of those. Richer countries purchase more high-quality varieties. They import more high-quality products from the most productive exporters. The paper then studies the impact of productivity and population changes on the quality composition of exports. It finally explains why countries import higher quality goods from more distant countries.

Keywords: vertical differentiation, horizontal differentiation, trade, income heterogeneity.
JEL codes: F12, F16, L11, L15.

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1 Introduction

In the past decade, researchers have highlighted important patterns in the quality of traded goods. In particular, countries import more high quality goods from higher productivity exporters while, compared to poorer countries, richer nations import a higher share of high-quality goods, specialize more in the production and exportation of high-quality goods. Also, higher quality goods are shown to be exported to more distant countries.[1] Those findings naturally call for a unified theoretical foundation that explains the quality of traded goods in the context of many countries, goods and quality standards.

This paper asks the question about how the quality of traded goods change with country productivity, populations and trade costs. The economic literature offers a series of formal explanations for the above distinct effects in separate models. A branch of the recent literature explains the product quality as a simple demand shifter of a horizontally differentiated product and without reference to income effects.[2] This perspective strongly differs from the earlier view of product quality grounded in the vertical differentiation approach, where producers supply several quality versions of the same good and consumers differ with respect to income and willingness to pay for each good.[3] Departure from this standard framework stems from the analytical complications that the non-homothetic preferences of vertical differentiation models bring in general equilibrium frameworks with many goods and countries.[4]

The main contribution of this paper is to present a tractable model of vertical differentiation with many goods and countries. Each country produces a continuous set of goods with high and low quality versions while consumers are endowed with non-homothetic preferences and purchase a single version of every good from every country. While higher quality versions give higher utility, they are more costly to produce. For each variety, consumers then compare the prices of each quality version with their marginal utility. The main innovation of the paper is to use a class of quality and cost profiles that makes consumer expenditures linear in the consumer's inverse marginal utility. As a result, the trade equilibrium is governed by a set of linear equations that can readily be solved and discussed.

The second contribution of the paper readily follows from the first as our theoretical model encompasses all the empirical patterns we mentioned above. In particular, average import prices are larger to countries with larger per capita income and for the goods shipped from more productive exporters. Also, richer countries trade more numerous high-quality goods with each other (Linder hypothesis 1961). It is further shown that a rise in a country's productivity entices this country to specialize in high quality goods. Productivity increases do not have exactly similar effects as population increases. Indeed, whereas a bigger population leads to

[1] See Fieler (2011), Hallak (2010), Choi et al., (2009), Dalgin et al. (2008), Hallak (2006), Hummels and Klenow (2005) and Schott (2004), Manova and Zhang (2012), Crozet et al. (2012), inter alia.

[2] See Jaimovic and Merella (2015), (2012), Comite et al. (2014), Picard (2015), Baldwin and Harrigan (2012), Fajgelbaum et al. (2011), and Verhoogen (2008), inter alia.

[3] Mussa and Rosen 1978; Gabszevicz and Thisse 1979; Shaked and Sutton 1982.

[4] Helpman and Flam 1987; Stokey, 1991; Matsuyama, 2000; Fieler 2012; and others.

wider consumption of local high-quality goods but it may lead to a narrower range of high quality imports.

The model is finally consistent with the empirical effects of trade costs and distance. A fall in *ad-valorem* (iceberg) trade cost entices countries to substitute domestic for foreign high-quality goods. It boosts exports of high quality goods, increases cif prices and finally raises utility everywhere. The model leads to a gravity equation consistent with the literature, where the fall in ad-valorem trade cost does not match the empirical effect on fob export prices. As discussed in Hummels and Skiba (2004), the reconciliation with empirics comes from the presence of *unit* trade cost that creates the Alchian and Allen effect (1964). According to this latter, unit trade costs lower the price of high quality goods relatively to low quality goods. Under such trade costs it is shown that exports can be biased towards high quality goods for more distant trading partners if consumers drop the low-quality low-costs goods from their consumption baskets.

Before proceeding further, it is important to highlight how this paper departs from the existing trade theory literature on product quality.

Related literature The paper is firstly linked to the general equilibrium studies of trade under vertical differentiation. Early papers discuss the endogenous quality spectrum of a single good, which makes them unsuitable to discuss intra-industry trade (Flam and Helpman, 1987; Stokey, 1991, etc.). By contrast, this paper considers a continuous set of goods with a few quality levels, which permits the study of intra-industry trade. Other papers explore vertical differentiation in a North-South setting where one country is endowed with a strong productivity advantage (Matsuyama, 2000). By contrast, this paper studies trade between a large number of not too asymmetric countries.

In contrast to this research lines, the present paper discusses trade properties using a novel and unexplored setting of costs and preferences. This includes a set of horizontally differentiated varieties *produced in several quality versions* in the spirit of the seminal vertical differentiation literature initiated by Mussa and Rosen (1978). This is the reason why our model elaborates on preferences close to those discussed in Matsuyama (2000), Tasarov's (2009, 2012) and Fieler (2012). Whereas Tasarov studies a continuous set of varieties versioned in one quality each and sold monopolisitic competition, this paper studies the same set of varieties versioned in a few quality levels and – for the sake of simplicity - sold under perfect competition.

In recent years, there has been a renewed interest in the analysis of quality in trade with the aim to explain product quality in micro-data on trade. Those studies discuss the relative prices, import penetration and export compositions. In contrast to this paper, Jaimovich and Merella (2012, 2015) assume the divisible rather than indivisible goods - as in the usual traditional differentiation literature with unit purchases. Jaimovich and Merella (2012) confirms the Linder hypothesis. Finally, in contrast to some papers, our analysis carries over a finite set of countries with finite population sizes and all general equilibrium effects through relative prices are present.

Like in this paper, research on product quality and trade is a search for a set of preferences that best reflects observed patterns. Towards this aim, Eaton and Fieler (2017) propose two-tier CES preferences nesting horizontal and vertical dimensions of goods. Their modeling differ from this paper as countries produce goods with a single quality level and goods are divisible. Matsuyama (2015) proposes two-tier Hannoch and CES preferences over goods with heterogenous income elasticities. Associating higher income elastic goods to higher quality ones, he shows that richer countries are net exporter of high quality goods. In contrast to those interesting research lines, the present paper discusses trade properties using another novel and unexplored setting of costs and preferences.

The remainder of the paper is organized as follows. Section 2 provides a short reminder of the existing empirical evidence on the role of quality on U.S. trade prices and values. Section 3 describes the model of vertical differentiation with many goods and countries and presents the role of linear real expenditure. The trade equilibrium and its properties are examined in Sections 4 and 5. Section 6 discusses the model with ad-valorem trade costs and elaborates on the gravity equation resulting from this model. Section 7 discusses the choice and generality of our model primitives and studies the conditions for the existence of an Alchian Allen effect. Section 8 shortly concludes. Appendices include mathematical details.

2 Empirical motivation

To illustrate the empirical relevance of our theoretical analysis, we first investigate the relationship between trade and quality in the U.S. trade data for the period 1990-2006 collated by Feenstra *et al.*(2002).[5] This dataset includes product values and quantities in transactions classified in the 6-digits new North American Classification System (NAICS).

We are interested in the relationship between product quality at U.S. customs and the trade partners' GDP per capita, distance to U.S. and remoteness. We first measure product quality as the transaction unit price p_{czit} (gross value divided by quantity q_{czit} of goods sold) for a trade transaction c, (NAICS 6 digit) product category z, trade partner i and year t. Unit prices are "free on board" (FOB) for exports and include "cost insurance & freight" (CIF) for imports. We also measure product quality as the average unit price in the product category $\overline{\mathrm{p}}_{zit} = (1/n_{zit}) \sum_{z=1}^{n_{zit}} \mathrm{p}_{czit}$ (Armington, 1969, Feenstra, 1994). We further investigate the difference of log of those average prices between country pairs for same good and year, which removes commodity- and year-specific variations (e.g. low quality cars are more expensive than high quality shirts). Properties about such differences are closely investigated in our theoretical approach. Countries' GDP per capita are obtained from World Development Indicators, while distances between countries' most populated cities are from CEPII (Mayer and Zignago, 2006). Each trade partner's remoteness indicator is computed as a weighted average of bilateral

[5]Data available on http://www.nber.org/data.

distance with all other countries, weighted by countries' GDP as in Manova and Zhang (2012). To complete our study, we extend the analysis to the relationship between trade values and those factors (gravity equation). Trade values are given by the gross values by product, year and country, $T_{zit} = \sum_z \mathrm{p}_{czit} q_{czit}$. We use a pooled OLS. The detailed empirical specifications are presented in the Appendix.

Table 1 summarizes the results for U.S. exports and imports. The estimated coefficient of trade partners' GDP per capita is positive and significant in all exercises, suggesting that trade partners with larger per capita income import U.S. goods of higher quality and the U.S. imports higher quality goods from the more productive exporters. Those results confirms the literature (e.g. Hummels and Klenow 2005). Indeed, columns (1) and (2) are consistent with Fieler (2012), who tests the log of import/export unit and average prices of many countries between the years 1995 and 2007. They corroborate Jaimovic and Merella's (2012) finding according to which the average quality of imports increases with importer's productivity. They support the Linder hypothesis, according to which higher income countries produce and trade a larger proportion of high quality goods. Finally, Table 1 shows that expenditures on traded goods increase with higher GDP per capita, which is in line with the literature on gravity equation.

Distance is positively and significantly related to unit prices and the ratio of average prices for exported goods. These results can be linked to the Alchian Allen (1964) conjecture, according to which an increase in per-unit trade cost decreases the relative price of high-quality goods, by increasing in turn their consumption share over low quality items. Such this effect has been highlighted for FOB export prices by Hummels and Skiba (2004). So, if trade cost increases with distance, unit prices and ratio of average prices should do the same, as it is shown in Table 1. However, the effect of distance on CIF import prices is positive when we look at transaction prices but negative when we consider average product prices. In the latter case, U.S. consumers choose lower prices and quality for more distant imports.

Finally, higher unit prices are associated with less remote partner countries. This result is consistent with Manova and Zhang (2012), Crozet et al. (2012) amongst others. By contrast, remoteness is positively associated with the ratio of average prices and expenditure.

Table 1: U.S. export/import , regression with year-product dummies

Dep. variable	unit price $\log \mathrm{p}_{czit}$ (1)	per-country av. price $\log \frac{\overline{\mathrm{p}}_{zit}}{\overline{\mathrm{p}}_{zjt}}$ (2)	expenditure $\log T_{zit}$ (3)
U.S. export	**(fob price)**		
GDP/c	0.01 *** (0.001)	0.03 *** (0.001)	0.04 *** (0.001)
distance	0.15 *** (0.001)	0.08 *** (0.001)	−0.55*** (0.001)
remoteness	−0.01*** (0.001)	0.04 *** (0.001)	0.23 *** (0.001)
R2	0.75	0.01	0.21
obs	2,473,818	34,618,481	454,001
U.S. import	**(cif price)**		
GDP/c	0.27 *** (0.001)	0.30 *** (0.001)	0.03 *** (0.001)
distance	0.02 *** (0.003)	−0.03*** (0.003)	−1.08*** (0.001)
remoteness	−0.01*** (0.001)	0.02 *** (0.001)	0.54 *** (0.001)
R2	0.64	0.09	0.30
obs	1,868,477	10,764,243	242,087

US unit prices 1990-2006, Feenstra et al.(2002). Standard errors in parentheses.
In Columns (2), dependent and control variables must be read as "ratio of".
We discard products with less than 30 observations per importer/year and transactions for less than $ 25,000.

These empirical results give the motivation for our theoretical analysis. Similar results have been presented in the literature using empirical strategies robust to many criticisms uncovered in this section. As the main goal of the paper is to offer an explanation with a model mixing horizontal and vertical differentiation, we now move to the description of our model.

3 Model

We consider an economy with N trading countries $i \in \{1, ..., N\}$ populated by a mass M_i of individuals who are each endowed with s_i labor units (skill). The share of country i's population in the world is denoted as $m_i = M_i/M$ where $M = \sum_i M_i$. Following Armington (1961), each country i produces a set of differentiated goods $z \in [0, n]$ where the mass of goods produced in a country is denoted by n. Goods produced in a country cannot be made in another. The world set of goods is given by $[0, n]^N$ and the world number of varieties is equal to Nn. The main assumption of this paper is that each good can be versioned with high or low quality, denoted by $k \in \{H, L\}$.

Production Following Armington (1961), each country produces a set of varieties $z \in [0, n]$ requires $a_H(z)$ and $a_L(z)$ labor units for the high and low quality versions of the variety. Under perfect competition and in the absence of trade cost, the price of variety z sold in country i is equal to its unit cost:

$$p_{ijk}(z) = a_k(z) w_j,\ k \in \{H, L\}, \tag{1}$$

where w_j is the wage (per labor unit) in the production country j. We assume that quality upgrades are more difficult to obtain for more costly varieties. Input functions a_H and $a_L : [0, n] \to \mathbb{R}^+$, $0 < n < 1$, follow the profiles $a_L(z) = (n - z)^2$ and $a_H(z) = (n - z)^2 + (n - z)^{-2}$. Low quality version of a good always costs less its high quality version. Ceteris paribus, the cost of low quality varieties z fall with larger index z while the cost of high quality varieties z increases for high enough z. It becomes prohibitive when $z \to n$ as $a_H(z) \to \infty$, which will play the role of Inada condition on quality choice.

Demands A variety z yields to the consumer a utility level $b_H(z) > 0$ for its high quality version and $b_L(z) > 0$ for its low quality version. For conciseness, we shall call $b_i(z)$ also product quality. The quality profiles are given by $b_L(z) = n - z$ and $b_H(z) = (n - z) + (n - z)^{-1}$. One readily checks that $b_H(z) > b_L(z)$. Product quality profiles have similar structures as their associated costs profiles. The product quality of a low quality good falls with larger index z while the product quality of high quality version of z increases for high enough z. Section 7 discusses the cost primitives in more details.

Every individual consume a unit of every variety z produced in every country j. An individual in country i maximizes her utility

$$U_i = \sum_{j=1}^{N} \int_0^n \left(\sum_{k=H,L} b_k(z)\, x_{ijk}(z) \right) \mathrm{d}z,$$

subject to her budget constraint

$$\sum_{j=1}^{N} \int_0^n \left(\sum_{k=H,L} p_{ijk}(z)\, x_{ijk}(z) \right) \mathrm{d}z = w_i s_i,$$

where $p_{ijk}(z) > 0$ is the (destination) consumer prices and $x_{ijk}(z) \in \{0, 1\}$ the unitary consumption decision of variety z ($x_{ijH} + x_{ijL} = 1$). In country i, each individual earns the income $w_i s_i$ by offering her s_i labor units. Replacing the prices by their values in (1), there exits a positive scalar μ_i such that the individual i buys the high quality version H of a variety z if

$$b_H(z) - \frac{1}{\mu_i} a_H(z) w_j \geq b_L(z) - \frac{1}{\mu_i} a_L(z) w_j, \tag{2}$$

and the low quality L otherwise. The scalar μ_i measures the inverse of the marginal utility of income and is equal to the inverse of the Lagrange multiplier of the budget constraint.

By (2), the set of high-quality varieties produced in country j consumed in country i is given by

$$\mathcal{H}\left(\frac{\mu_i}{w_j}\right) \equiv \left\{z : \frac{\mu_i}{w_j} \geq \ell(z)\right\}, \tag{3}$$

where

$$\ell(z) \equiv \frac{a_H(z) - a_L(z)}{b_H(z) - b_L(z)} = \frac{1}{n-z},$$

denotes the per-quality-unit labor input of upgrading variety z. For the sake of brevity, we shall call this the "per-quality input". Per-quality input monotonically rises from $1/n$ to infinity as the variety index z increases from 0 to n. Per-quality input has inverse function $\ell^{-1}(y) = n - 1/y$. The sets of the purchased low quality varieties is defined as $\mathcal{L}(\mu_i/w_j) = [0, n]\backslash\mathcal{H}(\mu_i/w_j)$. From the above definition, it is apparent that μ_i/w_i is a sufficient statistics for the mass of consumers' purchases of local high-quality varieties $\mathcal{H}(\mu_i/w_i)$ and μ_i/w_j for their consumption of high quality imports $\mathcal{H}(\mu_i/w_j)$.

To be valid, the above demands require the two following restrictions. First, every individual must buy a mix of high and low qualities. For this, it should be that $\mu_i/w_j \in [\ell(0), \ell(n)) = [1/n, \infty)$, $\forall i, j$, which implies $\mu_i/w_j \geq 1/n$. Second, individuals must buy all varieties. Such a restriction is fulfilled in many trade models by assuming Cobb-Douglas and CES preferences or by assuming high enough initial endowment (in good or labor) for other preferences like those stemming from linear quadratic utility function. In the industrial organization literature on vertical differentiation, it corresponds to the "full market coverage" condition. This restriction can be split in two conditions. The first condition is that a consumer who prefers a high quality over low quality good also chooses to purchase this high quality good. This implies that the per-quality input schedule ℓ lies above the schedule a_H/b_H, a condition that is always satisfied under the above primitives. The second condition ensures that low quality goods are always purchased: that is, μ_i/w_j lies above than the schedule a_L/b_L, which is satisfied under our primitives if $\mu_i/w_j \geq 1$, $\forall i, j$. To sum up, since $n < 1$, we simply need to impose the following simple restriction:

$$\frac{\mu_i}{w_j} \geq \frac{1}{n}. \tag{4}$$

As μ_i/w_j will be shown to be positively related to income, this condition expresses that consumers should have a high enough income to purchase all low quality varieties.

Figure 1 represents the per-quality input of varieties $z \in [0, n]$ produced in any country j. Consumptions of high and low quality varieties can readily be inferred for a consumer in country i. The latter has an inverse marginal utility μ_i and consumes the sets of high and low-quality varieties from j, $\mathcal{H}(\mu_i/w_j)$ and $\mathcal{L}(\mu_i/w_j)$. The first assumption imposes the equilibrium to lie below the highest value of ℓ while the second one constrains the equilibrium to lie above the highest curve $a_H(z)/b_H(z)$ and $a_L(z)/b_L(z)$.

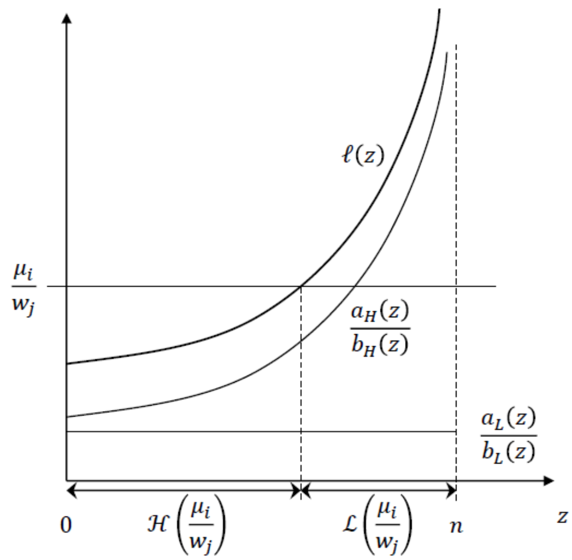

Figure 1: Country i' individual demand for high- and low-quality varieties from country j.

We denote the labor content of the set of varieties produced in country j and consumed by an individual in country i as

$$E\left(\frac{\mu_i}{w_j}\right) \equiv \int_{\mathcal{H}\left(\frac{\mu_i}{w_j}\right)} a_H(z)\mathrm{d}z + \int_{\mathcal{L}\left(\frac{\mu_i}{w_j}\right)} a_L(z)\mathrm{d}z.$$

This represents her *expenditure* on varieties imported from j in terms of importing country's wage. Using the above setting the function E successively reduces to

$$\begin{aligned} E(y) &= \int_{\mathcal{H}(y)} \left[a_H(z) - a_L(z)\right] \mathrm{d}z + \int_0^n a_L(z)\mathrm{d}z \\ &= \int_0^{\ell^{-1}(y)} \left[a_H(z) - a_L(z)\right] \mathrm{d}z + \int_0^n a_L(z)\mathrm{d}z \\ &= \int_0^{n-1/y} (n-z)^{-2}\,\mathrm{d}z + \int_0^n (n-z)^2\mathrm{d}z \\ &= y - n^{-1} + n^3/3 \end{aligned}$$

Hence,

$$E(y) = y - r,$$

where r is the constant

$$r = \frac{1 - n^3/3}{n} > 0.$$

Hence, the real expenditure function simplifies to a linear relationship. It is important to note that this linear relationship is not a knife edge case. As explained in Section 7, there indeed exists a large class of primitive functions yielding linear real expenditures. In this paper we have simply presented a convenient primitive. As we now show, such a linear relationship brings the important advantage to permit aggregation of countries' expenditures and allow for a closed-form linear solution of general equilibrium conditions. To our knowledge, the use of such properties is novel in the trade literature with non-homothetic preferences.

The total expenditure of an individual in country i simplifies to

$$E_i = \sum_{j=1}^{N} w_j E\left(\frac{\mu_i}{w_j}\right) = N\mu_i - r\left(\sum_{j=1}^{N} w_j\right). \tag{5}$$

Importantly, our model reduces to a set-up where wages w_j and inverse marginal utility of income μ_i appear in a linear way. Finally, to balance budget, expenditure E_i should equal to incomes $s_i w_i$. Using this in the above identity for real expenditure, we have

$$\mu_i = \frac{s_i w_i}{N} + \frac{r}{N}\sum_{l=1}^{N} w_l, \quad i \in \{1, ..., N\}. \tag{6}$$

The inverse marginal utility of income μ_i reflects the consumer's incentive to purchase an upgraded quality version of the good amongst her basket of low quality goods. Note that multiplying all prices by any constant scalar leads to multiply the value of μ_i by the same scalar. As a result μ_i/w_i and $\mathcal{H}(\mu_i/w_i)$ are invariant to global price increases. Demands for high- and low-quality goods are homogenous of degree zero.

To close the model, we express the trade balance condition for each country i, which equates the values of its imports and exports:

$$\sum_{l \neq i} m_i w_l E\left(\frac{\mu_i}{w_l}\right) = \sum_{l \neq i} m_l w_i E\left(\frac{\mu_l}{w_i}\right).$$

Adding $m_i w_i E\left(\mu_i/w_i\right)$ on both sides and substituting for E yields

$$\sum_{l=1}^{N} m_i\left(\mu_i - r w_l\right) = \sum_{l=1}^{N} m_l\left(\mu_l - r w_i\right), \quad i \in \{1, ..., N\}. \tag{7}$$

To sum up, our model is characterized by two sets of equations (6) and (7) that are linear in w_i and μ_i, $i \in \{1, ..., N\}$.

Finally, we establish three measures of interest for the sequel discussion. First, the average price of imports is given by

$$\overline{p}_{ij} \equiv \frac{1}{n}\left(\int_{\mathcal{H}(\mu_i/w_j)} w_j a_H(z)\mathrm{d}z + \int_{\mathcal{L}(\mu_i/w_j)} w_j a_L(z)\mathrm{d}z\right) = \frac{1}{n} w_j E\left(\frac{\mu_i}{w_j}\right). \tag{8}$$

Second, the share of high quality purchases in imported goods is equal to

$$\frac{\int_{\mathcal{H}(\mu_i/w_j)} \mathrm{d}z}{n} = \frac{\int_0^{\ell^{-1}\left(\frac{\mu_i}{w_j}\right)} \mathrm{d}z}{n} = 1 - \frac{1}{n}\frac{w_j}{\mu_i}$$

So, the ratio μ_i/w_j is a sufficient statistics for this share. Finally, the indirect utility simplifies to

$$\begin{aligned}
V_i &= \sum_{j=1}^{N}\left[\int_{\mathcal{H}\left(\frac{\mu_i}{w_j}\right)} b_H(z)\mathrm{d}z + \int_{\mathcal{L}\left(\frac{\mu_i}{w_j}\right)} b_L(z)\mathrm{d}z\right] \\
&= \sum_{j=1}^{N}\left[\int_0^{\ell^{-1}(\mu_i/w_j)} \left[b_H(z) - b_L(z)\right]\mathrm{d}z + \int_0^n b_L(z)\,\mathrm{d}z\right] \\
&= \sum_{j=1}^{N}\left[\int_0^{n-1/y} (n-z)^{-1}\,\mathrm{d}z + \int_0^n (n-z)\,\mathrm{d}z\right]
\end{aligned}$$

Hence, after simplifications,

$$V_i = \sum_{j=1}^{N} \ln\left(\frac{\mu_i}{w_j}\right) + N\left(\frac{n^2}{2} + \ln n\right). \tag{9}$$

As a result, the ratios μ_i/w_j are also sufficient statistics for utility. The second term in this expression highlights the presence of consumer's love for variety. Welfare indeed increases with each country's mass of varieties n and the number of such countries N. So, it increases with the total mass of varieties.

4 Equilibrium

A trade equilibrium is defined by the profiles of prices $p_H(z)$ and $p_L(z)$, $z \in [0, n]$, that make firms break even (condition (1)) in every country $j \in \{1, ..., N\}$, the vector of inverse marginal utility of income $\mu = (\mu_1,..., \mu_N)$ that matches individuals' optimal consumption choices at given prices (condition (6)), the vector of wages $w = (w_1, ..., w_N)$ that balances trade conditions (7). Finally, under condition (4), consumers buy all varieties and a mix of qualities at the equilibrium.

Since prices are directly derived from wages, it is sufficient to check the $2N$ conditions (7) and (6), which are linear in μ and w. Given demand homogeneity of degree zero and Walras law, the equilibrium is the solution of $2N-1$ equations and $2N-1$ values of w and μ. In the sequel we concentrate on the relative wage and marginal utility of income w_i/w_j and μ_i/w_j. Conditions (7) and (6) gives the following unique solution for relative wages

$$\frac{w_i}{w_j} = \frac{m_j s_j + r}{m_i s_i + r}. \tag{10}$$

The above first identity is remarkable because it is mainly expressed in terms of the countries's labor supply, $m_j s_j$. Relative wages between two countries w_i/w_j are inversely related to the ratio of their labor supplies. Very intuitively, larger labor supplies push the price of labor down.

Given the above, one gets the relative inverse marginal utility of income

$$\frac{\mu_i}{w_j} = \frac{1}{N}\left(\frac{w_i}{w_j}s_i + r\sum_{l=1}^{N}\frac{w_l}{w_j}\right). \tag{11}$$

Thus, the incentive to purchase high quality goods in country i from j, μ_i/w_j, increases with the individual's productivity s_i and relative wages w_i/w_j between countries i and j. The last identity can be written as function of the exogenous variables as

$$\frac{\mu_i}{w_j} = \frac{1}{N}\left(\frac{m_j s_j + r}{m_i s_i + r}s_i + r\sum_{l=1}^{N}\frac{m_j s_j + r}{m_l s_l + r}\right). \tag{12}$$

Hence, if it exists, the equilibrium is unique. The only restrictions for the existence is condition (4). For readability we focus on the existence of a trade equilibrium with symmetric countries where $m_i = m$ and $s_i = s$.

Proposition 1 *A symmetric country trade equilibrium exists and is unique for* $s \geq n^2 N/3$.

Proof. At the symmetric equilibrium $\mu_i/w_j = \mu^0 = s/N + r$. Condition (4) impose $\mu_i/w_j \geq 1/n$; that is, $s/N \geq 1/n - r = n^2/3$. ■

The symmetric country trade equilibrium exists for a large range of productivity levels. However, individual's productivity and therefore income must rise with the number of countries because, in this Armington model, consumers are required to purchase all varieties from each country. This contrasts to usual models with divisible goods. Finally, by continuity, trade equilibria exist for not too asymmetric country productivities.

In the sequel we assume a set of parameters such that a trade equilibrium exists. We now turn to the discussion of the properties of trade equilibria.

5 Properties

In this section, we discuss the equilibrium properties with respect to the countries' productivity and population sizes. We first consider the trade properties between country pairs because of their application in empirical studies.

5.1 Country pair properties

In this subsection, we compare the trade patterns of two countries with respect to a third trade partner. Such an approach is often used in econometric works to isolate the effects of each country's factors from the rest of the world. First note that, by (10), *a higher productivity s_i in country i reduces its wage relative to any other country.* This is because its labor supply rises while the mass of local variety does not change.

5.1.1 Exports from the same origin

Take two countries i and j importing from the same exporting country l ($l \neq i \neq j$). Then, by (11), we can write

$$\frac{\mu_i}{w_l} - \frac{\mu_j}{w_l} = \frac{1}{N} \frac{w_j s_j}{w_l} \left(\frac{w_i s_i}{w_j s_j} - 1 \right), \tag{13}$$

so that

$$\frac{\mu_i}{w_l} \geq \frac{\mu_j}{w_l} \iff \frac{w_i s_i}{w_j s_j} \geq 1.$$

Therefore, given that μ_i / w_l is a sufficient statistic for the larger share of high-quality varieties and its associated utility, the last condition states that *a country with larger per capita income imports a larger share of high-quality varieties from a same country l and gets a larger utility from its imports from country l.* By (8), it can further be shown that average import prices rank such as

$$\overline{p}_{il} \geq \overline{p}_{jl} \iff \frac{\mu_i}{w_l} \geq \frac{\mu_j}{w_l}.$$

Therefore, *the average import price is larger to the country with larger per capita income.* Empirically, one should finds a positive correlation between import prices and importer income per capita. Finally, by (10), the ratio of income per capita can be related to exogenous productivity parameters as

$$\frac{w_i s_i}{w_j s_j} = \frac{s_i / \left(m_i s_i + r \right)}{s_j / \left(m_j s_j + r \right)}$$

This implies that more productive countries import a larger share of high quality goods and have higher average import prices.

5.1.2 Imports from different origins

Take a country l that imports from two different exporting countries i and j ($l \neq i \neq j$). Then, by (11),

$$\frac{\mu_l}{w_i} - \frac{\mu_l}{w_j} = \frac{1}{N} \left(\frac{1}{w_i} - \frac{1}{w_j} \right) \left(w_l s_l + r \sum_{k=1}^{N} w_k \right).$$

So, we have

$$\frac{\mu_l}{w_i} \geq \frac{\mu_l}{w_j} \iff \frac{w_i}{w_j} \leq 1 \iff \frac{m_i s_i}{m_j s_j} \geq 1.$$

Therefore, country l imports a larger share of high-quality products from the country with higher labor supply. Controlling for exporter sizes, *country l imports a larger share of high quality varieties and thus have higher expenditures for the varieties manufactured by the more productive exporters.*

Using (8), one shows that average import prices rank such as

$$\overline{p}_{li} \geq \overline{p}_{lj} \iff w_i \leq w_j$$

Therefore, *the average import price to country l is larger for the goods shipped from more productive exporters.* Empirically, this should lead to an positive correlation between exporter income per capita and unit price.

5.1.3 Linder hypothesis

According to the Linder's (1961) hypothesis, richer countries trade more numerous high-quality goods with each other than poorer ones. To show this in the present model, consider three countries (i, j, l) with same size $(m_i = m_j = m_l)$ such that countries i and j have the same high productivity while country l is less productive $(s_i = s_j > s_l)$. Then, wages become

$$\frac{w_i}{w_j} = 1 > \frac{w_i}{w_l}.$$

The wage is lower in the more productive country because of its more abundant labor supply. This gives $w_i = w_j < w_l$. At the same time, from (11), the incentives to purchase high quality goods compare as follows:

$$\frac{\mu_i/w_j}{\mu_j/w_i} = 1 \quad \text{and} \quad \frac{\mu_i/w_j}{\mu_i/w_l} = \frac{w_l}{w_j} > 1.$$

From the first identity, we observe that the two more productive countries import the same range of high quality goods. From the second inequality, country i imports more numerous high-quality goods from the more productive country than from the lower productivity one. By symmetry, country j does the same. Hence, controlling for population sizes, *two high income countries specialize in the production of higher quality goods and trade more of those*, which confirms Linder (1961).

We now turn to the study of the effects of productivity and population size on the consumptions of high quality varieties.

5.2 Productivity changes

Consider an increase in the productivity s_i of country i. Then, its labor supply $m_i s_i$ rises and its wage falls relative to other countries as we compute

$$\frac{\mathrm{d}\left(w_i/w_j\right)}{\mathrm{d}s_i} = -\frac{m_i\left(m_j s_j + r\right)}{\left(m_i s_i + r\right)^2} < 0. \tag{14}$$

This depresses its relative prices and makes the country more competitive in international markets. As a result, every other country $j \neq i$ imports more numerous high-quality goods from country i, substituting for the trade of high quality goods with third countries $l \neq j \neq i$. Indeed, one can compute the changes in high quality imports into country j from countries i and $l \neq i$ as

$$\frac{\mathrm{d}\left(\mu_j/w_i\right)}{\mathrm{d}s_i} = m_i \frac{s_j + r\sum_{l=1,l\neq i}^{N} \frac{r+m_j s_j}{m_l s_l + r}}{N\left(r + m_j s_j\right)} > 0 \quad \text{and} \quad \frac{\mathrm{d}\mu_j/w_l}{\mathrm{d}s_i} = -\frac{r\left(m_l s_l + r\right)}{N\left(m_i s_i + r\right)^2} < 0.$$

At a given wage, country i's workers benefit from larger incomes and from cheaper production of local high-quality goods. But, although their relative wage falls and import prices become higher relative to their incomes, they import a wider range of high quality goods as indeed,

$$\frac{\mathrm{d}\left(\mu_i/w_j\right)}{\mathrm{d}s_i} = r\frac{\left(1 - m_i\right)\left(r + m_j s_j\right)}{N\left(r + m_i s_i\right)^2} > 0.$$

They however purchase a larger range of local high variety goods as

$$\frac{\mathrm{d}\left(\mu_i/w_i\right)}{\mathrm{d}s_i} = \frac{1}{N}\left(1 + r\sum_{l=1,l\neq i}^{N} \frac{m_i s_i + r}{m_l s_l + r}\right) > 0.$$

Proposition 2 *In the equilibrium of trade network with N countries, a rise in productivity of country i entices this country to specialize in high quality goods. Country i consumes a wider range of local and imported high quality varieties. Other countries import more high quality varieties from country i and less from each other.*

One consequence of the proposition is that the average quality of home imports increases when the home productivity rises. The result supports Jaimovic and Merella's (2012) study.

5.3 Population changes

Consider an infinitesimal increase in country i's population size, $\mathrm{d}M_i$. Keeping constant other countries' populations, this impacts the population ratios of all countries as follows:

$$\mathrm{d}m_i = \frac{M_i + \mathrm{d}M_i}{M + \mathrm{d}M_i} - \frac{M_i}{M} \simeq (1 - m_i)\frac{\mathrm{d}M_i}{M},$$
$$\mathrm{d}m_j = \frac{M_j}{M + \mathrm{d}M_i} - \frac{M_j}{M} \simeq -m_j\frac{\mathrm{d}M_i}{M}.$$

It increases country i's population ratio m_i and decreases other countries' m_j, $j \neq i$, in proportion to global population changes $\mathrm{d}M_i/M$ and initial population distributions. Combining this with the effects of population ratios on μ_i/w_j we can establish the following comparative statics properties. First, there is a decrease in wage for country i relative to other countries $j \neq i$. Indeed, we show in the Appendix that $\mathrm{d}\left(w_i/w_j\right)/\mathrm{d}M_i < 0$. This is because country i's population growth raises labor supply and decreases local production cost and product prices. As their local prices fall and import prices rise, individuals in country i have incentive to augment their consumption of local high-quality varieties. We indeed show that $\mathrm{d}\left(\mu_i/w_i\right)/\mathrm{d}M_i > 0$ while $\mathrm{d}\left(\mu_i/w_j\right)/\mathrm{d}M_i < 0$ if countries' labor supplies are close to symmetry ($s_l m_l \simeq s_j m_j$).

Proposition 3 *Consider a rise in the population of country i in a trade network with N country. This implies:*

- *a decrease in wage for country i relative to other countries $j \neq i$;*
- *a rise in country l's wage relative to country j's if l has a larger effective labor supply than j ($m_l s_l > m_j s_j$);*
- *a rise in country i's consumption of its local high-quality goods;*
- *a decrease in the range of high-quality imports consumed by country i's consumers, if countries are sufficiently symmetric.*

The first line of Proposition 3 is intuitive. A larger domestic population increases labor supply in country i and reduces local wages. Therefore, the growing country incurs a fall in its wage with respect to each other trade partner. By the same token, other countries have a rise in their wages relative to country i.

The terms of trade between each other countries also change: a country l has a rise in its wage compared to country j if it has a larger effective labor supply $m_j s_j > m_l s_l$. Moreover, the fall in wages negatively affect domestic consumers' purchasing power so that they buy fewer high-quality local goods.

The effects of a rise in country i population on high quality imports is unclear. The first part of (20) in the appendix, is always negative, reflecting the fall in wage due to the increase in supply in country i. The second effect in the second part of the equation is ambiguous, and it is determined by the differences in effective labor supplies of other countries, which affect the interplays of wages among countries. Suppose, for instance, that country j has the highest effective labor supply of the whole economy. Then, purchasing goods from country j becomes more expensive for country i consumers, who reduce the number of high-quality goods imported from j. If conversely, country j has a very low effective labor supply, the effect due by the difference in productivity of other countries might be positive for high quality import of country i and might also compensate the fall in wage.

Finally, if countries are symmetric, the increase in population depresses the range of high quality goods purchased by country i. In this case the effect of differences in productivity is nil, leaving the fall in purchasing power driven by the decrease in country i wages.

6 Ad-valorem trade costs

We consider the presence of symmetric ad valorem (iceberg) trade costs $\tau_{ij} \geq 1$ where a share $1/\tau_{ij}$ of each good arrives at destination i after shipment from country j. Trade costs are symmetric across countries and nil within countries: $\tau_{ji} = \tau_{ij}$ and $\tau_{ii} = 1$. Accordingly, the (destination) consumption price of an unit z imported from country j to country i is given by $p_{ijk}(z) = \tau_{ij} w_j a_k(z)$, $k = H, L$, and an individual in country i with inverse marginal utility μ_i will purchase all high-quality imports z if $\mu_i/(\tau_{ij} w_j) \geq \ell(z)$. Incentives to purchase high quality goods are then given by the statistics $\mu_i/(\tau_{ij} w_j)$: the higher this is, the wider the range of consumed high-quality imports. Hence, ceteris paribus, a higher τ_{ij} entices consumers to reduce their range of high quality goods. Using the same argument for (4), it can be shown that the requirement $\mu_i/(\tau_{ij} w_j) \geq 1/n$ entices every consumer to buy a mix of all goods with high and low quality versions.

Following the previous procedure and using the above definition of E, the expenditure of an individual in country i for goods produced in j is successively given by

$$\begin{aligned}
E_{ij} &\equiv \int_{\mathcal{H}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)} \tau_{ij} w_j a_H(z) dz + \int_{\mathcal{L}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)} \tau_{ij} w_j a_L(z) dz \\
&= \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) \\
&= \tau_{ij} w_j \left(\frac{\mu_i}{\tau_{ij} w_j} - r\right) \\
&= \mu_i - r\tau_{ij} w_j.
\end{aligned}$$

Her income is equal to her total expenditure: $w_i s_i = E_i \equiv \sum_{j=1}^{N} E_{ij}$. That is,

$$w_i s_i = N\mu_i - r\sum_{j=1}^{N} \tau_{ij} w_j.$$

This gives the incentives to purchase as a function of relative factor prices and trade costs:

$$\frac{\mu_i}{\tau_{ij} w_j} = \frac{1}{N} \frac{s_i}{\tau_{ij}} \frac{w_i}{w_j} + \frac{r}{N} \sum_{l=1}^{N} \frac{\tau_{il}}{\tau_{ij}} \frac{w_l}{w_j}. \tag{15}$$

In country i trade balances the value of imports and exports as

$$\sum_{j \neq i}^{N} m_i \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) = \sum_{j \neq i}^{N} m_j \tau_{ji} w_i E\left(\frac{\mu_j}{\tau_{ji} w_i}\right),$$

Given the linear expenditure function, the balanced trade condition simplifies to

$$\sum_{j=1}^{N} m_i \left(\mu_i - r \tau_{ij} w_j\right) = \sum_{j=1}^{N} m_j \left(\mu_j - r \tau_{ji} w_i\right).$$

It is useful to denote the country i's average ad-valorem trade cost $\overline{\tau}_i \equiv 1 + \sum_{j=1}^{N} m_j \left(\tau_{ij} - 1\right)$ where the second term measures the average trade cost of country i's exports weighted by the export destination populations. Hence, the relative factor prices and incentives to purchase high-quality goods simplify to

$$\frac{w_i}{w_j} = \frac{m_j s_j + r\overline{\tau}_j}{m_i s_i + r\overline{\tau}_i}, \tag{16}$$

$$\frac{\mu_i}{\tau_{ij} w_j} = \frac{1}{N\tau_{ij}} \left(\frac{m_j s_j + r\overline{\tau}_j}{m_i s_i + r\overline{\tau}_i} s_i + r \sum_{l=1}^{N} \tau_{il} \frac{m_j s_j + r\overline{\tau}_j}{m_l s_l + r\overline{\tau}_l} \right). \tag{17}$$

Those expressions compare to the ones without trade costs.

Finally, we recall our three measures of interest. The share of high quality purchases in imported goods is given by

$$\frac{\int_{\mathcal{H}(\mu_i/\tau_{ij} w_j)} \mathrm{d}z}{n} = 1 - \frac{1}{n} \frac{\tau_{ij} w_j}{\mu_i}.$$

The indirect utility in country i simplifies to

$$V_i = \sum_{j=1}^{N} \ln\left(\frac{\mu_i}{\tau_{ij} w_j}\right) + N\left(\frac{n^2}{2} + \ln n\right).$$

As a result, the ratios $\mu_i/\tau_{ij} w_j$ are also sufficient statistics for the share of high quality goods and the utility from imports. Because of trade costs, the average import prices must be distinguished by whether they are evaluated at origin or destination. Following international trade terminology, freight on board (fob) prices do not include trade costs while cost, insurance & freight (cif) prices include them. Exports are most generally reported in fob values at the borders of exporting countries and imports are denominated in cif prices at the gates of importing countries. As a result, we extend our earlier definition of average prices as

$$\overline{p}_{ij}^{\text{fob}} = \frac{1}{n} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) \quad \text{and} \quad \overline{p}_{ij}^{\text{cif}} = \tau_{ij} \overline{p}_{ij}^{\text{fob}} = \frac{1}{n} \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right). \tag{18}$$

6.1 Symmetric countries

To make thing clear, we firstly consider the case of symmetric countries and trade costs ($s_i = s$, $m_i = 1/N$, $\tau_{ij} = \tau$, $i \neq j$ while $\overline{\tau}_i \equiv \overline{\tau} = 1 + (\tau - 1)(N - 1)/N$). Equilibrium conditions simplify as

$$\frac{w_i}{w_j} = 1, \quad \frac{\mu_i}{w_i} = \frac{1}{N}\left[s + r + r\tau(N-1)\right] \quad \text{and} \quad \frac{\mu_i}{\tau_{ij} w_j} = \frac{1}{N}\left[\frac{s+r}{\tau} + r(N-1)\right].$$

Hence, a global fall in ad-valorem trade cost (lower τ) entices workers to consume fewer local high-quality goods (μ_i/w_i falls) and a larger share of high-quality imports ($\mu_i/(\tau_{ij} w_j)$ rises). The trade equilibrium exists if $\mu_i/(\tau_{ij} w_j) \geq 1/n$; that is, after simplifications, if $s \geq \tau N/n + r\tau(N-1) - r$. As trade costs rises, individuals' productivity s must be increased as to sustain consumption of all goods.

Denoting the wages by w, the average fob and cif prices compute as

$$\overline{p}_{ij}^{\text{fob}} = \frac{w}{nN}\left(\frac{s+r}{\tau} - r\right) \quad \text{and} \quad \overline{p}_{ij}^{\text{cif}} = \tau \overline{p}_{ij}^{\text{fob}} = \frac{w}{nN}(s + r - r\tau).$$

So, both average prices rise with the fall in trade cost. Lower trade costs indeed entice consumers to import a larger share of high-quality goods, which pushes up the average fob price. Interestingly, the average cif price rises. Consumers increase more their expenditure on import than what they save on trade cost. This is because they reduce their purchases of local high-quality goods. This can be expressed in the country utility, which successively computes as

$$\begin{aligned} V_i &= N \ln \frac{\mu_i}{w_i} - (N-1)\ln\tau + \text{constant}, \\ &= N \ln\left[s + r + r\tau(N-1)\right] - (N-1)\ln\tau + \text{constant} \end{aligned}$$

The first and second terms express the impact of local consumption and the effect of trade cost on imports. It can be shown that the utility falls with τ under the above trade equilibrium existence condition. By a continuity argument, the same properties apply for not too dissimilar countries.

Proposition 4 *Suppose symmetric countries with an identical ad-valorem trade cost. There exists a unique equilibrium for $s \geq \tau N/n + r\tau(N-1) - r$. A fall in this trade cost entices each country to consume as smaller share of high-quality goods from home and a larger one from* *abroad. It boosts exports of high quality goods, increases both average* fob *and* cif *prices and finally raises utility everywhere.*

This proposition highlights the trade-off between quality and trade cost for fixed number and quantity of goods consumed. It therefore complements the trade literature about the trade-offs between trade costs, intensive and extensive margins of trade.

Whereas the above text discusses the effect of a common value of bilateral trade cost, we now study the effect of discrepancies in such cost. We therefore consider the same country pairs as in Subsection 5.1 but add idiosyncratic bilateral trade costs.

6.2 Exports from the same origin

Take two countries i and j importing from the same exporter l ($l \neq i \neq j$). We know that high-quality import shares and utility from those imports depend on the incentives to buy high-quality goods $\mu_i / (\tau_{il} w_l)$ and $\mu_j / (\tau_{jl} w_l)$. Interestingly, the comparison of average fob import prices also depend on those ratios since, using (18), one gets

$$\overline{p}_{il}^{\text{fob}} \geq \overline{p}_{jl}^{\text{fob}} \iff \frac{\mu_i}{\tau_{il} w_l} \geq \frac{\mu_j}{\tau_{jl} w_l}.$$

Then, cross-country comparisons between high-quality import shares, utility and average fob import prices can be studied with the differences in incentives to buy high-quality goods. By (15), the latter computes as

$$\frac{\mu_i}{\tau_{il} w_l} - \frac{\mu_j}{\tau_{jl} w_l} = \frac{1}{N w_l} \left(\frac{s_i w_i + r \sum_{h=1}^{N} w_h + r \sum_{h=1}^{N} (\tau_{ih} - 1) w_h}{\tau_{il}} - \frac{s_j w_j + r \sum_{h=1}^{N} w_h + r \sum_{h=1}^{N} (\tau_{jh} - 1) w_h}{\tau_{jl}} \right), \tag{19}$$

which reduces to (13) in the absence of trade cost. The difference in high-quality import shares and average fob import prices depends on the difference between each term in the parentheses. Ceteris paribus, high-quality import share and average fob import prices in country i are larger when the first term becomes larger. This occurs if country i has higher per-capita income $s_i w_i$, lower bilateral trade cost τ_{li} and higher remoteness measured by the average trade cost $\sum_{h=1}^{N} (\tau_{ih} - 1) w_h$. The same hold for utility of imports. Note that the average trade cost depends on *all* bilateral trade costs and therefore has the same function as Anderson and Van Wincoop's (2003) "multilateral resistance". A rise in trade barriers with all trading partners raises this index.

To express the above condition as a function of exogenous parameters, it is convenient to define the following *average relative price:*

$$\omega_i \equiv \frac{w_i}{\frac{1}{N}\sum_j w_j} = \frac{\left(m_i s_i + r\overline{\tau}_i\right)^{-1}}{\frac{1}{N}\sum_j \left(m_j s_j + r\overline{\tau}_j\right)^{-1}}.$$

It is smaller in a country i that has higher supply of labor units $m_i s_i$ relative to other countries. This translates an average deterioration of its terms of trade. Ceteris paribus, it is also smaller for a relatively more remote country i (higher $\overline{\tau}_i$), which reflects a deterioration of terms of trade caused by a lower international demand for its exports. Substituting for μ_i and μ_j, we obtain

$$\frac{\mu_i}{\tau_{il}w_l} - \frac{\mu_j}{\tau_{jl}w_l} = \frac{1}{N\omega_l}\left[\frac{1}{\tau_{il}}\left(s_i\omega_i + rN + r\sum_{h=1}^{N}\left(\tau_{ih}-1\right)\omega_h\right) - \frac{1}{\tau_{jl}}\left(s_j\omega_j + rN + r\sum_{h=1}^{N}\left(\tau_{jh}-1\right)\omega_h\right)\right]$$

The structure of this condition is the same as (19) after substitution of w_i by ω_i. So, ceteris paribus, the high-quality import share and average fob import price $\overline{p}_{il}$ in country i are larger when the latter country has higher productivity s_i, average relative price ω_i, lower bilateral trade cost τ_{li} and higher remoteness measured by the average 'relative' trade cost $\sum_{h=1}^{N}\left(\tau_{ih}-1\right)\omega_h$. However, average relative price ω_i also falls with remoteness, as measured by $\overline{\tau}_i$. So, the impact of remoteness is a priori unclear.

These effects compare with ones found in the top panel and second column of Table 1. The main discrepancy between theoretical prediction and empirical observation appears in the effect of distance. As shown in Hummels and Skiba (2004), this discrepancy lies in the assumption of iceberg trade costs rather unit trade/transaction cost. Similar effects of distance and remoteness have been empirically verified by Zhang and Manova (2012), Crozet et al. (2012) and others.

6.3 Imports from different origins

Now, consider a country l that imports from two different exporters i and j ($l \neq i \neq j$). Using (18), we obtain the following conditions on the ranking of average cif price:

$$\overline{p}_{li}^{\text{cif}} \geq \overline{p}_{lj}^{\text{cif}} \iff \frac{w_i}{w_j} \leq \frac{\tau_{lj}}{\tau_{li}} \iff 1 \leq \frac{\left(m_i s_i + r\overline{\tau}_i\right)/\tau_{li}}{\left(m_j s_j + r\overline{\tau}_j\right)/\tau_{lj}}$$

Therefore, after controlling for productivity, population and remoteness ($m_i s_i = m_j s_j$ and $\overline{\tau}_i = \overline{\tau}_j$), the average cif price is higher in the importing country with lower bilateral trade barriers. Similarly, after controlling for productivity, population and bilateral trade barriers ($m_i s_i = m_j s_j$ and $\tau_{li} = \tau_{lj}$), the average cif price is higher in the importing country i facing a larger remoteness, defined as average trade cost $\overline{\tau}_i$. Those effects correspond to the one found in the bottom panel and second column of Table 1. Again, such effects of distance and remoteness have been empirically verified by Zhang and Manova (2012), Crozet et al. (2012) and others.

The shares of high quality imports and their contribution to utility increase with the ratios $\mu_l/(\tau_{il}w_i)$ and $\mu_l/(\tau_{jl}w_j)$. To compare high-quality shares and utility contributions of imports from various countries, we simply study the difference

$$\frac{\mu_l}{\tau_{li}w_i} - \frac{\mu_l}{\tau_{lj}w_j} = \frac{1}{N}\left(\frac{1}{\tau_{li}w_i} - \frac{1}{\tau_{lj}w_j}\right)\left(s_l w_l + r\sum_{h=1}^{N} w_h\right)$$
$$+ r\left(\frac{1}{\tau_{li}w_i} - \frac{1}{\tau_{li}w_j}\right)\frac{1}{N}\sum_{h=1}^{N}\left(\tau_{hl}-1\right)w_h.$$

When this expression is positive, exporter i ships a higher share of high-quality goods to importer l than exporter j. The first term in the RHS measures the direct effect of trade barrier and is equivalent to the expression obtained in the absence of trade costs. Accordingly, a higher bilateral trade barrier between i and l, relatively to that between j and l, entices exporter i to ship a smaller share of high-quality goods to the importing country l than what exporter j does. This in turn implies that country l gets a higher utility out of its imports from country i. The second term in the RHS adds the effect of remoteness of importing country l, measured by its average trade cost $\frac{1}{N}\sum_{h=1}^{N}\left(\tau_{hl}-1\right)w_k$. It can then be seen that higher remoteness amplifies the effects of bilateral trade costs on high-quality import shares and utility from imports.

6.4 Gravity

We end up with the discussion of the traditional gravity equation that expresses trade values as functions of local incomes and distances. Country j's export to country i is captured by the expenditure and number of high quality variety, which increases with the statistics $\mu_i/(\tau_{ji}w_j)$. The (nominal) expenditure on import from j to i (at cif prices) is given by

$$E_{ij}^{\text{cif}} = \tau_{ij}w_j E\left(\frac{\mu_i}{\tau_{ij}w_j}\right) = \frac{1}{N}s_i w_i - r\tau_{ij}w_j + \frac{r}{N^2}\sum_{l=1}^{N}\tau_{il}w_l.$$

From this expression, it comes that trade expenditure rises with importer's higher income per capita $s_i w_i$, higher exporter's wage w_j, lower bilateral trade cost τ_{ij} and higher remoteness, here measured by $\sum_{l=1}^{N}\tau_{il}w_l$.

The above gravity equation includes exporter's wage rather then income. We can substitute wage by income using the following procedure. Assuming that trade cost is paid in exporting country's labor, we note the national income is sequentially given by

$$
\begin{aligned}
Y_j &= \sum_{h=1}^{N} m_h E_{hj}^{\text{cif}} \\
&= \frac{1}{N}\sum_{h=1}^{N} m_h s_h w_h - r w_j \sum_{h=1}^{N} m_h \tau_{hj} + \frac{r}{N^2}\sum_{l=1}^{N}\sum_{h=1}^{N} m_h \tau_{hl} w_l \\
&= \frac{1}{N}\sum_{h=1}^{N} m_h s_h w_h - r w_j \overline{\tau}_j + \frac{r}{N^2}\sum_{l=1}^{N} \overline{\tau}_l w_l,
\end{aligned}
$$

So, we can extract the wage as

$$
r w_j = -\frac{Y_j}{\overline{\tau}_j} + \frac{1}{N}\frac{1}{\overline{\tau}_j}\sum_{h=1}^{N} m_h s_h w_h + \frac{r}{N^2}\frac{1}{\overline{\tau}_j}\sum_{l=1}^{N} \overline{\tau}_l w_l
$$

and compute the world income as

$$
\sum_{h=1}^{N} Y_h = \sum_{h=1}^{N} m_h s_h w_h - r\frac{N-1}{N}\sum_{h=1}^{N} w_h \overline{\tau}_h
$$

We finally plug this back to the gravity equation, which gives

$$
E_{ij} = \frac{1}{N} s_i w_i + \frac{\tau_{ij}}{\overline{\tau}_j}\left[Y_j - \frac{1}{N}\sum_{h=1}^{N} Y_h - \frac{r}{N}\sum_{h=1}^{N} w_h \overline{\tau}_h \right] + \frac{r}{N^2}\sum_{l=1}^{N} \tau_{il} w_l .
$$

The import expenditure rises with higher importer's income per capita $s_i w_i$ and now with higher exporter's national income Y_j. Note that the squared bracket term is negative if exporter j has a national income no higher than average or/and countries are close to symmetry. In that case, the import expenditure falls with bilateral trade cost τ_{ij} and increases with the remoteness indicators $\overline{\tau}_j$ of exporter j. The latter expresses again a "multilateral resistance" à la Anderson and Van Wincoop's (2003). Under those conditions, our theoretical results match our empirical evidence in the third column and bottom panel of Table 1. Finally, the last term $\sum_{l=1}^{N} \tau_{il} w_l$ is the remoteness indicator associated to importer i, which is pined down as the inward multilateral resistance in Anderson and Van Wincoop's (2003) and reflects a rise in expenditure due to importer's remoteness to its trade partners. This effect is empirically confirmed in the third line, third column and top panel of Table 1.

7 Discussion

Before concluding the paper, we find it important to discuss the choice of our primitives and the confirmation of the Alchian and Allen conjecture.

7.1 Primitives

The main methodological innovation of the paper has been to select the primitives on cost and product quality leading to real expenditures that are linear functions of inverse marginal utility. This facilitates the aggregations of individual budgets and trade balances. As a consequence, the general equilibrium is the solution of a set of linear conditions of inverse marginal utility. Some generality is lost in the process, but not too much. Indeed, the primitives (a_H, a_L, b_H, b_L) consist of a quadridimensional functional space while linear real expenditure imposes a single restriction on functionals:

$$E'(y) = 1,$$

where the RHS is set to one for simplicity. Hence, the primitives supporting this class of primitives consists of a tridimensional functional space. To be more precise, it can be shown that the above condition is equivalent to

$$\frac{(a'_H - a'_L)}{(a_H - a_L)} = (b_H - b_L) + \frac{(b'_H - b'_L)}{(b_H - b_L)},$$

where the primes $'$ denotes derivatives with respect to the product address z. This identity maps cost upgrades to product quality upgrades. Therefore, one can choose any low cost profile a_L and low quality profiles b_L as well as a positive cost upgrade profile $a_H - a_L$ that yields a positive quality upgrade profile $b_H - b_L$ according to this identity. The corresponding high quality profile b_H is obtained by summing the latter to b_L. Many primitives can be obtained with other functional forms.[6]

The above identity shows the restriction imposed by linear real expenditure on cost and quality upgrades. If one abstracts from the first term on the RHS, the identity suggests that acceptable cost and quality upgrades must rise at the same rate as z increases. In other words, cost upgrades should be large for goods that have large quality upgrades. This seems to be an acceptable and intuitive assumption on the primitives of the model.

7.2 Alchian Allen conjecture

The previous section discussed the role of ad-valorem trade cost in the quality composition of traded goods. Such trade costs do not explain the Alchian and Allen effect according to which exports are biased towards high quality goods for more distant trading partners. The effect is apparent in Table 1 where fob export prices rise with distance from US to its trade partners. Hummels and Skiba (2004) highlight this same effect in a very comprehensive way and emphasize an explanation through the existence of *unit* trade costs that accrue on each good

[6]For instance, one can expand our primitives to the following class: $a_H(z) = a_L(z) + (n-z)^{-2}$ and $b_H(z) = b_L(z) + (n-z)^{-1}$ where $a_L(z)$ and $b_L(z)$ are any functions of z. One can also use other combinations of power functions. For example, the primitives $a_H(z) = 2a_L(z) = 2\alpha(z+1)^{\alpha-1}$, $b_H(z) = (\alpha+\beta)/(z+1)$ and $b_L(z) = \beta/(z+1)$ with $\alpha > 1$ and $\beta > 0$ give the real expenditure function is $E(y) = y - r$ where $r = 2 - (1+n)^{\alpha}$.

independently on their value. When unit trade costs increase, consumers are enticed not only to purchase fewer goods in total but also to consume relatively fewer low quality goods. This is because a rise in unit trade cost has a relatively stronger impact on the low-cost low-quality version of a good than on its corresponding high-cost high-quality version.

To encompass the Alchian and Allen conjecture, we must change our model by allowing consumers to purchase fewer than n goods from each producing country. To fix ideas, we focus on Hummel and Skiba's (2004) partial equilibrium analysis by fixing relative prices w_j and inverse marginal utility μ_i. For the sake of generality, we resume to the model with general primitives $a_k(z)$ and $b_k(z)$, $k = L, H$, and assume an identical unit trade cost t so that consumer prices become $p_{ijk}(z) = (a_k(z) + t)\, w_j$. Under such a unit trade cost, the per-quality input $\ell(z)$ is independent of the unit trade cost t because the prices of high and low quality goods rise by the same cost amount tw_j and cancel at the numerator of $\ell(z)$. The choice for high quality over low quality therefore is driven by the exactly same condition as before: $\mu_i/w_j \geq \ell(z)$.

In this subsection, we are interested in the situation where consumers purchase only a subset of the low quality goods. That is, we consider the sets of high and low quality purchases $\mathcal{H}(\mu_i/w_j) = [0, \ell^{-1}(\mu_i/w_j)]$ and $\mathcal{L}(\mu_i/w_j) = (\ell^{-1}(\mu_i/w_j), \widetilde{n}(\mu_i/w_j, t)]$ where $\widetilde{n}(\mu_i/w_j, t) < n$ is the number of purchased goods. The latter solves the binding full market coverage condition: $\mu_i/w_j = (a_L(\widetilde{n}) + t)/b_L(\widetilde{n})$. Figure 2 depicts this situation where the consumer does not purchase all goods. To be in such a configuration, it is assumed that some low quality good are not purchased, $\mu_i/w_j < (a_L(n) + t)/b_L(n)$ and that high quality goods are purchased when they are preferred over low quality ones:

$$\ell(z) > \mu_i/w_j \Rightarrow \ell(z) \geq (a_H(z) + t)/b_H(z).$$

It is further assumed that the number of purchased goods falls with t; that is $\widetilde{n}_t \equiv \partial\widetilde{n}/\partial t = 1/\left[b_L'(\widetilde{n})(\mu_i/w_j) - a_L'(\widetilde{n})\right] < 0$.

We are now equipped to verify the existence of the Alchian and Allen conjecture according to which the average fob price increases with larger t. The fob price of good z is given by $p_{ijk}^{\text{fob}}(z) \equiv a_k(z)w_j$, $k = L, H$, while the average fob price is equal to

$$\overline{p}_{ij}^{\text{fob}} = \frac{1}{\widetilde{n}(\mu_i/w_j, t)}\left[\int_0^{\ell^{-1}(\mu_i/w_j)} a_H(z)w_j \mathrm{d}z + \int_{\ell^{-1}(\mu_i/w_j)}^{\widetilde{n}(\mu_i/w_j, t)} a_L(z)w_j \mathrm{d}z\right].$$

Since the function ℓ and its inverse ℓ^{-1} are independent of t, we get

$$\frac{\mathrm{d}\overline{p}_{ij}^{\text{fob}}}{\mathrm{d}t} = \left[a_L(\widetilde{n})w_j - \overline{p}_{ij}^{\text{fob}}\right]\frac{\widetilde{n}_t}{\widetilde{n}},$$

where $\widetilde{n}$ is evaluated at $(\mu_i/w_j, t)$. As a result, because $\widetilde{n}_t < 0$, a rise in unit trade cost increases

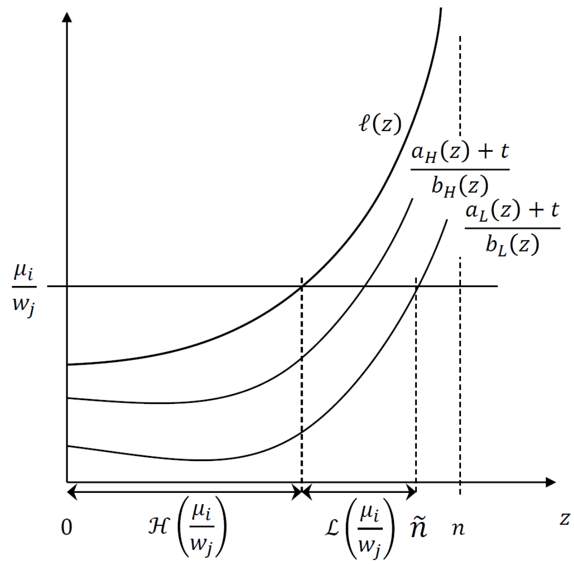

Figure 2: Country's individual demands when not all goods are consumed.

the average fob price if and only if

$$a_L(\widetilde{n})w_j = p_{ijL}^{\text{fob}}(z) \leq \overline{p}_{ij}^{\text{fob}}.$$

That is, each low quality good dropped by consumers has a price lower than the average price of the basket. As sufficient condition for this property is that a_L is a non-increasing function of z.

Proposition 5 *Suppose that relative prices and marginal utilities are constant, consumers purchase only a subset of the low quality goods and the number of purchased goods falls with larger unit trade cost t. Then, the average fob price increases with t if the low quality labor input profile a_L is a non-increasing function of z.*

Proof. Since $a_H > a_L$ and a_L is a non-increasing function, we successively have that $\overline{p}_{ij}^{\text{fob}} > \frac{w_j}{\widetilde{n}}\left[\int_0^{\widetilde{n}} a_L(z)\text{d}z\right] \geq w_j a_L(\widetilde{n})$, where $\widetilde{n}$ is evaluated at $(\mu_i/w_j, t)$ ∎

Hence, a sufficient condition is that the lowest quality goods dropped by consumers have lowest prices. The above Proposition is fulfilled by the primitives presented in Section 3 for small enough unit trade cost and high enough inverse marginal utility levels: $t < 1 - n^2$ and $\mu_i/w_j > 2\sqrt{t}$. Indeed, $a_L(z) = n - z$ is a decreasing function of z while the subset of low quality purchases is given by $\widetilde{n}(y,t) = n - \frac{1}{2}\left(y - \sqrt{y^2 - 4t}\right)$, which is a decreasing function of t (see details in Appendix).

8 Concluding remarks

In this paper we have analyzed a trade model where preferences are non-homothetic, each product is versioned in two different qualities and where a many countries exhibit different size and productivity. Once we derived the equilibrium, we have first examined the effects of differences in productivity among countries. We have shown that a rise in the productivity of one country implies a fall in domestic wage relative to other countries. Richest countries demand more high-quality varieties from abroad. Between two countries of same size, the more productive specializes in exporting goods of higher quality. Finally, high-income countries specialize in the production of high-quality goods and trade more of those, as suggested by the Linder hypothesis (1961).

We have then investigated the effects of changes in population and productivity in one country. An increase in population induces a decrease in relative prices and, subsequently, in the consumption of high quality goods. An rise in productivity favors the consumption of local high-quality goods only if the relative size of the country is sufficiently small, while high quality exports decrease. Finally, our results support the Alchian and Allen conjecture, which suggests that countries import higher quality goods from more distant countries. Our theoretical framework help explaining important empirical regularities in the trade literature.

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Appendix

Empirical specification

To highlight the role of quality on trade, we use a number of identification strategies. First, we test the logs of unit prices of imports p^m (exports p^x) on the logs of exporter (importer) GDP per capita (Y_i), distance (D_i) and remoteness (R_i) of country i. As standard, we use distance and remoteness represent a measure of trade costs. Moreover, we take into account of time and products using product (d_s) and time (d_t) dummies:

$$\log p^k_{zsit} = \beta_0 + \beta_1 \log Y_{it} + \beta_2 \log D_i + \beta_3 R_{it} + \beta_4 d_s + \beta_5 d_t + \varepsilon_{zsit}, \quad k = m, x.$$

For $k = m$, the dependent variable is the import unit-price p^m_{zsit} of a trade transaction z in the (NAICS 6 digit) category s of products originating from country i in year t. For $k = x$, the same specification holds with export prices p^m_{zsit} and destination country i.

Second, our dataset allows us to evaluate the effects of quality on imports from the same origin, using the data on U.S. export, and the results of imports from different origins, using the data on U.S. import. In addition, we are able to verify the validity of the Linder hypothesis in this setting. To do so, we aggregate the average prices of each product by country (Armington, 1969, Feenstra, 1994),

$$\overline{p}^k_{sit} = \frac{1}{N^k_{sit}} \sum_{z=1}^{N_{sit}} p^k_{zsit},$$

where $\overline{p}^k_{sit}$ denotes the average unit price of product s imported/exported by country i at year t, while N^k_{sit} denotes the number of transactions between the U.S. and country i. For each product s, we regress the ratio of average unit prices by countries, using the log of the ratio of GDP per capita, distance and remoteness as control variables, as well as years and products dummies:

$$\log \frac{\overline{p}^k_{sit}}{\overline{p}^k_{sjt}} = \gamma_0 + \gamma_1 \log \frac{Y_i}{Y_j} + \gamma_2 \log \frac{D_i}{D_j} + \gamma_3 \log \frac{R_i}{R_j} + \gamma_4 d_s + \gamma_5 d_t + v_{sit}.$$

In the last exercise, we test the validity of the gravity equation. We do so by evaluating the gross values of imports/exports by product, year and country,

$$E^k_{sit} = \sum_z p_{zsit} q_{zsit}.$$

The log of expenditure is then estimated over the usual log of distance, remoteness and product and year dummies:

$$\log E^k_{sit} = \beta_0 + \beta_1 \log Y_{it} + \beta_2 \log D_i + \beta_3 \log R_{it} + \beta_4 d_s + \beta_5 d_t + \sigma_{sit}.$$

For all the three exercises, we employ a pooled OLS. As a robustness check, we then perform the same analysis by restricting the attention to manufacturing goods (Fieler, 2012).

Population changes

Consider an absolute increase in the population size M_i of country i by $\mathrm{d}M_i$. This implies the simultaneous first order changes in relative population sizes

$$\begin{aligned}
\mathrm{d}m_i &= \frac{M_i + \mathrm{d}M_i}{M + \mathrm{d}M_i} - \frac{M_i}{M} \simeq (1 - m_i)\frac{\mathrm{d}M_i}{M}, \\
\mathrm{d}m_j &= \frac{M_j}{M + \mathrm{d}M_i} - \frac{M_j}{M} \simeq -m_j\frac{\mathrm{d}M_i}{M}.
\end{aligned}$$

Hence, for any variable X, an increase in the population size M_i implies

$$\frac{\mathrm{d}X}{\mathrm{d}M_i} = \frac{\partial X}{\partial m_i}\frac{\mathrm{d}m_i}{\mathrm{d}M_i} + \sum_{k\neq i}\frac{\partial X}{\partial m_k}\frac{\mathrm{d}m_k}{\mathrm{d}M_i} = \frac{1}{M}\left[(1-m_i)\frac{\partial X}{\partial m_i} - \sum_{k\neq i} m_k\frac{\partial X}{\partial m_k}\right]. \tag{20}$$

Relative factor prices For $i \neq j \neq l$,

$$\frac{\partial w_i/w_j}{\partial m_i} = -\frac{s_i\left(m_j s_j + r\right)}{\left(m_i s_i + r\right)^2} < 0, \quad \frac{\partial w_j/w_i}{\partial m_i} = \frac{s_i}{m_j s_j + r} > 0 \quad \text{and} \quad \frac{\partial w_l/w_j}{\partial m_i} = 0.$$

Hence, we have

$$\begin{aligned}
\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} &= \frac{1}{M}\left[(1-m_i)\frac{\partial w_i/w_j}{\partial m_i} - \sum_{k\neq i} m_k\frac{\partial w_i/w_j}{\partial m_k}\right] \\
&= \frac{1}{M}\left[(1-m_i)\frac{\partial w_i/w_j}{\partial m_i} - m_j\frac{\partial w_i/w_j}{\partial m_j}\right] \\
&= -\frac{1}{M}\frac{m_j s_j + r}{m_i s_i + r}\left[\frac{(1-m_i)\,s_i}{(m_i s_i + r)} + \frac{m_j s_j}{(m_j s_j + r)}\right] < 0.
\end{aligned} \tag{21}$$

So, the more populated country incurs a fall in its wage with respect to each other trade partner. Also,

$$\begin{aligned}
\frac{\mathrm{d}w_j/w_i}{\mathrm{d}M_i} &= \frac{1}{M}\left[(1-m_i)\frac{\partial w_j/w_i}{\partial m_i} - m_j\frac{\partial w_j/w_i}{\partial m_j} - \sum_{k\neq i\neq j} m_k\frac{\partial w_j/w_i}{\partial m_k}\right], \\
&= \frac{1}{M}\left[(1-m_i)\frac{\partial w_j/w_i}{\partial m_i} - m_j\frac{\partial w_j/w_i}{\partial m_j}\right], \\
&= \frac{(m_i s_i + r)}{M\,(m_j s_j + r)}\left[\frac{(1-m_i)\,s_i}{m_i s_i + r} + \frac{m_j s_j}{m_j s_j + r}\right] > 0.
\end{aligned} \tag{22}$$

So, the other countries have a rise in their wages with respect to the more populated country. Finally,

$$\begin{aligned}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i} &= \frac{1}{M}\left((1-m_i)\frac{\partial w_l/w_j}{\partial m_i} - \sum_{k\neq i} m_k \frac{\partial w_l/w_j}{\partial m_k}\right),\\ &= -\frac{1}{M}\left(m_l \frac{\partial w_l/w_j}{\partial m_l} + m_j \frac{\partial w_l/w_j}{\partial m_j}\right),\\ &= \frac{1}{M}\frac{r\left(m_l s_l - m_j s_j\right)}{\left(m_l s_l + r\right)^2}. \end{aligned} \tag{23}$$

This is positive for $m_l s_l > m_j s_j$. A country l has a rise in its wage compared to country j if it has a larger effective labor supply. In turn

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) &= \sum_{l=1}^{N}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i},\\ &= \frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} + \sum_{l\neq i}^{N}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i}.\end{aligned}$$

By (21) and (23), this is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) &= -\frac{1}{M}\frac{m_j s_j + r}{m_i s_i + r}\left[\frac{(1-m_i)\, s_i}{(m_i s_i + r)} + \frac{m_j s_j}{(m_j s_j + r)}\right]\\ &+ \frac{1}{M}\sum_{l\neq i}^{N}\frac{(m_j s_j + r)\, m_l s_l - (m_l s_l + r)\, m_j s_j}{\left(m_l s_l + r\right)^2}\\ &= -s_i \frac{m_j s_j + r}{M\left(m_i s_i + r\right)^2} + \frac{1}{M}\sum_{l}^{N}\frac{r\left(m_l s_l - m_j s_j\right)}{\left(m_l s_l + r\right)^2}\end{aligned} \tag{24}$$

The first part is negative. A sufficient condition of negativity of the second part is $m_j s_j < m_l s_l$ for all $l \neq j$. The expression is also negative if countries' labor supply are close to symmetry $m_l s_l \to m_j s_j$.

Country i local consumption By (11), the incentives to consume local high quality goods are given by

$$\frac{\mathrm{d}\mu_i/w_i}{\mathrm{d}M_i} = \frac{r}{N}\left(\sum_{l=1}^{N}\frac{\mathrm{d}w_l/w_i}{\mathrm{d}M_i}\right) = \frac{r}{N}\left(\sum_{l\neq i}^{N}\frac{\mathrm{d}w_l/w_i}{\mathrm{d}M_i}\right),$$

which is positive by (22).

Country i imports from country j Differentiating μ_i/w_j in (11) with respect to M_i yields:

$$\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} = \frac{1}{N}\left(s_i\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right)\right).$$

By (21) and (24), the first term is negative while the second is negative if $m_j s_j < m_l s_l$ for all $l \neq j$ or if countries' labor supply are close to symmetry $m_l s_l \to m_j s_j$.

After some simplifications we get

$$\begin{aligned}\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}\mu_i/w_k}{\mathrm{d}M_i} &= \frac{1}{N}s_i\left(\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}w_i/w_k}{\mathrm{d}M_i}\right) + \frac{r}{N}\left(\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) - \frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_k\right)\right)\\ &= -\frac{1}{MN}\left(m_j s_j - m_k s_k\right)\left[s_i\frac{2r+s_i}{\left(r+m_i s_i\right)^2} + \sum_{l}^{N}\frac{r^2}{\left(m_l s_l + r\right)^2}\right]\end{aligned}$$

Therefore, a rise in country i's population entices this country to replace its high quality imports from high labor supply countries by high quality imports from low labor supply countries ($\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}\mu_i/w_k}{\mathrm{d}M_i} > 0 \iff m_j s_j < m_k s_k$).

Country j imports from country l Differentiating μ_j/w_l in (11) with respect to M_i yields:

$$\frac{\mathrm{d}\mu_l/w_j}{\mathrm{d}M_i} = \frac{1}{N}\left(s_l\frac{\mathrm{d}}{\mathrm{d}M_i}\frac{w_l}{w_j} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\sum_{k=1\neq i\neq j}^{N}\frac{w_k}{w_j} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\frac{w_i}{w_j}\right)$$

The last term is always negative. The first and second terms are negative if $m_j s_j < m_l s_l$ for all $l \neq j$ or $m_l s_l \to m_j s_j$. So, under the latter condition, the expression is negative.

Alchian Allen effect

In this Appendix, we are interested in the situation where consumers purchase only a subset of the low quality goods. That is, we consider the sets of high and low quality purchases $\mathcal{H}(\mu_i/w_j) = [0, \ell^{-1}(\mu_i/w_j)] = [0, n - (\mu_i/w_j)^{-1}]$ and $\mathcal{L}(\mu_i/w_j) = [n - (\mu_i/w_j)^{-1}, \widetilde{n}\left(\mu_i/w_j, t\right)]$ where $\widetilde{n}\left(y,t\right) = \widetilde{n}\left(y,t\right) = n < n$ is the number of purchased goods, which solves the binding full market coverage constraint

$$y = \left(a_L\left(\widetilde{n}\right) + t\right)/b_L(\widetilde{n}) \iff \widetilde{n}\left(y,t\right) = n - \frac{1}{2}\left(y - \sqrt{y^2 - 4t}\right)$$

For a solution $\widetilde{n}$, it should be that $\mu_i/w_j = y > 2\sqrt{t}$. To be in the Alchian Allen configuration, it is assumed that high quality goods are purchased when they are preferred over low quality ones,

$$
\begin{aligned}
&\ell(z) > \mu_i/w_j \Rightarrow \ell(z) \geq \left(a_H(z) + t\right)/b_H(z) \\
&\qquad \Longleftrightarrow \\
&\quad z \in \left[0, n - (\mu_i/w_j)^{-1}\right] \Rightarrow \ell(z) \geq \left(a_H(z) + t\right)/b_H(z) \\
&\qquad \Longleftrightarrow \\
&\quad z \in \left[0, n - (\mu_i/w_j)^{-1}\right] \Rightarrow z > n - \sqrt{1-t} \;\text{ and } t < 1
\end{aligned}
$$

This holds for if $t < 1 - n^2$. It is also assumed that some low quality goods are not purchased, that is

$$\mu_i/w_j < \left(a_L\left(n\right) + t\right)/b_L(n) \iff \mu_i/w_j < \infty$$

which always holds. It is further assumed that the number of purchased goods falls with t; that is $\widetilde{n}_t \equiv \partial\widetilde{n}/\partial t = 1/\left[b'_L(\widetilde{n})\left(\mu_i/w_j\right) - a'_L\left(\widetilde{n}\right)\right] < 0$. We compute

$$\widetilde{n}_t \equiv \partial\widetilde{n}/\partial t = -\frac{1}{\sqrt{\left(\mu_i/w_j\right)^2 - 4t}} < 0$$

which holds. To sum up the Alchian Allen effect holds under the present primitives for $t < 1-n^2$ and $\mu_i/w_j > 2\sqrt{t}$.

Part I

Supplementary material

Linear trade costs

In this section, we consider the presence of linear trade costs. Alchian and Allen's (1964) postulate that a per unit transactions cost lowers the relative price of high quality goods and raises the relative demand for them. Hummels and Skiba (2004) confirm this hypothesis by showing that exporters charge destination prices that vary positively with per unit linear shipping costs and negatively with ad valorem tariffs. We therefore start with linear trade cost.

We consider a trade cost $t_{ij}(z)$ for shipment of good z in country i from country j. For the sake of simplicity, we assume that the trade cost is incurred in the destination country i and while it can depend on the nature of each good but not its quality version. For instance, transport costs and tariffs are usually paid according to the quantity rather than the quality of watches, cars, etc... Therefore, the total price of an imported unit z of quality $k = H, L$, from country j into country i amounts to the sum of the mill price $w_j a_k(z)$ and trade cost $w_i t_{ij}(z)$. There is no trade cost within a same country: $t_{ii}(z) = 0$, $z \in [0, n]$. Since trade costs are the same for high and low qualities, per-quality input $\ell(z)$ is independent of trade costs. As a consequence, the consumer makes the same choice between high and low quality if she faced the same inverse marginal utility μ_i and wages w_i as without trade costs. The point is that the inverse marginal utility and wages and therefore the product portfolio will change because of higher prices.

Since consumers import all goods in high or low quality version, they pay trade costs on all goods. As a consequence, only the total trade cost matters in their consumption decisions. It is therefore useful to define their total trade costs paid on imports in country i from country j, as $t_{ij} = \int_0^n t_{ij}(z)dz$, and, their total trade cost on all their imports in country i as $t_i = \sum_{j=1}^N t_{ij}$.

Using those definitions, the (nominal) expenditure writes as

$$E_i = \sum_{l=1}^{N} \left(\int_{\mathcal{H}\left(\frac{\mu_i}{w_l}\right)} w_l a_H(z)dz + \int_{\mathcal{L}\left(\frac{\mu_i}{w_l}\right)} w_l a_L(z)dz + w_i \int_0^n t_{il}(z)dz \right),$$

and simplifies to

$$E_i = w_i \left[t_i + \sum_{l=1}^{n} \frac{w_l}{w_i} E\left(\frac{\mu_i}{w_l}\right) \right],$$

where $E(y)$ is defined as before. Balanced trade imposes that the values of exports and imports equate at the mill, "before" payment of trade costs (those are taken in charge by the consumers at destination). That is,

$$\sum_{l\neq i}^{n}\frac{w_l}{w_i}E\left(\frac{\mu_i}{w_l}\right)=\sum_{l\neq i}^{n}\frac{w_i}{w_l}E\left(\frac{\mu_l}{w_i}\right),$$

which is the same identity as before. The average prices are given by

$$\overline{p}_{ij}^{\text{fob}}=\frac{1}{n}w_jE\left(\frac{\mu_i}{w_j}\right)\quad\text{and}\quad\overline{p}_{ij}^{\text{cif}}=\frac{1}{n}w_jE\left(\frac{\mu_i}{w_j}\right)+\frac{1}{n}w_it_{ij}.$$

The indirect utility V_i is still defined as in (9) as a function of the ratios μ_i/w_j, which may now depend on linear trade costs.

In the equilibrium, balance trade is satisfied as well as budget balance $E_i = w_i s_i$. The equilibrium is then the same as without trade cost, except that s_i should be replaced by $s_i - t_i$. Therefore, in this framework, *a lower import linear cost is equivalent to a rise in productivity,* s_i. If one interprets s_i as a country fixed 'work time', then t_i is simply the number of hours spent in transporting goods to home. A lower t_i allows workers to supply more time for production, which allows to increase their output and income. Hence, using (11), the incentive to purchase high quality goods in country i from j is given by

$$\frac{\mu_i}{w_j}=\frac{1}{N}\left(\frac{w_i}{w_j}\left(s_i-t_i\right)+r\sum_{l=1}^{N}\frac{w_l}{w_j}\right).\qquad(25)$$

In term exogenous variables, the relative price writes as

$$\frac{w_i}{w_j}=\frac{m_j\left(s_j-t_j\right)+r}{m_i\left(s_i-t_i\right)+r}.\qquad(26)$$

A country with higher import cost has higher relative price because a higher share of its labor supply is shifted from production to import activities. *Ceteris paribus*, the country becomes less competitive in international markets. This gives the following incentive to purchase high quality goods:

$$\frac{\mu_i}{w_j}=\frac{1}{N}\left(\frac{m_j\left(s_j-t_j\right)+r}{m_i\left(s_i-t_i\right)+r}\left(s_i-t_i\right)+r\sum_{l=1}^{N}\frac{m_j\left(s_j-t_j\right)+r}{m_l\left(s_l-t_l\right)+r}\right).\qquad(27)$$

In this linear trade cost setting, incentives to purchase high quality goods relate to each country's trade costs on all its imports, t_i. Specific import costs t_{ij} matter only through its effect on t_i.

Symmetric countries

Consider symmetric countries and symmetric trade costs. On the one hand, we suppose that $m_i = 1/N$ and $s_i = s$. On the other hand, we suppose that each country incurs the same total import cost t so that $t_{ii} = 0$ and $t_{ij} = t/(N-1)$, $j \neq i$. The trade equilibrium is then the same as without trade cost, except that s should be replaced by $s - t$. From (12), we have that $\mu_i/w_j = \frac{1}{N}(s - t + rN)$. So, a fall in trade cost t increases the share of high quality goods purchased in the import and local markets. From Proposition 1, an equilibrium exists if $s > nN + t$. Lower trade cost reduces the requirement on productivity that guarantees a purchase for all imported goods. Wages are symmetric and, say, equal to w. The average prices are computed by

$$\overline{p}_{ij}^{\text{fob}} = \frac{w}{nN}(s-t) \quad \text{and} \quad \overline{p}_{ij}^{\text{cif}} = \frac{w}{nN}s.$$

Average fob prices increase with the fall in trade cost. This is because consumers import higher shares of high-quality goods. By contrast, average cif prices are unresponsive to trade cost fall: the latter indeed fully dampens the price increase related to the higher shares of high quality imports. Finally, utility can be computed as $V_i = N \ln(s - t + rN)$ +constant, which increases with lower trade cost.

Exports from the same origin

Take two countries i and j importing from the same exporter l ($l \neq i \neq j$) with total trade costs t_{il} and t_{il}. We know that high-quality import shares and utility from those imports depend on the incentives to buy high-quality goods μ_i/w_l and μ_j/w_l. Average fob import prices also rank according to those ratios as one can check that $\overline{p}_{il}^{\text{fob}} \geq \overline{p}_{jl}^{\text{fob}} \iff \mu_i/w_l \geq \mu_j/w_l$. One readily checks that

$$\frac{\mu_i}{w_l} > \frac{\mu_j}{w_l} \iff \frac{w_i(s_i - t_i)}{w_j(s_j - t_j)} > 1 \iff \frac{(s_i - t_i)/(m_i s_i + r)}{(s_j - t_j)/(m_j s_j + r)} > 1$$

Hence, country i imports a larger share of high quality goods and pays higher average fob import price from country l if it has lower total import trade cost t_i. Note that $t_i = \sum_h t_{ih}$ is an indicator of remoteness and specific import costs t_{il} do not appear in isolation. Hence, higher remoteness reduces average fob import prices. To conclude, average fob import prices rise with lower remoteness.

Imports from different origins

Now, consider a country l that imports from two different exporters i and j ($l \neq i \neq j$) with total trade costs t_{li} and t_{lj}. We know that high-quality import shares and utility from those imports depend on the incentives to buy high-quality goods μ_l/w_j and μ_l/w_j. We then get

$$\frac{\mu_l}{w_i} \geq \frac{\mu_l}{w_j} \iff \frac{w_i}{w_j} \leq 1 \iff \frac{m_i(s_i - t_i)}{m_j(s_j - t_j)} \geq 1.$$

All other things being the same, a larger total import cost t_i in country i reduces the country l's incentive to purchase a high quality good from it. This is because import cost reduces the labor supply available for the productive sector, which in turn raises wages and product prices.

Finally, average cif import prices rank as

$$\begin{aligned}
&\overline{p}_{li}^{\text{cif}} > \overline{p}_{lj}^{\text{cif}} \\
&\quad \Longleftrightarrow r\left(\frac{w_i}{w_l} - \frac{w_j}{w_l}\right) \leq t_{li} - t_{lj} \\
&\quad \Longleftrightarrow \frac{r}{m_i\left(s_i - t_i\right) + r} - \frac{r}{m_i\left(s_j - t_j\right) + r} \leq \frac{t_{li} - t_{lj}}{m_l\left(s_l - t_l\right) + r}
\end{aligned}$$

In the absence of specific trade costs (t_{li}, t_{lj}), the average cif import price is larger for imports from the country with the lower wage. A low wage indeed makes high quality goods cheaper and entices country l's consumers to buy a higher share of them. To have a lower equilibrium wage, a country i must have either higher labor supply $m_i s_i$ or lower import activity $m_i t_i$. Since t_i is an indicator of remoteness, average cif import prices are larger from imports from less remote countries. The specific trade costs or bilateral distances (t_{li}, t_{lj}) may however alter this conclusion as they are passed through average import cif prices. *Ceteris paribus*, the latter are higher for imports from farther countries. To conclude, average cif import price increase with higher bilateral distance and lower remoteness.