Abstracts

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The conference was organized by the algebra group of the Institute of Discrete Mathematics and Geometry of Technische Universität Wien.

Invited talks

Marcin Kozik
A contribution of CSP to universal algebra

I will present a number of results and research directions, in universal algebra, that are motivated by the connection with Constraint Satisfaction Problems. I will argue that both questions and solutions that originate in the connection are very often natural and elegant.

Dragan Mašulović
Structural Ramsey theory
from the point of view of category theory

Generalizing the classical results of F. P. Ramsey from the late 1920s, the structural Ramsey theory originated at the beginning of 1970s. We say that a class K of finite structures has the *Ramsey property* if the following holds: for any number $k \geq 2$ of colors and all $A, B \in K$ such that A embeds into B there is a $C \in K$ such that no matter how we color the copies of A in C with k colors, there is a *monochromatic* copy B' of B in C (that is, all the copies of A that fall within B' are colored by the same color).

Showing that the Ramsey property holds for a class of finite structures K can be an extremely challenging task and a slew of sophisticated methods have been proposed in literature. These methods are usually constructive: given $A, B \in K$ and $k \geq 2$ they prove the Ramsey property directly by constructing a structure $C \in K$ which is Ramsey for B, A and k.

In this talk we explicitly put the Ramsey property and the dual Ramsey property in the context of categories of finite structures. We use the machinery of category theory to provide new Ramsey and dual Ramsey statements for some classes of finite algebras and related combinatorial structures.

Miroslav Olšák
The story of loop conditions

An interesting by product of joint research on universal algebra and constraint satisfaction problem is the fact that a finite algebra has a term $s(x, x, y, y, z, z) \approx s(y, z, z, x, x, y)$

if and only if it has a term $s(r, a, r, e) \approx s(a, r, e, a)$. We studied such type of Maľtsev conditions in detail, calling them loop conditions. It turned out that the equivalence is true in general. We explain a bit of the background, how our research evolved over time, and show how to apply the theory of loop conditions for reproving a loop lemma for finite strongly connected graphs.

Matt Valeriote
FINITE ALGEBRAS AND TREE LANGUAGES

By regarding deterministic finite state automata as finite multi-unary algebras, and vice versa, one can define the notion of a regular language in algebraic terms. From this perspective, a multi-unary algebra is a string processor, where strings correspond to unary terms, and the algebra will accept or reject a string/term, depending on the value the term produces when applied to a designated initial element of the algebra. In a similar fashion, a finite algebra of arbitrary type can be regarded as a tree processor, with trees corresponding to the terms of the algebra. In this way, the notion of a regular language of strings can be extended to that of a regular tree language.

In my talk I will discuss several definability questions for regular tree languages and connect them with some decision problems for classes of finite algebras that are closed under operations, that from a computational perspective, seem quite natural. One of the operations of interest is the matrix power construction and another is the wreath product.

Mikhail Volkov
IDENTITIES OF KAUFFMAN MONOIDS:
FINITE AXIOMATIZATION AND ALGORITHMS

Kauffman monoids were introduced by Temperley and Lieb in their studies on some graph-theoretical problems in statistical mechanics. Later, they were independently rediscovered as purely geometric objects by Kauffman in his work on knot theory. Over the past few years it turned out that algebraic properties of Kauffman monoids are of much interest as well. The talk presents results on equational theories of Kauffman monoids found by the speaker and his coauthors. We have discovered that, even though these theories admit no finite axiomatization, there are certain cases in which the identities of Kauffman monoids can be recognized by polynomial time algorithms.

Contributed talks

Paolo Aglianò

Three (or four?) properties of divisible residuated semilattices

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We will show that divisible residuated semilattices have some specific algebraic properties, namely:

1. for divisible residuated semilattices representability is equivalent to the existence of a join term;
2. any integral divisible residuated semilattice is distributive;
3. a finite divisible residuated semilattice is integral and commutative.

All these properties were previously known to hold for divisible residuated *lattices* and (in some case) for commutative residuated semilattices.

Next we will connect these properties with a very old result of J.C. Varlet (*On distributive residuated groupoids*, **Semigroup Forum** 6 (1973), 80–85) that deals with a weakened form of distributivity.

Delbrin Ahmed

Geometric constructibility of polygons lying on a circular arc

(joint work with Gábor Czédli and Eszter K. Horváth, University of Szeged, Bolyai Institute)

For a positive integer n, an n-sided polygon lying on a circular arc or, shortly, an n-fan is a sequence of $n+1$ points on a circle going counterclockwise such that the "total rotation" δ from the first point to the last one is at most 2π. We prove that for $n \geq 3$, the n-fan cannot be constructed with straightedge and compass in general from its central angle δ and its central distances, which are the distances of the edges from the center of the circle. Also, we prove that for each fixed δ in the interval $(0, 2\pi]$ and for every $n \geq 5$, there exists a concrete n-fan with central angle δ that is not constructible from its central distances and δ. The present work generalizes some earlier results published by G. Czédli and Á. Kunos on the particular cases $\delta = 2\pi$ and $\delta = \pi$.

Erhard Aichinger

Polynomial maps on supernilpotent algebras

We derive properties of polynomial maps on finite supernilpotent algebras in congruence modular varieties, and use these properties to solve systems of polynomial equations over such algebras.

Florian Aichinger

New proofs for the absence of antichains in certain partially ordered sets

(joint work with Erhard Aichinger, Johannes Kepler University)

It has been shown that the following partially ordered sets have no infinite antichain:

(1) $(\mathbb{N}, \leq)$ (obvious)

(2) $(\mathbb{N}_0^m, \leq)$ (Dickson 1931)

(3) $(A^*, \leq_\mathrm{e})$, ordered by embedding (Higman 1952)

(4) monomial ideals of $\mathbb{Q}[x_1, \ldots, x_n]$ (Maclagan 2001, Nash-Williams 1964)

(5) upward closed subsets of $(A^*, \leq_\mathrm{e})$ (Nash-Williams 1964)

We present simple proofs for (4) and (5).

Delaney Aydel

Classifying actions of $T_n \otimes T_n$ on path algebras of quivers

Let T_n denote the nth Taft algebra. We fully classify inner-faithful actions of $T_n \otimes T_n$ on four-vertex Schurian quivers as extensions of the actions of $\mathbb{Z}_n \times \mathbb{Z}_n$. One example will be presented in full, with the remaining results briefly given.

Libor Barto

A loop lemma for nonidempotent cores

(joint work with Marcin Kozik, Jagiellonian University)

The "loop lemma" states that each digraph satisfying mild structural conditions, which is compatible with a finite idempotent equationally non-trivial clone, has a loop. This theorem, proved together with M. Kozik and T. Niven, has interesting consequences in the finite domain constraint satisfaction problem (CSP) and universal algebra. Some fundamental questions in the infinite domain CSP motivate the search for "loop lemmata" that do not require finiteness and idempotency. While satisfactory infinite versions are still not available, we have recently made a progress toward removing the idempotency assumption: when the structural assumptions on the digraph are a bit strengthened, then, instead of idempotency, it is enough to require that the clone is a core. In the talk I will present this result, its background, and consequences.

Mike Behrisch

On a characterization of automatic homeomorphicity

(joint work with Edith Vargas-García, ITAM)

Reconstruction deals with the problem of determining a (usually relational) first order structure up to some sort of equivalence from some isomorphism class of its automorphism group. A common method is to prove that the automorphism group of the structure under consideration has the small index property thus ensuring reconstruction up to first order bi-interpretability in the countable $\aleph_0$-categorical case. When generalizing this type of question from the automorphism group to transformation monoids (or even clones) naturally associated with the structure, such as the endomorphism monoid, the monoid of self-embeddings or the polymorphism clone, the notion of index is not available any more. Hence, a more suitable way of getting reconstruction results is by showing a property of the monoid or clone called automatic homeomorphicity, or the even stronger automatic action compatibility.

The talk will focus on a new characterization of automatic homeomorphicity for the case when the automorphism group is dense in the transformation monoid, which is particularly important for applications with respect to constraint satisfaction problems in theoretical computer science. This characterization was recently used to provide new automatic action compatibility (and hence automatic homeomorphicity) results for clones associated with $(\mathbb{Q}, <)$, the random graph, the random digraph, the random tournament, the countable universal k-uniform hypergraph for $k \geq 2$, the countable universal homogeneous $\mathbb{K}_n$-free graph for $n \geq 3$ and the countable universal homogeneous Henson digraphs.

Vijay Kumar Bhat

Vertex connectivity of line graph of $\Gamma(\mathbf{Z}_n)$

(joint work with Pradeep Singh)

Line graph of zero divisor graph of $\mathbf{Z}_n$ was introduced in 2005 and is denoted by $L(\Gamma(\mathbf{Z}_n))$. Vertex connectivity of $L(\Gamma(\mathbf{Z}_n))$ for different values of n has been discussed by several authors. In this article, we derive a relation to find the vertex connectivity of $L(\Gamma(\mathbf{Z}_n))$ for $n = p^m$, where p is a prime number. We prove the following:

The vertex connectivity of $L(\Gamma(\mathbf{Z}_{p^m}))$ is:

1. $2(p^{\frac{m}{2}} - 3)$, when m is even.

2. $p^{\frac{m+1}{2}} + p^{\frac{m-1}{2}} - 5$, when m is odd.

Bekalu Tarekegn Bitew

On fuzzy ideals and fuzzy filters of an almost distributive fuzzy lattice

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We introduce the notion of fuzzy ideal and fuzzy filters in an ADFL (R, A) as a fuzzy set μ in terms of a fuzzy partial order relation which agrees with the notion of ideal and filters of an almost distributive lattice, and we investigate some basic properties that coincide with the properties of ideals and filters of an ADFL. In addition we characterize a fuzzy ideal and filter μ of an ADFL by its support set $S(\mu)$. We also define the smallest fuzzy ideal and filter of an ADFL induced by any arbitrary non-zero fuzzy set. We study the properties of operations (intersection and union) on fuzzy ideals and fuzzy filters of a given ADFL. Moreover, we study the properties of a homomorphism on fuzzy ideals and fuzzy filters of an ADFL, and obtain some basic results.

Manuel Bodirsky

No weakest height 1 strong Maľtsev condition

(joint work with Antoine Mottet, Miroslav Olšák, Jakub Opršal, Michael Pinsker, Ross Willard)

We show that there is no weakest height 1 strong Maľtsev condition, even when restricted to oligomorphic clones. In fact, for each non-trivial height 1 strong Maľtsev condition Σ there exists a structure $\mathbb{A}$ such that $\mathbb{A}$ has finite relational signature, $\mathbb{A}$ is a first-order reduct of a finitely bounded homogeneous structure, $\mathbb{A}$ has a polynomial-time tractable CSP, Pol($\mathbb{A}$) satisfies some non-trivial height 1 strong Maľtsev condition (equivalently, does not possess a minor-preserving map to the clone of projections on a two-element set), and Pol($\mathbb{A}$) does not satisfy Σ. It follows that polynomial-time tractability of CSPs of first-order reducts of finitely bounded homogeneous structures cannot be described by a height 1 strong Maľtsev condition. In contrast, polynomial-time tractability of finite-domain CSPs can be described by such a condition.

Bertalan Bodor
CSP dichotomy for finite coverings of unary structures

The *constraint satisfaction problem* over a structure $\mathfrak{B}$ with finite relational signature is the decision problem whether a given finite structure $\mathfrak{A}$ with the same signature as $\mathfrak{B}$ has a homomorphism to $\mathfrak{B}$. Using concepts and techniques from universal algebra, A. A. Bulatov and D. N. Zhuk recently proved that if $\mathfrak{A}$ is finite, then the CSP over $\mathfrak{A}$ is either in **P** or **NP**-complete. Following this result, it is a natural question to ask when and how this dichotomy can be generalized for infinite structures. One of the results in this direction is presented in a recent paper by M. Bodirsky and A. Mottet where the same dichotomy is proven for reducts of unary structures. In this talk I will talk about a further generalization of this result which can be formulated as follows.

Let $\mathcal{K}$ denote the class of those structures $\mathfrak{A}$ for which there exist constants c, d with $d < 1$ such that the number of injective n-orbits of $\mathfrak{A}$ is at most cn^{dn}. It turns out that this class can also be characterized as the class of finite coverings of reducts of unary structures. Our main result is that the CSP dichotomy holds for the class $\mathcal{K}$. During this talk I would like to present some of the tools and ideas I used in the proof, such as *polymorphisms, pseudo-identities, minions, minion homomorphisms, canonical functions,* and *canonical consistency.*

Marco Bonatto
Mal'tsev classes of left quasigroups

We investigate some Mal'tsev classes of varieties of left quasigroups. We prove that several Mal'tsev classes coincide with the strong Mal'tsev class of varieties admitting a Taylor term. Then we specialize to the setting of quandles for which we prove that the meet semidistributive varieties are those which do not have the finite model property.

Victor Bovdi
Group algebras whose groups of normalized units have exponent 4
(joint work with Mohamed A. Salim)

It is well known that there does not exist a reasonable description of finite groups of prime square exponent p^2 (not even in the case when the exponent is 4). However Z. Janko (see for example On finite nonabelian 2-groups all of whose minimal nonabelian subgroups are of exponent 4. *J. Algebra*, 315(2):801–808, 2007; Finite nonabelian 2-groups all of whose minimal nonabelian subgroups are metacyclic and have exponent 4. *J. Algebra*, 321(10):2890–2897, 2009; Finite p-groups of exponent p^e all of whose cyclic subgroups of order p^e are normal. *J. Algebra*, 416:274–286, 2014) was able to characterize these groups under certain additional restrictions on their structure. In this way he obtained interesting classes of finite p-groups.

In our talk we give a full description of locally finite 2-groups G such that the normalized group of units $V(FG)$ of the group algebra FG over a field F of characteristic 2 has exponent 4 (see A. A. Bovdi. The group of units of a group algebra of characteristic p. *Publ. Math. Debrecen*, 52(1–2):193–244; V. A. Bovdi and M. Salim. Group algebras whose groups of normalized units have exponent 4. *Submitted for publ.*, 1–10, 2015).

Ivan Chajda

Residuated operators in complemented posets

(joint work with Helmut Länger, TU Wien)

Using the operators of taking upper and lower cones in a poset with a unary operation, we define operators $M(x,y)$ and $R(x,y)$ in the sense of multiplication and residuation, respectively, and we show that these operators, a general modification of residuation can be introduced. A relatively pseudocomplemented poset can be considered as a prototype of such an operator residuated poset. As main results we prove that every Boolean poset as well as every pseudo-orthomodular poset can be organized into a (left) operator residuated structure. Some results on pseudo-orthomodular posets are presented which show the analogy to orthomodular lattices and orthomodular posets.

Hassan Cheriha

Descartes' rule of signs, Rolle's Theorem and sequences of admissible pairs

(joint work with Yousra Gati and Vladimir Petrov Kostov)

Given a real univariate degree d polynomial P, the numbers pos_k and neg_k of positive and negative roots of $P^{(k)}$, $k = 0, \ldots, d-1$, must be admissible, i.e. they must satisfy certain inequalities resulting from Rolle's theorem and from Descartes' rule of signs. For $1 \leq d \leq 5$, we give the answer to the question for which admissible d-tuples of pairs $(pos_k,\ neg_k)$ there exist polynomials P with all nonvanishing coefficients such that for $k = 0, \ldots, d-1$, $P^{(k)}$ has exactly pos_k positive and neg_k negative roots all of which are simple.

Codruța Chiș

On the chromatic number of the van Kampen graph associated with a group and a generator set

(joint work with Mihai Chiș, West University of Timișoara, Romania)

We investigate the van Kampen graph associated with a finite group and a generator set. We determine the chromatic number of this graph for some particular classes of finite groups.

Mihai Chiș

On the chromatic polynomial of the van Kampen graph associated to a finite group and a generator set

(joint work with Codruța Chiș, Banat University of Agricultural Sciences "King Mihai I of Romania" Timișoara)

We investigate the chromatic polynomial associated to the van Kampen graph defined for a finite group and a generator set of the group. We consider some particular classes of groups and determine the corresponding chromatic polynomials.

Alfredo Costa

Free profinite semigroups, symbolic dynamics, and codes

A symbolic dynamical system $\mathcal{X}$ over the alphabet A is a nonempty subset $\mathcal{X}$ of $A^{\mathbb{Z}}$ that is topologically closed (where we consider the discrete topology of A and the product topology of $A^{\mathbb{Z}}$) which moreover satisfies $\sigma(\mathcal{X}) = \mathcal{X}$ for the shift mapping, defined by $\sigma((x_i)_{i\in\mathbb{Z}}) = (x_{i+1})_{i\in\mathbb{Z}}$. The language of finite blocks of $\mathcal{X}$ is the set $B(\mathcal{X}) = \{x_i x_{i+1} \cdots x_{i+n} \mid i \in \mathbb{Z},\, n \geq 0\}$ of finite nonempty words that appear in elements of $\mathcal{X}$.

A major motivation for the development of semigroup theory was, and still is, its applications to the study of formal languages. Therefore, it is not surprising that the correspondence $\mathcal{X} \mapsto B(\mathcal{X})$ entails a connection between symbolic dynamics and semigroup theory. Free profinite semigroups constitute a tool of major importance for (finite) semigroup theory and its connections with formal languages. About fifteen years ago, Jorge Almeida begun to investigate the consequences of considering the topological closure of $B(\mathcal{X})$ on the free profinite semigroup $\widehat{A^+}$ generated by A, when $\mathcal{X}$ is a symbolic dynamical system over the alphabet A.

The structure of the free semigroup A^+ is very poor, but, in contrast, that of the free profinite semigroup $\widehat{A^+}$ is very rich. The aforementioned connection between symbolic dynamical systems and free profinite semigroups led to new results that clarified structural features of the latter. For example, while free semigroups do not have subgroups (i.e., subsemigroups with group structure), this connection with symbolic dynamics was used to show that the free profinite semigroup over an alphabet with at least two letters contains all finitely generated free profinite groups as maximal subgroups (a maximal subgroup is a subgroup that is maximal for the inclusion). On the other hand, the same type of connection was used to obtain the first examples of maximal subgroups of $\widehat{A^+}$ that are not relatively free profinite semigroups.

More recently, in a joint work of the speaker with Jorge Almeida, Revekka Kyriakoglou and Dominique Perrin, this research on the maximal subgroups of $\widehat{A^+}$ was applied to obtain a result on code theory which apparently has nothing to do with free profinite semigroups, in what can be considered a new and interesting application of (profinite) universal algebra to another field. This result gives necessary and sufficient conditions for the group of a maximal bifix code Z to be isomorphic to the F-group of the code $Z \cap F$, where F is a recurrent language.

In this talk, we review some of the main aspects of the connection between symbolic dynamics and free profinite semigroups, with an emphasis on the latest applications to codes.

Jimmy Devillet

Characterizations and classifications of quasitrivial semigroups

(joint work with Jean-Luc Marichal, University of Luxembourg; Bruno Teheux, University of Luxembourg)

A semigroup (X, F) is said to be *quasitrivial* if $F(x, y) \in \{x, y\}$ for all $x, y \in X$. We investigate classifications of quasitrivial semigroups defined by certain equivalence relations. The subclass of quasitrivial semigroups that are order-preserving for a given total ordering is also investigated. In particular, we characterize the latter class in terms of single-plateauedness which was introduced by Duncan Black in social choice theory. In the special case of finite semigroups, we address and solve several related enumeration problems.

Stefano Fioravanti

Closed sets of finitary functions from $\mathbb{Z}_q$ to $\mathbb{Z}_p$

We investigate the finitary functions from $\mathbb{Z}_q$ to $\mathbb{Z}_p$, for two distinct prime numbers p and q. A (p, q)-linear closed clonoid is a subset of these functions which is closed under composition from the left and from the right with linear mappings.

We give a characterization of these subsets of functions through the invariant subspaces of the vector space $\mathbb{Z}_p^a$ via a certain linear transformation of order $q - 1$. Furthermore we prove that each of these subsets of functions it is completely determined by its unary functions.

Heghine Ghumashyan
ON WEAKLY HYPERASSOCIATIVE SEMIGROUPS

The present talk is devoted to the necessary and sufficient conditions of semigroups, which polynomially satisfy the following hyperidentities (1)–(4):

$$F(F(x,x,y),x,x) = F(x,x,F(y,x,x)) \tag{1}$$
$$F(F(x,x,x),x,x) = F(x,x,F(x,x,x)) \tag{2}$$
$$F(F(x,x,x),x,x) = F(x,x,F(x,y,x)) \tag{3}$$
$$F(F(x,x,x),x,y) = F(x,x,F(x,x,y)) \tag{4}$$

Pierre Gillibert
PSEUDO-LOOP CONDITIONS
(joint work with J. Jonušas, TU Wien; M. Pinsker, TU Wien)

A pseudo-loop condition is a set of identities of the form:

$$u_1 \circ t(x_1^1, \dots, x_n^1) = u_2 \circ t(x_1^2, \dots, x_n^2) = \dots = u_m \circ t(x_1^m, \dots, x_n^m)\,,$$

where the x_i^j belong to a set of variables, t is an n-ary operation symbol, and $u_1, \dots, u_m$ are unary operation symbols.

For example, the *pseudo-Siggers* pseudo-loop condition is:

$$u \circ t(x,y,x,z,y,z) = v \circ t(y,x,z,x,z,y)\,.$$

More generally consider a set of variables V of cardinality $k \geq 2$. Set $n = k^m - k$, enumerate all non constant m-ary tuples of V

$$\begin{pmatrix} x_1^1 \\ x_1^2 \\ \vdots \\ x_1^m \end{pmatrix}, \begin{pmatrix} x_2^1 \\ x_2^2 \\ \vdots \\ x_2^m \end{pmatrix}, \dots, \begin{pmatrix} x_n^1 \\ x_n^2 \\ \vdots \\ x_n^m \end{pmatrix}.$$

The pL_k^m pseudo-loop condition is:

$$u_1 \circ t(x_1^1, \dots, x_n^1) = u_2 \circ t(x_1^2, \dots, x_n^2) = \dots = u_m \circ t(x_1^m, \dots, x_n^m)\,.$$

We shall discuss the satisfiability of pseudo-loop conditions in closed oligomorphic core clones. We prove that every closed oligomorphic core clone satisfying a pseudo-Taylor identity also satisfies pL_4^2.

Johannes Greiner

New tractability results for combinations of qualitative CSPs

(joint work with Manuel Bodirsky)

Given two countably infinite omega-categorical structures $\mathfrak{A}$ and $\mathfrak{B}$ we want to combine the algorithms for the constraint satisfaction problem (CSP) of $\mathfrak{A}$ and $\mathfrak{B}$ to solve a combined problem in polynomial time. More precisely, we study the CSP of the union of the first-order theories of $\mathfrak{A}$ and $\mathfrak{B}$ when their signatures are disjoint. We are interested in the computational complexity of this combined CSP, in particular in cases where the combined problem is polynomial time tractable, assuming that CSP($\mathfrak{A}$) and CSP($\mathfrak{B}$) are polynomial time tractable. The first important publication, leading to a polynomial time algorithm for some cases, appeared in 1979, written by Greg Nelson and Derek Oppen. Apart from their approach, not much is known about algorithms with polynomial runtime in this setup.

We present two new procedures with polynomial runtime that apply to cases not covered by Nelson and Oppen's theorem. The first procedure is based on a technique known as 'sampling' and the second derives and exchanges equalities and disjunctions of disequalities of variables.

Mehsin Jabel Atteya

Commutativity with permuting n-derivations of semiprime rings

(joint work with Ajda Fošner, Faculty of Management, University of Primorska,
Cankarjeva 5, SI-6000 Koper, Slovenia)

Our purpose in this paper is to construct the formula of the composition of n-derivations of any set and investigate some properties of permuting n-derivations for prime rings and semiprime rings. In fact, we divided this article into two sections, where in the first section we emphasize on commutativity with centralizer permuting n-derivations for prime rings and semiprime rings. Actually, in this section we exam the action of some identities on prime rings and semiprime rings while in the second section we focus on the effect of the trace of permuting n-derivations satisfying certain identities on the previous type of rings. In addition to that, we provide some examples to illustrate some results which required that.

Throughout this paper, $\mathbb{R}$ will denote an associative ring with center $Z(\mathbb{R})$. For any $x, y \in \mathbb{R}, xy - yx$ (resp. $xy + yx$) will denote the commutator $[x, y]$ (resp. anti-commutator $x \circ y$). A ring $\mathbb{R}$ is said to be prime (resp. semiprime) if $a\mathbb{R}b = 0$ implies that either $a = 0$ or $b = 0$ (resp. $a\mathbb{R}a = 0$ implies $a = 0$. A map $f : \mathbb{R} \longrightarrow \mathbb{R}$ is said to be centralizing (resp. commuting) on a nonempty subset $\mathbb{S}$ of $\mathbb{R}$ if $[f(x), x] \in Z(\mathbb{R})$ (resp. $[f(x), x] = 0$) holds for all $x \in \mathbb{S}$. An additive mapping $d : \mathbb{R} \longrightarrow \mathbb{R}$ is called a derivation on $\mathbb{R}$ if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in \mathbb{R}$.

For a fixed positive integer n, a map $\triangle$: $\mathbb{R}^n \longrightarrow \mathbb{R}$ is said to be n-additive if $\overset{n}{\triangle}(x_1, x_2, \ldots, \grave{x_i} + x_i, \ldots, x_n) = \overset{n}{\triangle}(x_1, x_2, \ldots, \grave{x_i}, \ldots, x_n) + \overset{n}{\triangle}(x_1, x_2, \ldots, x_i, \ldots, x_n)$ holds for all $\grave{x_i}, x_i \in \mathbb{R}$. The notion of permuting n-derivation was defined by Park as follows: a permuting map $\triangle : \mathbb{R}^n \longrightarrow \mathbb{R}$ is said to be a permuting n-derivation if $\triangle$ is n-additive and

$$\overset{n}{\triangle}(x_1, x_2, \ldots, \grave{x_i}x_i, \ldots, x_n) = \overset{n}{\triangle}(x_1, x_2, \ldots, \grave{x_i}, \ldots, x_n)x_i + \grave{x_i}\overset{n}{\triangle}(x_1, x_2, \ldots, x_i, \ldots, x_n)$$

holds for all for all $\grave{x_i}, x_i \in \mathbb{R}$. A map $\triangle : \mathbb{R}^n \longrightarrow \mathbb{R}$ defined by $\delta(x) = \triangle(x, x, \ldots, x)$ is called the trace of $\triangle$.

Some results:

Theorem 1. *For any fixed integer $n \geqslant 2$, let $\mathbb{R}$ be a semiprime ring and $\mathbb{U}$ be an ideal of $\mathbb{R}$. Suppose that $\mathbb{R}$ admits a permuting n-derivation $\triangle : \mathbb{R}^n \longrightarrow \mathbb{R}$. If $\triangle$ satisfies $\overset{n}{\triangle}(x \circ y) \mp (x \circ y) \in Z(\mathbb{R})$ for all $x, y \in \mathbb{R}$. Then*

(i) $\overset{n}{\triangle}$ is a centralizer mapping (resp. commuting mapping) of $\mathbb{R}$.

(ii) $\mathbb{R}$ is commutative ring.

Corollary 2. *For any fixed integer $n \geqslant 2$, let $\mathbb{R}$ be a semiprime ring and $\mathbb{U}$ be an ideal of R has $1 \neq 0$. Suppose that $\mathbb{R}$ admits a non-zero permuting n-derivation $\triangle : \mathbb{R}^n \longrightarrow \mathbb{R}$ and with trace $\delta : \mathbb{R} \longrightarrow \mathbb{R}$ satisfying $[\overset{n}{\triangle}(x), \delta(y)] \in Z(\mathbb{R})$ for all $x, y \in \mathbb{R}$. Then R contains weakly semiprime ideal.*

Přemysl Jedlička

Yang-Baxter equation and a congruence of biracks

(joint work with Agata Pilitiowska, Anna Zamojska-Dzienio)

Set-theoretic solutions of Yang-Baxter equation are simplifications of a topic studied in particle physics. Their universal algebraic counterparts are called biracks; an algebra $(X, \circ, \bullet, \backslash, /)$ is called a birack if $(X, \circ, \backslash)$ is a left quasigroup, $(X, \bullet, /)$ is a right quasigroup and the operations satisfy the following identities:

$$\begin{aligned} x \circ (y \circ z) &= (x \circ y) \circ ((x \bullet y) \circ z), \\ (x \circ y) \bullet ((x \bullet y) \circ z) &= (x \bullet (y \circ z)) \circ (y \bullet z), \\ (x \bullet y) \bullet z &= (x \bullet (y \circ z)) \bullet (y \bullet z). \end{aligned}$$

A birack is said to be involutive if it satisfies one of the two equivalent identities

$$(x \circ y) \circ (x \bullet y) = x \quad \Leftrightarrow \quad (x \circ y) \bullet (x \bullet y) = y.$$

The following congruence

$$x \sim y \quad \equiv \quad x \circ z = y \circ z \text{ for all } z \in X$$

of an involutive birack is often studied in the literature. We present a generalization of the congruence for non-involutive biracks.

Julius Jonušas

A weakest non-trivial idempotent identity

(joint work with Pierre Gillibert, TU Wien; Michael Pinsker, TU Wien)

In this talk I will introduce the notion of a loop condition, an identity of a certain form which may hold in a given algebra. This notion can then be used to give an alternative and elementary proof for the result by Olšák, which states that every non-trivial idempotent algebra satisfies a particular fixed identity.

Éva Jungabel

On some homomorphism-homogeneous point-line geometries

A relational structure is homomorphism-homogeneous if every homomorphism between finite substructures extends to an endomorphism of the structure. A point-line geometry is a non-empty set of elements called points, together with a collection of subsets called lines in a way that every line contains at least two points and any pair of points is contained in at most one line. A line which contains more than two points is called a regular line. Point-line geometries can alternatively be formalised as relational structures. We show that there is an equivalence between the point-line geometries defined in this way and the point-line geometries defined as first-order structures (i.e. that the class of point-line geometries corresponds to a subclass of 3-uniform hypergraphs). We characterise the homomorphism-homogeneous point-line geometries with two regular non-intersecting lines. Homomorphism-homogeneous point-line geometries containing two regular intersecting lines have already been classified by Mašulović.

Joanna Kaleta

Equivalence of finite effect algebras and x-sets

(joint work with Grzegorz Bińczak)

We define the category EA_x whose objects are pairs $(E,(a_1,\dots,a_m))$, where $x=(n_1,\dots,n_m)\in\mathbb{N}^m$, E is a finite effect algebra and $a_1,\dots,a_m\in E$ are atoms such that $\operatorname{ord}(a_i)=n_i$. We define also the category C_x of x-sets. The set $Y\subseteq\mathbb{N}^m$ is an x-set if it consists of m-tuples $(y_1,\dots,y_m)$ such that $0\le y_i\le n_i$ where $x=(n_1,\dots,n_m)$ and Y satisfies some other conditions. Finally we show that EA_x is equivalent to C_x.

Balázs Kaprinai

Lazy groupoids

(joint work with Hajime Machida, Hitotsubashi University; Tamás Waldhauser, University of Szeged)

A binary operation $f(x, y)$ is said to be lazy if every operation that can be obtained from f by composition is equivalent to $f(x, y)$, $f(x, x)$ or x. This means that operations created by composing f by itself depend on at most two variables despite having arbitrarily many variables. For example, in a *rectangular band*, i.e., a semigroup satisfying the identities $xx \approx x$ (idempotence) and $xyz \approx xz$, an arbitrarily long product of variables can be reduced to at most 2 variables, namely the product of the first and the last variables. We describe all other lazy operations by identities (i.e., we determine all varieties of lazy groupoids), and we also characterize lazy groupoids up to isomorphism similarly to the well-known description of the rectangular bands.

Alexandr Kazda

The interpretability lattice of clonoids is distributive

(joint work with Matthew Moore)

A clonoid (AKA minion AKA minor closed set) is a family of operations closed under taking minors (renaming variables, identifying variables, and adding dummy variables). A clonoid homomorphism is a mapping from one clonoid to another that is compatible with minor-taking.

For example, say $\mathcal{C}$ is the clonoid of all operations $\{0,1\}^n \longrightarrow \mathbb{N}$ (where n ranges over $\mathbb{N}$) and $\mathcal{D}$ is the clonoid that for each $n \in \mathbb{N}$ contains one constant operation $t_n(x_1, \ldots, x_n) = 97$. Then the mapping that sends all n-ary operations of $\mathcal{C}$ to t_n is a clonoid homomorphism $\mathcal{C} \longrightarrow \mathcal{D}$.

The existence of homomorphisms between clonoids gives us a quasiorder on clonoids. If we identify clonoids that are homomorphically equivalent, we get a partial order; in fact, we get a lattice – the interpretability lattice of clonoids.

This interpretability lattice is large (it is a proper class) and complicated. However, we show that the lattice of clonoids is distributive. That is, for every clonoids $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{E}$ we have (modulo homomorphic equivalence)

$$\mathcal{C} \wedge (\mathcal{D} \vee \mathcal{E}) = (\mathcal{C} \wedge \mathcal{D}) \vee (\mathcal{C} \wedge \mathcal{E}).$$

Simon Knäuer

Constraint satisfaction over the random tournament

(joint work with Manuel Bodirsky, TU Dresden)

The random tournament $\mathbb{T}$ is the Fraïssé limit of the class of all finite tournament graphs. In this talk we study the complexity of the constraint satisfaction problem for first-order reducts of $\mathbb{T}$. We will present a proof of the complexity dichotomy for CSPs of first-order expansions of $\mathbb{T}$ by injective relations.

Michael Kompatscher
Solving equations in finite groups extended by their commutator

The equation solvability problem of an algebra $\mathbf{A}$ is the computational problem of deciding whether a (single) input equation has a solution in $\mathbf{A}$ or not. For finite groups, the complexity of such problems is not fully classified yet, since it is unknown for most solvable, non-nilpotent groups.

However (unlike in solving systems of equations) the complexity is not stable under changes of the signature: Solving equations in the alternating group A_4 is in P, while it is NP-complete in its extension by the commutator operation $[x, y]$. We generalize the latter hardness result to a big class of solvable, non-nilpotent groups. By our result we can classify the equation solvability problems of almost all finite groups extended by their commutator, essentially leaving the complexity open only for the dihedral groups D_{2p}.

Kavitha Koppula
On prime strong ideals of seminearrings
(joint work with Kedukodi Babushri Srinivas, MIT, Manipal Academy of Higher Education; Kuncham Syam Prasad, MIT, Manipal Academy of Higher Education)

The concept prime ideals and corresponding radicals play an important role in the study of nearrings. In this paper, we define different prime strong ideals of a seminearring S and study corresponding prime radicals. In particular, we prove that $P_e = \{S \mid P_e(S) = S\}$ is a Kurosh-Amitsur radical class where $P_e(S)$ denotes the intersection of equiprime strong ideals of S.

Miroslav Korbelář
Divisible factors of commutative semirings

Let $F = \mathbb{N}_0[x_1, \ldots, x_n]$ be the semiring of polynomials over the natural numbers $\mathbb{N}_0$. We confirm that a semiring S is additively idempotent provided that S is an additively divisible factor of a subsemiring T of F that is generated by some set of monomials. In particular, every finitely generated additively divisible commutative semiring with a unit is also additively idempotent. This work was supported by project CAAS CZ.02.1.01/0.0/0.0/16_019/0000778.

Sebastian Kreinecker

THE LATTICE OF INTEGER POLYNOMIAL NEARRINGS

(joint work with Erhard Aichinger)

We consider the polynomial nearring $(\mathbb{Z}[x], +, \circ)$, where the nearring multiplication is the composition of polynomials. We investigate the lattice of its subnearrings. In this talk we present some results about subnearring generation and give an infinite independent subset of $(\mathbb{Z}[x], +, \circ)$, which yields infinite ascending chains, infinite descending chains, uncountable linearly ordered subsets and uncountable antichains in the subnearring lattice.

Ganna Kudryavtseva

F-INVERSE MONOIDS IN ENRICHED SIGNATURE

(joint work with Karl Auinger, University of Vienna; Maria B. Szendrei, Bolyai Institute, University of Szeged)

Every F-inverse monoid can be equipped with a second unary operation $a \mapsto a^{\mathtt{m}}$. Here $a^{\mathtt{m}}$ denotes the greatest element in the σ-class of a. In this enriched signature, the class of all F-inverse monoids forms a variety of algebraic structures. We describe universal objects in several classes of F-inverse monoids, in particular free F-inverse monoids. More precisely, for every X-generated group G we describe the initial object in the category of all X-generated F-inverse monoids F for which $F/\sigma = G$.

Jan Kühr

DERIVATIONS ON POCRIMS AND PMV-ALGEBRAS

(joint work with J. Rachůnek; D. Šalounová)

We characterise the so-called derivations on bounded pocrims (partially ordered commutative residuated integral monoids) and on PMV-algebras (MV-algebras with product).

Helmut Länger

LATTICES OF SUBSPACES OF VECTOR SPACES

(joint work with Ivan Chajda)

For arbitrary vector spaces closed and splitting subspaces are introduced in a purely algebraic way and corresponding algebraic structures are presented.

Erkko Lehtonen

Associative spectra of graph algebras

(joint work with Tamás Waldhauser, University of Szeged)

The associative spectrum was introduced by Csákány and Waldhauser in 2000 as a measure of non-associativity of a binary operation. Let us denote by B_n the set of all bracketings of size n, i.e., groupoid terms obtained by inserting valid pairs of parentheses into $x_1 x_2 \dots x_n$. If $\mathbf{A} = (A; \cdot)$ is a groupoid, then the equational theory of $\mathbf{A}$ induces an equivalence relation $\sigma_n(\mathbf{A})$ on B_n. The sequence $(\sigma_n(\mathbf{A}))_{n=1}^{\infty}$ is called the *fine associative spectrum* of $\mathbf{A}$. The *associative spectrum* of $\mathbf{A}$ is the sequence $(s_n(\mathbf{A}))_{n=1}^{\infty}$, where $s_n(\mathbf{A}) = |B_n/\sigma_n(\mathbf{A})|$. Intuitively, the faster the associative spectrum grows, the less associative the operation is.

Introduced by Shallon in 1979, the *graph algebra* of a digraph $G = (V, E)$ is the algebra $\mathbb{A}(G) = (V \cup \{\infty\}; \cdot, \infty)$ of type $(2, 0)$, where ∞ is a new element distinct from the vertices, and $x \cdot y = x$ if $(x, y) \in E$, and $x \cdot y = \infty$ otherwise.

In this talk, we focus on associative spectra of (groupoid reducts of) graph algebras. We show that for an undirected graph G, the associative spectrum of $\mathbb{A}(G)$ is one of the just three possible sequences: $(1)_{n=1}^{\infty}$, $(\lceil 2^{n-2} \rceil)_{n=1}^{\infty}$, $(C_{n-1})_{n=1}^{\infty}$, where C_n is the n-th Catalan number. The situation is drastically more complicated for arbitrary digraphs G. We can, however, describe the fine associative spectrum of $\mathbb{A}(G)$ with the help of a few simple structural parameters of G.

Xianhua Li

Finite groups and synchronizing semigroups

(joint work with Wangwei Li)

The concept of synchronization comes from the theory of automata. From an algebraic viewpoint, an automaton is a subsemigroup of the full transformation semigroup on a finite set with a prescribed set of generators. Černý's conjecture (see Mat.-Fyz. Čas. Slovensk. Akad. Vied. 14 (1964), 208–216) in automata theory has attracted many experts in group and semigroup theory, they have done a lot of work. In this talk, I will introduce some of our recent work on finite groups and synchronizing semigroups.

Hajime Machida

Minimal study of minimal clones generated by binary functions

(joint work with Mike Behrisch, Technische Universität Wien)

A great Hungarian mathematician, B.C., studied minimal clones on a 3-element set. Another, a bit younger, great Hungarian mathematician, P^3, studied minimal clones generated by binary functions. One more, much younger, great Hungarian mathematician, T.W., studied minimal clones generated by majority functions. So, the topic of minimal clones is truly a Hungarian topic.

> "Are you also related to Hungary?"
>
> "Me? No. ... But, my co-author once lived in Szeged for a few months, which justifies our study of minimal clones."
>
> "Sure, it certainly does."

In this talk we introduce the *pr-distance* for a binary function, intending to reflect the nearness of the function to the projections. Then we show, for example, the following:

Theorem. *For an idempotent binary function f defined over any finite set A ($|A| > 1$), if the pr-distance of f is* 1 *then f generates a minimal clone.*

Peter Mayr

Commutators of subdirect products

(joint work with Keith Kearnes, CU Boulder; Nik Ruškuc, University of St Andrews)

In 2017 Derek Holt asked on mathoverflow whether any solvable quotient on a subdirect product of perfect groups is necessarily abelian. Recall that a group (algebra) is perfect if it does not have any non-trivial abelian quotient.

To answer Holt's question we first show that in any congruence modular variety, solvable quotients of subdirect products of finitely many perfect algebras are supernilpotent. Using higher commutator relations we then construct examples of finite subdirect products of perfect groups of arbitrary nilpotency class.

Antoine Mottet

Topology is relevant (in the dichotomy conjecture for infinite-domain constraint satisfaction problems)

(joint work with Manuel Bodirsky, TU Dresden; Jakub Opršal, Durham University; Mirek Olšák, Charles University; Michael Pinsker, TU Wien; Ross Willard, University of Waterloo)

The algebraic dichotomy conjecture for Constraint Satisfaction Problems (CSPs) of reducts of (infinite) finitely bounded homogeneous structures states that such CSPs are polynomial-time tractable when the polymorphism clone of the template locally satisfies some non-trivial height 1 identities. We show that local satisfaction and global satisfaction of non-trivial height 1 identities differ for ω-categorical structures with less than double exponential orbit growth, thereby resolving one of the main open problems in the algebraic theory of such structures.

Nebojša Mudrinski

The Ramsey property and finite distributive lattices

(joint work with Dragan Mašulović, University of Novi Sad)

The class of finite distributive lattices does not have the Ramsey property. In 2012 Kechris and Sokić have proved that no expansion of the class of finite distributive lattices by linear orders satisfies the Ramsey property. We have proved that the class of finite distributive lattices does not have the dual Ramsey property either. However, we are able to derive a dual Ramsey theorem for finite distributive lattices endowed with a particular linear order. Both results are consequences of the recently observed fact that categorical equivalence preserves the Ramsey property.

Henri Mühle

The core-label order of a congruence-uniform lattice

A (finite) lattice L is congruence-uniform if and only if it can be obtained from the singleton lattice by a sequence of interval doublings. If we record which edge of the poset diagram of L is created at which step in this process, then we obtain a natural edge-labeling of L. This set of edge labels can be used to define an alternate partial order on the elements of L: the core-label order of L.

The main source of examples for this construction comes from lattices of biclosed sets. In many cases, when these lattices of biclosed sets can be realized combinatorially, the corresponding core-label order is a lattice, too. In general, however, it is an open problem to characterize the congruence-uniform lattices whose core-label orders are lattices again. We provide an equivalent condition for the lattice property of the core-label order and we discuss constructions that either preserve or destroy the lattice property.

Finally we show how the core-label order can be used to characterize finite Boolean and finite distributive lattices.

Claudia Mureșan

VARIETIES OF PBZ*-LATTICES

(joint work with Roberto Giuntini and Francesco Paoli)

PBZ*-lattices are the paraorthomodular Brouwer–Zadeh lattices in which every pair of an element and its Kleene complement satisfies the Strong De Morgan condition; more precisely, a *PBZ*-lattice* is an algebra $(L, \vee, \wedge, \cdot', \cdot^{\sim}, 0, 1)$ of type $(2, 2, 1, 1, 0, 0)$ such that:

- $(L, \vee, \wedge, \cdot', 0, 1)$ is a bounded involution lattice, whose involution $\cdot'$ is called *Kleene complement*, satisfying, for all $a, b \in L$, the *Kleene condition*: $a \wedge a' \leq b \vee b'$, and *paraorthomodularity*: $a \leq b$ and $a' \wedge b = 0$ imply $a = b$;

- $\cdot^{\sim}$ is an order-reversing unary operation on L, called *Brouwer complement*, that satisfies, for all $a \in L$: $a \wedge a^{\sim} = 0$, $a \leq a^{\sim\sim} = a^{\sim\prime}$ and *condition* $(*)$:

$$(a \wedge a')^{\sim} = a^{\sim} \vee a'^{\sim}. \qquad (*)$$

We denote the variety of the PBZ*-lattices by $\mathbb{PBZL}^*$. The *sharp elements* of a PBZ*-lattice are its elements having their Kleene complements as bounded lattice complements. When endowed with a Brouwer complement equalling their Kleene complement, orthomodular lattices become exactly the PBZ*-lattices whose every element is sharp. We denote by $\mathbb{OML}$ the variety of orthomodular lattices, which includes the variety $\mathbb{BA}$ of Boolean algebras, considered with this extended signature. PBZ*-lattices whose only sharp elements are their lattice bounds are called *antiortholattices*; they form a proper universal class, denoted by $\mathbb{AOL}$, and the variety $\mathcal{HSP}(\mathbb{AOL})$ they generate intersects $\mathbb{OML}$ at the single atom $\mathbb{BA}$ of the lattice of subvarieties of $\mathbb{PBZL}^*$. For any non-trivial orthomodular lattice A and any non-trivial PBZ*-lattice B, the horizontal sum $A \boxplus B$ of the underlying bounded lattices of A and B becomes a PBZ*-lattice having A and B as subalgebras.

We investigate the structure of the lattice of subvarieties of $\mathbb{PBZL}^*$ and provide axiomatic bases for some of its members. For instance, we point out an infinite ascending chain of varieties of distributive PBZ*-lattices, which are thus subvarieties of $\mathcal{HSP}(\mathbb{AOL})$, we prove that the horizontal sums of non-trivial orthomodular lattices with non-trivial antiortholattices generate the subvariety $\mathcal{HSP}(\mathbb{OML} \boxplus \mathbb{AOL})$ relatively axiomatized w.r.t. $\mathbb{PBZL}^*$ by: **J2** $: x \approx (x \wedge (y \wedge y')^{\sim}) \vee (x \wedge (y \wedge y')^{\sim\sim})$, **S2** $: (x \wedge (y \wedge y')^{\sim})^{\sim} \approx x^{\sim} \vee (y \wedge y')^{\sim\sim}$ and **S3** $: (x \wedge (y \wedge y')^{\sim\sim})^{\sim} \approx x^{\sim} \vee (y \wedge y')^{\sim}$, while the join $\mathbb{OML} \vee \mathcal{HSP}(\mathbb{AOL})$ is the subvariety axiomatized w.r.t. $\mathcal{HSP}(\mathbb{OML} \boxplus \mathbb{AOL})$ by **WSDM** $: (x \wedge y^{\sim})^{\sim} \approx x^{\sim} \vee y^{\sim\sim}$ and thus w.r.t. $\mathbb{PBZL}^*$ by **J2** and **WSDM**.

Cyrus F. Nourani

Product models and ultrafilter categories: a preview

(joint work with Patrik Eklund, Umeå University)

This abstract is an overview towards newer models for algebraic topology with ultraproducts and ultrafilters on fragment categories for n-types. Towards filter monads we consider newer models since (Nourani VSL Vienna) on product topologies with n-type ultrafilters. (Nourani-Eklund 2015-MAA) was a glimpse on characterizing product models with monads.

Newer ultrafilter functor categories (Nourani 2018) are applied towards model category applications in a forthcoming volume on term functor categories (Nourani-Eklund 2019). The first author has defined a discrete topology on the Kiesler fragment on infinitary logic presented at ASL Summer Colloquium several years ago for projective set saturated models. New techniques have been developed since for term functor monads and ultrafilter categories. Newer theorems on ultrafilter functor categories, for example, prove that there are embedding functors on direct product categories realizing a filter for the product algebras that are with filter monad. New sheaves natural transformations are further new applications for topological structure models that are cardinal saturation models in the Kiesler ultrafilter order sense. The new areas are presented in a 2019 volume by Cyrus Nourani and Patrik Eklund entitled 'Term Functors, Filter Computability, and Realizability Morphisms', Düsseldorf.

Anvar Nurakunov

On complexity of the structure of quasivarieties and quasivariety lattices

(joint work with Alexander Kravchenko, Sobolev Institute of Mathematics, Siberian Branch RAS; Marina Schwidefsky, Sobolev Institute of Mathematics, Siberian Branch RAS)

We find a sufficient condition for a quasivariety to have continuum many subquasivarieties having an independent quasi-equational basis but for which the quasi-equational theory and the finite membership problem are undecidable. We present a number of applications.

Jakub Opršal

Some very weak height 1 identities

(joint work with Manuel Bodirsky, TU Dresden; Antoine Mottet, Charles University, Prague; Mirek Olšák, Charles University, Prague; Michael Pinsker, TU Wien and Charles University, Prague; Ross Willard, University of Waterloo)

There have been a few celebrated results showing that a certain property of algebras can be expressed in a very elegant manner using so-called strong Maľcev conditions, i.e., conditions that require existence of finitely many terms satisfying some finite set of identities. Two most notable results in this field are: a Siggers term that describes all finite idempotent algebras that satisfy a non-trivial Maľcev condition, and an Olšák term that describes all idempotent algebras that satisfy a non-trivial Maľcev condition.

Notably both of these conditions are of height 1 (they do not involve composition of functions). Height 1 identities play a big role in the complexity of CSPs. In particular, using Siggers's result and a simple trick of reducing to a core, the Bulatov-Zhuk dichotomy theorem can be formulated as follows: If a finite relational structure has a polymorphism satisfying the Siggers identity, its CSP is solvable in polynomial time, if not, its CSP is NP-complete. Motivated by this fine dichotomy and CSPs over infinite domain relational structures, we ask whether, similarly to the finite domain case, there is a weakest strong height 1 Maľcev condition.

Péter P. Pálfy

Some self-commuting operations

(joint work with Hajime Machida)

An n-ary operation f is self-commuting, if it satisfies the identity

$$\begin{aligned}&f(f(x_{11},x_{12},\dots,x_{1n}),f(x_{21},x_{22},\dots,x_{2n}),\dots,f(x_{n1},x_{n2},\dots,x_{nn}))\\=&f(f(x_{11},x_{21},\dots,x_{n1}),f(x_{12},x_{22},\dots,x_{n2}),\dots,f(x_{1n},x_{2n},\dots,x_{nn})).\end{aligned}$$

An operation is called conservative if every subset is closed under the operation. We determined all binary conservative self-commuting operations. We also found all self-commuting operations (of arbitrary arity) over the 2-element set.

Bojana Pantić

On homomorphism-homogeneous and polymorphism-homogeneous metric spaces

(joint work with Maja Pech, University of Novi Sad, Faculty of Sciences, Department of Mathematics and Computer Sciences)

The notion of homomorphism-homogeneous structures was first brought to light by P. J. Cameron and J. Nešetřil in 2002, as a relaxed version of homogeneity. We say that a structure is homomorphism-homogeneous if every homomorphism between finitely generated substructures extends to an endomorphism of the structure. Another generalisation – polymorphism-homogeneity, ensued in 2014 by C. Pech and M. Pech, with structures possessing this property in case any local polymorphism may be extended to a global polymorphism of the structure in question. The classification of such structures, in both cases, is still a rather challenging problem, even for finite structures.

The aim of this talk is to report on the current results from the ongoing research on metric spaces, with respect to these two properties.

This is joint work with Maja Pech.

Jan Paseka

Injectivity in ordered semicategories

There are quite a lot of papers investigating injective hulls for algebras. Here we only mention some of them which our current work is related to. Injective hulls for posets were studied by Banaschewski and Bruns (1967) where they got that the injective hull of a poset is its MacNeille completion. After that, Bruns and Lakser (1970), and independently Horn and Kimura (1971) constructed injective hulls of semilattices, and their results were soon applied into S-systems over a semilattice by Johnson, Jr., and McMorris (1972). By the conclusion of Schein (1974) that there are no non-trivial injectives in the category of semigroups, it took a long time to make further development for the theory of injective hulls on both discrete and ordered (general) semigroups. In 2012, Lambek, Barr, Kennison and Raphael studied a kind of category of pomonoids in which the usual category of pomonoids is its subcategory, and found that injective hulls for pomonoids are exactly unital quantales. Later on, Zhang and Laan generalized their results first to the posemigroup case (2014), and later to S-posets (2015) and ordered Ω-algebras (2016). In 2017, Xia, Zhao, and Han obtained almost the same constructions as Zhang and Laan, but they described it in a different way.

In our lecture we explain how some of the preceding results admit far-reaching generalizations in the framework of semicategories. We put the results on S-posets by Zhang and Laan into that wider perspective. We consider here the “multi-signature” version of modules over posemigroups which turns out to be modules over ordered semicategories. Our approach sheds a new light on applications of a new kind of fuzzy-like structure.

Gonçalo Pinto

Pointed principally ordered regular semigroups

(joint work with T. S. Blyth, Mathematical Institute, University of St. Andrews, Scotland)

An ordered semigroup is said to be *principally ordered* if, for every $x \in S$ there exists $x^* = \max\{y \in S \mid xyx \leq x\}$. Here we investigate those principally ordered regular semigroups that are *pointed* in the sense that the classes modulo Green's relations R, L, have biggest elements which are idempotent. Such semigroups are necessarily idempotent-generated. In particular, we describe the structure of the subalgebra of $(S; {}^*)$ generated by a pair of comparable idempotents that are D-related.

Nikolay L. Polyakov

On a variant of the Rudin–Keisler preorder defined via finitary maps

(joint work with Denis I. Saveliev, Russian Academy of Sciences)

The classical notion of the Rudin–Keisler preorder on ultrafilters over an infinite set X is defined by letting $u \leq_{\mathrm{RK}} v$ iff there exists a map $f : X \longrightarrow X$ such that $\tilde{f}(u) = v$ for all $u, v \in \beta X$, where βX is the set of ultrafilters over X endowed with the standard topology, and $\tilde{f} : \beta X \longrightarrow \beta X$ is the continuous extension of f (see e.g. W. W. Comfort, S. Negrepontis, The theory of ultrafilters, Springer, 1974; N. Hindman, D. Strauss, Algebra in the Stone–Čech compactification, 2nd ed., de Gruyter, 2012). By using right continuous extensions of maps of any finite arities (introduced in D. I. Saveliev, Ultrafilter extensions of models, Lecture Notes in AICS 6521 (2011), 162–177), we expand this notion as follows. Let $u \; R_n \; v$ iff there exists an n-ary map $f : X^n \longrightarrow X$ such that $\tilde{f}(u, \ldots, u) = v$ for all $u, v \in \beta X$ where $\tilde{f} : (\beta X)^n \longrightarrow \beta X$ is the right continuous extension of f. Let also $\leq$ be the union of all the R_n, $n \in \omega$. Clearly, R_1 coincides with $\leq_{\mathrm{RK}}$, and $R_m \subseteq R_n$ whenever $m \leq n$.

We show that $R_m \circ R_n = R_{m \cdot n}$, whence it follows that $\leq$ is a preorder. We prove that this preorder properly extends the Rudin–Keisler preorder, i.e., there are $u, v \in \beta X$ such that $u \leq v$ but not $u \leq_{\mathrm{RK}} v$. As is well known, under the Continuum Hypothesis (CH) there exist non-principal ultrafilters over ω that are $\leq_{\mathrm{RK}}$-minimal (see Comfort, Negrepontis, op. cit.). We show that an analogous fact is true for the stronger preorder $\leq$. To prove this, we state the following generalization of the Ramsey theorem, which may be of an independent interest: for any $n \in \omega$ and n-ary map $f : \omega^n \longrightarrow \omega$ there exist $m \leq n$, $K \in [n]^m$, and an infinite $A \subseteq \omega$ such that for increasing n-tuples $x_0 < \ldots < x_{n-1}$ and $y_0 < \ldots < y_{n-1}$ in A we have $f(x_0, \ldots, x_{n-1}) = f(y_0, \ldots, y_{n-1})$ iff $x_k = y_k$ for all $k \in K$. (The case $m = 0$ gives a constant map.)

Finally, we expand the hierarchy from the relations R_n to R_α for arbitrary ordinals α, and strengthen some of the obtained results. In particular, we show that the R_α are increasing under inclusion as α grows, and included in the Comfort preorder. On $\beta\omega$, all R_α with $\alpha < \omega_1$ are distinct, and $R_{<\alpha} = \bigcup_{\xi<\alpha} R_\xi$ is a preorder whenever α is a multiplicatively indecomposable ordinal.

The second author was supported by the RFBR grant 17-01-00705.

Reinhard Pöschel

Graph quasivarieties

(joint work with Erkko Lehtonen)

Graph algebras were introduced by C. R. Shallon in 1979 and establish a useful connection between graph theory and universal algebra. Given a graph $G = (V, E)$, the corresponding *graph algebra* is $A(G) = \langle V \cup \{\infty\}, \cdot, \infty\rangle$ where $\infty \notin V$ is a constant and "$\cdot$" is a binary operation given by $x \cdot y = x$ if $(x, y) \in E$ is a (directed) edge, otherwise $x \cdot y = \infty$. The class of graph algebras does not form a variety nor a quasivariety, but nevertheless one may ask how to characterize classes of graphs definable by identities or quasiidentities (called *graph varieties* or *graph quasivarieties*). Graph varieties were characterized by R. Pöschel in 1990, graph quasivarieties for (finite) undirected graphs by R. Pöschel and W. Wessel in 1987. Here we present a "Birkhoff-style" theorem for graph quasivarieties of arbitrary (finite) graphs: *A class of graphs is a graph quasivariety if and only if it is closed under isomorphic copies and so-called strong pointed subproducts* (which shall be introduced in the talk).

Tom Quinn-Gregson

CSPs of homogeneous ω-categorical algebras

Given an algebra A, we are concerned with the following computational complexity problem:

- Instance: a finite list $\mathcal{E}$ of equations and disequalities over A with variables from a finite set V.
- Question: is there an assignment $\varphi : V \to A$ such that $\mathcal{E}$ holds in A?

For example, if A is a semigroup then an instance could be $\{xy = z, zy = t, y \neq t\}$. We are chiefly concerned with ω-categorical homogeneous algebras. The constraint satisfaction problem (CSP) for ω-categorical structures is well-studied, and we shall show that the algebras we consider give rise to CSPs with 'well behaved' templates.

A number of algebras have already been considered; the atomless Boolean algebra is $\mathbb{NP}$-hard (Bodirsky, Hils, Krimkevitch) while the infinite dimensional vector space over $\mathbb{F}_q$ is tractable (Bodirsky, Chen, Kára, von Oertzen). We extend the latter work by classifying the tractable ω-categorical abelian groups. General methods for showing the complexity of an algebra are also discussed, which allow a classification of tractable homogeneous ω-categorical semilattices. The following open problem is vital to further study: does there exist a tractable ω-categorical non-abelian group?

This work is ongoing.

Jakub Rydval

Temporal constraint satisfaction problems in fixed point logic

(joint work with Manuel Bodirsky, TU Dresden; Wied Pakusa, RWTH Aachen University)

The constraint satisfaction problem (CSP) for a fixed structure $\mathfrak{A}$ with finite relational signature is the computational problem of deciding whether a given finite structure $\mathfrak{A}$ homomorphically maps to $\mathfrak{B}$. A temporal constraint language (TCL) is a structure over the rational numbers $\mathbb{Q}$ whose relations have a first-order definition in $(\mathbb{Q}; <)$. The computational complexity of CSPs for TCLs has been classified: each of them is either in P or NP-complete. In contrast to finite domain structures, there are TCLs whose CSP cannot be solved by any Datalog program but that can be expressed in fixed point logic (FP). The logic FP and its counting extension FP+C are important formalisms in the quest for finding a logic for P; a substantial fragment of problems in P can be expressed in these logics. In this talk we present a complete classification of those temporal constraint languages whose CSP is expressible in FP or in FP+C.

Mohamed A. Salim

Neighborhood radius estimation for Arnold's miniversal deformations of complex and p-adic matrices

(joint work with V. Bovdi, UAEU; V. Sergeichuk, IM Kiev)

Arnold (Matrices depending on parameters. *Uspehi Mat. Nauk*, 26(2(158)):101–114, 1971) constructed a simple normal form B_{arn} to which all complex matrices B in a neighborhood U of a given square matrix A can be reduced by similarity transformations that smoothly depend on the entries of B. This result was extended to real matrices by Galin (Real matrices depending on parameters. *Uspehi Mat. Nauk*, 27(1(163)):241–242, 1972).

We (Neighborhood radius estimation for Arnold's miniversal deformations of complex and p-adic matrices. *Linear Algebra Appl.*, 512:97-112, 2017) extend Arnold's normal form of complex matrices to matrices over any field that is complete with respect to a nontrivial absolute value, in particular, over the field $\mathbb{Q}_p$ of p-adic numbers and the field $\mathbb{F}((T))$ of Laurent series over a field $\mathbb{F}$. Over such a field, we construct a smooth similarity transformation that transforms all $B \in U$ to B_{arn}. Another method for constructing this transformation for complex matrixes was developed by Mailybaev (Transformation to versal deformations of matrices. *Linear Algebra Appl.*, 337:87–108, 2001).

Also we find the neighborhood U in an explicit form, which is important for applications. As far as we know, the estimate of the radius of U was unknown even for complex matrices.

Dana Šalounová

A lattice-theoretical approach to extensions of filters in algebras of certain fuzzy logics

(joint work with Jiří Rachůnek)

Commutative bounded integral residuated lattices (residuated lattices, in short) form a large class of algebras which can be interpreted as algebraic counterparts of certain propositional fuzzy logics. Among others, MTL-algebras, Rℓ-monoids, BL-algebras, MV-algebras and Heyting algebras can be considered as such algebras.

In our talk we will deal with the so-called extended filters of filters associated with subsets of residuated lattices. We will use the fact that if B is a subset of a residuated lattice M and F is a filter of M, then the extended filter associated with B coincides with the filter associated with the filter $[B]$ of M generated by B. We will show that this enables us to investigate, without loss of generality, only extended filters within the Heyting lattice of filters of M, using the technique of Heyting algebras.

Kritsada Sangkhanan

Green's relations and regularity for semigroups of transformations with restricted range that preserve double direction equivalence relations

(joint work with Utsithon Chaichompoo, Chiang Mai University)

Let $T(X)$ be the full transformation semigroup on a set X. For an equivalence E on X and a nonempty subset Y of X, let

$$T_{E^*}(X,Y) = \{\alpha \in T(X) : X\alpha \subseteq Y \text{ and } \forall x,y \in X, (x,y) \in E \Leftrightarrow (x\alpha, y\alpha) \in E\}.$$

In this work, we give a necessary and sufficient condition for $T_{E^*}(X,Y)$ to be a subsemigroup of $T(X)$ under the composition of functions and study the regularity of $T_{E^*}(X,Y)$. Finally, we characterize Green's relations on this semigroup.

Denis I. Saveliev

IDEMPOTENT ULTRAFILTERS ARE NOT WEAKLY SELECTIVE

We show that over weakly cancellative semigroups there exist no uniform ultrafilters which would be idempotent and selective. Moreover, if such a semigroup S has cardinality $\kappa > \omega$, then for any $\lambda < \kappa$ there exist no uniform idempotent (λ, ω)-selective ultrafilters over S.

This result improves earlier results by Protasov and Hindman stating that there exist no idempotent σ-complete ultrafilters over groups. A classical fact of algebra of ultrafilters is that the ultrafilter extension of any semigroup has an idempotent, and even a uniform idempotent whenever the semigroup is weakly cancellative. As ultrafilters satisfying weaker versions of selectivity can exist over cardinals below the first measurable, the improvement is essential.

The author was supported by the RFBR grant 17-01-00705.

Stefan Schmidt

ON DIRECTED DISTANCE, PROXIMITY, AND FUZZY ORDER

(joint work with Stefan Schmidt)

The talk discusses the relationship between three fundamental concepts of measurement. In particular, we provide supporting arguments for the significance of the concepts of directed distance and directed proximity. Examples of the latter occur as material implications in the sense of residuation theory, also relevant in fuzzy formal concept analysis and in quantitative concept analysis (in the sense of Dusko Pavlovic). Finally, we point out the intimate relationship between the concepts of directed proximity and fuzzy order. We conclude that fuzzy mathematics has implicitly been around longer than usually believed.

Friedrich Martin Schneider

RANDOM WALKS ON AUTOMORPHISM GROUPS

(joint work with Andreas Thom, TU Dresden, Institute of Geometry)

The study of random walks on discrete (resp., locally compact) groups, initiated in the 1960s by the seminal work of Furstenberg, Kesten, and others, is a central topic in geometric group theory. In my talk, I will explain the basics of an emerging theory of random walks on large (i.e., not necessarily locally compact) topological groups, with particular focus on automorphism groups of countably infinite discrete structures, and describe a natural connection between structural Ramsey theory and the theory of

Liouville actions. The main result to be presented is a general topological version of Furstenberg's boundary conjecture. I will illustrate how this result can be combined with Ramsey theory to gain new insights into the geometry of Thompson's group F and answer some recent questions by Kate Juschenko.

Aishwarya Shivananda

On (σ, τ)-derivations in nearrings

(joint work with Kedukodi Babushri Srinivas, MIT, Manipal Academy of Higher Education; Kuncham Syam Prasad, MIT, Manipal Academy of Higher Education)

We study (σ, τ)-derivations on nearrings and define a similarity relation on the set of (σ, τ)-derivations of a nearring. We prove that similarity is preserved under nearring automorphisms and show that it is not necessarily preserved under group automorphisms. Finally, we obtain commutativity results in nearrings using (σ, τ)-derivations.

Olʹga Sipachëva

Independent and large sets in Boolean groups, ultrafilters, and topology

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A topological group is a group with a topology with respect to which both group operations (multiplication and inversion) are continuous. We shall consider only nondiscrete Hausdorff group topologies, because only they are of interest. The existence of such a topology with certain properties imposes constraints on the algebraic structure. Thus, there are groups which do not admit any group topology at all, a compact group topology can exist only on residually finite groups, and so on. One of the topological properties in worst agreement with the group structure is extremal disconnectedness (a topological space is extremally disconnected if the closure of any open set is open, or, equivalently, if its Boolean algebra of clopen subsets is complete), and the class of groups admitting group topologies with the most diverse properties is that of Boolean groups. The question of the existence in ZFC of an extremally disconnected topological group, which was asked by Arhangelʹskii in 1967, is still open, but it is known that any extremally disconnected group contains an open Boolean subgroup (Malyhin, 1975). This question has proved closely related to the existence of special ultrafilters on ω, on the one hand, and of nonclosed discrete sets in topological groups, on the other hand.

Boolean groups have the simplest structure; in particular, each Boolean group is a vector space over the field $\mathbb{F}_2 = \{0, 1\}$ and is freely generated by any basis of this space. Thus, any Boolean group is the free Boolean group $B(X)$ on a basis X (that is, the set $[X]^{<\omega}$ of all finite subsets of X under the operation $\triangle$ of symmetric difference), and any nondiscrete topology on X induces the nondiscrete free Boolean group topology on $B(X)$ (the weakest group topology such that any continuous map of X to any Boolean topological group extends to a continuous homomorphism). However, this is of little help in solving the extremal disconnectedness problem:

Theorem. *If there exists a nondiscrete topological space X for which $B(X)$ is extremally disconnected, then there exists a Ramsey ultrafilter on ω.* (The nonexistence of Ramsey ultrafilters is consistent with ZFC.)

One of the most natural topologies on a set X is that generated by a filter: we take any point $* \in X$ and any free filter $\mathcal{F}$ on $Y = X \setminus \{*\}$; the neighborhoods of $*$ are the elements of $\mathcal{F}$, and all points of Y are isolated. We denote the topological space thus obtained by $X_{\mathcal{F}}$. The free Boolean topological group $B(X_{\mathcal{F}})$ is topologically isomorphic to the quotient $B(X_{\mathcal{F}})/\{0, *\}$; we denote this quotient by $B(\mathcal{F})$.

Theorem. *An ultrafilter $\mathcal{U}$ on a countable Boolean group is Ramsey iff it contains a linearly independent set X and $B(\mathcal{U}{\restriction}X)$ is extremally disconnected.*

If an extremally disconnected countable Boolean group contains a nondiscrete linearly independent set, then there exists a P-point ultrafilter on ω. (The nonexistence of P-point ultrafilters is consistent with ZFC.)

If there exists a Ramsey ultrafilter on a cardinal κ, then there exists a (nondiscrete) Boolean topological group of cardinality κ in which all bases (and hence all independent sets) are closed and discrete.

On the other hand, the existence of only one closed discrete basis in a countable Boolean topological group does not require additional set-theoretic assumptions: *Any countable Boolean topological group has a closed discrete basis.*

Zelenyuk proved that the existence of any (not necessarily independent) nonclosed discrete countable set in an extremally disconnected group implies that of P-point ultrafilters. On the other hand, the nonexistence of (nondiscrete) countable topological groups without such sets is consistent with ZFC:

Theorem. *If there exist no rapid (ultra)filters on ω, then*

(i) *any countable nondiscrete topological group contains a discrete set with precisely one limit point;*

(ii) *any countable nondiscrete Boolean topological group contains two disjoint discrete subsets for each of which zero is the only limit point.*

It is unknown whether there exists a model of ZFC in which there are neither P-point nor rapid ultrafilters; however, an extremally disconnected group cannot contain two disjoint sets specified in (ii). Therefore, the nonexistence of countable extremally disconnected groups is consistent with ZFC.

The key role in the proof of the last theorem is played by a new class of large sets in groups introduced by Reznichenko and the author and called vast sets. Various notions of large sets in groups and semigroups naturally arise in dynamics and combinatorial number theory. Most familiar are those of syndetic, thick (or replete), and piecewise syndetic sets. (A set in a group is syndetic if finitely many translates of this set cover the group, a set is thick if it intersects all syndetic sets, and piecewise syndetic sets are the intersections of syndetic sets with thick ones.) It is hard to say which is more interesting, these sets themselves or the interplay between them. Thus, piecewise syndetic sets in $\mathbb{N}$ are partition regular, contain arbitrarily long arithmetic progressions, and admit an ultrafilter characterization; the difference set of a syndetic set in a countable Abelian group almost (up to a set of upper Banach density zero) contains a set open in the Bohr topology, and so on. Vast sets are unique in that they form a filter (although they can generate a group topology only under additional set-theoretic assumptions); on the other hand, vast sets in Boolean groups are very close to Δ_n^*-sets introduced by Bergelson, Furstenberg, and Weiss for $\mathbb{Z}$. We construct new examples distinguishing between various kinds of large sets and characterize certain (in particular, arrow and Ramsey) ultrafilters on arbitrary infinite sets in terms of vast sets in Boolean groups.

Dmitry Skokov

On modular elements of the lattice of semigroup varieties

(joint work with Viacheslav Shaprynskii, Ural Federal University, Institute of Natural Sciences and Mathematics)

During the last 25 years, a number of articles appeared where special elements of different types in the lattice SEM of all semigroup varieties were examined. Special elements of several types (for instance, neutral, distributive, lower-modular elements) in SEM are completely determined.

One of the most interesting types of special elements are modular elements. First results about modular elements in SEM were obtained by Ježek and McKenzie in 1993. Further achievements are due to Vernikov (2007) and Shaprynskii (2012). Nevertheless, the problem of a complete description of modular elements in the lattice SEM is still open so far. We are going to talk about recent results and some constructions concerning modular elements in SEM.

The reported study was funded by RFBR according to the research projects No.18-31-00443 and No.17-01-00551, and by the Ministry of Education and Science of the Russian Federation (project 1.6018.2017/8.9).

Florian Starke

Smooth digraphs modulo pp-constructability

(joint work with Albert Vucaj, Technische Universität Dresden, Institut für Algebra)

We call a digraph smooth if every vertex has at least one incoming and one outgoing edge. We consider the set of all finite smooth digraphs ordered by pp-constructability, i.e., $A \geq B$ iff A is pp-constructable from B. In a recent result Barto, Kozik, and Niven showed that the core of a smooth digraph with a WNU is a disjoint union of directed cycles. Therefore we will mostly talk about disjoint unions of cycles. In this talk we will present results about the structure of this quasi-order. We will show that every finite smooth digraph with a WNU is equivalent to a disjoint union of cycles of lengths $l_1, \ldots, l_n$, such that no l_i is a multiple of another and for every prime number p that divides some of the l_i there is a j where p^2 does not divide l_j. In particular, every directed cycle is equivalent to a directed cycle whose length is square-free. Furthermore, we show that the induced poset has an infinite chain, no atoms, and infinitely many coatoms.

Teerapong Suksumran

Left regular representation of gyrogroups

In this work, we examine a subspace $L^{\mathrm{gyr}}(G)$ of the complex vector space

$$L(G) = \{f \colon f \text{ is a function from } G \text{ into } \mathbb{C}\},$$

where G is a nonassociative group-like structure called a gyrogroup. We construct a linear action of G so that $L^{\mathrm{gyr}}(G)$ becomes a representation space for G, consisting of complex-valued functions invariant under certain permutations of G. In the case when G is finite, we give a (basis-dependent) lower bound for the dimension of $L^{\mathrm{gyr}}(G)$ in terms of numbers of fixed points:

$$\dim\left(L^{\mathrm{gyr}}(G)\right) \geq \frac{1}{|G|}\sum_{a\in G}|X_a|,$$

where X is a union of orbits induced by the action of G that cover a basis of $L^{\mathrm{gyr}}(G)$ and $X_a = \{f \in X \colon a \cdot f = f\}$ for all $a \in G$.

Csaba Szabó

Test-enhanced learning in studying university algebra

(joint work with Csilla Bereczky-Zámbó, Anna Muzsnay, Janka Szeibert)

This is not an algebra talk. This is about *teaching* algebra.

Retrieving information from memory after an initial learning phase enhances long-term retention more than restudying the material. Test-enhanced learning is a method which uses active recall of the information during the learning process. It has been proved to be efficient concerning texts or foreign words, these experiments were principally carried out in laboratory-environment.

The topic of our presentation is about an experiment on the efficacy of test-enhanced learning at university level in mathematics. The aim of the experiment was to understand whether test-enhanced learning is efficient regarding complex, abstract material which requires deductive thinking. (Previous results of laboratory-experiments on complex material are contradictory.) We carried out an experiment for pre-service math teachers during their first three semesters in the Algebra and Number Theory course at Eötvös Loránd University. Each semester, half of the students – the experimental group – learned the subject using the testing effect and the other half – the control group – were taught in the usual manner. Results of the first semester of the experiment were presented at SSAOS 56th edition. In this talk we show the all-over results of the experiment.

The experimental group showed a significantly better performance in the midterms and after-tests on the topic of the first semester. The 2nd and 3rd semester included more abstract material. Although there was no difference between the midterm scores of the control and experimental groups during these semesters, the after-tests showed a huge difference for the experimental group. Our results prove the long-term efficacy of testing effect in university-level mathematics.

Bruno Teheux

Order-theoretic properties of profinite completions of median algebras

(joint work with Georges Hansoul, University of Liège)

Recall that a ternary algebra $(A, \mathbf{m})$ is a *median algebra* if it satisfies the equational theory of the median term $m(x, y, z) := (x \vee y) \wedge (x \vee z) \wedge (y \vee z)$ in the variety of bounded distributive lattices. Every median algebra $\mathbf{A}$ is residually finite, and has a profinite completion $\hat{\mathbf{A}}$.

Given an element a of a median algebra $(A, \mathbf{m})$, the relation $\leq$ defined on A by

$$b \leq c \quad \text{if} \quad \mathbf{m}(\mathrm{a}, \mathrm{b}, \mathrm{c}) = b \qquad \text{(ord)}$$

is a meet-semilattice order. Semilattices obtained from a median algebra by (ord) are called *median semilattices*.

In the talk, we investigate the properties of $\hat{\mathbf{A}}$ with regard to the median semilattices associated with $\mathbf{A}$ and $\hat{\mathbf{A}}$. If time permits, we will also investigate ways to extend maps between median algebras to their profinite completions, and show some preservation results for expansions of median algebras.

Endre Tóth

Solution sets of systems of equations over finite algebras

(joint work with Tamás Waldhauser, University of Szeged)

Solution sets of systems of homogeneous linear equations over fields are characterized as being subspaces, i.e., sets that are closed under linear combinations. Our goal is to characterize solution sets of systems of equations over arbitrary finite algebras by a similar closure condition. We show that solution sets are always closed under the centralizer of the clone of term operations of the given algebra; moreover, the centralizer is the only clone that could characterize solution sets. If every centralizer-closed set is the set of all solutions of a system of equations over a finite algebra, then we say that the algebra has Property (SDC) (SDC stands for Solution sets are Definable by closure under the Centralizer). Our main results are the following. We show that every two-element algebra has Property (SDC), and we also describe finite lattices and semilattices with Property (SDC): we prove that a finite lattice has Property (SDC) if and only if it is a Boolean lattice, and a finite semilattice has Property (SDC) if and only if it is distributive.

Edith Vargas-García

The number of clones determined by disjunctions of unary relations

(joint work with Mike Behrisch, Dmitriy Zhuk)

We consider finitary relations (also known as crosses) that are definable via finite disjunctions of unary relations, i.e. subsets, taken from a fixed finite parameter set Γ. In this talk we will see how to prove that whenever Γ contains at least one non-empty relation distinct from the full carrier set, there is a countably infinite number of polymorphism clones determined by relations that are disjunctively definable from Γ. In particular we answer the open question of determining the exact cardinality of the set of all clones described by clausal relations, so-called *C-clones*.

Boris Vernikov

Cancellable elements of varietal lattices

(joint work with Vyacheslav Shaprynskii, Ural Federal University; Dmitry Skokov, Ural Federal University)

There are a number of articles where special elements in the lattice of all semigroup varieties and certain related varietal lattices are examined. By related varietal lattices we mean certain sublattices of the lattice of semigroup varieties (such as the lattices of all commutative or all overcommutative varieties) as well as varieties of certain types of algebras related to semigroups (such as epigroups or monoids). Here we continue these researches. An element x of a lattice $\langle L; \vee, \wedge \rangle$ is called *cancellable* if, for all $y, z \in L$, the equalities $x \vee y = x \vee z$ and $x \wedge y = x \wedge z$ imply that $y = z$. We completely determine all cancellable elements in three varietal lattices, namely in the lattice of all semigroup varieties, the lattice of overcommutative semigroup varieties and the lattice of all epigroup varieties. In particular, we verify that in the lattice of overcommutative varieties the property to be a cancellable element is equivalent to the property to be a neutral element.

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Thomas Vetterlein

Quotients in projective geometry

Quantum physical processes are commonly described within a Hilbert space over the complex numbers. In order to understand why precisely this model fulfils its purpose, the question is natural whether we might reduce it to a structure of algebraic, relational, or category-theoretic type (cf. K. Engesser et al., “Handbook of Quantum Logic and Quantum Structures”). A well-known idea is to focus on the ortholattice of closed subspaces. Furthermore, the collection of one-dimensional subspaces of a Hilbert space gives rise to a so-called orthogonality space, whose characteristic features can be captured be means of assumptions on the existence of certain automorphisms.

Whereas the latter approach works quite well in infinite dimensions, the finite-dimensional case requires particular additional efforts (see Th. V., Orthogonality spaces of finite rank and the complex Hilbert spaces). It is comparably easy to construct a suitable orthomodular space (generalised Hilbert space) over an involutorial division ring, endowed with an order in the sense of Baer. In order to single out the canonical case, however, it remains to ensure the Archimedean property. This turns out to be surprisingly difficult.

We present a hypothesis about orthogonality spaces that lacks elegancy but has the desired effect. Namely, we exclude explicitly the existence of certain non-trivial quotients. When an orthogonality space does allow such a quotient, the question arises which structure-preserving map should correspond to it on the side of the projective geometry associated with the representing space. Certainly, the usual choice of morphisms between projective spaces (see C.-A. Faure, A. Frölicher, Modern projective geometry) is not useful in the present context. We propose an alternative notion, involving a conditional preservation of collinearity.

Albert Vucaj

Two-element structures modulo pp-constructability

We study a partially ordered set which naturally arises from recent work by Barto, Opršal and Pinsker. This poset can be defined in three equivalent ways: from a relational point of view, we define $\mathbb{A}$ to be smaller than $\mathbb{B}$ if $\mathbb{A}$ pp-constructs $\mathbb{B}$; this corresponds to studying classes of algebras that are not only closed under homomorphic images, subalgebras and direct products, but also under taking reflections. This in turn is equivalent to assert that there is a minor-preserving map from $\mathrm{Pol}(\mathbb{A})$ to $\mathrm{Pol}(\mathbb{B})$. In this talk, we consider the restriction $\mathfrak{L}$ to the case of clones on a two-element set; the poset turns out to be a lattice in this case. Following the celebrated classification of Post, we explicitly provide the pp-constructions or the minor-preserving maps that insure collapses and, on the other side, we provide the height 1 conditions which guarantee separations. We display a picture of $\mathfrak{L}$ and in this way we give a modern treatment of the applications to Complexity Classification of Boolean Constraint Satisfaction Problems described by Allender, Bauland, Immerman, Schnoor and Vollmer.

Tamás Waldhauser

Categorical equivalence of finite rings

(joint work with Kalle Kaarli and Oleg Košik, University of Tartu)

Two algebras **A** and **B** are called *categorically equivalent*, if there is a categorical equivalence between the varieties they generate sending **A** to **B**. A special case of categorical equivalence is weak isomorphism: **A** and **B** are called *weakly isomorphic* if **A** is term equivalent to an isomorphic copy of **B**.

There are several results describing categorical equivalence in certain classes of algebras:

- all primal algebras are categorically equivalent to each other (Hu, 1969);
- the finite fields $\mathbf{F}_{p^k}$ and $\mathbf{F}_{p^\ell}$ are categorically equivalent if and only if $k = \ell$ (Bergman and Berman, 1996);
- two finite groups are categorically equivalent if and only if they are weakly isomorphic (Zádori, 1997);
- two finite semigroups are categorically equivalent if and only if they are weakly isomorphic (Behrisch and Waldhauser, 2012);
- two lattices are categorically equivalent if and only if they are isomorphic or dually isomorphic (Košik, 2012).

We present some results towards a description of categorical equivalence of finite rings with identity. Every finite ring can be represented as a direct product of p-rings for different primes p. First we reduce the problem to these p-components.

Theorem 3. *Finite rings* $\mathbf{R}$ *and* $\mathbf{S}$ *are categorically equivalent if and only if there is a one-to-one correspondence between their p-components such that the corresponding p-components are categorically equivalent.*

It remains to describe all categorical equivalences between a finite p-ring $\mathbf{R}$ and a finite q-ring $\mathbf{S}$ for possibly different primes p and q. The case $p \neq q$ is completely settled by the following theorem.

Theorem 4. *Let* $\mathbf{R}$ *be a finite p-ring and let* $\mathbf{S}$ *be a finite q-ring for distinct primes p and q. Then* $\mathbf{R}$ *and* $\mathbf{S}$ *are categorically equivalent if and only if* $\mathbf{R} \cong \mathbf{F}_{p^{k_1}} \times \cdots \times \mathbf{F}_{p^{k_m}}$ *and* $\mathbf{S} \cong \mathbf{F}_{q^{k_1}} \times \cdots \times \mathbf{F}_{q^{k_m}}$ *for some positive integers* $k_1, \ldots, k_m$.

The case when $\mathbf{R}$ and $\mathbf{S}$ are both p-rings for the same prime p is still open, but we have some partial results suggesting that perhaps $\mathbf{R}$ and $\mathbf{S}$ must be weakly isomorphic in this case.

Theorem 5. *If* $\mathbf{R}$ *and* $\mathbf{S}$ *are finite categorically equivalent p-rings, then* $\mathbf{R}$ *and* $\mathbf{S}$ *have isomorphic additive groups.*

Theorem 6. *If* $\mathbf{R}$ *and* $\mathbf{S}$ *are finite categorically equivalent p-rings such that* $|R| \leq p^3$, *then* $\mathbf{R}$ *and* $\mathbf{S}$ *are weakly isomorphic (but not necessarily isomorphic).*

Alexander Wires

Higher dimensional rectangulation

We develop higher dimensional centralizer relations and commutators extending the binary rectangular commutator. There are possible generalizations of binary rectangulation which yield different notions of higher rectangulation for arbitrary algebras, both infinite and finite. We consider to what degree a natural choice arises in varieties satisfying a nontrivial idempotent Maľcev condition. Along the way we observe that in such varieties, nilpotence in the binary rectangular commutator and rectangulation (abelian) in the higher commutator are independent phenomenona; as a result, "supernilpotence" may be poor terminology for a general theory of commutators.

Marlen Yolchyan

On idempotent and hyperassociative algebras

(joint work with Yuri Movsisyan, Yerevan State University, University of Bergen)

A binary algebra $(Q;\Sigma)$ is called hyperassociative, if it satisfies the following hyperidentity of associativity:

$$X(x, Y(y,z)) = Y(X(x,y),z). \qquad (\mathrm{ass}_1)$$

In this talk we characterize the structure of idempotent and hyperassociative algebras. In particular, we prove the following result: If $(Q;\Sigma)$ is an idempotent and hyperassociative algebra satisfying the following hyperidentity

$$Y(X(x,y), Y(z,x)) = Y(X(x,z), Y(y,x)), \qquad (\mathrm{n}_2)$$

then the relation $\theta = \{(x,y) \in Q \times Q \mid X(x,y) = X(y,x); \forall X \in \Sigma\}$ is a congruence relation of the algebra $(Q;\Sigma)$ and the corresponding quotient algebra is a rectangular semigroup, and in each equivalence classes any operation of the set Σ is semilattice operation.

Samir Zahirović

A study of enhanced power graphs of finite groups

(joint work with Ivica Bošnjak, University of Novi Sad; Rozália Madarász, University of Novi Sad)

The enhanced power graph $\mathcal{G}_e(\mathbf{G})$ of a group $\mathbf{G}$ is the graph with vertex set G such that two vertices x and y are adjacent if they are contained in a same cyclic subgroup. We prove that finite groups with isomorphic enhanced power graphs have isomorphic directed power graphs. We show that any isomorphism between power graphs of finite groups is an isomorhism between enhanced power graphs of these groups, and we find all finite groups $\mathbf{G}$ for which $\mathrm{Aut}(\mathcal{G}_e(\mathbf{G})$ is abelian, all finite groups $\mathbf{G}$ with $|\mathrm{Aut}(\mathcal{G}_e(\mathbf{G})|$ being prime power, and all finite groups $\mathbf{G}$ with $|\mathrm{Aut}(\mathcal{G}_e(\mathbf{G})|$ being square free. Also we describe enhanced power graphs of finite abelian groups. Finally, we give a characterization of finite nilpotent groups whose enhanced power graphs are perfect, and we present a sufficient condition for a finite group to have weakly perfect enhanced power graph.

This is joint work with Ivica Bošnjak (Novi Sad) and Rozália Madarász (Novi Sad).

Anatolii Zhuchok

On free rectangular doppelsemigroups

A nonempty set D equipped with two binary associative operations $\dashv$ and $\vdash$ satisfying the axioms

$$(x \dashv y) \vdash z = x \dashv (y \vdash z), \quad (x \vdash y) \dashv z = x \vdash (y \dashv z)$$

for all $x, y, z \in D$ is called a *doppelsemigroup* (A.V. Zhuchok. Free products of doppelsemigroups. Algebra Univers. **77**, no. 3, 361–374 (2017)). A doppelalgebra (B. Richter. Dialgebren, Doppelalgebren und ihre Homologie. Diplomarbeit, Universität Bonn (1997). Available at `https://www.math.uni-hamburg.de/home/richter/publications.html`) is just a linear analog of a doppelsemigroup. More information on doppelsemigroups can be found in (A.V. Zhuchok. Relatively free doppelsemigroups. Monograph series Lectures in Pure and Applied Mathematics. Germany, Potsdam: Potsdam University Press. 5, 86 p. (2018)).

A semigroup S is called *rectangular* if $xyz = xz$ for all $x, y, z \in S$. For example, rectangular bands ($xyx = x$) and zero semigroups ($xy = zu$) are rectangular semigroups. A doppelsemigroup will be called *rectangular* if both its semigroups are rectangular. The class of all rectangular doppelsemigroups is a subvariety of the variety of doppelsemigroups. A doppelsemigroup which is free in the variety of rectangular doppelsemigroups will be called a *free rectangular doppelsemigroup.*

Let X be an arbitrary nonempty set and $B = \{a, b\}$. Define operations $\dashv$ and $\vdash$ on $X \cup (B \times X \times X \times B)$ by

$$\begin{gathered}(a_1, b_1, c_1, d_1) * (a_2, b_2, c_2, d_2) = (a_1, b_1, c_2, d_2),\\ x \dashv (a_1, b_1, c_1, d_1) = (a, x, c_1, d_1), \quad (a_1, b_1, c_1, d_1) \dashv x = (a_1, b_1, x, a),\\ x \vdash (a_1, b_1, c_1, d_1) = (b, x, c_1, d_1), \quad (a_1, b_1, c_1, d_1) \vdash x = (a_1, b_1, x, b),\\ x \dashv y = (a, x, y, a), \qquad\qquad x \vdash y = (b, x, y, b)\end{gathered}$$

for all $(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2) \in B \times X \times X \times B$, $x, y \in X$ and $* \in \{\dashv, \vdash\}$. The obtained algebra will be denoted by $FRDop(X)$.

Theorem 7. *$FRDop(X)$ is the free rectangular doppelsemigroup.*

Corollary 8. *The free rectangular doppelsemigroup $FRDop(X)$ generated by a finite set X is finite. Specifically, if $|X| = n$, then $|FRDop(X)| = n + 4n^2$.*

We also study different properties of $FRDop(X)$.

Dmitriy Zhuk

An exponential lower bound on the size of primitive positive definition

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We define a concrete n-ary relation on a 3-element set and a relational basis consisting of two relations. We prove that any positive primitive definition of this relation is of exponential size.

Pavol Zlatoš

The Bohr compactification of an abelian group as a quotient of its Stone-Čech compactification

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For any abelian group G, the canonical (surjective and continuous) mapping $\beta G \longrightarrow \mathfrak{b}G$ from the Stone-Čech compactification βG of G to its Bohr compactfication $\mathfrak{b}G$ is a homomorphism with respect to the semigroup operation on βG, extending the multiplication on G, and the group operation on $\mathfrak{b}G$. We will show that the Bohr compactification $\mathfrak{b}G$ is canonically isomorphic (both in the algebraic and topological sense) to the quotient of βG with respect to the least closed congruence relation on βG merging all the *Schur ultrafilters* on G (a notion to be defined within the talk) into the unit of G.

Pasha Zusmanovich

Hom-Lie structures

Hom-Lie structures on Lie algebras are endomorphisms ("twists") satisfying a certain twisted variant of the Jacobi identity. The impetus comes from physics, where similar structures started to appear long time ago in a constant quest for deformed, in that or another sense, Lie algebra structures bearing a physical significance.

We will survey some recent results about Hom-Lie structures on various kinds of Lie algebras, and discuss two (related) questions: when Hom-Lie structures form a Jordan algebra, and connections with Yang-Baxter equations.

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