# Conviviality Measures 

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#### Abstract

Conviviality has been introduced as a social science concept for multiagent systems to highlight soft qualitative requirements like user friendliness of systems. In this paper we introduce formal conviviality measures for dependence networks using a coalitional game theoretic framework, which we contrast with more traditional efficiency and stability measures. Roughly, more opportunities to work with other people increases the conviviality, whereas larger coalitions may decrease the efficiency or stability of these involved coalitions. We first introduce assumptions and requirements, then we introduce a classification, and finally we introduce the conviviality measures. We use a running example from robotics to illustrate the measures.


## Categories and Subject Descriptors

I.2.11 [Artificial Intelligence]: Distributed Artificial Intelligence -Multiagent Systems

## General Terms

Theory, Measurement, Human Factors

## Keywords

Agent Societies and Societal Issues, Artificial Social Systems, Dependence Networks

## 1. INTRODUCTION

Computer systems have to be user friendly and convivial, a concept from the social sciences defined by Illich as "individual freedom realized in personal interdependence" [10]. Multiagent systems technology can be used to realize tools for conviviality when we interpret "freedom" as choice [5]. For example, if there is only one supply store in your building, then you depend on it for your supplies, but if there are several stores, then you do not depend on a single store. We say that there is more choice, and thus it is more convivial. The challenge of measuring conviviality breaks down into the following research questions:

1. How to define conviviality measures?

[^0]2. How to classify conviviality?
3. What are the assumptions and requirements?
4. Why do we need a new measure?
5. How to use the measures in multiagent systems?

We measure conviviality by counting the possible ways to cooperate, indicating degree of choice or freedom to engage in coalitions. Our coalitional theory is based on dependence networks $[6,19]$, labeled directed graphs where the nodes are agents, and each labeled edge represents that the former agent depends on the latter one to achieve some goal.

To explain the need for the conviviality measures, we show the difference with stability and efficiency measures. Tools for conviviality are concerned in particular with dynamic aspects of conviviality, such as the emergence of conviviality from the sharing of properties or behaviors whereby each member's perception is that their personal needs are taken care of [10]. In such dynamic circumstances, the stability of the coalitions is an important criterion. Moreover, traditional coalition formation and game theoretic methods have been focused on the efficiency of coalitions.

The focus on dependence networks and more specifically on their cycles, is a reasonable way of formalizing conviviality as something related to the freedom of choice of individuals plus the subsidiary relations -interdependence for task achievement- among fellow members of a social system. However, this freedom of choice view is not the only view of conviviality, not even the most pertinent one. For example, in earlier work we define conviviality masks based on Taylor's idea that conviviality "masks the power relationships and social structures that govern societies." [20] A conviviality mask is a transformation of social dependencies by hiding power relations and social structures to facilitate social interactions, and conviviality mask measures can be defined to measure these transformations.

In this paper we do not consider Polanyi's notion of empathy, which needs trust, shared commitments and mutual efforts to build up and maintain conviviality, or the many definitions and relations with other social concepts discussed in the conviviality literature, referring to qualities such as trust, privacy and community identity.

The layout of this paper is as follows. In Section 2 we introduce a running example from coalition formation in robotics, in Section 3 we discuss stability and efficiency measures for dependence networks, in Section 4 we discuss the assumptions and requirements of conviviality measures, in Section 5 we introduce a conviviality classification, and in Section 6 we introduce the conviviality measures.

## 2. RUNNING EXAMPLE: NAO ROBOTS

We shall now give a scenario where we can discuss how our system works. In an office building, there are assistant robots to human being workers. The workers need office materials, which are scare and are to be shared, in order to accomplish their jobs. A worker may need materials which are not currently available at their desks, e.g., someone else in the building is using it. It is considered waste of time and unproductive for a worker to ring everyone else to find out where the needed materials are and leave his desk to collect those materials himself. Instead, the worker can submit a request to the robots to get and/or deliver the needed materials for him while he can continue with other works at his desk. We shall refer to a request submitted to the robots as a main task, which can be split into a number of tasks.

The communication between the workers and the robots can be done via a simple web-based application, which will transmit the request of the worker to the robots as well as keeping track of their status. However, the robots have limited computational resources. They only keep track of what they have done recently. They rely on each other to provide information about finding the location of a material. Basically, the last robot which dealt with it will know. We assume the existence of such an application as well as the communication network is stable and reliable. In general, the robots then travel from place to place during the working hours. A depiction of this scenario is presented in Figure 1.


Figure 1: A depicted scenario of robots in office building.

In our example here, we assume that there is a set of 4 Nao robots, $N=\left\{n_{1}, n_{2}, n_{3}, n_{4}\right\}$ and there are two main tasks: $T_{A}=\left\{t_{1}, t_{2}\right\}$ and $T_{B}=\left\{t_{3}, t_{4}\right\}$, where $t_{1}$ is to deliver a pen to desk $\mathrm{A}, t_{2}$ is to deliver a piece of paper to desk A , $t_{3}$ is to deliver a tube of glue to desk B , and $t_{4}$ is to deliver a cutter to desk B. Note that executing a task involves detailed actions, such as grabbing and dropping the pen, which are beyond the scope of this paper. These tasks are given to robots, which need to complete them in minimal use of power. Therefore, they need to minimize their travel time. A robot has incentives to perform as many tasks as possible as well as to save its battery life. ${ }^{1}$.

Upon receiving the tasks, robots need to form coalitions to finish them. Due to limited resources in the robots, not all of the robots know about the tasks. There are mul-

[^1]tiple steps to carry out all the tasks from start to finish. First, the information known by each robot is who has the information about the sources and the destinations of the resources needed to accomplish the tasks. The actual coordinates, involving the present location of each material and the respective desk, are revealed only after an agreement on a coalition among the robots has been made. This involves interdependency among robots. Second, robots need to decide how they form coalitions, i.e., which ones will join to carry out each main task. Third, for each possible coalition, each robot needs to plan for their optimal route to carry out the assigned task.

At the start, robots get the information concerning the material locations and the distances between the materials and destinations. For example, robot $n_{1}$, regarding task $t_{1}$, knows i) nothing about the source of the pen, i.e., where it currently is, and ii) the destination of the pen, i.e., where it must be delivered. Regarding task $t_{2}$, robot $n_{1}$ knows where the paper is but knows nothing about its destination. Table 1 presents the knowledge of the robots about the tasks and the current distances among the robots, the materials and the destinations.

Table 1: Robots' knowledge (top); Distances (bottom).

| Robot | $n_{1}$ |  |  |  | $n_{2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Task | $t_{1}$ | $t_{2}$ | $t_{3}$ | $t_{4}$ | $t_{1}$ | $t_{2}$ | $t_{3}$ | $t_{4}$ |
| Source |  | $\checkmark$ |  |  | $\checkmark$ |  |  |  |
| Destination | $\checkmark$ |  | $\checkmark$ |  |  |  |  | $\checkmark$ |
| Robot | $n_{3}$ |  |  |  | $n_{4}$ |  |  |  |
| Task | $t_{1}$ | $t_{2}$ | $t_{3}$ | $t_{4}$ | $t_{1}$ | $t_{2}$ | $t_{3}$ | $t_{4}$ |
| Source |  |  |  | $\checkmark$ |  |  | $\checkmark$ |  |
| Destination |  | $\checkmark$ |  |  |  |  |  |  |
| Distances among locations |  |  |  |  |  |  |  |  |
| Robot |  | Pen | Pap |  | Glue |  | utter |  |
| $n_{1}$ |  | 10 | 15 |  | 9 |  | 12 |  |
| $n_{2}$ |  | 14 | 8 |  | 11 |  | 13 |  |
| $n_{3}$ |  | 12 | 14 |  | 10 |  | 7 |  |
| $n_{4}$ |  | 9 | 12 |  | 15 |  | 11 |  |
| Destination |  | Pen | Pap |  | Glue |  | utter |  |
| Desk A |  | 11 | 16 |  | 9 |  | 8 |  |
| Desk B |  | 14 | 7 |  | 12 |  | 9 |  |

Upon receiving information about the tasks, robots form coalitions to execute them. We refer to a coalition as a group of robots executing a main task, i.e., either $T_{A}$ or $T_{B}$. Robots joining the coalition are to execute the task, e.g., deliver the pen to desk A. For example to accomplish all the tasks $t_{1}, t_{2}, t_{3}, t_{4}$, the following coalitions may be formed: $C_{0}:\left\{\left(n_{1}, t_{3}\right),\left(n_{2}, t_{2}\right),\left(n_{3}, t_{4}\right),\left(n_{4}, t_{1}\right)\right\}, C_{1}:\left\{\left(n_{1}, t_{1}\right)\right.$, $\left.\left(n_{2}, t_{2}\right),\left(n_{3}, t_{3}\right),\left(n_{4}, t_{4}\right)\right\}$ and $C_{2}:\left\{\left(n_{1}, t_{3}\right),\left(n_{2}, t_{4}\right),\left(n_{3}, t_{2}\right)\right.$ $\left.\left(n_{4}, t_{1}\right)\right\}$. For agents to execute their tasks, they need to know an optimal plan such that they can minimize their costs for executing the task. Given the knowledge, they are capable of computing for an optimal route for getting the assigned materials and for delivering $i t^{2}$. Therefore, the robots can generate plans for themselves after they have been given tasks. However, discussing the details about generating plans for the robots is out of the scope of this paper.

[^2]
## 3. EFFICIENCY AND STABILITY

### 3.1 Definitions

There are many ways to define efficiency. Generally speaking, efficiency in a coalition is a relation between what agents can achieve as part of the organization compared to what they can do alone or in different coalitions. In this section and to give an illustration on our example, we recall two definitions of efficiency: the cost efficiency (Def. 3.1), and the economic efficiency (Def. 3.2).

Definition 3.1 (Cost Efficiency). Let the set of agents $N=\left\{n_{1}, \ldots, n_{j}\right\}, T$ be the set of tasks (or goals), and $C \subseteq N$ a coalition. Let Cost : $2^{N} \rightarrow \mathbb{R}$ be the function that associates to a coalition the cost of achieving all tasks of $T$. Then, $C$ is cost efficient if and only $\forall n_{i} \in C$, $\left(\operatorname{Cost}\left(n_{i}\right)-\operatorname{Cost}(C)\right)>0$.

Definition 3.2 (Economic Efficiency). A coalition is economically efficient iff i) no one can be made better off without making someone else worse off, ii) no additional output can be obtained without increasing the amount of inputs, iii) production proceeds at the lowest possible per-unit cost [14].

Stability of coalitions is related to the potential gain in staying in the coalition or quitting the coalition for more profit (i.e., free riding). Hence, several elements come to play for the evaluation of a coalition's stability.

First, the coalition outcome should be greater than the individual ones cumulated. This is usually computed via a characteristic function such as proposed by [13]. Therefore, a necessary condition to stability is that the characteristic function is positive, i.e., acting as a group is overall more beneficial than acting individually.

Second, the distribution of benefits should be fair. Several functions, named sharing rules where proposed such as Shapley value [16], nucleolus [15], and Satisfactory Nucleolus [12]. The leading idea is to take the individual contribution and the free rider's value into account when sharing the benefits.

For the purpose of illustration, we introduce the concept of core to check the stability of a coalition. Indeed, it is relatively (computably) simple to check if a coalition is in the core. Informally, a coalition is in the core iff no sub-coalition is more profitable. Formally, the core follows Def. 3.3.

Definition 3.3 (Core). Let $x \in \mathbb{R}^{N}$ be a pay-off allocation vector, $\nu: 2^{N} \rightarrow \mathbb{R}$ be the characteristic function (pay-off function), and $C \subseteq N$ a coalition. Then, $x$ is in the core iff $\sum_{i \in N} x_{i}=\nu(N)$ and $\sum_{i \in C} x_{i} \geq \nu(C)$.

### 3.2 Efficiency computation

Let us apply the above definitions to our example. From Table 1 of Sect. 2 we can compute the distance for each robot to do each task, as displayed on Table 2:

Using this table we can compute the cost of executing tasks in a given coalition by adding up the costs of each robot to the assigned task. For instance, the cost of $C_{1}:\left\{\left(n_{1}, t_{1}\right)\right.$, $\left.\left(n_{2}, t_{2}\right),\left(n_{3}, t_{3}\right),\left(n_{4}, t_{4}\right)\right\}$ is $\operatorname{Cost}\left(C_{1}\right)=87$, whereas the cost of $C_{2}:\left\{\left(n_{1}, t_{3}\right),\left(n_{2}, t_{4}\right),\left(n_{3}, t_{2}\right)\left(n_{4}, t_{1}\right)\right\}$ is $\operatorname{Cost}\left(C_{2}\right)=93$, and the cost of $C_{3}:\left\{\left(n_{1}, t_{1}\right),\left(n_{1}, t_{3}\right),\left(n_{2}, t_{2}\right),\left(n_{4}, t_{4}\right)\right\}$ is $\operatorname{Cost}\left(C_{3}\right)=86$.

Table 2: Distances between robots and their tasks.

|  | $t_{1}$ | $t_{2}$ | $t_{3}$ | $t_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $n_{1}$ | $10+11=21$ | $15+16=31$ | $9+12=21$ | $12+9=21$ |
| $n_{2}$ | $14+11=25$ | $8+16=24$ | $11+12=23$ | $13+9=22$ |
| $n_{3}$ | $12+11=23$ | $14+16=30$ | $10+12=22$ | $7+9=16$ |
| $n_{4}$ | $9+11=20$ | $12+16=28$ | $15+12=27$ | $11+9=20$ |

These costs have to be compared to the costs of each robot doing all tasks on their own, which are respectively 94 for $n_{1}$ and $n_{2}, 91$ for $n_{3}$, and 95 for $n_{4}$. As a conclusion, we can say that $C_{1}$ and $C_{3}$ seem efficient for all robots, whereas $C_{2}$ is a bad option with respect to efficiency for $n_{3}$ only.

We can see that $C_{3}$ is more cost efficient than $C_{1}$. However, we should note that $C_{1}$ is not economically efficient. Indeed, there is a coalition $C_{0}:\left\{\left(n_{1}, t_{3}\right),\left(n_{2}, t_{2}\right),\left(n_{3}, t_{4}\right)\right.$, $\left.\left(n_{4}, t_{1}\right)\right\}$ where at least one agent is better off without making anyone worse off (actually, this applies for all of them), all the rest been equal. If we compare $\operatorname{Cost}\left(C_{0}\right)=81$ to $\operatorname{Cost}\left(C_{3}\right)=86$, we conclude that $C_{0}$ is economically efficient and more cost efficient than $C_{3}$.

### 3.3 Stability computation

As explained earlier, we will check the stability of the coalitions according to the core definition (Def. 3.3).

We can see that $C_{1}$ is not in the core, hence not stable, because there exist at least a sub-coalition which is more profitable, e.g., $C_{3}$. Indeed, in the context of $C_{1}$ the robot $n_{1}$ can threaten $n_{3}$ to do the task $t_{3}$ for the same outcome but less cost. The two other robots agree since their respective pay-off is unchanged. The coalition $C_{2}$ is also not in the core, since $n_{2}$ can be threatened by all agents and $n_{3}$ can be threatened by $n_{2}$ and $n_{4}$.

In contrast, $C_{3}$ and $C_{0}$ are in the core. In fact, in $C_{3}$, even if $n_{4}$ has a lower cost than $n_{1}$ for the task $t_{1}$, neither $n_{2}$ nor $n_{4}$ can handle the task $t_{3}$ without decreasing their global pay-off, i.e., they are satisfied with this coalition.

The coalition $C_{0}$ is stable, according to the core definition, and it also involves all the robots, whereas $C_{3}$ leaves one robot idle $\left(n_{3}\right)$ and gives additional work to another one $\left(n_{1}\right)$. As a preliminary conclusion, for efficiency and stability, as well as for the sake of balancing the workload (which was also an objective of the main goal achievement), the coalition $C_{0}$ seems to be the best.

### 3.4 Need for other coalition measures

Efficiency and stability metrics are commonly used to evaluate coalitions. The former giving an assurance on the economical gain reached by being in the coalition, the later giving a certainty that the coalition is viable on the long term. Therefore, the positive evaluation of a coalition against these two metrics is often considered to be a prerequisite for the coalition formation.

However, depending on the application domain, other functional and non-functional requirements, e.g., security, userfriendliness or conviviality, may play an important role in the choice of a coalition. Requirements may be considered in a trade-off at the same level as efficiency and stability, or as a further filtering criterion, to select among otherwise efficient and stable coalitions. This highlights the need for further metrics, such as the proposed conviviality metrics.

## 4. ASSUMPTIONS AND REQUIREMENTS

According to [3], conviviality may be measured by the number of reciprocity based coalitions that can be formed. Some coalitions, however, provide more opportunities for their participants to cooperate with each other than others, being thereby more convivial. To represent the interdependencies among agents in the coalitions, we use dependence networks. First, we present definition 4.1 [3], illustrated with our running example. Then, we review our assumptions and requirements for the conviviality measures we define.

Recalling Section 2, two steps are needed to achieve each task. To each step, we associate a goal for a robot to reach. For example, to perform task $t_{1}$, deliver pen to desk $A$, robots must have the goals $g_{1 S}$, get the pen from its source, and $g_{1 D}$ deliver it to its destination. Abstracting from tasks and plans we define a dependence network as in 4.1 [3]:

Definition 4.1 (Dependence networks). A dependence network is a tuple $\langle A, G$, dep, $\geq\rangle$ where: $A$ is a set of agents, $G$ is a set of goals, dep $: A \times A \rightarrow 2^{G}$ is a function that relates with each pair of agents, the sets of goals on which the first agent depends on the second, and $\geq: A \rightarrow 2^{G} \times 2^{G}$ is for each agent a total pre-order on sets of goals occurring in his dependencies: $G_{1}>_{(a)} G_{2}$.

In our example Section 3, robots form the coalitions $C_{0}, C_{1}$ and $C_{2}$. Let $D N_{0}, D N_{1}$ and $D N_{2}$, visualized in Figure 2 (a), (b) and (c), be three dependence networks respectively corresponding to these coalitions, where:
Nao robots $N=\left\{n_{1}, n_{2}, n_{3}, n_{4}\right\}$,
Goals $G=\left\{g_{1 S}, g_{1 D}, g_{2 S}, g_{2 D}, g_{3 S}, g_{3 D}, g_{4 S}, g_{4 D}\right\}$,
where dependencies are built from Table 1 and preferences are the following:

- for $D N_{0}: \operatorname{dep}\left(n_{1}, n_{4}\right)=\left\{g_{3 S}\right\}, \operatorname{dep}\left(n_{2}, n_{1}\right)=\left\{g_{2 S}\right\}$, $\operatorname{dep}\left(n_{2}, n_{3}\right)=\left\{g_{2 D}\right\}, \operatorname{dep}\left(n_{3}, n_{2}\right)=\left\{g_{4 D}\right\}$, $\operatorname{dep}\left(n_{4}, n_{1}\right)=\left\{g_{1 D}\right\}, \operatorname{dep}\left(n_{4}, n_{2}\right)=\left\{g_{1 S}\right\} ;$
Robot $n_{4}$ prefers to deliver pen to desk A than to get it : $\left\{g_{1 D}\right\}>_{\left(n_{2}\right)}\left\{g_{1 S}\right\}$;
- for $D N_{1}: \operatorname{dep}\left(n_{1}, n_{2}\right)=\left\{g_{1 S}\right\}, \operatorname{dep}\left(n_{2}, n_{1}\right)=\left\{g_{2 S}\right\}$, $\operatorname{dep}\left(n_{2}, n_{3}\right)=\left\{g_{2 D}\right\}, \operatorname{dep}\left(n_{3}, n_{4}\right)=\left\{g_{3 S}\right\}$, $\operatorname{dep}\left(n_{3}, n_{1}\right)=\left\{g_{3 D}\right\}, \operatorname{dep}\left(n_{4}, n_{3}\right)=\left\{g_{4 S}\right\}$, $\operatorname{dep}\left(n_{4}, n_{2}\right)=\left\{g_{4 D}\right\} ;$
Robot $n_{4}$ prefers to get cutter than deliver it to desk B: $\left\{g_{4 S}\right\}>{ }_{\left(n_{2}\right)}\left\{g_{4 D}\right\}$, and $n_{3}$ prefers to get glue than deliver it to desk B: $\left\{g_{3 S}\right\}>_{\left(n_{1}\right)}\left\{g_{3 D}\right\}$;
- for $D N_{2}: \operatorname{dep}\left(n_{2}, n_{3}\right)=\left\{g_{4 S}\right\}, \operatorname{dep}\left(n_{1}, n_{4}\right)=\left\{g_{3 S}\right\}$,
$\operatorname{dep}\left(n_{3}, n_{1}\right)=\left\{g_{2 S}\right\}, \operatorname{dep}\left(n_{4}, n_{1}\right)=\left\{g_{1 D}\right\}$,
$\operatorname{dep}\left(n_{4}, n_{2}\right)=\left\{g_{3 S}\right\} ;$
Robot $n_{3}$ prefers to deliver pen to desk A than get glue: $\left\{g_{1 D}\right\}>{ }_{\left(n_{2}\right)}\left\{g_{3 S}\right\}$.


### 4.1 Assumptions

In this work, the cycles identified in a dependence network are considered as coalitions. These coalitions are used to evaluate conviviality in the network. Cycles denote the smallest graph topology expressing interdependence, i.e, conviviality, and are considered as atomic relations conveying interdependence. When referring to cycles, we are implicitly signifying simple cycles (as defined in [7]), also discarding self-loops. Moreover, when referring to conviviality, we always refer to potential interaction not actual interaction.

In our second assumption, we consider the conviviality of a dependence network to be evaluated in a bounded domain, i.e., over a $[\min ; \max ]$ interval. This allows to read the values obtained by any evaluation method.

### 4.2 Requirements

The first requirement for our conviviality measures concerns the size of coalitions. This requirement is captured by the statement that larger coalitions are more convivial than smaller ones. We express this requirement through the following two cases. First case, a dependence network $D N_{i}$ with a coalition of size $n$ is better for conviviality than a $D N_{j}$ with coalition of size $m=(n-\alpha)$, where $m<n$. For example, consider a coalition for peace in the world. The more countries participate, the better it is. Second case, a dependence network $D N_{i}$ with a coalition of size $n$ is better for conviviality than a dependence network $D N_{j}$ with two coalitions, one of size $k$ and the other of size $l$, such as that $k+l \leq n$, all else being equal. This is motivated by the fact that having one large coalition eliminates the risk of being exposed to potential competition from other coalitions, which may be looking for the same resources.

Our second requirement concerns the number of coalitions. It is captured by the statement that the more coalitions in the dependence network, the higher the conviviality measure (all else being equal). This requirement is motivated by the fact that a large number of coalitions indicates more interactions among agents, which is positive in term of conviviality according to our definition based on interdependence.


Figure 2: Dependence networks $D N_{0}, D N_{1}$ and $D N_{2}$.

## 5. CONVIVIALITY CLASSIFICATION

Based on the requirements outlined in Section 4, we now propose a conviviality classification that allows an intuitive grasp of conviviality measures through a ranking of the dependence networks. First, we introduce the five definitions of conviviality classes, from the absolute best to the absolute worst convivial networks.

### 5.1 Definitions

Definition 5.1 (P). A dependence network $D N$ is $P$ convivial (most convivial), iff all agents in $D N$ belong to all cycles, i.e., $\forall a_{i} \in A$ and $\forall c_{k} \in C, a_{i}$ is s.t. $a_{i} \in c_{k}$, where $C=\left\{c_{1}, \ldots, c_{l}\right\}$ is the set of all cycles.

Definition 5.2 (APE). A dependence network $D N$ is APE convivial, iff all agents in $D N$ belong to at least one cycle, i.e., $\forall a_{i} \in A, \exists c_{k} \notin C$, s.t. $a_{i} \in c_{k}$, where $C=$ $\left\{c_{1}, \ldots, c_{l}\right\}$ is the set of all cycles.

Definition 5.3 ( N ). A dependence network $D N$ is $N$ convivial, iff there exists at least one cycle in $D N$, and there is at least one agent not in a cycle, i.e., $\exists a, b \in A$ s.t. $\quad a, b \in$ $c_{k}$, where $c_{k} \notin C$, and $\exists d \in A$ s.t. $\quad d \notin c_{i}, \forall c_{i} \in C$, where $C=\left\{c_{1}, \ldots, c_{l}\right\}$ is the set of all cycles.

Definition 5.4 (AWE). A dependence network $D N$ is $A W E$ convivial, iff there is no cycle in $D N$, i.e., $C=\{\emptyset\}$, and s.t. $\exists \operatorname{dep}(a, b)=\left\{g_{i}\right\}$, where $a, b \in A$ and $g_{i} \in G$.

Definition 5.5 (W). A dependence network $D N$ is $W$ convivial (worst convivial), iff there is no dependency between the agents in $D N$, i.e., $\exists \operatorname{dep}(a, b)=\left\{g_{i}\right\}$, where $a, b \in$ $A$ and $g_{i} \in G$.

Figure 3, illustrates the different types of dependence networks that correspond to each conviviality class. The arrow on the top of the figure depicts the direction of increasing conviviality. The scale goes from the worst case (no conviviality) to the best case (maximal conviviality).

### 5.2 Examples

Consider the three dependence networks $D N_{0}, D N_{1}$, and $D N_{2}$ respectively corresponding to robots coalitions $C_{0}, C_{1}$, and $C_{2}$ illustrated Figure 2. All robots belong to at least one cycle. Hence, from Definition $5.2, C_{0}, C_{1}$, and $C_{2}$ belong to the $A P E$ conviviality class. They are said to be Almost Perfectly convivial. All robots are engaged in reciprocal dependence relations: each one gives to the coalition and receives from it. All robots are pursuing goals and cooperate with at least one other robot to achieve their tasks.

With a different initial knowledge, the potential coalitions formed may belong to other conviviality classes. For instance, if in the initial knowledge table, the destination of task $t_{2}$ is known by $n_{4}$ instead of $n_{3}$, then coalition $C_{01}$ is represented by the dependence network $D N_{01}$ depicted on Figure 4 . We note that $n_{3}$ depends on another robot $\left(n_{2}\right)$, but that this dependency is not reciprocated, leaving $n_{3}$ out of any coalition. Hence, $n_{3}$ being isolated, the corresponding coalition belongs to the $N$ conviviality class.


Figure 4: Conviviality class $N$.
Consider now that in $C_{1}$, each robot knows the information, i.e., source and destination, about one task only, and is assigned the task it knows about. Then, not a single robot depends on another, since each robot knows exactly what to do on its own. There is no cooperation among the robots, each is isolated. The corresponding network consists of four nodes and no dependencies. Therefore, this coalition belongs to the $W$ conviviality class. Similarly, if all robots know all the information about all tasks, then any task assignment results in a coalition corresponding to a network of conviviality class $W$, as all robots may perform any task by themselves without having to cooperate with any other robots to obtain the information concerning the source and destination of the office supplies they have to move.

### 5.3 Preliminary distinctions among measures

Returning to the efficiency and stability measures presented in Section 3, we can already see a major distinction between conviviality and the two former metrics. Indeed, in order to evaluate conviviality, we need to perform an analysis of the dependencies between the agents, i.e., we must consider the topological aspects of the task (or goal) dependencies in the graph. This is not the case in efficiency and stability metrics, which only compare coalitions to subcoalitions or individuals in terms of global pay-off. Therefore, we cannot rely on similar functions to evaluate conviviality. Finally, conviviality is orthogonal to efficiency and stability, and trade-off situations are to be expected.


Figure 3: Conviviality classes.

## 6. CONVIVIALITY MEASURES

We now propose two indices based on graph properties and built on three measures; the number of: agents in the network, agents that belong to at least one cycle, and cycles.

We define the in-the-loop index $\rho_{D N_{i}}$ as the ratio of the number of agents in cycles in relation to the total number of agents in the network. For example, computing the in-theloop index for coalitions $C_{0}$ and $C_{01}$, respectively represented by $D N_{0}$ depicted Figure 3a and $D N_{01}$ depicted Figure 4 yields: $\rho_{D N_{0}}=1$ and $\rho_{D N_{0}}=0.75$. Although useful this metric and its inverse ( $\beta=1-\rho$ ) do not allow to differentiate between the number of coalitions present in the network, e.g., we obtain the same value for the networks depicted Figure 3a, 3b and 3c: $\rho_{D N_{0}}=\rho_{D N_{1}}=\rho_{D N_{2}}=1$.

We therefore set a second index, the connectivity index $\delta_{D N_{i}}$, defined as the average length of cycles in the network, and reflecting when coalitions are larger, i.e. more convivial. Computing the connectivity index for coalitions $C_{0}, C_{01}, C_{1}$ and $C_{2}$ we obtain: $\delta_{D N_{0}}=1.333$ and $\delta_{D N_{01}}=1, \delta_{D N_{1}}=1$ and $\delta_{D N_{2}}=2$. Clearly, this result satisfies the requirement of Section 4 that the larger the cycle, the more convivial the network, all else being equal, however, it fails to distinguish between $D N_{01}$ and $D N_{1}$ even though $D N_{1}$ contains more cycles. Moreover, intuitively, and per our classification Section $5, D N_{01}$ containing one isolated node is less convivial than $D N_{1}$, in which each node belongs to at least one cycle.

Combining the two indices, as well as defining other measures based on global graph properties, does not seem to create more accurate measures, i.e., satisfying our requirements, hence highlighting the need to capture the network topologies more precisely.

Therefore, we propose a conviviality measure constructed on our assumption Section 4 that conviviality measures are based on the coalitions the agents form with each other. As at least two agents are needed to form a coalition, the measure is based on pairs of agents. More specifically, what is measured is the number of coalitions to which any two given agents in the dependence network belong, the evaluation being performed over the whole network. Furthermore, to allow comparisons between dependence networks of various sizes and to increase its usefulness, the measure must be defined over a bounded space, such as $[0 ; 1]$.

### 6.1 Bounding evaluations

Our first step is to define a function that evaluates conviviality over one pair of agents - denoting a partial measure of conviviality. Let $\operatorname{coal}_{D N_{i}}(a, b) \in \mathbb{N}$, be the number of cycles that contain both $a$ and $b$ in a dependence network $D N_{i}$, where $a, b \in A$ and $a \neq b$. Then, based on $\operatorname{coal}(a, b)$, we construct a bounded conviviality measure. We start by determining the maximum number of cycles that contain any two agents. We note that the number of cycles containing two agents, $\operatorname{coal}(a, b)$, can neither be more than the maximum number of cycles possible containing two (given) agents nor less than no cycle at all. Let $\Theta$ be the maximum number of cycles between two agents, we write:

$$
\begin{equation*}
0 \leq \operatorname{coal}(a, b) \leq \Theta \tag{1}
\end{equation*}
$$

In order to determine the maximum number of cycles, let us first assume that the set of goals is reduced to only one goal, i.e., $|G|=1$, and the DN is a clique on all goals. We note that the maximal number of cycles is the summation of the maximal number of cycles for each cycle length. We call
$L$ the cycle length. In addition, as stated in Section 4, we do not consider self-loops in the evaluation. So, the smallest cycle to consider is $L=2$, and that can happen iff the set of agents $A$ has a cardinality greater than or equal to 2 , i.e., $|A| \geq 2$. Trivially, when $|G|=1$ there can be at most 1 cycle between two agents such that $L=2$.

To have a cycle of length $L=3$, we must have at least 3 agents in the DN, i.e., $|A|=3$. We can already generalize, saying that the maximal cycle length $L$ in a DN with $|A|$ number of agents is $L=|A|$.

Furthermore, given two agents $a, b \in A, a \neq b$, a cycle of length $L=3$ is found if there is a agent $c \in A, c \neq a, c \neq b$ such that there is an edge from $a$ (resp. $b$ ) to $c$ and an edge from $c$ to $b$ (resp. a). The maximum number of cycle of length $L=3$ is then obtained by choosing one agent $c$ among the agents which are neither $a$ nor $b$, without repetition and with order. Since there are $|A|-2$ such $c$ agents, the maximal number of cycle of length $L=3$ can be expressed by the permutation $P(|A|-2,1)$, where $P(n, k)$ is the usual permutation defined in combinatorics by: $P(n, k)=\frac{n!}{(n-k)!}$, where $n$ is the number of elements available for selection and $k$ is the number of elements to be selected $(0 \leq k \leq n)$

For length $L \geq 3$, applying a similar reasoning, we obtain the maximal number of cycles of length $L$ by choosing $L-2$ agents among $|A|-2$, without repetition and with order, hence given by the expression $P(|A|-2, L-2)$.

Finally, as noted above, the maximum number of cycles is the summation of the maximal number of cycles for each cycle length. Hence for $|G|=1$, the maximum number of cycles, $\Theta_{|G|=1}$, is:

$$
\begin{equation*}
\Theta_{|G|=1}=\sum_{L=2}^{L=|A|} P(|A|-2, L-2) \tag{2}
\end{equation*}
$$

Now, for $|G| \geq 1$, we can choose for each edge one goal among $|G|$. Since the number of edges for a cycle is defined by its length $L$, we have a maximum of $|G|^{L}$ cycles of length $L$. Therefore, the maximum number of cycles, $\Theta$, is expressed as follows:

$$
\begin{equation*}
\Theta=\sum_{L=2}^{L=|A|} P(|A|-2, L-2) \times|G|^{L} \tag{3}
\end{equation*}
$$

### 6.2 Combining conviviality measures

In Equation 2 we obtain bounds for a pairwise evaluation. We now need to sum up all these pairwise evaluations. Let $\sum \operatorname{coal}(a, b)$ be this summation. As there are $|A|(|A|-1)$ pairs of agents to consider in the whole network:

$$
\begin{equation*}
0 \leq \sum \operatorname{coal}(a, b) \leq|A|(|A|-1) \times \Theta \tag{4}
\end{equation*}
$$

If we want to bound our conviviality measure conv over $[0 ; 1]$, i.e., $0 \leq \operatorname{conv} \leq 1$, then we get the following Equation 5:

$$
\begin{equation*}
0 \leq \frac{\sum \operatorname{coal}(a, b)}{A(A-1) \times \Theta} \leq 1 \tag{5}
\end{equation*}
$$

We can now write Equation 6 to express the pairwise conviviality measure of a dependence network $D N$ :

$$
\begin{equation*}
\operatorname{conv}(D N)=\frac{\sum \operatorname{coal}(a, b)}{\Omega} \tag{6}
\end{equation*}
$$

where we write $\Omega=A(A-1) \times \Theta$ for the sake of readability, for the remainder of the paper.

### 6.3 Conviviality computation

In Table 3, we present the conviviality evaluation for each dependence network, illustrated in Figure 2. As expected, the value for the maximum number of cycles is a large number, $\Omega=A(A-1) \times \Theta=111360$. The evaluations are performed using the pairwise measure defined in Equation 6. The results return $\operatorname{conv}\left(D N_{1}\right)=0.000143>\operatorname{conv}\left(D N_{2}\right)=$ $0.000125>\operatorname{conv}\left(D N_{0}\right)=0.0000897$, indicating that $D N_{1}$ is the most convivial network, followed by $D N_{2}$ and that $D N_{0}$ is the least convivial.

We observe that $\operatorname{conv}\left(D N_{1}\right)>\operatorname{conv}\left(D N_{0}\right)$, coincides with our intuition as clearly, $D N_{1}$ contains more cycles than $D N_{0}$. This result satisfies our requirements Section 4 namely, that the more coalitions in the dependence network, the higher the conviviality measure (all else being equal). $D N_{1}$ is more convivial than $D N_{0}$ as more cooperation may occur among the robots in coalition $C_{1}$. Similarly, the computation returns $\operatorname{conv}\left(D N_{1}\right)>\operatorname{conv}\left(D N_{2}\right)$ as $D N_{1}$ contains more cycles than $D N_{2}$. The result $\operatorname{conv}\left(D N_{2}\right)>\operatorname{conv}\left(D N_{0}\right)$ reflects the fact that $D N_{2}$ contains a cycle larger than the largest cycle in $D N_{0}$. In this grand coalition ( $n_{1}, g_{3 D}, n_{4}, g_{1 S}, n_{2}, g_{4 S}$, $n_{3}, g_{2 S}, n_{1}$ ), all four robots may cooperate. As per our requirements Section 4, such a coalition is more convivial as the potential conflicts that may arise among several smaller coalitions is reduced.

Computing $\operatorname{conv}\left(D N_{01}\right)$ returns, as expected, the smaller value $\left(\frac{8}{\Omega}=0.0000718\right)$ highlighting the lesser conviviality of coalition $D N_{01}$.

In our running example, we measured conviviality by counting the possible ways for robots to cooperate, indicating the degree of choice or freedom to engage in coalitions. Indeed, the conviviality measures allow to compare the coalitions and select the most appropriate one(s) for the multiagent system. If a high level of cooperation is needed in the system, then coalitions involving the highest number of agents and cycles will be preferred. Of course, trade-offs must be made among the system requirements, including user-friendliness and conviviality as well as efficiency and stability. However, the conviviality measures allow to provide an indicator for the level of cooperation among the agents and their degree of choice to engage in coalitions. More opportunities to work together with other agents increases the conviviality. As stated by Bradshaw et al. [11], the success of future human-agent teams relies in such sophisticated interdependence among human-agent team members.

Table 3: Measures based on dependencies.

| Fig. | Pairs in 1 cycle | Pairs in 2 cycles | Conviviality <br> $\left(=\frac{\Sigma \text { coal }(a, b)}{\Omega}\right)$ |
| :---: | :--- | :--- | :--- |
| $D N_{0}$ | $\left(n_{1}, n_{2}\right),\left(n_{2}, n_{1}\right)$, <br> $\left(n_{2}, n_{3}\right),\left(n_{2}, n_{4}\right)$, <br> $\left(n_{3}, n_{2}\right),\left(n_{4}, n_{2}\right)$ | $\left(n_{1}, n_{4}\right),\left(n_{4}, n_{1}\right)$ | $\frac{6 \times 1+2 \times 2}{\Omega}$ <br> $=\frac{10}{\Omega}$ |
| $D N_{1}$ | $\left(n_{1}, n_{3}\right),\left(n_{2}, n_{4}\right)$, <br> $\left(n_{3}, n_{1}\right),\left(n_{4}, n_{2}\right)$ | $\left(n_{1}, n_{2}\right),\left(n_{2}, n_{1}\right)$, <br> $\left(n_{2}, n_{3}\right),\left(n_{3}, n_{2}\right)$, <br> $\left(n_{3}, n_{4}\right),\left(n_{4}, n_{3}\right)$ | $\frac{4 \times 1+6 \times 2}{\Omega}$ <br> $\frac{16}{\Omega}$ |
| $\Delta N_{2}$ | $\left(n_{1}, n_{2}\right),\left(n_{1}, n_{3}\right)$, <br> $\left(n_{2}, n_{1}\right),\left(n_{2}, n_{3}\right)$, <br> $\left(n_{2}, n_{4}\right),\left(n_{3}, n_{1}\right)$, <br> $\left(n_{3}, n_{2}\right),\left(n_{3}, n_{4}\right)$, <br> $\left(n_{4}, n_{2}\right),\left(n_{4}, n_{3}\right)$ | $\left(n_{1}, n_{4}\right),\left(n_{4}, n_{1}\right)$ | $\frac{10 \times 1+2 \times 2}{\Omega}$ |

## 7. RELATED RESEARCH

This paper builds on our previous work, Caire et al. [5] in which, conviviality has been proposed as a social concept to develop multi-agent systems. Indeed, the intuitions behind the term conviviality are significant for social IT-enabled systems, and has been very little studied so far. However, conviviality is likely to become a core design feature for such systems in the future.

In "Conviviality Measure for Early Requirement Phase", Caire and van der Torre [4] introduce three conviviality models using dependence networks. First, temporal dependence networks model the evolution of dependence networks and conviviality over time. Second, epistemic dependence networks combine the viewpoints of stakeholders, and third normative dependence networks model the transformation of social dependencies by hiding power relations and social structures to facilitate social interactions. The authors show how to use these visual languages in design. The description level of the paper is methodologies and languages, and conviviality measures were not defined.

The approach we use in this paper brings novelty by operationalizing an elusive intuition and proposing a way to measure one type of conviviality. Furthermore, we provide an original approach to measuring one aspect of robustness of coalitions of agents. We present two kinds of measures: a conviviality classification that captures a hierarchical structure of the dependence networks, and a pairwise measure, based on the interdependencies among robots, that provide a total order on conviviality dependence networks.

This work builds on the notion of social dependence introduced by Castelfranchi along with concepts like groups and collectives [6]. Castelfranchi brings such concepts from social theory to agent theory to enrich agent theory and develop experimental, conceptual and theoretical new instruments for social sciences . The present work takes as a starting point an abstract notion of dependence graphs initially elaborated by Conte and Sichman [19]. The notions of dependence graphs and dependence networks were further developed by the authors [19], and with a more abstract representation similar to ours, in Boella et al. [1] and Caire et al. [5].

Dependence based coalition formation is analyzed by Sichman [18], while other approaches are developed in [17, 8, 2].

The clustering coefficient provides global and local measures in social networks to indicate respectively the overall clustering of the network and the embeddedness of single nodes. Although an interesting measure, the clustering coefficient was not used in our paper as it does not include the notion of cycle fundamental to our conviviality model. The literature concerning efficiency and stability in coalition is vast and referred to in Section 3. Particularly relevant to conviviality are the works related to the fairness of sharing the benefits of coalitions as in $[14,16,15,12]$.

Similarly to Grossi and Turrini [9], our approach brings together coalitional theory and dependence theory in the study of social cooperation within multiagent systems. However, our approach differs as it does not hinge on agreements.

Finally, works emphasizing agents' interdependence as a critical feature of multiagent systems, particularly for the design of systems involving joint interaction among humanagent systems such as in Johnson and Bradshaw et al. "coactive" design [11].

## 8. SUMMARY

Conviviality has been introduced as a social science concept for multiagent systems to highlight soft qualitative requirements like user friendliness of systems. In this paper we introduce formal conviviality measures for dependence networks using a coalitional game theoretic framework, which we contrast with more traditional efficiency and stability measures. Roughly, more opportunities to work with other people increases the conviviality.

We classify conviviality by five degrees of conviviality, from most convivial or fully connected to least convivial or unconnected. The assumptions of our conviviality measures are a bounded domain given by a [min; max] interval, and coalitions are represented by simple cycles. The requirements of our conviviality measures are that larger coalitions are more convivial than smaller ones, that coalitions based on mutual dependence are more convivial than coalitions based on reciprocal dependence, and that more possible coalitions indicate a higher conviviality, all else being equal. We need a new measure, since more traditional measures like efficiency or stability measures are different. More opportunities to work with other people increases the conviviality, whereas larger coalitions may decrease the efficiency or stability of these involved coalitions. Conviviality measures may be seen as a particular kind of robustness measures, since more convivial systems have more opportunities for agents to choose their partners, and therefore are also more robust when partnerships break up. However, in contrast to robustness measures, conviviality measures do not say anything about the stability of the coalitions. Note that intuitively, these measures may be related, for example that more stable coalitions may be more convivial, but in this paper we have disentangled these measures as much as possible. We illustrate how to use the conviviality measures in multiagent systems by discussing an example from robotics.

In further research we contemplate the need to come up with different notions of conviviality when one wants to say that a "goal-directed" system is convivial (e.g., a G2C portal) as opposed to when one claims that an "open interaction platform" is convivial (e.g., Facebook or LinkedIn). While in the first case there is an owner of the system (the city government or the tax authority) that imposes a certain way of doing things in order to reach some goals that may be convivial or dictatorial, in the second place one may think of functionalities that make the platform prone to a conviviality that is closer to the intuitions operationalized in this paper (e.g. artifacts that facilitate bringing friends into the platform and doing interesting things with them thanks to the platform). We will also look into the "conviviality as mask" intuition where conviviality appears to be more a matter of etiquette and discretion, than a matter of task interdependence. We expect that the proposed measures do not apply in a straightforward way, but that new measures will be needed to capture further views of conviviality.
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## 9. REFERENCES

[1] G. Boella, L. Sauro, and L. van der Torre. Power and dependence relations in groups of agents. In

International Conference on Intelligent Agent Technology, p. 246-252, 2004.
[2] G. Boella, L. Sauro, and L. van der Torre. Algorithms for finding coalitions exploiting a new reciprocity condition. Logic Journal of the IGPL, 17(3):273-297, 2009.
[3] P. Caire. New Tools for Conviviality: Masks, Norms, Ontology, Requirements and Measures. PhD thesis, Luxembourg University, Luxembourg, 2010.
[4] P. Caire and L. van der Torre. Conviviality measure for early requirement phase. In Normative Multi-Agent Systems. Dagstuhl Seminar Proceedings, volume 09121, 2009.
[5] P. Caire, S. Villata, G. Boella, and L. van der Torre. Conviviality masks in multiagent systems. In 7th International Joint Conference on Autonomous Agents and Multiagent Systems, volume 3, p. 1265-1268, 2008.
[6] C. Castelfranchi. The micro-macro constitution of power. Protosociology, 18:208-269, 2003.
[7] T.H. Cormen, C.E. Leiserson, R.L. Rivest, and C. Stein. Introduction to Algorithms. The MIT Press, 2nd edition, 2001.
[8] A. Gerber and M. Klusch. Forming dynamic coalitions of rational agents by use of the dcf-s scheme. In International Joint Conference on Autonomous Agents and Multiagent Systems, p. 994-995, 2003.
[9] D. Grossi and P. Turrini. Dependence theory via game theory. In International Joint Conference on Autonomous Agents and Multiagent Systems, p. 1147-1154, 2010.
[10] I. Illich. Tools for Conviviality. Marion Boyars Publishers, London, August 1974.
[11] M. Johnson, J.M. Bradshaw, P.J. Feltovich, C.M. Jonker, M. Sierhuis, and B. van Riemsdijk. Toward coactivity. In International Conference on Human-Robot Interaction, p. 101-102, 2010.
[12] L.G. Kronbak and M. Lindroos. Sharing rules and stability in coalition games with externalities. Marine Resource Economics, 22:137-154, 2007.
[13] M. Mesterton-Gibbons. An Introduction to Game theoretic Modelling. Addison-Wesley, CA, USA, 1992.
[14] A. O'Sullivan and S.M. Sheffrin. Economics: Principles in Action. Pearson Prentice Hall, 2006.
[15] D. Schmeidler. The nucleolus of a characteristic functional game. SIAM journal of Applied Mathematics, 17:1163-1170, 1969.
[16] L.S. Shapley. A value for n-person games. Annals of Mathematical Studies, 28:307-317, 1953.
[17] O. Shehory and S. Kraus. Methods for task allocation via agent coalition formation. Artificial Intelligence, 101(1-2):165-200, 1998.
[18] J.S. Sichman. Depint: Dependence-based coalition formation in an open multi-agent scenario. J. Artificial Societies and Social Simulation, 1(2), 1998.
[19] J.S. Sichman and R. Conte. Multi-agent dependence by dependence graphs. In First International Joint Conference on Autonomous Agents $\& 3$ Multiagent Systems, p. 483-490. ACM, 2002.
[20] M. Taylor. Oh no it isn't: Audience participation and community identity. Trans, Internet journal for cultural sciences, 1(15), 2004.


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[^1]:    ${ }^{1}$ This is common for goal-oriented agents. However, the model can also be applicable to other types of agents.

[^2]:    ${ }^{2}$ Planning for an optimal route is a typical shortest path finding algorithm, whose implementations are available and can be deployed on the robots.

