# Algorithmic Decision Theory

## Lecture 2: Who wins the election? Choosing from multiple opinions

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# Introduction

1. Uninominal election
   - Uninominal elections
   - Two stage uninominal elections
   - Sequential pairwise elections
2. Aggregating all voters' opinion
   - CONDORCET's method
   - BORDA's method
   - Some theoretical results
3. Voting and Complexity
   - Complexity of determining winner
   - Complexity of manipulation
   - Other types of manipulation

# Voting may be complicated

## Examples (Differences and similarities)

- UK parliament member voting: 650 single-seat constituencies, one representative elected in each constituency, one vote per voter, simple majority of votes for being elected.
- French parliament member voting: A single constituency for each seat. The candidate who obtains more than 50% of the votes is elected, otherwise there will be organized a second election stage with all candidates who obtained previously more than 12.5%. Eventually elected will be the candidate with majority of votes.
- French presidential election: Each voter may vote for one of the running candidates. If a candidate obtains more than 50%, the person is elected. Otherwise, a second election stage is organized with both candidates who obtained most of the votes in the first stage. In the second stage, the candidate with the majority of the votes is eventually elected.

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# Uninominal election: definition

## Principle

We suppose that each voter ranks all potential candidates from the best to the worst, without ties, and communicates this ranking without cheating.

In an uninominal election each voter votes for his best ranked candidate.

## Example

- Let $a, b$, and $c$ be three candidates at an election.
- Suppose that a voter prefers $a$ to $b$ and $b$ to $c$. We simply denote this information as $abc$.
- In this case, the voter will vote for candidate $a$.

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# Uninominal election: properties

## Example (Majority dictatorship)

- Let $\{a, b, c, ..., , y, z\}$ be the set of 26 candidates for a 100 voters election. Suppose that:
  - 51 voters have preferences $abc...yz$, and
  - 49 voters have preferences $zbc...ya$.
- 51 voters will vote for $a$ and 49 for $z$.

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- *In all uninominal election systems, candidate $a$ will be elected. Is $a$ really a good candidate ?*
- *No. Nearly half of the voters see candidate $a$ as their worst choice! Whereas candidate $b$ could be an unanimous second best candidate !*
- *Simple majority allows dictatorship of majority and does not favor consensual solutions.*

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## Example (Not respecting the majority of voters)

The voting system in the UK is plurality voting: The election is uninominal and the result is determined by a simple majority of votes.

- Let $\{a, b, c\}$ be the set of candidates for a 21 voters election. Suppose that:
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# Two stage uninominal elections

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- Same setting as before, but we suppose this time a two stage election as in France.
- After the first stage, $a$ obtains 10, $b$ 6 and $c$ 5 votes.
- Hence, no absolute majority ($> 50\%$) and there will be a second stage without candidate $c$.
- Suppose the voters do not change their preferences.
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- *Candidate $b$ will win the election with 15 out of 21 votes.*
- *Neither $a$, nor $c$, are preferred to $b$ by a majority of voters this time.*
- *Is this two stage voting system therefore always more satisfactory ?*

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## Example (Not-respecting the majority of voters)

- Let $\{a, b, c, d\}$ be the set of candidates for a 21 voters election. Suppose that:
  - 10 voters have preferences *bacd*,
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## Example (Not-respecting the majority of voters – continue)

- The previous result is clearly different from what a majority of voters prefer:
- Remind that:
  - 10 voters have preferences $bacd$,
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- Indeed, an absolute majority (11 out of 21) apparently prefers $a$ and $d$ over $b$ !

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- Same setting as before, but we suppose that the 6 voters who have previously voted in favor of $c$ are going to cheat, and vote instead for $a$, their second best choice.
- In this case, $a$ obtains 11, and $b$ 10 votes.

## Comment

- *Thus, candidate $a$ is elected with absolute majority right at the first stage.*
- *By cheating, these voters obtain a better result than if they were voting following their preferences.*
- *An election system which favors this kind of strategic (cheating) votings is called manipulable.*
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## Example (monotonicity violation in the two-stage voting system)

- Let $\{a, b, c\}$ be the set of candidates for a 17 voters election. Suppose that a pre-election survey reveals that:
  - 6 voters will have preferences $abc$,
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- After the first stage: $a$ will obtain 6, $b$ 6 and $c$ 5 votes. There probably will be a second stage with running candidates $\{a, b\}$.
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# Violating monotonicity of apparent preferences

## Example (Continue)

- Suppose that, following the survey, candidate $a$ wants to increase his lead over $b$ and strengthens his election campaign against $b$. Suppose that he succeeds in winning the two last voters for him (preferences $bac$ become $abc$).
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## Example (Favoring strategic abstentions)

- Let $\{a, b, c\}$ be the set of candidates for a 11 voters election. Suppose that:
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- As $c$ is their worst candidates, 2 out of 4 voters from the first group decide not to go for voting in the first stage.

## Exercise(s)

*Show that this strategic abstention is profitable for these two voters.*

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# Sequential pairwise elections: properties

## Example (Influence of the agenda)

- Let $\{a, b, c\}$ be the set of candidates for a 3 voters election. Suppose that:
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- *In this example, any candidate can be elected, it only depends on the agenda. A sequential election system always lacks neutrality of the candidates.*
- *Note that sequential voting is very frequent in parliaments, where the amendments to a bill are considered in a predefined sequence.*
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# Finding the winner by aggregating marginal rankings

1. Each voter ranks again without ties the potential candidates from his best to his worst candidate and communicates without cheating this ranking.
2. The election result is computed by aggregating directly these marginal rankings into a global consensual one.

### Comment

*Two seminal aggregation methods, quite different in their spirit, have been proposed in the 18th century by two French scientists:*

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# Condorcet's method

## Principle (Condorcet 18th century)

- In 1785, Condorcet suggests to compare pairwise all the potential candidates.
- Candidate $a$ is preferred to candidate $b$ when the number of voters who rank $a$ before $b$ is higher than the number of voters who ranks $b$ before $a$.
- A candidate, who is thus preferred to all the others, wins the election and is called Condorcet winner.

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## Example (The CONDORCET winner)

- Let $\{a, b, c, d, e, f, g, x, y\}$ be the set of candidates for a 101 voters election. Suppose that:
  - 19 voters have preferences $yabcdefgx$,
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*Write a Python program for computing the* CONDORCET *winner when given the results of an $n$ voters election with $p$ candidates.*

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## Comment

- *Let us compare the election results for candidates $x$ and $y$ by counting the voters who have ranked these candidates at rank $k = 1$ to 9.*

| | | | | | $k$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $x$ | 0 | 30 | 0 | 21 | 0 | 31 | 0 | 0 | 19 |
| $y$ | 50 | 0 | 30 | 0 | 21 | 0 | 0 | 0 | 0 |

1. *Candidate $y$ seams to be globally much better appreciated than the sc Condorcet winner $x$ !*
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1. *Candidate $y$ seams to be globally much better appreciated than the sc Condorcet winner $x$ !*
2. *There may not exist a CONDORCET winner !*

## Exercise(s)

*Find an example of election where there is no CONDORCET winner.*

# Weaknesses of Condorcet's method

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- *Let us compare the election results for candidates $x$ and $y$ by counting the voters who have ranked these candidates at rank $k = 1$ to 9.*

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# An election without any CONDORCET winner

- Suppose three voters show the following preferences:
  - 1 voter adopts the ranking *abc*,
  - 1 voter adopts the ranking *cba*, and
  - 1 voter adopts the ranking *bca*.
- No candidate does beat the other two candidates with a mojority of votes.
- And we observe a cyclic social preference $a \rightarrow b \rightarrow c \rightarrow a$ (An *inadmissible paradox* !? for number-obsessed people like some dubious mathematical economists for instance).

# BORDA's method

Contemporary with CONDORCET, BORDA invented his nowadays famous scoring method for computing the winner of an election:

## Principle (BORDA, 18th century)

In the marginal ordering of each voter, every candidate appears at a certain rank: 1 fort the first, 2 for second, etc.

The sum of marginal ranks obtained by each candidate is called its Borda score.

A candidate showing the smallest BORDA score wins the election and is called Borda winner.

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# BORDA's method –continue

## Comment

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- BORDA*'s methods, besides determining the* BORDA *winner(s), renders by the way a weak ranking –a ranking with possible ties– of the candidates.*

## Exercise(s)

*Write a Python program for computing the* BORDA *winner and ranking the candidates when given the results of an $n$ voters election with $p$ candidates. See https://digraph3.readthedocs.io/en/latest/*

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# Comparing CONDORCET's with BORDA's method

## Example

- Let $\{a, b, c, d\}$ be the set of candidates for a 3 voters election. Suppose that:
  - 2 voters have preferences *bacd*, and
  - 1 voter has preferences *acdb*
- The BORDA score of $a$ is $2 \times 2 + 1 \times 1 = 5$
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CONDORCET's *method, being pairwise, naturally verifies this property!*

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# Some theoretical results

## Definition (Useful properties of election systems)

- universality: The election system must be applicable to all possible voting outcomes.
- unanimity: If all voters rank candidate $a$ before candidate $b$ then $a$ must also be ranked before $b$ in the global consensus.
- transitivity: The global consensus gives a transitive ordering, possibly with ties.
- independence (IIA): – The difference in the global ranks of two candidates only depends on their respective marginal ranks.
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## Theorem (K. Arrow, 1963)

*When the number of candidates is at least three, there is no aggregation method of marginal rankings that can satisfy at the same time:* **universality, unanimity, transitivity, independence and non-dictatorship.**

## Comment

- BORDA's *method verifies: universality, unanimity, transitivity, and non-dictatorship.*
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## Comment

- *The French and British election systems verify universality and non-dictatorship.*
- *Therefore they are inevitably manipulable.*

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# Who wins the elections?

**Exercise(s) (Computing CONDORCET and BORDA winner(s))**

*Suppose that four candidates* $\{a, b, c, d\}$ *run for a position and five voters show the following preferences:*

$dabc$

$bcda$

$cdab$

$dacb$

$abdc$

*Who among the candidates is the* CONDORCET, *respectively the* BORDA, *winner ?*

# Complexity of determining winner

- How quickly can we determine the result under a certain voting rule?
  - $n$ candidates; $v$ voters.
  - Plurality: $O(n)$
  - CONDORCET and BORDA winners: $O(nv)$
- Even low order polynomials would be a problem in real elections: U.S. presidential elections with an $O(v^3)$ algorithm?
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  - $n$ candidates; $v$ voters.
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- **Dodgson's (Lewis Caroll) rule**: The winner of an election is the candidate who requires the fewest preference switches (adjacent) to become the CONDORCET winner.
- **Theorem** (Bartholdi, Tovey, and Trick, BTT 1989):

  It is NP-hard to determine the winner of an election under Dodgson's Method.
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- **Definition.** A voting system satisfies neutrality if it is symmetric in its treatment of the candidates.
- **Definition.** A voting system satisfies consistency if, when two disjoint sets of voters agree on a candidate $c$, the union of voters will also choose $c$.
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# Electing the Doge of Venice

1. Thirty members of the Great Council, chosen by lot, were reduced by lot to nine.
2. The nine chose forty and the forty were reduced by lot to twelve, who chose twenty-five.
3. The twenty-five were reduced by lot to nine and the nine elected forty-five.
4. Then the forty- five were once more reduced by lot to eleven.
5. And the eleven finally chose the forty-one,
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# Complexity of manipulation

- Can it ever be hard to manipulate?

  Yes
- **Definition.** The Copeland score of a candidate is the number of pairwise contests won minus the number lost.
- **Definition.** The Second order Copeland score of a candidate is the sum of the Copeland scores of each defeated candidate.
- Theorem (BTT 1989).

  It is NP-complete for a voter to determine how to manipulate an election under second order Copeland score.
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- Changing the Candidates
  1. Adding Candidates
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- Many "*fairness conditions*" address the question of whether a voting rule is vulnerable to these sort of manipulations.

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