



Algorithmic Decision Theory

Lecture 4: Evaluation models Measure and aggregate performances

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FSTC - CSC/ILIAS

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US Grade Point Average GPA

Aggregating à la Condorcet



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What is a grade ?

Definition

A grade is an evaluation of the performance of a student in a given course; an indication to which level a student fulfils the objectives of the course.

Comment

- *A grade should always be interpreted with respect to the objectives of the course.*
- *A grade may have several pedagogical functions such as certifying a certain performance level or being a hint indicating the student's strengths and weaknesses.*
- *A grade is also a public sign addressed to the parents, the University administration, future employers, etc.*



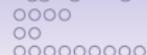
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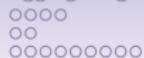
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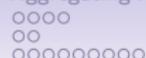
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On grading

Grading students copies relies on a number of conventions like :

- Grading scale : 0-20 (France, Belgium & Luxembourg), 0-30 (Italy), 6-1 (Germany), 0-100 (USA), $\{F, E, D, C, B, A\}$ (USA & Asia),
- The **model solution** giving the repartition of points per question,
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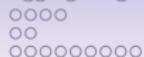
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Required properties of the grading

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Empirical properties of the grading

- In mathematics, a **difference in grades of 2 points** on a 0 – 20 scale may be commonly observed for similar copies. Motivated grading differences of up to 9 points do occur.
- In 50% of the cases, a **second grading** by the same corrector leads to a **significantly different result** than the first one.
- The grades show a high **auto-correlation** with the apparent level of the student : similar copies from presumably good and presumably weak students commonly obtain dissimilar grades in favour of the good ones.



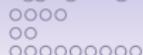
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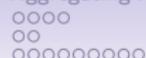
Empirical properties of the grading – continue

- The **order of the copies** has an incidence on the grading result. The spread of the grades given by the same corrector commonly augments with time for instance.
- There appear **anchorage phenomenas** : It is always better to be graded after a weak copy than after an excellent one.
- The **overall presentation of a copy** –writing, cleanliness – has certainly an influence on the grading result, even if the corrector is supposed to do not care about.



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Interpreting grades

- In Europe, grades give generally the impression that they are **numerical measures**.
- Yet, there is a problem with the minimum grade **0**. It does not signify that a student does know nothing !
- There is also a problem with the maximum grade **20**. Two excellent students getting 20/20 are not necessarily equivalent !
- What is the genuine **scale type** of exam grades :
ratio, interval, only ordinal ?



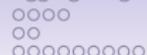
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Interpreting grades – continue

- The preceding problems give arguments to the promoters of Anglo-Saxon alphabetical – i.e. ordinal – grades : generally *E* or *F* to *A* (best grade).
- As a consequence, a large majority of students are often given a *neutral* grade like *B* or *C*.
- In order to better discriminate the effective performances, one introduces then *qualitative decorations* like + and – : *B+* signifying a grade slightly inferior to *A*, *B–* a grade slightly better than *C*.
- It is worthwhile noticing that all these ordinal grades are translating a certain range of *number of points or percentages* obtained in fact in the underlying exams !
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Rules for aggregating grades

Weighted average grades

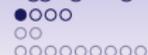
Methodological problems

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Ordinal measurement scales

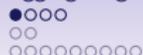
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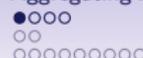
Rules for aggregating grades

- In order to validate a programme or a degree, it is common usage to aggregate grades obtained in the same and even in different courses.
- Three principles for aggregating are generally used :
 - Conjunctive aggregation
 - Weighted mean
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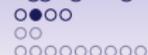
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Conjunctive aggregation

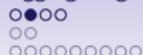
- The students must simply **validate all** their exams in a given time in order to get their degree.
- **Advantage** : No commensurability hypothesis concerning the individual grades is required.
- **Disadvantages** :

• Many students risk to eventually fail their degree.

• This is not a good way to encourage students to study.

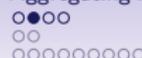
• Example in order to not discourage and possibly stimulate students to enhance performance in learning.

• An alternative can be implemented: the students can pass a simulated exam giving their best.



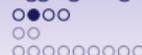
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 - There are only two types of results : **pass** and **fail**.
- Grading ?
- Aggregating ?
- Ordinal grades
- Conclusions



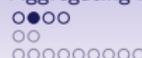
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 - **Many** students risk to eventually **fail** their degree.
 - There are only two types of results : **valid** and **invalid**.
 - **No formative** results may be expressed : slightly insufficient for example in order to not discourage and positively stimulate a student to enhance his performance for instance.
 - **No distinction** can be expressed : The students are not stimulated towards giving their best.



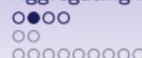
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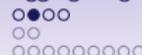
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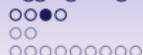
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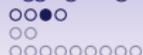
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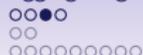
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- To validate a study programme or degree, this weighted average grade is then compared to standard values like 10/20, or 14/20, 16/20 etc. to attribute a distinction.
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- **Required minimal thresholds** for validating a course or a whole programme are commonly introduced in order to avoid full compensation between individual grades (a 0/20 grade being compensated by a 20/20 grade for instance).
- Sometimes, the average grade has to **reach a certain level** (14/20) before compensating is allowed.
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- We suppose that all grades are expressed on a 0 – 20 scale.
- We denote $g_i(a)$ the grade obtained by a student a in the course i ($i = 1$ to n).
- We denote w_i the (strictly positive) weight allocated to course i in the evaluation of the final grade.
- The final grade $g(a)$ of student a is computed as follows :

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$$g(a) = \sum_{i=1}^n w_i \cdot g_i(a)$$

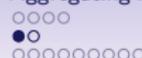


Weighted average grade : Notations

Definition

- We suppose that all grades are expressed on a 0 – 20 scale.
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Weighted average grade – continue

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- The weights w_i are commonly expressed as *integer numbers* (number of lectures, hours, lessons, or ECTS ...).
- The weights w_i may always be *normalised* without loss of generality as follows :

$$w'_i = \frac{w_i}{\sum_{i=1}^n w_i}$$

- Normalised weights w'_i – *rational numbers* – are thus confined between 0 and 1 and $\sum_{i=1}^n w'_i = 1$.
- The average grade, computed with normalised weights, will be expressed on the same scale (0 – 20 for instance) as the individual courses' grades.



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Methodological problems

Example (1. An undesirable effect of the compensation)

Consider four students $\{a, b, c, d\}$ enrolled in a study programme consisting of two courses $\{g_1, g_2\}$ of same weight and where they have obtained the following grades :

	g_1	g_2
a	11	11
b	5	19
c	20	4
d	4	6

Student a shows satisfactory results in both courses, whereas student d shows very weak results. On the contrary, b and c are both excellent students in one course and weak in the other. Usually a student is ranked according to the both courses span below.



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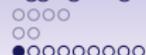
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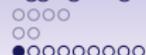
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Example (1) – continue

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Aggregating the four students grades with a weighted average results in following figures :

	g
b	12
c	12
a	11
d	5

Students b and c are ranked before student a . One may even verify that no other weighting of the two courses will allow to rank a before b and c . This is not the case if we consider the weighted average of the two courses. This is a sign of grouping that captures that d is really very good in all courses.



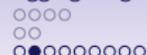
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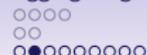
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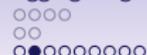
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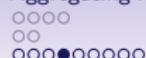


Methodological problems – continue

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Practical consequences of unlimited compensation :

- *Using a weighted average as rule for aggregating grades may turn students towards concentrating their efforts on a **limited** number of courses only by relying on the compensation mechanism for getting a sufficient final grade.*
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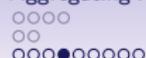


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Example (2. Interactions between performances to aggregate ?)

Consider four students $\{a, b, c, d\}$ enrolled in a programme consisting in statistics (S), mathematics (M) and economics (E). They got the following grades :

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Student a should be ranked before student b in an engineering study programme. b is, even more, weak in maths, which is convenient neither for an engineering nor an economics degree. With a similar reasoning, d is much better than c when considering an economics degree.



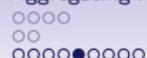
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Interactions between performances :

- *Whereas the preceding rankings seem quite reasonable, they are however not compatible with the weighted average rule.*
- *When the statistics results are excellent, the weight of mathematics outranks the one of economics (a outranks b).*
- *However, showing weak grades in statistics leads to consider that the weight of economics outranks the one of mathematics (d outranks c)*
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Consider two students enrolled in a programme with two courses of same weight. The grading is done on a 0 – 20 scale and a final grade of at least 10 is required in order to validate the programme.

	g_1	g_2
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b	12	9

Both students obtain the same average grade 10.5 and validate equivalently the programme. The difference between 12 and 11 in the first course exactly compensates the difference between 10 and 9 shown in the second course.



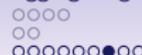
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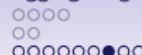
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Methodological problems – continue

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Incommensurable differences between grades :

- *As 10 is the threshold for validating the programme, one may suppose that the difference observed in the first course is more important than that observed in the second one.*
- *Consequently, student **a** must in fact have better validated the programme than student **b** ?*
- *Indeed, **a** was conjointly **successful in both courses**, whereas **b** failed one of the two courses.*
- *With the weighted average rule, a difference of one point is required to have uniformly the same signification all along the scale.*

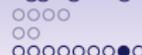


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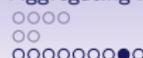


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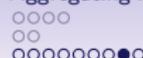


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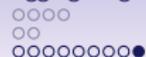


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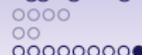
Methodological problems – continue

Example (4. Incommensurable differences between grades?)

Reconsider the three students enrolled in the same programme as in Example (3) :

Comment

	g_1	g_2
a		
b	14	14
c		



Methodological problems – continue

Example (4. Incommensurable differences between grades?)

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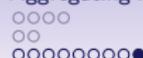
Comment

	g_1	g_2
a	$14 - x$	$14 + x$
b	14	14
c	$14 + x$	$14 - x$

The three students obtain the same average of 14 (for $x = 1, 2, \dots, 5$) and validate equivalently the programme with a final grade 14 (good).

If $x = 1$, this result is acceptable.

If $x = 5$, this result is no more acceptable.



Methodological problems – continue

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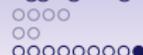
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a	$14 - x$	$14 + x$
b	14	14
c	$14 + x$	$14 - x$

The three students obtain the same average of 14 (for $x = 1, 2, \dots, 5$) and validate equivalently the programme with a final grade 14 (good).

If $x = 1$, this result is acceptable.

If $x = 5$, this result is no more acceptable.



Methodological problems – continue

Example (4. Incommensurable differences between grades?)

Reconsider the three students enrolled in the same programme as in Example (3) :

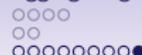
Comment

	g_1	g_2
a	13	15
b	14	14
c	15	13

The three students obtain the same average of 14 (for $x = 1, 2, \dots, 5$) and validate equivalently the programme with a final grade 14 (good).

*If $x = 1$, this result **is acceptable**.*

If $x = 5$, this result is no more acceptable.



Methodological problems – continue

Example (4. Incommensurable differences between grades?)

Reconsider the three students enrolled in the same programme as in Example (3) :

Comment

	g_1	g_2
a	9	19
b	14	14
c	19	9

The three students obtain the same average of 14 (for $x = 1, 2, \dots, 5$) and validate equivalently the programme with a final grade 14 (good).

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1. Grading students

What is a grade ?

The grading process

Interpreting grades

2. Aggregating performances

Rules for aggregating grades

Weighted average grades

Methodological problems

3. How to aggregate ordinal grades ?

Ordinal measurement scales

US Grade Point Average GPA

Aggregating à la Condorcet



How to aggregate ordinal grades ?

Example (5. grading on an ordinal scale)

Consider three students enrolled in a study programme consisting of three courses graded from 0 to 20 points and where a grade of 10/20 is required for succeeding the programme. If the grading scale is purely ordinal, the following grades will show the same result for each student.

	g_1	g_2	g_3		g_1	g_2	g_3
a	12	5	13	a	11	4	12
b	13	12	5	b	13	13	6
c	5	13	12	c	4	14	11

In the first case, all three students validate, whereas, in the second case, only b validates the programme.



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How to aggregate ordinal grades

Example (6. The US Grade Point Average GPA)

As the courses are graded on alphabetical levels from E to A, one has to numerically encode these levels. A common conversion schema is the following :

Comment

level	grade	mention
<i>A</i>	4	(excellent)
<i>B</i>	3	(very good)
<i>C</i>	2	(good)
<i>D</i>	1	(satisfactory)
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- *A constant difference between two adjacent levels is assumed.*
- *Obtaining an excellent level A is supposed to be 4 times as performing as obtaining a satisfactory level D (1).*

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Example (6) Computing the GPA – continue

Exams in the US are generally graded from 0 to 100 %. Suppose that three student obtained the following grades in three courses :

	g_1	g_2	g_3	Conversion schema :		
				level	interval	grade
a	90	69	70	A	90 – 100%	4
b	79	79	89	B	80 – 89%	3
c	100	70	69	C	70 – 79%	2
				D	60 – 69%	1
				E	0 – 59%	0

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Example (6) Computing the GPA – continue

Converting the results :

	g_1	g_2	g_3
a	A	D	C
b	C	C	B
c	A	C	D

Computing the GPA :

	g_1	g_2	g_3	GPA
a	4	1	2	2.33
b	2	2	3	2.33
c	4	2	1	2.33

Comment

All three students obtain the same GPA value 2.33.



Example (6) Computing the GPA – continue

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Other conversion schema :

level	interval	grade
A+	98 – 100%	10
A	94 – 97%	9
A–	90 – 93%	8
B+	87 – 89%	7
B	83 – 86%	6
B–	80 – 82%	5
C+	77 – 79%	4
C	73 – 76%	3
C–	70 – 72%	2
D	60 – 69%	1
E	0 – 59%	0

Conversion results :

	g_1	g_2	g_3
<i>a</i>	A–	D	C–
<i>b</i>	C+	C+	B+
<i>c</i>	A+	C–	D

Computing the GPA :

	g_1	g_2	g_3	GPA
<i>a</i>	8	1	2	3.66
<i>b</i>	4	4	7	5.00
<i>c</i>	10	2	1	4.33

Student *b* obtains now clearly a better result.



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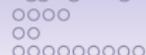
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Aggregating ordinal performances

Example (Condorcet's method)

Consider three students enrolled in a study programme consisting in three courses of same weight and who obtained the grades shown here :

	g_1	g_2	g_3
a	13	12	11
b	11	13	12
c	14	10	12

Comment



Aggregating ordinal performances

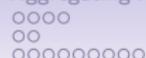
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Comment

- *The three students obtain the same average grade 12.*
- *Consider now that a difference of **one point** on the grading scale is **not really significant** for warranting an effective performance difference.*
- *Student a shows at least as good grades as b and c in all the courses.*
- *However, students b are c are only in two out of three courses at least as good as student a .*



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Exercise(s)

Here the table of grades obtained by four students : a , b , c , and d , in five courses : C_1 , C_2 , C_3 , C_4 and C_5 .

course	C_1	C_2	C_3	C_4	C_5
ECTS	2	3	4	2	4
a	11	13	9	15	11
b	12	9	13	10	13
c	8	11	14	12	14
d	15	10	12	8	13

An award is granted to the best amongst these four students.

1. Who would you nominate ?
2. Explain and motivate your selection algorithm.



Concluding

- Grading accurately someones performances is generally a **difficult task** in practice.
- Grading procedures are in general quite **complex** and must not be seen as simple as physical weight, time and length measures.
- Aggregating grades needs taking into account potential **imprecision**, uncertainty as well as known cognitive **biases**.
- Aggregating rules have to be analyzed with great attention. The simplests and evidents do not necessarily give the expected results.



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