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Generating random outranking digraphs

MICS: Algorithmic Decision Theory

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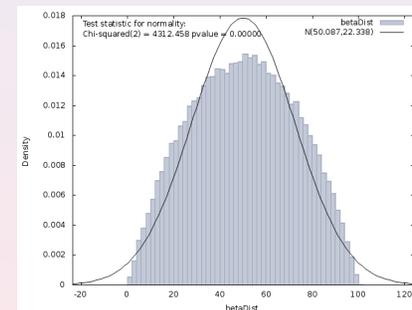
April 28, 2020

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Beta Performance Generator – 1

- The **beta** performance generator delivers random performance measures within a given performance scale following a **Beta(α, β)** probability law.

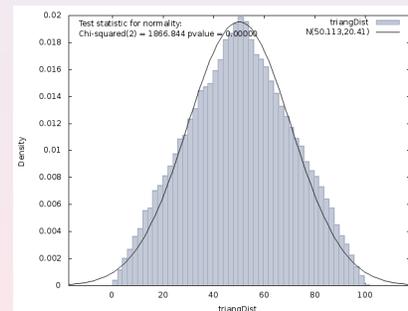
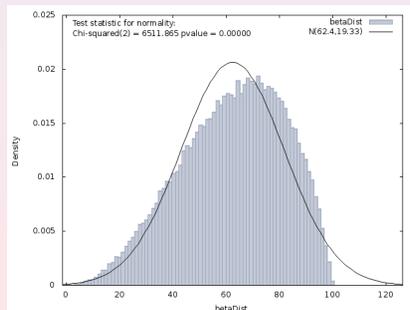
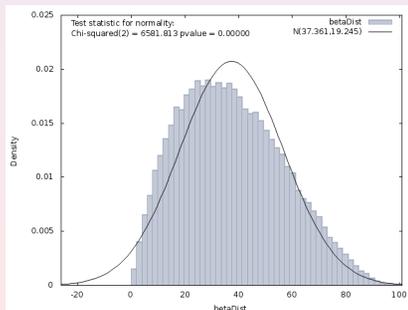


- In the default case, ($\alpha = 2.0, \beta = 2.0$), the **mode** xm is situated in the middle (50.0) of the performance scale [0.0, 100.0] and the **probability is equally distributed** on both sides, i.e. xm represents the median performance, and the standard deviation $sd \approx 15.0$.

Triangular Performance Generator – 1

Beta Performance Generator – 2

- We consider two variants with equal standard deviation $sd = 15$:
 - low performances: $xm = 25$ ($\alpha = 2.0, \beta = \frac{1.0}{1.0-xm}$),
 - high performances: $xm = 75$ ($\alpha = \frac{1.0}{1.0-xm}, \beta = 2.0$),



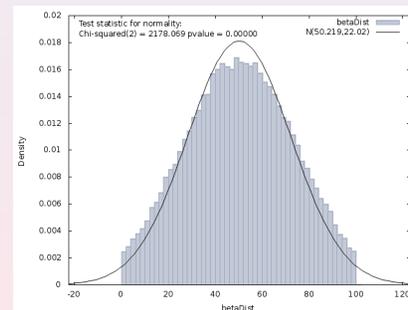
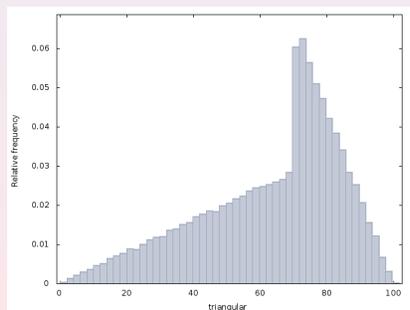
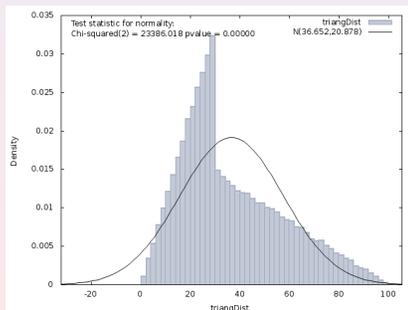
- The **triangular** performance generator delivers random performance measures within a given performance scale following an extended triangular alert $Tr(xm, r)$ probability law with **mode** xm and **probability repartition** r lower or equal xm .

- In the default case, the mode xm is situated in the middle (50.0) of the performance scale and the probability is **equally distributed** on both sides, i.e. $r = 0.5$ and xm represents the median performance measure.

Truncated Gaussian Performance Generator – 1

Triangular Performance Generator – 2

- We consider two variants with fixed repartition $r = 0.5$:
 - low** performances: $xm = 30$,
 - high** performances: $xm = 70$,



- The **truncated Gaussian** performance generator delivers random performance measures within a given performance scale following a truncated $\mathcal{N}(\mu, \sigma)$ probability law with **mean** μ and **standard deviation** σ .

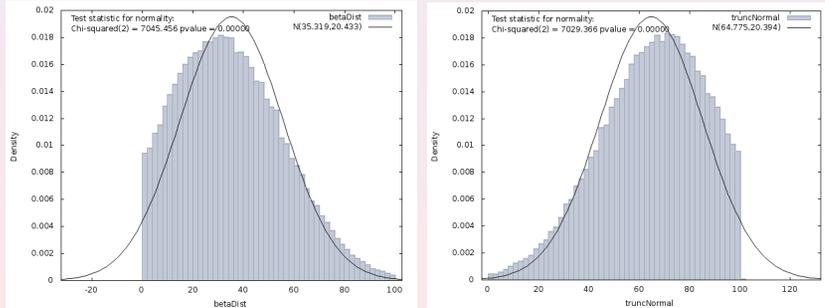
- In the default case, the mode xm is situated in the middle (50.0) of the performance scale and the standard deviation is a fourth (25.0) of the scale scope.

See the Digraph3 <RandomNumbers> module.



Truncated Gaussian Performance Generator – 2

- We consider two variants:
 - low performances: $xm = 30$,
 - high performances: $xm = 70$,



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A Standard Random Performance Tableau

- 20 decision actions; low variant: 13; high variant: 50;
- 13 criteria; low variant: 7; high variant: 21;
- All criteria are by default **equi-significant** (same weight 1); uniform random weights may be generated within a given weight scale;
- All criteria use a same cardinal performance measurement scale; from 0.0 to 100.0 by default: user provided scale limits may be given;
- Individual performances are by default generated with a beta law: $Beta(2, 2)$. Two variants are provided:
 - a **uniform** law: $\mathcal{U}(a, b)$ with a and b the performance measurement scale limits;
 - an **extended triangular** law: $\mathcal{T}(xm, r)$, where xm is the mode and r the percentile of xm .

See the **Digraph3** RandomPerformanceTableau class description



Fixed Discrimination Thresholds

- On each criterion, the **default discrimination thresholds** are chosen in percentages of the amplitude of the criterion performance measurement scales:
 - indifference** threshold equals 2.5% of the potential performance amplitude;
 - preference** threshold equals 5.0% of the potential performance amplitude;
 - veto** threshold equals 80.0% of the potential performance amplitude.
- Note: Ordinal criteria admit by default solely a preference threshold of one unit.

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The Digraph3 <RandomPerformanceTableau> class

Example Python session:

```
>>> from randomPerfTabs import RandomPerformanceTableau
>>> t = RandomPerformanceTableau(numberOfActions=13,\
    numberOfCriteria=7,weightDistribution='random',\
    weightScale=(0.0,10.0),seed=100,\
    missingDataProbability=0.03)
>>> t
*----- PerformanceTableau instance description -----*
Instance class: RandomPerformanceTableau
Seed      : 100
Instance name : randomperftab
# Actions   : 13
# Criteria  : 7
Attributes  : ['weightPreorder','BigData','criteria',
    'missingDataProbability','commonScale',
    'evaluation','digits','name','sumWeights',
    'commonMode','randomSeed','actions']
>>> t.showHTMLPerformanceTableau(Transposed=True,\
    title='Standard performance tableau')
```

Standard performance tableau

crit	a01	a02	a03	a04	a05	a06	a07	a08	a09	a10	a11	a12	a13
g1	76.12	27.99	58.55	44.59	43.58	53.48	75.81	87.68	43.83	38.12	25.36	23.61	94.28
g2	53.54	34.36	31.88	31.00	44.85	48.34	46.76	34.34	NA	79.97	29.50	53.59	88.90
g3	70.84	95.04	30.74	83.81	56.50	63.09	42.17	51.07	33.63	54.63	24.58	50.33	7.41
g4	41.59	73.65	78.28	75.01	69.26	77.84	76.50	28.84	22.32	64.32	35.38	40.52	73.26
g5	NA	35.61	77.25	60.77	74.02	34.83	64.77	84.78	53.76	43.09	15.31	43.67	49.74
g6	16.39	26.25	NA	81.29	63.48	50.86	73.49	48.74	17.26	40.48	47.92	67.83	43.61
g7	46.69	61.65	7.12	72.87	20.62	65.10	67.77	64.11	3.46	73.21	52.46	69.45	67.01

On each criterion *g1* to *g7*, the performances of the seven decision alternatives are generated on a common 0.00 to 100.00 satisfaction scale. The light green marked cells indicate the best performance obtained on this criterion, whereas the light red marked cells indicate the weakest performance obtained on this criterion. On criterion *g1*, for instance, alternative *a12* show the weakest and *a13* the best performance.

Notice by the way the three missing evaluations: one for alternative *a1* on criterion *g5*, one for *a03* on criterion *g6* and one for *a09* on criterion *g2*.

<RandomPerformanceTableau> class

Example Python session –continue:

```
>>> t.showCriteria(IntegerWeights=True)
*---- criteria ----*
g1 'RandomPerformanceTableau()' instance'
Scale = (0.0, 100.0)
Weight = 8
Threshold ind : 2.50 + 0.00x ; percentile: 0.06
Threshold pref : 5.00 + 0.00x ; percentile: 0.09
Threshold veto : 80.00 + 0.00x ; percentile: 1.0
...
...

```

On criterion *g1*, 6% of the performance differences are insignificant, 9% are below the preference discrimination threshold, and no considerable performance difference is observed.

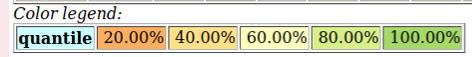
We may visualize a ranked heatmap of the performance tableau with the `t.showHTMLPerformanceHeatmap` method.

Example Python session –continue:

```
>>> t.showHTMLPerformanceHeatmap(Transposed=True,\
    pageTitle='Ranked heatmap of the decision alternatives',
    rankingRule='Copeland',colorLevels=5)
```

Ranked heatmap of the decision alternatives

criteria	weight	a07	a04	a08	a05	a06	a01	a13	a03	a12	a10	a02	a09	a11
g5	+7.00	64.77	60.77	84.78	74.02	34.83	NA	49.74	77.25	43.67	43.09	35.61	53.76	15.31
g6	+7.00	73.49	81.29	48.74	63.48	50.86	16.39	43.61	NA	67.83	40.48	26.25	17.26	47.92
g1	+6.00	75.81	44.59	87.68	43.58	53.48	76.12	94.28	58.55	23.61	38.12	27.99	43.83	25.36
g3	+6.00	42.17	83.81	51.07	56.50	63.09	70.84	7.41	30.74	50.33	54.63	95.04	33.63	24.58
g2	+5.00	46.76	31.00	34.34	44.85	48.34	53.54	88.90	31.88	53.59	79.97	34.36	NA	29.50
g4	+2.00	76.50	75.01	28.84	69.26	77.84	41.59	73.26	78.28	40.52	64.32	73.65	22.32	35.38
g7	+2.00	67.77	72.87	64.11	20.62	65.10	46.69	67.01	7.12	69.45	73.21	61.65	3.46	52.46



The criteria appear ordered by decreasing significance weight, whereas the decision alternatives are ranked following the *Copeland* ranking rule. See the Digraph3 tutorial on ranking with multiple incommensurable criteria.

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Random *Cost-Benefit* Performance Tableau – I

- 20 decision actions; low variant: 13; high variant: 50.
- 13 criteria; low variant: 7; high variant: 20.
- A criteria is with equal probability either to be minimized (**cost** criteria) or to be maximized (**benefit** criteria).
- All criteria either support an **ordinal** or a **cardinal** performance scale; the cost criteria being mostly cardinal (2/3) and the benefit ones mostly ordinal (2/3).
- Ordinal performances are represented on **integer** scales: $\{1, 2, \dots, 10\}$.
- Cardinal performances are represented on a **decimal** scale: $[0.0; 100.0]$ with a precision of 2 digits.

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Random *Cost-Benefit* Performance Tableau – II

- In the Cost-Benefit model the decision actions are divided randomly into three categories: **cheap**, **neutral**, **advantageous**.
- An action is called:
 - **cheap** when the performances are generated with $\mathcal{T}(xm = 30, r = 0.5)$ (default), $\mathcal{N}(\mu = 30, \sigma = 25)$, or $\text{Beta}(\alpha = 2.62203, \beta = 5.8661)$, i.e. $(xm = 25, sd = 15)$.
 - **advantageous** when the performances are generated with $\mathcal{T}(xm = 70, r = 0.5)$ (default), $\mathcal{N}(\mu = 70, \sigma = 25)$, or $\text{Beta}(\alpha = 5.8661, \beta = 2.62203)$, i.e. $(xm = 75, sd = 15)$
 - and **neutral** when the performances are generated with $\mathcal{T}(xm = 50, r = 0.5)$ (default), $\mathcal{N}(\mu = 50, \sigma = 25)$, or $\text{Beta}(\alpha = 5.055, \beta = 5.055)$, i.e. $(xm = 50, sd = 15)$

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Random *Cost-Benefit* Performance Tableau – II

Fixed Percentile Discrimination Thresholds:

On each cardinal criterion, the **default performance discrimination thresholds** are chosen such that the:

- **indifference** threshold equals the percentile 5 of all generated performance differences;
- **preference** threshold equals the percentile 10 of all generated performance differences;
- **veto** threshold equals the percentile 95 of all generated performance differences.

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<RandomCBPerformanceTableau> class – I

```
>>> from randomPerfTabs import RandomCBPerformanceTableau
>>> t = RandomCBPerformanceTableau(numberOfActions=7,\
    numberOfCriteria=11,commonPercentiles=\
    {'ind':0.05, 'pref':0.10, 'veto':0.95},\
    missingDataProbability=0.05,seed=109)
>>> t.showCriteria(IntegerWeights=True)
c1 'Costs/random cardinal cost criterion'
  Scale = (0.0, 100.0)
  Weight = 7
  Threshold ind : 7.64 + 0.00x ; percentile: 0.048
  Threshold pref : 8.11 + 0.00x ; percentile: 0.14
  Threshold veto : 62.25 + 0.00x ; percentile: 0.95
...
b2 'Benefits/random ordinal benefit criterion'
  Scale = (0, 10)
  Weight = 4
...
```

In this example we notice, for instance, a cardinal *Costs* criterion *c1* of weight 7 with default performance discrimination thresholds and an ordinal *Benefits* criterion *b2* of weight 4.

<RandomCBPerformanceTableau> class – II

Continue –

```
>>> t.showActions()
*----- show decision action -----*
key: a1
  short name: a1n
  name:      random neutral decision action
key: a2
  short name: a2c
  name:      random cheap decision action
...
key: a5
  short name: a5a
  name:      random advantageous decision action
...
>>> t.showHTMLPerformanceTableau(\
    title='Cost-Benefit Performance Tableau')
```

Contents	Random Generators	Standard Tableaux	Special Tableaux
o	ooo	oo	ooooo
	oo	oooo	ooooo
	oo	oooo	ooooo

Cost-Benefit Performance Tableau

criteria	b01	b02	b03	b04	b05	b06	b07	c01	c02	c03	c04
weight	4.00	4.00	4.00	4.00	4.00	4.00	4.00	7.00	7.00	7.00	7.00
a1n	59.11	4.00	8.00	31.64	4.00	68.21	NA	-66.27	-58.09	-44.53	-62.89
a2c	28.32	3.00	5.00	57.22	5.00	46.25	29.96	-48.44	-55.27	-38.31	-27.74
a3n	46.03	9.00	5.00	61.17	5.00	84.36	61.91	-56.55	-59.05	-49.73	-71.00
a4n	27.69	8.00	5.00	55.57	7.00	75.17	22.04	-26.20	-15.65	-59.28	-47.26
a5a	NA	8.00	8.00	57.59	9.00	25.83	41.54	-77.72	-67.53	-83.83	NA
a6c	82.24	5.00	5.00	12.67	2.00	11.80	65.86	-89.75	-36.31	-27.42	-76.57
a7c	26.72	3.00	3.00	21.20	2.00	NA	39.16	-64.58	-43.29	-35.74	-49.99

The sum of weights of the *Benefits* criteria ($7 \times 4 = 28$) equals the sum of weights of the *Costs* criteria ($4 \times 7 = 28$). We observe 3 cheap actions (*a2*, *a6*, *a7*), three neutral actions (*a1*, *a3*, *a4*) and one advantageous action (*a5*). As costs must be mimized, the performances registered on the *Costs* criteria are all negative. Cheap actions *a6* and *a7* show five, respectively four weakest performances, whereas neutral action *a3* shows three best performances.

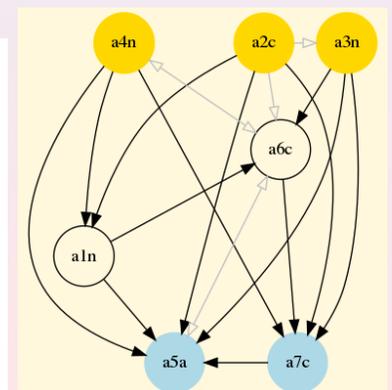
The random outranking digraph

```
>>> from outrankingDigraphs import BipolarOutrankingDigraph
>>> g = BipolarOutrankingDigraph(t,Normalized=True)
>>> g.showRelationTable()
>>> # strict(codual) outranking digraph drawing
>>> (~(-g)).exportGraphViz(bestChoice=['a2','a3','a4'],\
    worstChoice=['a5','a7'])
```

Normalized Relation

r(x S y)	a1	a2	a3	a4	a5	a6	a7
a1	-	-0.23	0.11	-0.25	0.30	0.14	0.11
a2	0.36	-	0.29	0.07	0.23	1.00	0.54
a3	0.27	0.00	-	0.11	0.52	1.00	0.18
a4	0.39	0.29	0.04	-	1.00	0.00	0.54
a5	-0.16	-0.09	-0.52	-1.00	-	0.00	-0.02
a6	-0.14	0.00	-1.00	0.00	0.00	-	1.00
a7	0.14	-0.14	-0.05	-0.29	0.16	-1.00	-

Valuation domain: [-1.00; +1.00]



Rubis Python Server (graphviz), R. Bisdorff, 2008

Generating random public policies

- we consider three decision objectives: **economical aspects**, **environmental aspects** and **societal aspects**.
- Every performance criteria is affected **randomly** to one of the three objectives.
- The three objectives are equally important and the criteria in each objective are equally significant.
- Each random potential public policy is allocated on each objective to one of three performance categories: **low** performances (−), **medium** performances (~) or **high** performances (+).
- When generating now the performances of a policy on a criterion, the random generator is modulated following the performance profile of the policy respective to each decision objective.

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<Random3ObjectivesPerformanceTableau> class

```
>>> from randomPerfTabs import \
    Random3ObjectivesPerformanceTableau
>>> t = Random3ObjectivesPerformanceTableau(\
    numberOfActions=7,numberOfCriteria=13,seed=100)
>>> t.showObjectives()
*----- show objectives -----*
Eco: Economical aspect
    ec01 criterion of objective Eco 24
    ec04 criterion of objective Eco 24
    ...
Total weight: 72.00 (3 criteria)
Soc: Societal aspect
    so02 criterion of objective Soc 12
    so05 criterion of objective Soc 12
    ...
Total weight: 72.00 (6 criteria)
Env: Environmental aspect
    en03 criterion of objective Env 18
    en08 criterion of objective Env 18
    ...
Total weight: 72.00 (4 criteria)
>>> ...
```

<Random3ObjectivesPerformanceTableau> class

Continue –

```
>>> t.showActions()
*----- show decision action -----*
key: p1
  short name: p1
  name: random decision action Eco+ Soc~ Env~
  profile: {'Eco':'good', 'Soc':'fair', 'Env':'fair'}
key: p2
  short name: p2
  name: random decision action Eco+ Soc- Env~
  profile: {'Eco':'good', 'Soc':'weak', 'Env':'fair'}
...
key: p6
  short name: p6
  name: random decision action Eco- Soc~ Env~
  profile: {'Eco':'fair', 'Soc':'fair', 'Env':'good'}
...
>>> t.showHTMLPerformanceHeatmap(\
  pageTitle='Performance heatmap of random public policies',\
  colorLevels=5,Correlations=True)
```

Performance heatmap of random public policies

criteria	ec07	en13	ec04	ec01	so06	so02	en10	so11	en03	so09	en08	so12	so05
weights	+24.00	+18.00	+24.00	+24.00	+12.00	+12.00	+18.00	+18.00	+12.00	+18.00	+12.00	+12.00	+12.00
tau(*)	+0.48	+0.45	+0.40	+0.40	+0.38	+0.24	+0.21	+0.19	+0.14	+0.07	+0.00	-0.12	-0.12
p6	55.57	68.83	53.77	75.31	66.21	49.05	78.60	NA	88.48	50.92	44.62	47.54	49.69
p2	87.02	77.66	70.30	72.54	35.52	15.33	21.83	53.73	77.19	NA	70.32	77.56	36.31
p4	64.37	25.14	80.40	85.67	43.54	89.12	17.94	56.36	20.79	62.18	85.20	37.60	55.60
p5	36.15	54.60	86.59	NA	3.85	82.36	20.30	81.22	46.78	34.65	91.94	72.54	63.58
p1	72.59	37.14	29.25	37.78	59.35	35.11	52.12	67.40	34.71	49.39	18.86	87.32	82.68
p7	26.95	27.04	8.96	46.75	31.22	7.42	50.04	7.31	70.45	9.83	72.53	13.94	29.52
p3	28.41	41.37	9.71	16.33	19.02	37.39	9.17	41.98	70.11	60.27	61.70	77.81	51.00

Color legend:



(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation

Ranking rule: NetFlows

Ordinal (Kendall) correlation between global ranking and global outranking relation: +0.815

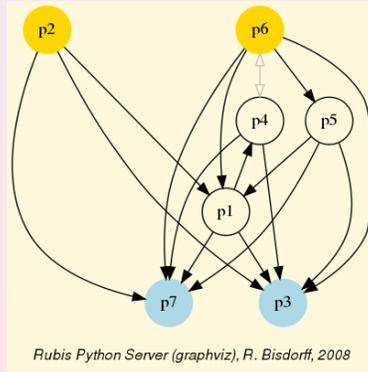
The performance criteria are ordered in decreasing marginal correlation with the default 'NetFlows' ranking of the seven potential public policies. Overall best performing policy appears to be policy p6, followed by p2. Weakest policy is p3. The three criteria, supporting the economic decision objective (ec07, ec04 and ec01), appear most correlated with the proposed ranking.

```
>>> from outrankingDigraphs import BipolarOutrankingDigraph
>>> g = BipolarOutrankingDigraph(t, Normalized=True)
>>> g.showHTMLRelationTable()
>>> # strict (codual) outranking digraph drawing
>>> (~(-g)).exportGraphViz(bestChoice=['p2', 'a6'], \
    worstChoice=['p3', 'p7'])
```

Normalized Relation

r(x S y)	p1	p2	p3	p4	p5	p6	p7
p1	-	-0.22	0.56	0.17	-0.06	-0.33	0.56
p2	0.28	-	0.72	0.06	0.11	0.22	1.00
p3	-0.22	-0.44	-	-1.00	-0.28	-1.00	0.33
p4	-0.06	0.28	1.00	-	0.08	0.00	0.67
p5	0.06	0.11	0.44	0.39	-	-0.11	0.44
p6	0.50	0.08	1.00	0.00	0.11	-	0.78
p7	-0.28	-1.00	0.28	-0.50	-0.33	-0.78	-

Valuation domain: [-1.00; +1.00]



Random academic performance tableaux – I

The randomPerfTabs.RandomAcademicPerformanceTableau class generates temporary performance tableaux with random grades for a given number of students in different courses.

Parameters:

- Number of students and number of Courses,
- weightDistribution := equisignificant — random (default),
- weightScale := (1, 1 — numberOfCourses (default when random)),
- IntegerWeights := Boolean (True = default),
- commonScale := (0,20) (default), ndigits := 0,
- WithTypes := Boolean (False = default),
- commonMode := ('triangular', xm=14, r=0.25) (default),
- commonThresholds := 'ind':(0,0), 'pref':(1,0) (default),
- missingDataProbability := 0.0 (default).

Random academic performance tableaux – II

When parameter WithTypes is set to True, the students are randomly allocated to one of the four categories: *weak* (1/6), *fair* (1/3), *good* (1/3), and *excellent* (1/3), in the bracketed proportions.

In the default 0-20 grading range, the random grading range of a weak student is 0-10, of a fair student 4-16, of a good student 8-20, and of an excellent student 12-20.

The random grading generator follows a *double triangular probability law* with mode (xm) equal to the middle of the random range and median repartition (r = 0.5) of probability each side of the mode (see the documentation of randomNumbers module).

<RandomAcademicPerformanceTableau> class

```
>>> from randomPerfTabs import \
    RandomAcademicPerformanceTableau
>>> t = RandomAcademicPerformanceTableau(\
    numberOfStudents=13, numberOfCourses=7, \
    missingDataProbability=0.03, \
    WithTypes=True, seed=100)
>>> t
*----- PerformanceTableau instance description -----*
Instance class: RandomAcademicPerformanceTableau
Seed           : 100
Instance name  : randstudPerf
# Actions      : 13
# Criteria     : 7
Attributes     : ['randomSeed', 'name', 'actions',
                  'criteria', 'evaluation', 'weightPreorder']
>>> t.showHTMLPerformanceHeatmap(Transposed=True, \
    colorLevels=5, ndigits=0)
```

The random outranking relation

```
>>> from outrankingDigraphs import BipolarOutrankingDigraph
>>> g = BipolarOutrankingDigraph(t, Normalized=True)
>>> g.showHTMLRelationTable()
```

Heatmap of Performance Tableau 'randstudPerf'

criteria	weight	s06	s08	s09	s05	s07	s04	s01	s12	s02	s03	s10	s11	s13
g6	+4.00	14	12	14	13	12	13	13	14	10	10	9	10	5
g2	+3.00	17	16	14	14	14	13	14	11	13	10	13	10	10
g5	+3.00	13	13	12	NA	12	13	9	10	12	10	NA	10	NA
g1	+2.00	16	14	15	13	NA	13	13	13	10	16	11	10	10
g3	+2.00	16	14	12	12	12	11	12	11	12	15	13	10	10
g4	+1.00	14	14	11	19	14	13	13	13	16	13	NA	10	10
g7	+1.00	14	15	9	17	12	11	12	12	18	13	11	10	10

Color legend:

quantile	20.00%	40.00%	60.00%	80.00%	100.00%
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The Courses (criteria) appear again ordered by decreasing significance weight, whereas the students are ranked following the 'Netflows' (default) ranking rule with student a06 first-ranked and student s13 last-ranked.

Normalized Relation

r(x S y)	s01	s02	s03	s04	s05	s06	s07	s08	s09	s10	s11	s12	s13
s01	-	0.38	0.00	0.62	0.56	-1.00	0.38	-0.50	-0.12	0.50	0.62	0.12	0.81
s02	-0.12	-	0.50	-0.12	-0.44	-0.75	0.00	-0.75	-0.12	0.25	1.00	0.25	0.81
s03	0.12	0.00	-	-0.25	-0.31	-0.75	-0.50	-0.50	-0.25	0.38	1.00	0.12	0.81
s04	0.25	0.50	0.38	-	-0.06	-0.62	0.00	-0.12	-0.38	0.50	1.00	0.38	0.81
s05	0.81	0.69	0.31	0.81	-	-0.56	0.69	-0.06	0.06	0.50	0.81	0.31	0.81
s06	1.00	0.75	1.00	1.00	0.56	-	0.88	0.88	1.00	0.75	1.00	1.00	0.81
s07	0.38	0.62	0.50	0.00	-0.06	-0.75	-	-0.25	0.38	0.38	0.88	0.38	0.69
s08	0.50	0.75	0.50	0.50	0.06	-0.38	0.88	-	0.25	0.75	1.00	0.50	0.81
s09	0.75	0.75	0.25	0.38	0.56	-0.50	0.62	-0.25	-	0.38	0.88	0.75	0.69
s10	-0.50	0.12	-0.38	0.00	-0.50	-0.75	-0.38	-0.75	-0.38	-	0.25	-0.12	0.75
s11	-0.62	-0.25	0.25	-1.00	-0.81	-1.00	-0.88	-1.00	-0.88	-0.25	-	-0.62	0.81
s12	0.38	-0.25	0.38	0.25	-0.06	-0.50	-0.25	-0.50	-0.25	0.12	1.00	-	0.81
s13	-0.81	-0.56	-0.44	-0.81	-0.81	-0.81	-0.69	-0.81	-0.69	-0.75	0.31	-0.81	-

Valuation domain: [-1.00; +1.00]

The best choice recommendation

```
>>> g.showBestChoiceRecommendation()
*****
Rubis choice recommendation(s)
(in decreasing order of determinateness)
Credibility domain: [-1.00,1.00]
=== >> potential best student(s)
* choice : ['s06']
independence : 1.00
dominance : 0.38
absorbency : -1.00
covering (%) : 100.00
determinateness (%) : 76.92
- most credible action(s) = { 's06': 0.56, }
# Condorcet winner
=== >> potential weakest students(s)
* choice : ['s11', 's13']
independence : 0.31
dominance : -0.81
absorbency : 0.44
covered (%) : 95.45
determinateness (%) : 81.49
- most credible action(s) = { 's13': 0.69, 's11': 0.31, }
```