

MICS 2 : Algorithmic Decision Theory

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Summer Semester 2020

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International ADT activities

- The International Conferences on *Algorithmic Decision Theory* : ADT'2009 (IT), ADT'2011 (US), ADT'2013 (BE), ADT'2015 (US), ADT'2017 (LU), ADT'2019 (US)
- The workshops DA2PL on *Multiple Criteria Decision Aid and Preference Learning* : 2012 (FR), 2014 (BE), 2016 (DE), 2018 (PL), and 2020 (IT)
- The GRAPHS&DECISIONS conference 2014 (LU)
- EURO working groups on *Multiple Criteria Decision Aid and Preference Handling*
- The DIMACS Special Focus on *Algorithmic Decision Theory*
- The International Workshops on *Computational Social Choice*
- *Smart Cities and Policy Analytics* Workshops
- The DECISION DECK project

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Content

1. Algorithmic Decision Theory
 - Activities and online resources
 - Types of decision problems
 - The Decision Aiding Process
2. Preference modelling with outranking situations
 - Selecting the evaluation model
 - Modelling preferential performance situations
 - Bipolar-valued outranking digraphs
3. Constructing decision recommendations
 - Selecting k -best or -worst choice
 - Constructing Rankings
 - k -Rating

Online ADT Documentation Resources

Tutorials and course materials on <http://www.algodec.org>.

44 contributions on **Algorithmic Decision Theory** contain videos and presentation materials originating from the tutorials and courses who took place at the meetings and doctoral schools organised by the COST Action IC0602 Algorithmic Decision Theory.

ALGORITHMIC DECISION THEORY

INTRODUCTION

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VIDEO & INTRODUCTION

Today's decision makers in fields ranging from engineering to psychology to medicine to economics to homeland security are faced with remarkable new technologies, huge amounts of information to help them in reaching good decisions, and the ability to share information at unprecedented speeds and quantities. These tools and resources should lead to better decisions. Yet, the tools bring with them daunting new problems: the massive amounts of data available are often incomplete or unreliable or distributed and there is great uncertainty in them; interdependent/distributed decision makers and decision-making devices need to be coordinated; many sources of data need to be fused into a good decision; information sharing under non-cooperation/competition arrangements; security problems. When faced with such issues, there are few highly efficient algorithms available to support decisions. This action's objective is to improve the ability of decision makers to perform in the face of these new challenges and problems through the use of methods of theoretical computer science, in particular algorithmic methods. The primary goal of the project is to explore and develop algorithmic approaches to decision problems arising in a variety of applications areas. Since many of the decision problems investigated arise in Artificial Intelligence, an important sub-goal is to explore the cross-fertilisation of Decision Theory and Artificial Intelligence.

Examples of such mutual benefits include, but are not limited to:

- Computational tractability/intractability of consensus functions.
- Improvement of decision support and recommender systems.
- Development of automatic decision devices including on-line decision procedures.
- Robust Decision Making.
- Learning for Multi-Agent Systems and other on-line decision devices.

This site contains materials originating from the tutorials and courses who took place at the meetings and doctoral schools organised by the COST Action IC0602 Algorithmic Decision Theory. It will be further updated as new materials will be produced by the COST Action.

Alexis Tsoukas

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Types of Decision Problems : Notation

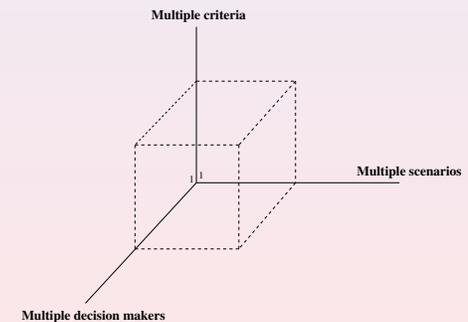
A decision problem will be a tuple $\mathcal{P} = (D, A, O, F, \Omega)$ where

1. D is a group of $d = 1, \dots$ **decision makers** ;
2. A is a set of $n = 2, \dots$ **decision alternatives** ;
3. O is a set of $o = 1, \dots$ **decision objectives** ;
4. F is a set of $m = 1, \dots$ **attributes** or **performance criteria** ; each one to be maximised or minimised with respect to a given decision objectives $obj \in O$;
5. Ω is a set of $\omega = 1, \dots, p$ potential states of the world or **context scenarios**.

Types of Decision Problems – continue

We may distinguish different types of decision problems along three directions :

- Single or multiple objectives/criteria,
- Single or multiple decision makers,
- Single or multiple context scenarios.



Decision aiding process

Timeline : →

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
	Decision Objects	Decision Result	Evaluation model	Tuning the parameters	
Actors	Objectives	Ranking	Value Functions	directly	Graph kernel extraction
Stakes	Alternatives	Choice	Performance Indicators	indirectly by learning	sorting algorithms
Resources	Performance Criteria	Rating	Preference modelling		quantiles estimation

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Formulating decision objectives and criteria

- Identifying the **strategic objectives** of the decision making problem,
- Identifying all **objective consequences** of the potential decision actions, measured on :
 - Discrete ordinal scales?
 - Numerical, discrete or continuous scales?
 - Interval or ratio scales?
- Each consequence, measured on a **performance criterion**, is associated with a strategic objective
 - to be **minimized** (Costs, environmental impact, energy consumption, etc);
 - to be **maximised** (Benefits, energy savings, security and reliability, etc).
- Verifying the coherence –**universal, minimal and separable**– of the family of criteria.

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Modelling the performance tableau

- Let X be a finite set of p decision alternatives.
- Let F be a finite set of n criteria (voters, experts, ...) supporting an increasing real performance scale from 0 to M_j ($j = 1, \dots, n$).
- Let $0 \leq \text{ind}_j < \text{pr}_j < v_j \leq M_j + \epsilon$ represent resp. the **indifference**, the **preference**, and the **considerable large performance difference** discrimination threshold observed on criterion j .
- Let w_j be the **significance** of criterion j .
- Let W be the sum of all criterion significances.
- Let x and y be two alternatives in X .
- Let x_j be the performance of x observed on criterion j

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Performing marginally “at least as good as”

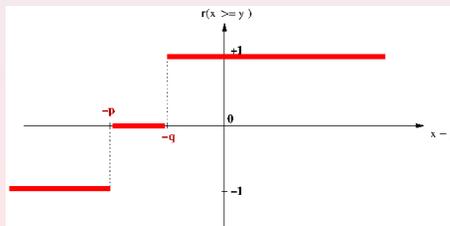
Each criterion j is characterizing a double threshold order \succeq_j on A in the following way :

$$r(x \succeq_j y) = \begin{cases} +1 & \text{if } x_j - y_j \geq -\text{ind}_j \\ -1 & \text{if } x_j - y_j \leq -\text{pr}_j \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

+1 signifies x is *performing at least as good as* y on criterion j ,

-1 signifies that x is *not performing at least as good as* y on criterion j .

0 signifies that it is *unclear* whether, on criterion j , x is performing at least as good as y .



Performing globally “at least as good as”

Each criterion j contributes the significance w_j of his “at least as good as” characterisation $r(\succeq_j)$ to the characterisation of a global “at least as good as” relation $r(\succeq)$ in the following way :

$$r(x \succeq y) = \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \succeq_j y) \right] \quad (2)$$

$1.0 \geq r(x \succeq y) > 0.0$ signifies x is *globally performing at least as good as* y ,

$-1.0 \leq r(x \succeq y) < 0.0$ signifies that x is *not globally performing at least as good as* y ,

$r(x \succeq y) = 0.0$ signifies that it is *unclear* whether x is globally performing at least as good as y .

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Performing marginally and globally “less than”

Each criterion j is characterising a double threshold order \prec_j (less than) on A in the following way :

$$r(x \prec_j y) = \begin{cases} +1 & \text{if } x_j + pr_j \leq y_j \\ -1 & \text{if } x_j + ind_j \geq y_j \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

And, the *global less than* relation (\prec) is defined as follows :

$$r(x \prec y) = \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \prec_j y) \right] \quad (4)$$

Property (Coduality principle)

The global “less than” relation \prec is the *dual* ($\not\prec$) of the global “at least as good as” relation \succeq .

X : Finite set of n alternatives

$x \succsim y$: Alternative x **outranks** alternative y if

1. there is a (weighted) **majority** of criteria (voters, experts, ...) supporting that x **performs at least as good as** y , and
2. **no considerable negative** performance difference between x and y is observed on a discordant criterion.

$x \not\succeq y$: Alternative x **does not outrank** alternative y if

1. there is a (weighted) majority of criteria (voters, experts, ...) supporting that x **does not perform at least as good as** y , and
2. **no considerable positive** performance difference between x and y is observed on a discordant criterion.

$r(x \succsim y)$ represents a bipolar, i.e. *concordance versus discordance*, valuation in $[-1, 1]$ that characterises the **epistemic truth** of affirmative assertion $x \succsim y$.

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Epistemic truth semantics of the r -valuation

Let $x \succsim y$ and $x' \succsim y'$ be two preferential assertions :

$r(x \succsim y) = +1$ means that assertion $x \succsim y$ is **certainly valid**,

$r(x \succsim y) = -1$ means that assertion $x \succsim y$ is **certainly invalid**,

$r(x \succsim y) > 0$ means that assertion $x \succsim y$ is more **valid** than invalid,

$r(x \succsim y) < 0$ means that assertion $x \succsim y$ is more **invalid** than valid,

$r(x \succsim y) = 0$ means that

validity of assertion $x \succsim y$ is **indeterminate**,

$r(x \succsim y) > r(x' \succsim y')$ means that

assertion $x \succsim y$ is **more valid** than assertion $x' \succsim y'$,

$r(x \not\succeq y) = -r(x \succsim y)$

logical (strong) negation by **changing sign**,

$r(x \succsim y \vee x' \succsim y') = \max(r(x \succsim y), r(x' \succsim y'))$

logical disjunction via the *max* operator,

$r(x \succsim y \wedge x' \succsim y') = \min(r(x \succsim y), r(x' \succsim y'))$

logical conjunction via the *min* operator.

Coherence of the bipolar-valued outranking concept

Properties :

1. The bipolar outranking relation \succsim is trivially **reflexive**,
2. The bipolar outranking relation \succsim is **weakly complete**, i.e. $r(x \succsim y) < 0$ implies $r(y \succsim x) \geq 0$.
3. The dual ($\not\succeq$) of the bipolar outranking relation \succsim is identical to the strict converse outranking $\not\succeq$ relation.

However, other properties, like being *acyclic* or even *transitive* are usually are not fulfilled.

Bipolar-valued outranking digraphs

Definition

- We denote $\tilde{G}(X, r(\succsim))$ the **bipolar-valued** digraph modelled by $r(\succsim)$ on the set X of potential decision alternatives.
 $\tilde{G}(X, \succsim)$ actually **minimizes the sum of the Kendall distances** with all marginal –single criterion based– outranking digraphs.
- The average absolute value of the r -valuation is called the **epistemic determination** of $\tilde{G}(X, r(\succsim))$.
- We denote $G(X, \succsim)$ the associated **Condorcet or median cut digraph**, i.e. the crisp digraph associated with \tilde{G} where we retain all arcs such that $r(x \succsim y) > 0$.
- $G(X, \succsim)$ has usually, except from being trivially reflexive, no other relational properties.

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Selecting k -best or -worst choice

Timeline : \longrightarrow

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
	Decision Objects	Decision Result	Evaluation model	Tuning the parameters	
Actors	Objectives	Ranking	Value Functions	directly	Kernel extraction
Stakes	Alternatives	Choice	Performance Indicators	indirectly by learning	Sorting algorithms
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The Best Choice Problematique

- A choice problem traditionally consists in the search for a **single best** alternative;
- Pragmatic Best Choice Recommendation - **BCR** - principles :
 - P_1 : Non retainement for well motivated reasons ;
 - P_2 : Recommendation of minimal size ;
 - P_3 : Stable (irreducible) recommendation ;
 - P_4 : Effectively best choice ;
 - P_5 : Recommendation maximally supported by the given preferential information.
- The decision aiding process **progressively** uncovers the best single choice via more and more refined choice recommendations ;
- The process stops when the decision maker is ready to make her final decision.

References : Roy & Bouyssou (1993), Bisdorff, Roubens & Meyer (2008).

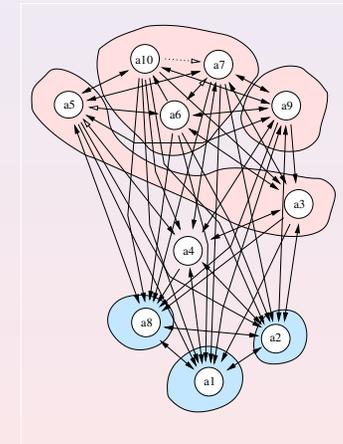
Useful choice qualifications

Let Y be a non-empty subset of X , called a **choice**.

- Y is said to be **outranking** (resp. **outranked**) when $x \notin X \Rightarrow \exists y \in Y : r(y \succsim x) > 0$ (resp. $r(x \succsim y)$).
- Y is said to be **independent** (resp. **weakly independent**) when for all $x \neq y$ in Y we have $r(x \succsim y) < 0$ (resp. $r(x \succsim y) \leq 0$).
- Y is called an **outranking kernel** (resp. **prekernel**) when it is an *outranking* and *independent* (resp. *weakly independent*) choice.
- Y is called an **outranked kernel** (resp. **prekernel**) when it is an *outranked* and *independent* (resp. *weakly independent*) choice.

Translating BCR principles into choice qualifications

- P_1 : Non-retainment for well motivated reasons.
A BCR is an **outranking choice**.
- P_{2+3} : Minimal size & stable.
A BCR is a **prekernel**.
- P_4 : Effectivity.
A BCR is a **strictly more outranking than outranked choice**.
- P_5 : Maximal epistemic support.
A BCR has **maximal determinateness**.



Property (BCR Decisiveness)

Any bipolar strict outranking digraph without chordless odd circuit contains at least one outranking and one outranked prekernel.

Constructing Rankings

Timeline : \longrightarrow

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
	Decision Objects	Decision Result	Evaluation Model	Tuning the parameters	
Actors	Objectives	Ranking	Value Functions	directly	Kernel extraction
Stakes	Alternatives	Choice Rating	Performance Indicators	indirectly by learning	Sorting algorithms
Resources	Performance Criteria	Clustering	Preference modelling		Quantiles estimation

The Ranking Problem

- A ranking problem traditionally consists in the search for a **linear ordering** of the set of alternatives ;
- A particular ranking is computed with the help of a **ranking rule** which aggregates preferences over all decision makers and/or criteria into a global (weak) order based, either on (rank) scoring (**Borda**), or, on (pairwise) voting procedures (**Kemeny, Slater, Copeland, Kohler, Ranked Pairs**) ;
- Characteristic properties of ranking rules :
 - A ranking rule is called **Condorcet-consistent** when the following holds :
If the majority relation is a linear order, then this linear order is the unique solution of the ranking rule ;
 - A ranking rule is called **B-ordinal** if its result only depends on the order of the majority margins B ;
 - A ranking rule is called **M-invariant** if its result only depends on the majority relation M .

Reference : Cl. Lamboray (2007,2009,2010)

A classification of ranking rules by Cl. Lamboray

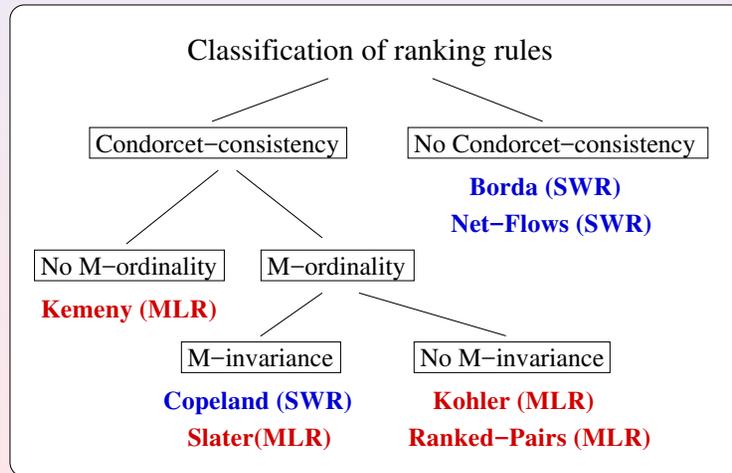


FIGURE – Legend : SWR : single weak ranking, MLR : multiple linear rankings

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What ranking rule should one use ?

1. Kemeny's and Slater's ranking-by-scoring rules, besides potentially delivering multiple weak rankings, are furthermore **computationally difficult** problems and exact ranking results are only computable for tiny outranking digraphs (order < 20).
2. Similarly, the ranking-by-choosing and their dual, the ordering-by-choosing rules, are unfortunately not scalable to outranking digraphs of larger orders (> 100).
3. Only Copeland's and the NetFlows ranking rules, with a polynomial complexity $\mathcal{O}(n^2)$, where n is the order of the outranking digraph, remain scalable for outranking digraphs with several hundred or thousand decision alternatives.

See the Digraph3 tutorial on *Ranking with multiple incommensurable criteria* (<https://digraph3.readthedocs.io/en/latest/index.html>).

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k-Rating

Timeline : →

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
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Stakes	Alternatives	Choice	Performance Indicators	indirectly by learning	Sorting algorithms
Resources	Performance Criteria	Rating	Preference modelling		Quantiles estimation

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The k-Rating Problem

- A rating problem consists in a **supervised partitioning** of the set of alternatives into $k = 2, \dots$ **ordered categories**.
- Usually, a rating procedure is designed to deal with an **absolute evaluation model**, whereas choice and ranking algorithms essentially rely on relative evaluation models.
- A crucial problem, hence, lies in the definition of the given categories, i.e., of the **evaluation norms** that define each sort category.
- Two type of such norms are usually provided :
 - **Relative quantiles** estimated from the given performance tableau ;
 - **Absolute quantile norms** learned from historical performance records.

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q -tiles sorting with bipolar outrankings

Property

In a **multiple criteria outranking** approach, the bipolar-valued characteristic of x belonging to upper-closed q -tiles class q^k (resp. lower-closed class q_k) may be assessed as follows :

$$r(x \in q^k) = \min [-r(q(p_{k-1}) \succsim x), r(q(p_k) \succsim x)]$$

$$r(x \in q_k) = \min [r(x \succsim q(p_{k-1})), -r(x \succsim q(p_k))]$$

The bipolar outranking relation \succsim , being weakly complete, verifies the **coduality principle** (Bisdorff 2013). Hence :

$$-r(q(p_{k-1}) \succsim x) = r(q(p_{k-1}) \succ x) = r(q(p_{k-1}) \prec x),$$

$$-r(x \succsim q(p_k)) = (x \succ q(p_k)) = r(x \prec q(p_k)).$$

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Algorithmic Decision Theory Software Resources

- The **DIGRAPH3** Linux & MacOS software collection provides practical tools for practical *Algorithmic Decision Theory* Applications.
- **Download** options :
 1. By using a github clone :
...\$ git clone https ://github.com/rbisdorff/Digraph3
 2. Or a sourceforge clone :
...\$ git clone https ://git.code.sf.net/p/digraph3/code Digraph3
- **Tutorials** and Reference Manual :
<https://digraph3.readthedocs.io/en/latest/>

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Properties of q -tiles sorting result

1. **Coherence** : Each object is always sorted into a non-empty subset of adjacent q -tiles classes.
2. **Uniqueness** : If the q -tiles classes represent a discriminated partition of the measurement scales on each criterion and $r \neq 0$, then every object is sorted into exactly one q -tiles class.
3. **Independence** : The sorting result for object x , is independent of the other object's sorting results.

Comment

The independence property gives us access to efficient **parallel processing** of class membership characteristics $r(x \in q^k)$ for all $x \in X$ and q^k in Q .

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