MICS 2 : Algorithmic Decision Theory

Raymond Bisdorff

Emeritus Professor of
Applied Mathematics and Computer Science
University of Luxembourg
http://lepold-loewenheim.uni.lu/bisdorff/

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Content

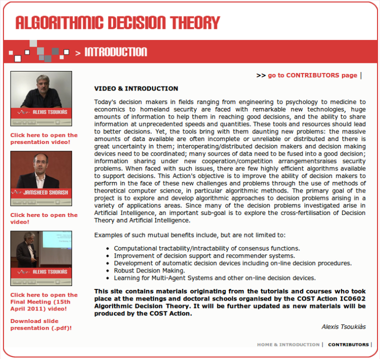
International ADT activities

- The International Conferences on *Algorithmic Decision Theory* : ADT'2009 (IT), ADT'2011 (US), ADT'2013 (BE), ADT'2015 (US), ADT'**2017** (LU), ADT'2019 (US)
- The workshops DA2PL on *Multiple Criteria Decision Aid and Preference Learning* : 2012 (FR), 2014 (BE), 2016 (DE), 2018 (PL), and 2020 (IT)
- The GRAPHS&DECISIONS conference 2014 (LU)
- EURO working groups on *Multiple Criteria Decision Aid* and on *Preference Handling*
- The DIMACS Special Focus on *Algorithmic Decision Theory*
- The International Workshops on *Computational Social Choice*
- *Smart Cities and Policy Ananlytics* Workshops
- The DECISION DECK project

Online ADT Documentation Resources

Tutorials and course materials on `http://www.algodec.org`.

44 contributions on Algorithmic Decision Theory contain videos and presentation materials originating from the tutorials and courses who took place at the meetings and doctoral schools organised by the COST Action IC0602 Algorithmic Decision Theory.



Types of Decision Problems : Notation

A decision problem will be a tuple $\mathcal{P} = (D, A, O, F, \Omega)$ where

1. D is a group of $d = 1, \ldots$ decision makers :
2. A is a set of $n = 2, \ldots$ decision alternatives ;
3. O is a set of $o = 1, \ldots$ decision objectives ;
4. F is a set of $m = 1, \ldots$ attributes or performance criteria ; each one to be maximised or minimised with respect to a given decision objectives $obj \in O$;
5. Ω is a set of $\omega = 1, \ldots, p$ potential states of the world or context scenarios.

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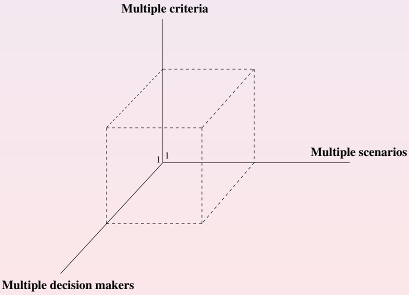
Types of Decision Problems – continue

We may distinguish different types of decision problems along three directions :

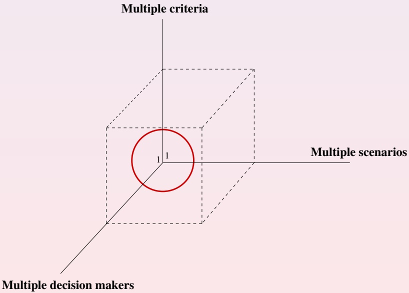
- Single or multiple objectives/criteria,
- Single or multiple decision makers,
- Single or multiple context scenarios.



Types of Decision Problems – continue

We may distinguish different types of decision problems along three directions :

Standard single criterion optimization problems, $\mathcal{P} = (1, n, 1, 1, 1)$.
Issues : Evaluation model, imprecision, incommensurability, n big.



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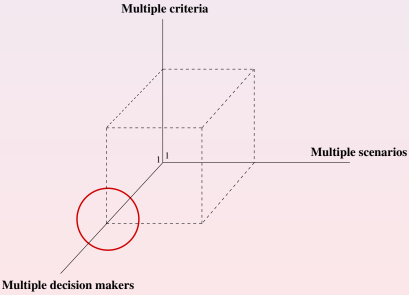
We may distinguish different types of decision problems along three directions :

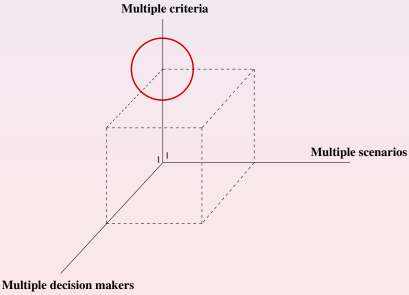
Social choice and consensus problems, $\mathcal{P} = (d, n, 1, 1, 1)$.
Issues : Voting, privacy and security, abstention, manipulability, auctions, fair division, d big.



Types of Decision Problems – continue

We may distinguish different types of decision problems along three directions :

Compromise decision problems, $\mathcal{P} = (1, n, o, m, 1)$.
Issues : Business analytics, evaluation models, objectives' importance, criteria significance, preference aggregation, incommensurability, imprecision, missing data.

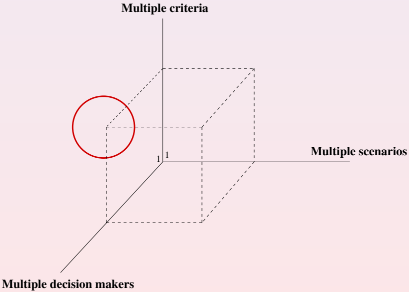


Types of Decision Problems – continue

We may distinguish different types of decision problems along three directions :

Social or group compromise decision problems, $\mathcal{P} = (d, n, o, m, 1)$.

Issues : Policy analytics, conflicting value systems, mediation.



Decision aiding process

Timeline : $\longrightarrow$

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
	Decision Objects	Decision Result	Evaluation model	Tuning the parameters	
Actors	Objectives	Ranking	Value Functions	directly	Graph kernel extraction
Stakes	Alternatives	Choice	Performance Indicators		sorting algorithms
Resources	Performance Criteria	Rating	Preference modelling	indirectly by learning	quantiles estimation

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Formulating decision objectives and criteria

- Identifying the strategic objectives of the decision making problem,
- Identifying all objective consequences of the potential decision actions, measured on :
  - Discrete ordinal scales ?
  - Numerical, discrete or continuous scales ?
  - Interval or ratio scales ?
- Each consequence, measured on a performance criterion, is associated with a strategic objective
  - to be minimized (Costs, environmental impact, energy consumption, etc) ;
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- Verifying the coherence –universal, minimal and separable– of the family of criteria.

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Modelling the performance tableau

- Let X be a finite set of p decision alternatives.
- Let F be a finite set of n criteria (voters, experts, ...) supporting an increasing real performance scale from 0 to M_j ($j = 1, ...n$).
- Let $0 \leqslant ind_j < pr_j < v_j \leqslant M_j + \epsilon$ represent resp. the indifference, the preference, and the considerable large performance difference discrimination threshold observed on criterion j.
- Let w_j be the significance of criterion j.
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Performing marginally "*at least as good as*"

Each criterion j is characterizing a double threshold order $\succsim_j$ on A in the following way :

$$r(x \succsim_j y) = \begin{cases} +1 & \text{if} \quad x_j - y_j \geqslant -ind_j \\ -1 & \text{if} \quad x_j - y_j \leqslant -pr_j \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

- $+1$ signifies x is *performing at least as good as* y on criterion j,
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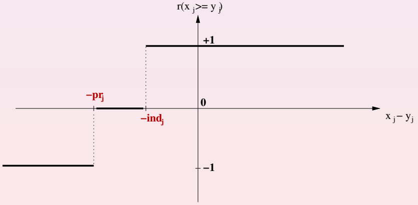

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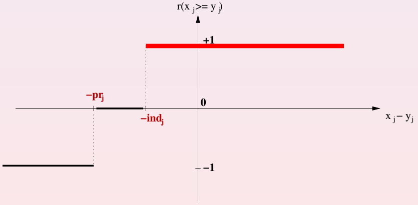

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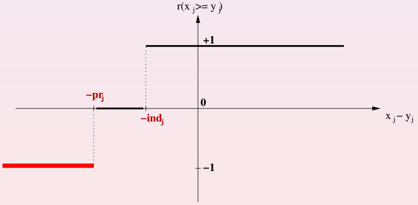

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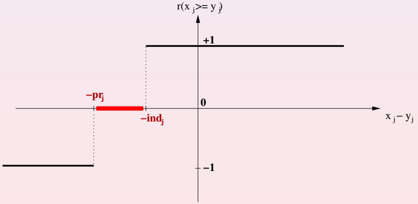

Performing globally "*at least as good as*"

Each criterion j contributes the significance w_j of his "*at least as good as*" characterisation $r(\succsim_j)$ to the characterisation of a global "*at least as good as*" relation $r(\succsim)$ in the following way :

$$\boxed{r(x \succsim y) \; = \; \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \succsim_j y) \right]} \qquad (2)$$

$1.0 \geqslant r(x \succsim y) > 0.0$ signifies x is *globally performing at least as good as* y,

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Performing marginally and globally "*less than* "

Each criterion j is characterising a double threshold order $\prec_j$ (*less than*) on A in the following way :

$$r(x \prec_j y) \quad = \quad \begin{cases} +1 & \text{if} \quad x_j + pr_j \leqslant y_i \\ -1 & \text{if} \quad x_j + ind_j \geqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

And, the *global less than* relation ($\prec$) is defined as follows :

$$\boxed{r(x \prec y) = \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \prec_j y) \right]} \tag{4}$$

Property (Coduality principle)

The global "less than" relation $\prec$ is the dual ($\not\succcurlyeq$) of the global "at least as good as" relation $\succcurlyeq$.

Performing marginally and globally "*less than* "

Each criterion j is characterising a double threshold order $\prec_j$ (*less than*) on A in the following way :

$$r(x \prec_j y) \quad = \quad \begin{cases} +1 & \text{if} \quad x_j + pr_j \leqslant y_i \\ -1 & \text{if} \quad x_j + ind_j \geqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

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Modelling outranking situations

X : Finite set of n alternatives

$x \succsim y$: Alternative x outranks alternative y if

1. there is a (weighted) majority of criteria (voters, experts, ...) supporting that x performs at least as good as y, and
2. no considerable negative performance difference between x and y is observed on a discordant criterion.

$x \not\succsim y$: Alternative x does not outrank alternative y if

1. there is a (weighted) majority of criteria (voters, experts, ...) supporting that x does not perform at least as good as y, and
2. no considerable positive performance difference between x and y is observed on a discordant crterion.

$r(x \succsim y)$ represents a bipolar, i.e. *concordance versus discordance*, valuation in $[-1, 1]$ that characterises the epistemic truth of affirmative assertion $x \succsim y$.

Epistemic truth semantics of the r-valuation

Let $x \succsim y$ and $x' \succsim y'$ be two preferential assertions :

$r(x \succsim y) = +1$ means that assertion $x \succsim y$ is certainly valid,

$r(x \succsim y) = -1$ means that assertion $x \succsim y$ is certainly invalid,

$r(x \succsim y) > 0$ means that assertion $x \succsim y$ is more valid than invalid,

$r(x \succsim y) < 0$ means that assertion $x \succsim y$ is more invalid than valid,

$r(x \succsim y) = 0$ means that
validity of assertion $x \succsim y$ is indeterminate,

$r(x \succsim y) > r(x' \succsim y')$ means that
assertion $x \succsim y$ is more valid than assertion $x' \succsim y'$,

$r(x \not\succsim y) = -\, r(x \succsim y)$
logical (strong) negation by changing sign,

$r(x \succsim y \vee x' \succsim y') = \max(r(x \succsim y), r(x' \succsim y'))$
logical disjunction via the *max* operator,

$r(x \succsim y \wedge x' \succsim y') = \min(r(x \succsim y), r(x' \succsim y'))$
logical conjunction via the *min* operator.

Coherence of the bipolar-valued outranking concept

Properties :

1. The bipolar outranking relation $\succsim$ is trivially reflexive,
2. The bipolar outranking relation $\succsim$ is weakly complete, ie $r(x \succsim y) < 0$ implies $r(y \succsim x) \geq 0$.
3. The dual ($\not\succsim$) of the bipolar outranking relation $\succsim$ is identical to the strict converse outranking $\precnsim$ relation.

However, other properties, like being *acyclic* or even *transitive* are usually are not fulfilled.

Bipolar-valued outranking digraphs

Definition

- We denote $\widetilde{G}(X, r(\succsim))$ the bipolar-valued digraph modelled by $r(\succsim)$ on the set X of potential decision alternatives.
  $\widetilde{G}(X, \succsim)$ actually minimizes the sum of the Kendall distances with all marginal –single criterion based– outranking digraphs.
- The average absolute value of the r-valuation is called the epistemic determination of $\widetilde{G}(X, r(\succsim))$.
- We denote $G(X, \succsim)$ the associated Condorcet or median cut digraph, i.e. the crisp digraph associated with $\widetilde{G}$ where we retain all arcs such that $r(x \succsim y) > 0$.
- $G(X, \succsim)$ has usually, except from being trivially reflexive, no other relational properties.

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Selecting *k*-best or -worst choice

Timeline : $\longrightarrow$

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
	Decision Objects	Decision Result	Evaluation model	Tuning the parameters	
Actors	Objectives	Ranking	Value Functions	directly	Kernel extraction
		Choice			
Stakes	Alternatives	Rating	Performance Indicators		Sorting algorithms
Resources	Performance Criteria	Clustering	Preference modelling	indirectly by learning	Quantiles estimation

The Best Choice Problematique

- A choice problem traditionally consists in the search for a single best alternative ;
- Pragmatic Best Choice Recommendation - BCR - principles :

  P_1 : Non retainement for well motivated reasons ;
  P_2 : Recommendation of minimal size ;
  P_3 : Stable (irreducible) recommendation ;
  P_4 : Effectively best choice ;
  P_5 : Recommendation maximally supported by the given preferential information.

- The decision aiding process progressively uncovers the best single choice via more and more refined choice recommendations ;
- The process stops when the decision maker is ready to make her final decision.

References : Roy & Bouyssou (1993), Bisdorff, Roubens & Meyer (2008).

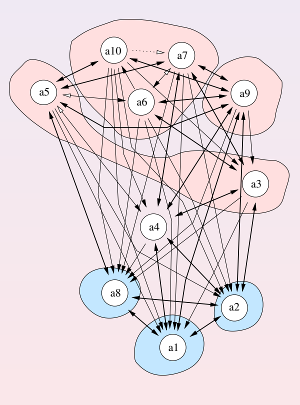
Useful choice qualifications

Let Y be a non-empty subset of X, called a choice.

- Y is said to be outranking (resp. outranked) when $x \notin X \Rightarrow \exists y \in Y : r(y \succsim x) > 0 \quad \big(resp. r(x \succsim y) \big)$.
- Y is said to be independent (resp. weakly independent) when for all $x \neq y$ in Y we have $r(x \succsim y) < 0)$ (resp. $r(x \succsim y) \leqslant 0)$).
- Y is called an outranking kernel (resp. prekernel) when it is an *outranking* and *indendent* (resp. *weakly independent*) choice.
- Y is called an outranked kernel (resp. prekernel) when it is an *outranked* and *indendent* (resp. *weakly independent*) choice.

Translating BCR principles into choice qualifications

- P_1 : Non-retainment for well motivated reasons.
  A BCR is an outranking choice.
- P_{2+3} : Minimal size & stable.
  A BCR is a prekernel.
- P_4 : Effectivity.
  A BCR is a stricly more outranking than outranked choice.
- P_5 : Maximal epistemic support.
  A BCR has maximal determinateness.



Property (BCR Decisiveness)

Any bipolar strict outranking digraph without chordless odd circuit contains at least one outranking and one outranked prekernel.

Constructing Rankings

Timeline : $\longrightarrow$

Situating the problem	Formulating the problem		Selecting the evaluation model		Constructing recommendations
	Decision Objects	Decision Result	Evaluation Model	Tuning the parameters	
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		Rating		indirectly by learning	
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The Ranking Problem

- A ranking problem traditionally consists in the search for a linear ordering of the set of alternatives ;
- A particular ranking is computed with the help of a ranking rule which aggregates preferences over all decision makers and/or criteria into a global (weak) order based, either on (rank) scoring (Borda), or, on (pairwise) voting procedures (Kemeny, Slater, Copeland, Kohler, Ranked Pairs) ;

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- Characteristic properties of ranking rules :
  1. A ranking rule is called Condorcet-consistent when the following holds : If the majority relation is a linear order, then this linear order is the unique solution of the ranking rule ;
  2. A ranking rule is called B-ordinal if its result only depends on the order of the majority margins B ;
  3. A ranking rule is called M-invariant if its result only depends on the majority relation M.

Reference : Cl. Lamboray (2007,2009,2010)

A classification of ranking rules by Cl. Lamboray

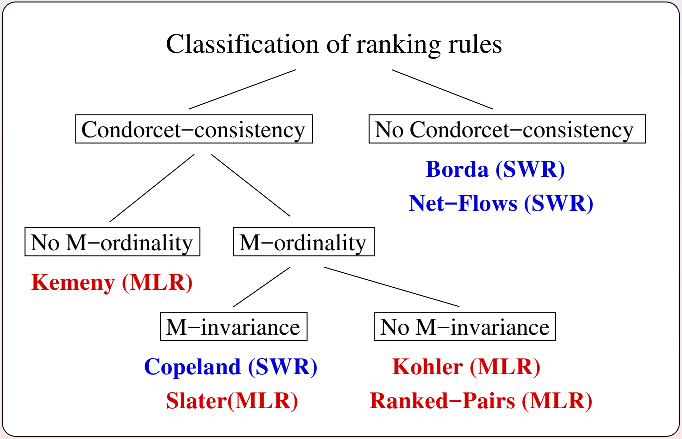

FIGURE – Legend : *SWR* : single weak ranking, *MLR* : multiple linear rankings

What ranking rule should one use ?

1. Kemeny's and Slater's ranking-by-scoring rules, besides potentially delivering multiple weak rankings, are furthermore computationally difficult problems and exact ranking results are only computable for tiny outranking digraphs (order < 20).
2. Similarly, the ranking-by-choosing and their dual, the ordering-by-choosing rules, are unfortunately not scalable to outranking digraphs of larger orders (> 100).
3. Only Copeland's and the NetFlows ranking rules, with a polynomial complexity $\mathcal{O}(n^2)$, where n is the order of the outranking digraph, remain scalable for outranking digraphs with several hundred or thousand decision alternatives.

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k-Rating

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The k-Rating Problem

- A rating problem consists in a supervised partitioning of the set of alternatives into $k = 2, \ldots.$ ordered categories.
- Usually, a rating procedure is designed to deal with an absolute evaluation model, whereas choice and ranking algorithms essentially rely on relative evaluation models.
- A crucial problem, hence, lies in the definition of the given categories, i.e., of the *evaluation norms* that define each sort category.
- Two type of such norms are usually provided :
  - Relative quantiles estimated from the given performance tableau ;
  - Absolute quantile norms learned from historical performance records.

q-tiles sorting with bipolar outrankings

Property

In a multiple criteria outranking approach, the bipolar-valued characteristic of x belonging to upper-closed q-tiles class q^k (resp. lower-closed class q_k) may be assessed as follows :

$$r(x \in q^k) = \min\Big[\, -r\big(\mathrm{q}(p_{k-1}) \succsim x\big),\; r\big(\mathrm{q}(p_k) \succsim x\big)\Big]$$

$$r(x \in q_k) = \min\Big[\, r\big(x \succsim \mathrm{q}(p_{k-1})\big),\; -r\big(x \succsim \mathrm{q}(p_k)\big)\Big]$$

The bipolar outranking relation $\succsim$, being weakly complete, verifies the coduality principle (Bisdorff 2013). Hence :

$$-r\big(\mathrm{q}(p_{k-1}) \succsim x\big) \;=\; r\big(\mathrm{q}(p_{k-1}) \not\succsim x\big) \;=\; r\big(\mathrm{q}(p_{k-1}) \prec x\big),$$

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Properties of q-tiles sorting result

1. *Coherence* : Each object is always sorted into a non-empty subset of adjacent q-tiles classes.
2. *Uniqueness* : If the q-tiles classes represent a discriminated partition of the measurement scales on each criterion and $r \neq 0$, then every object is sorted into exactly one q-tiles class.
3. *Independence* : The sorting result for object x, is independent of the other object's sorting results.

Comment

The independence property gives us access to efficient parallel processing of class membership characteristics $r(x \in q^k)$ for all $x \in X$ and q^k in $\mathcal{Q}$.

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Algorithic Decision Theory Software Resources

- The DIGRAPH3 Linux & MacOS software collection provides practical tools for practical *Algorithmic Decision Theory* Applications.
- Download options :
  1. By using a github clone :
     ...$ git clone https ://github.com/rbisdorff/Digraph3
  2. Or a sourceforge clone :
     ...$ git clone https ://git.code.sf.net/p/digraph3/code Digraph3
- Tutorials and Reference Manual :
  https://digraph3.readthedocs.io/en/latest/

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(See https://leopold-loewenheim.uni.lu/bisdorff/publications.html)

Thank you for your attention