Algorithmic Decision Theory

Lecture 4: Evaluation models
Measure and aggregate performances

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FSTC - CSC/ILIAS

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Content

What is a grade ?

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A grade is an evaluation of the performance of a student in a given course ; an indication to which level a student fulfils the objectives of the course.

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- *A grade should always be interpreted with respect to the objectives of the course.*
- *A grade may have several pedagogical functions such as certifying a certain performance level or being a hint indicating the student's strengths and weaknesses.*
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On grading

Grading students copies relies on a number of conventions like :

- Grading scale : 0-20 (France, Belgium & Luxembourg), 0-30 (Italy), 6-1 (Germany), 0-100 (USA), $\{F, E, D, C, B, A\}$ (USA & Asia),
- The model solution giving the repartition of points per question,
- The weight of different exams in the final grade,
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- In mathematics, a difference in grades of 2 points on a $0 - 20$ scale may be commonly observed for similar copies. Motivated grading differences of up to 9 points do occur.
- In 50% of the cases, a second grading by the same corrector leads to a significantly different result than the first one.
- The grades show a high auto-correlation with the apparent level of the student : similar copies from presumably good and presumably weak students commonly obtain dissimilar grades in favour of the good ones.

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Empirical properties of the grading – continue

- The order of the copies has an incidence on the grading result. The spread of the grades given by the same corrector commonly augments with time for instance.
- There appear anchorage phenomenas : It is always better to be graded after a weak copy than after an excellent one.
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Interpreting grades

- In Europe, grades give generally the impression that they are numerical measures.
- Yet, there is a problem with the minimum grade 0. It does not signify that a student does know nothing !
- There is also a problem with the maximum grade 20. Two excellent students getting 20/20 are not necessarily equivalent !
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- If a grading scale is supposed to be of ratio type, all grading differences must in theory be commensurable.
- Yet, very high and low grades for instance do not verify in practice this hypothesis.
- The same is also commonly the case when there exists a validating threshold grade (10/20 for instance). Grading differences, even small, around such a threshold level become consequently more significant : the difference between 10 and 11 is not the same as the one between 18 and 19 for instance.
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Interpreting grades – continue

- The preceding problems give arguments to the promoters of Anglo-Saxon alphabetical – i.e. ordinal – grades : generally E or F to A (best grade).
- As a consequence, a large majority of students are often given a neutral grade like B or C.
- In order to better discriminate the effective performances, one introduces then qualitative decorations like $+$ and $-$: $B+$ signifying a grade slightly inferior to A, $B-$ a grade slightly better than C.
- It is worthwhile noticing that all these ordinal grades are translating a certain range of number of points or percentages obtained in fact in the underlying exams !
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Rules for aggregating grades

- In order to validate a programme or a degree, it is common usage to aggregate grades obtained in the same and even in different courses.
- Three principles for aggregating are generally used :
  - Conjunctive aggregation
  - Weighted mean
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- The students must simply validate all their exams in a given time in order to get their degree.
- Advantage : No commensurability hypothesis concerning the individual grades is required.
- Disadvantages :
  - Many students risk to eventually fail their degree.
  - There are only two types of results : valid and invalid.
  - No formative results may be expressed : slightly insufficient for example in order to not discourage and positively stimulate a student to enhance his performance for instance.
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- To validate a study programme or degree, this weighted average grade is then compared to standard values like 10/20, or 14/20, 16/20 etc. to attribute a distinction.
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- We suppose that all grades are expressed on a $0 - 20$ scale.
- We denote $g_i(a)$ the grade obtained by a student a in the course i ($i = 1$ to n).
- We denote w_i the (strictly positive) weight allocated to course i in the evaluation of the final grade.
- The final grade $g(a)$ of student a is computed as follows :

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$$w_i' = \frac{w_i}{\sum_{i=1}^{n} w_i}$$

- *Normalised weights w_i' – rational numbers – are thus confined between 0 and 1 and $\sum_{i=1}^{n} w_i' = 1$.*
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Methodological problems

Example (1. An undesirable effect of the compensation)

Consider four students $\{a, b, c, d\}$ enrolled in a study programme consisting of two courses $\{g_1, g_2\}$ of same weight and where they have obtained the following grades :

	g_1	g_2
a	11	11
b	5	19
c	20	4
d	4	6

Student a shows satisfactory results in both courses, whereas student d shows very weak results. On the contrary, b and c are both excellent students in one course and weak in the other. Globally, a should be ranked before b and c, and both ranked again before d.

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Aggregating the four students grades with a weighted average results in following figures :

	g
b	12
c	12
a	11
d	5

Students b and c are ranked before student a. One may even verify that no other weighting of the two courses will allow to rank a before b and c ! Use a weighted average is in fact incompatible with the idea of promoting those students that do reasonably good in all courses.

Example (1) – continue

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Aggregating the four students grades with a weighted average results in following figures :

	g
b	12
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Students b and c are ranked before student a. One may even verify that no other weighting of the two courses will allow to rank a before b and c ! Use a weighted average is in fact incompatible with the idea of promoting those students that do reasonably good in all courses.

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Exercise(s) (1. An undesirable effect of the compensation)

Show that, when aggregating with a weighted average the grades above, there does not exist any possible weighting of both courses such that a is ranked before b and c

	g_1	g_2
a	11	11
b	5	19
c	20	4

- $g(a) > g(b)$ iff $11w_1 + 11w_2 > 5w_1 + 19w_2$ iff $w_1 > 4/3w_2$.
- $g(a) > g(c)$ iff $11w_1 + 11w_2 > 20w_1 + 4w_2$ iff $w_1 < 7/9w_2$.
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Methodological problems – continue

Comment

Practical consequences of unlimited compensation :

- *Using a weighted average as rule for aggregating grades may turn students towards concentrating their efforts on a limited number of courses only by relying on the compensation mechanism for getting a sufficient final grade.*
- *Requiring minimal threshold grades may limit, but not completely inhibit, this undesirable effect.*

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Methodological problems – continue

Example (2. Interactions between performances to aggregate ?)

Consider four students $\{a, b, c, d\}$ enrolled in a programme consisting in statistics (S), mathematics (M) and economics (E). They got the following grades :

	g_S	g_M	g_E
a	18	12	6
b	18	7	11
c	5	17	8
d	5	12	13

Student a should be ranked before student b in an engineering study programme. b is, even more, weak in maths, which is convenient neither for an engineering nor an economics degree. With a similar reasoning, d is much better than c when considering an economics degree.

Methodological problems – continue

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Methodological problems – continue

Comment

Interactions between performances :

- *Whereas the preceding rankings seam quite reasonable, they are however not compatible with the weighted average rule.*
- *When the statistics results are excellent, the weight of mathematics outranks the one of economics (a outranks b).*
- *However, showing weak grades in statistics leads to consider that the weight of economics outranks the one of mathematics (d outranks c)*
- *These interactions between course subjects, despite the fact of being quite common in practice, are not compatible with the weighted average rule.*

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Methodological problems – continue

Example (3. Incommensurable differences between grades ?)

Consider two students enrolled in a programme with two courses of same weight. The grading is done on a $0-20$ scale and a final grade of at least 10 is required in order to validate the programme.

	g_1	g_2
a	11	10
b	12	9

Both students obtain the same average grade 10.5 and validate equivalently the programme. The difference between 12 and 11 in the first course exactly compensates the difference between 10 and 9 shown in the second course.

Methodological problems – continue

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Incommensurable differences between grades :

- *As 10 is the threshold for validating the programme, one may suppose that the difference observed in the first course is more important than that observed in the second one.*
- *Consequently, student a must in fact have better validated the programme than student b ?*
- *Indeed, a was conjointly successful in both courses, whereas b failed one of the two courses.*
- *With the weighted average rule, a difference of one point is required to have uniformly the same signification all along the scale.*

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Methodological problems – continue

Example (4. Incommensurable differences between grades ?)

Reconsider the three students enrolled in the same programme as in Example (3) :

Comment

	g_1	g_2
a		
b	14	14
c		

Methodological problems – continue

Example (4. Incommensurable differences between grades ?)

Reconsider the three students enrolled in the same programme as in Example (3) :

Comment

	g_1	g_2
a	$14 - x$	$14 + x$
b	14	14
c	$14 + x$	$14 - x$

The three students obtain the same average of 14 (for $x = 1, 2, ..., 5$) and validate equivalently the programme with a final grade 14 (good).

If $x = 1$, this result is acceptable.

If $x = 5$, this result is no more acceptable.

Methodological problems – continue

Example (4. Incommensurable differences between grades ?)

Reconstruct the three students enrolled in the same programme as in Example (3) :

Comment

	g_1	g_2
a	$14 - x$	$14 + x$
b	14	14
c	$14 + x$	$14 - x$

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Example (4. Incommensurable differences between grades ?)

Reconsider the three students enrolled in the same programme as in Example (3) :

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	g_1	g_2
a	13	15
b	14	14
c	15	13

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Reconsider the three students enrolled in the same programme as in Example (3) :

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	g_1	g_2
a	9	19
b	14	14
c	19	9

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If $x = 1$, this result is acceptable.

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How to aggregate ordinal grades ?

Example (5. grading on an ordinal scale)

Consider three students enrolled in a study programme consisting of three courses graded from 0 to 20 points and where a grade of 10/20 is required for succeeding the programme. If the grading scale is purely ordinal, the following grades will show the same result for each student.

	g_1	g_2	g_3
a	12	5	13
b	13	12	5
c	5	13	12

	g_1	g_2	g_3
a	11	4	12
b	13	13	6
c	4	14	11

In the first case, all three students validate, whereas, in the second case, only b validates the programme.

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Example (6. The US Grade Point Average GPA)

As the courses are graded on alphabetical levels from E to A, one has to numerically encode these levels. A common conversion schema is the following :

level	grade	mention
A	4	(excellent)
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The choice of grades 4 to 0 is

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- *A constant difference between two adjacent levels is assumed.*
- *Obtaining an excellent level A is supposed to be 4 times as performing as obtaining as satisfactory level D ! ? !*

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Example (6) Computing the GPA – continue

Exams in the US are generally graded from 0 to 100 %. Suppose that three student obtained the following grades in three courses :

	g_1	g_2	g_3
a	90	69	70
b	79	79	89
c	100	70	69

Conversion schema :

level	interval	grade
A	90 – 100%	4
B	80 – 89%	3
C	70 – 79%	2
D	60 – 69%	1
E	0 – 59%	0

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Example (6) Computing the GPA – continue

Converting the results :

	g_1	g_2	g_3
a	A	D	C
b	C	C	B
c	A	C	D

Computing the GPA :

	g_1	g_2	g_3	GPA
a	4	1	2	2.33
b	2	2	3	2.33
c	4	2	1	2.33

Comment

All three students obtain the same GPA value 2.33.

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Example (6) Computing the GPA – continue

Other conversion schema :

level	interval	grade
$A+$	98 – 100%	10
A	94 – 97%	9
$A-$	90 – 93%	8
$B+$	87 – 89%	7
B	83 – 86%	6
$B-$	80 – 82%	5
$C+$	77 – 79%	4
C	73 – 76%	3
$C-$	70 – 72%	2
D	60 – 69%	1
E	0 – 59%	0

Conversion results :

	g_1	g_2	g_3
a	$A-$	D	$C-$
b	$C+$	$C+$	$B+$
c	$A+$	$C-$	D

Computing the GPA :

	g_1	g_2	g_3	GPA
a	8	1	2	3.66
b	4	4	7	5.00
c	10	2	1	4.33

Student b obtains now clearly a better result.

Example (6) Computing the GPA – continue

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Conversion results :

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b	$C+$	$C+$	$B+$
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Aggregating ordinal performances

Example (Condorcet's method)

Consider three students enrolled in a study programme consisting in three courses of same weight and who obtained the grades shown here :

	g_1	g_2	g_3
a	13	12	11
b	11	13	12
c	14	10	12

Comment

- *The three students obtain the same average grade 12.*
- *Consider now that a difference of one point on the grading scale is not really significant for warranting an effective performance difference.*

Aggregating ordinal performances

Example (Condorcet's method)

Consider three students enrolled in a study programme consisting in three courses of same weight and who obtained the grades shown here :

	g_1	g_2	g_3
a	13	12	11
b	11	13	12
c	14	10	12

Comment

- *The three students obtain the same average grade 12.*
- *Consider now that a difference of one point on the grading scale is not really significant for warranting an effective performance difference.*
- *Student a shows at least as good grades as b and c in all the courses.*
- *However, students b are c are only in two out of three courses at least as good as student a.*

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Exercise(s)

Here the table of grades obtained by four students : a, b, c, and d, in five courses : C_1, C_2, C_3, C_4 and C_5.

course	C_1	C_2	C_3	C_4	C_5
ECTS	2	3	4	2	4
a	11	13	9	15	11
b	12	9	13	10	13
c	8	11	14	12	14
d	15	10	12	8	13

An award is granted to the best amongst these four students.

1. *Who would you nominate ?*
2. *Explain and motivate your selection algorithm.*

Concluding

- Grading accurately someones performances is generally a difficult task in practice.
- Grading procedures are in general quite complex and must not be seen as simple as physical weight, time and length measures.
- Aggregating grades needs taking into account potential imprecision, uncertainty as well as known cognitive biases.
- Aggregating rules have to be analyzed with great attention. The simplests and evidents do not necessarily give the expected results.

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