Lecture 5: Choosing with multiple criteria: The Multiple Attribute Value Theory (MAVT) approach

Algorithmic Decision Theory

MICS 2 semester

FSTC-CSC

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Potential alternatives

Example (Choose a new office site for a SME)

- A SME specialized in printing and copy services has to move into new offices.
- The CEO of the SME has gathered seven potentials sites :

Site	Code	Annual rent
Avenue de la liberté	(A)	30 000 €
Bonnevoie	(B)	15 000 €
Cessange	(C)	5 000 €
Dommeldange	(D)	12 000 €
Esch-Belval	(E)	30 000 €
Fentange	(F)	15 000 €
Avenue de la **G**are	(G)	10 000 €

Decision objectives

Example (Choose a new office site for a SME – continue)

The CEO has identified three objectives to guide the choice of the new site. He wishes to :

1. minimize the yearly costs induced by the moving,
2. maximize the future turnover of the SME,
3. maximize the new working conditions.

Decision consequences to take into account

Example (Choose a new office site for a SME – continue)

1. minimize the yearly costs induced by the moving :
   - 1.1 Annual rent
   - 1.2 Functional costs (electricity, water, ..)
   - 1.3 Cleaning costs
2. maximize the future turnover of the SME
   - 2.1 Standing of the building
   - 2.2 Visibility of the office
   - 2.3 Proximity with the customers
3. maximize the new working conditions
   - 3.1 Working space
   - 3.2 Working Confort
   - 3.3 Parking facilities

Measuring the costs

Example (Choose a new office site for a SME – continue)

Annual total renting, functioning and cleaning costs (in €) :

alternative	Rent	Cleaning	Functional
Avenue de la liberté	30 000	3 000	2 000
Bonnevoie	15 000	2 000	800
Cessange	5 000	1 000	700
Dommeldange	12 000	1 000	1 100
Esch-Belval	30 000	2 500	2 300
Fentange	15 000	1 000	2 600
Avenue de la **G**are	10 000	1 100	900

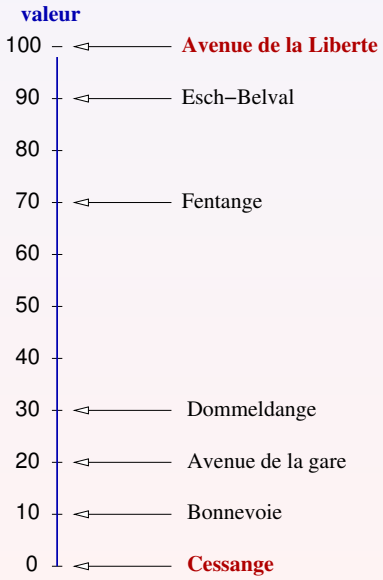
Measuring preferences on a qualitative consequence

The CEO of the SME may rank the seven potential sites from the best to the worst from the point of view of the standing of the building :

rank	alternative
1st	**A**venue de la liberté
2nd	**E**sch-Belval
3rd	**F**entange
4th	**D**ommeldange
5th	Avenue de la **G**are
6th	**B**onnevoie
7th	**C**essange

The CEO is furthermore invited to place the individual sites along a 0–100 axis such that the numerical positions represent the apparent differences in standing he observes between the potential site buildings.

Measuring preferences on a qualitative consequence



Positioning of the potential sites :

1. A grade of 100 is given to the best site and a grade of 0 is given to the worst site.
2. The CEO then positions the other sites such that the numerical positions represent the apparent differences in standing he observes between the potential site buildings.

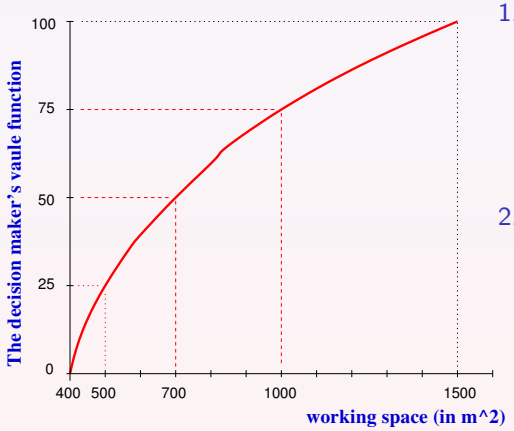
Measuring preferences on a quantitative consequence

Let us now consider a qualitative consequence : working space, contributing to the evaluation of the performance of an alternative to objective : maximize working conditions.

alternative	Working space (in m^2)
Avenue de la liberté	1000
Bonnevoie	550
Cessange	400
Dommeldange	800
Esch-Belval	1500
Fentange	400
Avenue de la **G**are	700

An increasae from 500 to 1000 m^2 is very attractive. The same increase from 1000 to 1500 m^2 is however not anymore so attractive.

Measuring preferences on a quantitative consequence



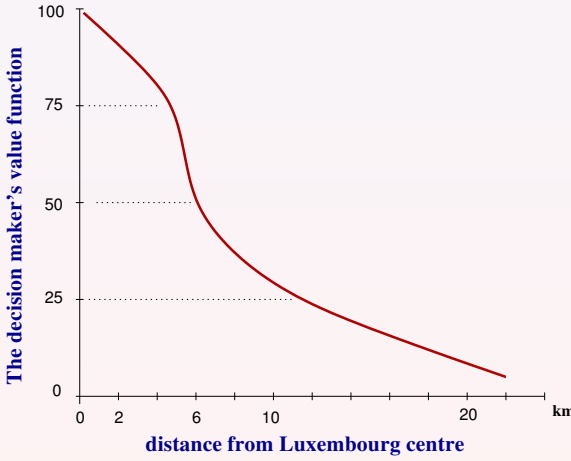
Here the conversion the CEO is proposing.

1. A working space of 700 m^2 is considered to be situated right in the middle between the maximum ($1500m^2 = 100$ pts) and the minimum available surface ($400m^2 = 0$ pts).
2. A working space of 1000 m^2 is considered to be situated right in the middle between the previous middle ($700m^2 = 50$ pts) and the maximum available surface ($1500m^2 = 100$ pts).

The preference value of the working surface thus becomes an interval scale.

Measuring preferences on a quantitative consequence

A similar procedure allows to measure the preference value of the customers proximity consequence :



The preference value is maximal = 100 in the city centre.

The preference value of the consequence decreases with the distance to the city centre, smoothly in the beginning, then sharply in the middle before decreasing smoothly again at the end.

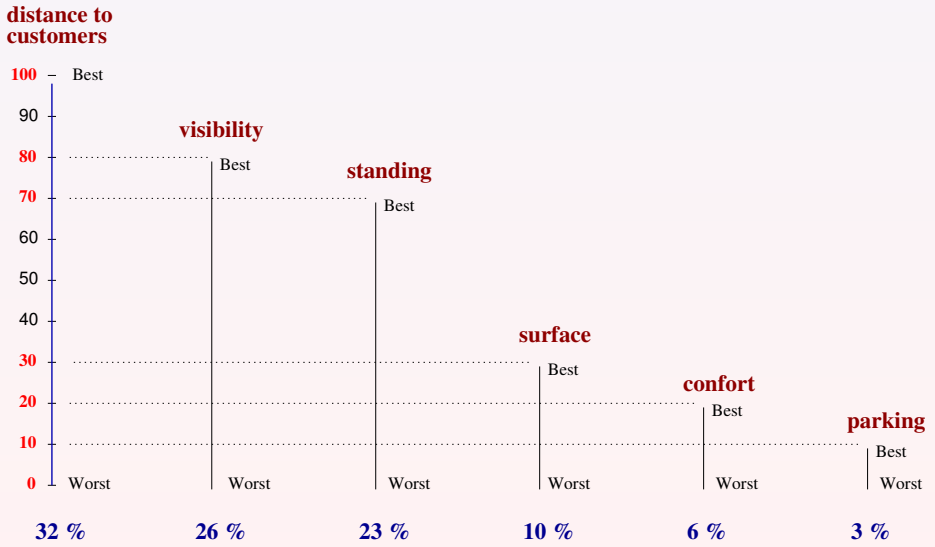
Aggregating costs

All the costs categories taken into account : rent, cleaning and functional, may simply be summed up, as they are all expressed on the same commensurable preference scale, i.e. annual amounts of Euros.

site	Rent	Cleaning	Functional	Total
Avenue de la liberté	30 000	3 000	2 000	35 000
Bonnevoie	15 000	2 000	800	17 800
Cessange	5 000	1 000	700	6 700
Dommeldange	12 000	1 000	1 100	14 100
Esch-Belval	30 000	2 500	2 300	34 800
Fentange	15 000	1 000	2 600	18 600
Avenue de la **G**are	10 000	1 100	900	12 000

Weighing the benefits

A common method is called "swing weights" :



Weighing the benefits – continue

Comment

- *The consequence : customer proximity is considered to be the most important consequence.*
- *The regret to switch from the best to the worst on the consequence visibility is judged to be 80% of the regret to switch from the best to the worst on the most important consquence.*
- *These decreasing regret percentages (80%, 70%, etc) are then normalised on a* 0 *to* 100 *scale.*

Maximize future business turnovers

The performance of the potential alternatives along this objective results from the aggregation of the following consquences : standing, visibility, and customer proximity :

conseq.	weight	Alternatives A	B	C	D	E	F	G
standing	23 %	100	10	0	30	90	70	20
visibility	26 %	60	80	70	50	60	0	100
proximity	32 %	100	20	80	70	40	0	60
total	81 %	70.6	29.5	43.8	42.3	49.1	16.1	49.8

Maximize the working conditions

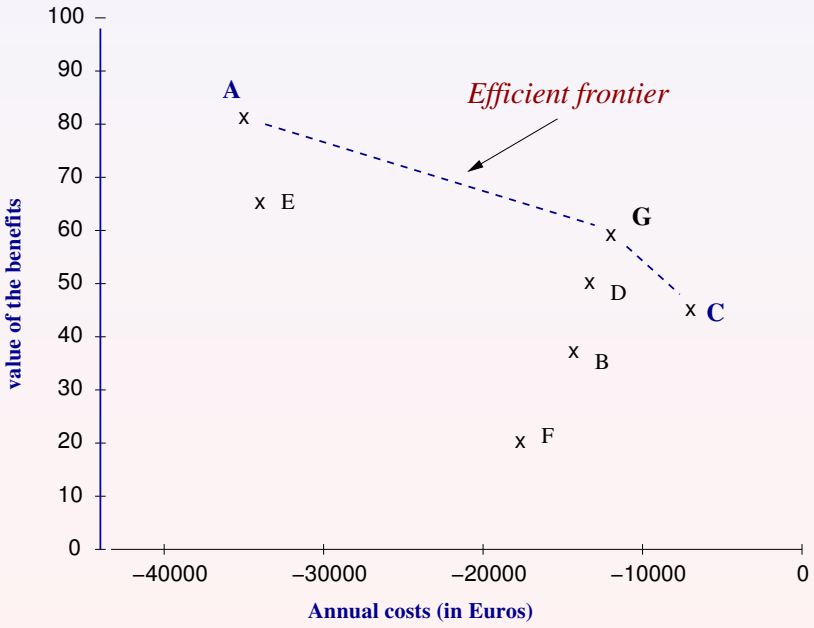
The performance of the potential alternatives along this objective results from the aggretaion of the following consequences : working space, comfort, parking facilities :

conseq.	weight	Alternatives A	B	C	D	E	F	G
space	10 %	75	30	0	55	100	0	50
comfort	6 %	0	100	10	30	60	80	50
parking	3 %	90	30	100	90	70	0	80
total	19 %	10.2	9.9	3.6	10	15.7	4.8	10.4

Performances wrt the three objectives

Rank	Costs	Turnover	Work. Cond.	Total Benefit
1st	C (6 700 €)	A (70.6)	E (15.7)	A (80.8)
2nd	G (12 000 €)	G (49.8)	G (10.4)	E (64.8)
3rd	D (14 100 €)	E (49.1)	A (10.2)	G (60.2)
4th	B (17 800 €)	C (43.8)	D (10)	D (52.3)
5th	F (18 600 €)	D (42.3)	B (9.9)	C (47.4)
6th	E (34.800 €)	B (29.5)	F (4.8)	B (39.4)
7th	A (35 000 €)	F (16.1)	C (3.6)	F (20.9)

How to compensate between costs and benefits ?



How to compensate between costs and benefits ?

Comment

- *Three sites : Avenue de la liberté (A), Avenue de la Gare (G) et Cessange (C) appear non dominated. They represent potential candidates for the best choice.*
- *Consider first switching from alternative C to alternative G. We notice an increase in benefits from 47.4 to 60.2 points, whereas the costs increase consists in 5 300€. The marginal increase in benefits is hence* $5300/12.8 = 414$*€.*

How to compensate between costs and benefits ?

Comment

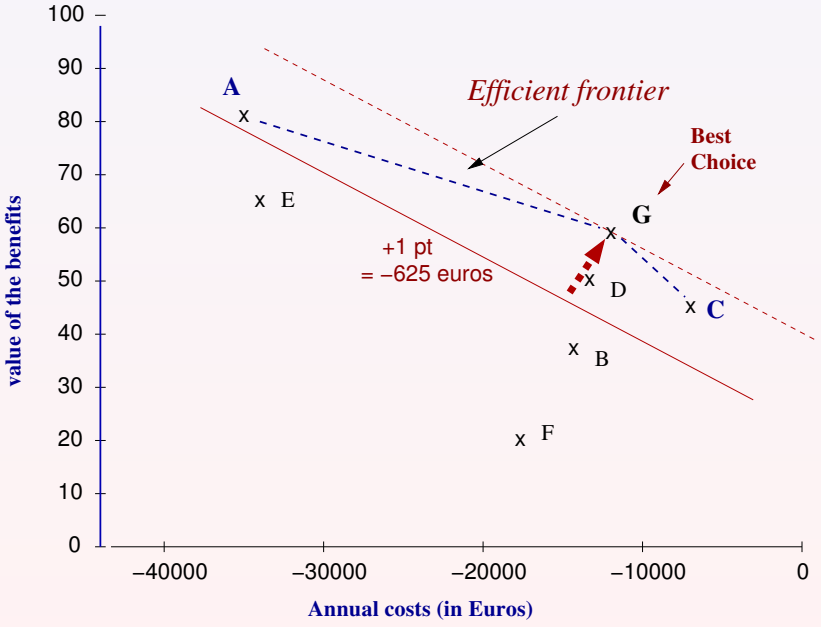
- *Similarly, switching from alternative G to alternative A increases the benefits by 20.6 points and the costs by 23 000€. The marginal increase in benefits costs here* $23000/20.6 = 1117$*€.*
- *If the CEO considers that a benefit point is worth :*
  - *less than 414€ he will prefer the site Cessange (C),*
  - *between 414€ and 1117€ he will prefer the site Avenue de la Gare (G),*
  - *more than 1117€ he will prefer the site Avenue de la liberté (A).*

How to compensate between costs and benefits ? – continue

Example (Evaluate the marginal benefit increase in Euros)

- The decision maker is asked how much he would agree to invest in order to increase the standing of the new office from the worst to the best.
- Suppose he declares that he would for this purpose spend up 15 000€.
- The consequence "standing" representing 23% of the value of the benefits, the decision maker is thus ready to invest 15 000€ for getting an increase of 23 points in the benefits. He is hence ready to invest 652€ per point.
- On this base, the best choice is given with alternative Avenue de la Gare (G).

How to compensate between costs and benefits ? – continue



Multiple Attributes Value Theory (MAVT)

Comment

- *In this lecture we illustrate a best choice decision algorithm which consists in constructing a commensurable numerical representation of all the consequences to be taken into account for choosing the best alternative.*
- *The Multiple Attribute Value Theory (MAVT), was initiated in 1976 by two American scientists : Ralph Keeney and Howard Raiffa.*

Principles of MAVT

Principle (Complete comparability)

The potential alternatives'performances may be numerically measured on all the consequences, the objectives, and globally(after aggregation).

Comment

- *We supposed for instance that the increase in benefits switching from site Cessange (C) to site Avenue de la Gare (G), was greater than the one from site Bonnevoie (B) to site Cessange (C).*
- *It may however happen that the decision maker is not able (not available for instance) to give a precise numerical value to such eventualities.*

Principles of MAVT – continue

Principle (Additivity)

If the decision maker prefers alternative A over alternative B, and alternative B over alternative C, then the difference in preference between A and C has to be greater than the differences between A and B, and, between B and C.

Comment

- *The differences in preference have to respect the rankings of the alternatives on each consequence, on each objective and globally.*
- *The appreciation of a preference difference between two alternatives must result from the comparison of their respective values on each consequence.*

Principles of MAVT – continue

Principle (Linear (geometrical) perception of the performances)

The decision maker is always able to cut into half the difference in preference he observes when considering a performance difference on a consequence.

Comment

- *Concerning the working space for instance, the CEO was able to say that the increase from 400 to 700 m^2 is équivalent in preference to an increase in value from 700 to 1500m^2.*
- *This principle is commonly not verified with essentially qualitative consequences like the standing of the office building.*

Principles of MAVT – continue

Principle (Finite value scales)

No alternative may admit an infinite positive nor negative value.

Comment

- *All performances being in principle commensurable, there may not be any infinitely valued consequence.*

Principles of MAVT – continue

Principle (Transitivity)

If the decision maker prefers alternative A over alternative B, and alternative B over alternative C, then he must prefer alternative A over alternatice C.

Comment

- *This principle is essential when representing preference with numerical values. All common number sets (integers, rationals, floats, reals, etc) verify this transitivity principle.*
- *Aggregating global preferences based on pairwise majority margins à la Condorcet does however not satify this principle.*

Principles of MAVT – continue

Principle (Mutual preferential independance between the consequences taken into account)

Each consequence has to measure a specific performance which must be independent from the performances on the other consequences in order to avoid overlapping (and hence overweighing) of the performance in the global aggregation.

Comment

- *For instance, the consequence office visibility may only become relevant when the proximity with the customers is sufficiently small. Otherwise, this consequece should not count.*
- *This principle is usually not verified when evaluating consequences from common socio-ecomonic indicators.*
- *However, one may control this principle with an ad hoc construction of relevant consequences measures.*

Concluding on MAVT

- MAVT points to the necessity to follow a consistent and systematic method for evaluating alternatives on multiple attributes in a choice decision problem.
- MAVT may take into account quantitative costs and qualitative benefits.
- It is a quantitative (value measured) and transparent (explicit algorithm) best choice method.
- MAVT requires, however, complete comparability, commensurability, transitivity, etc, all principles that may be difficult to verify in a given real decision problem.