Best multiple criteria compromise choice: the Rubis outranking approach

MICS: AlgorithmicDecision Theory

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April 27, 2020

Introduction

1. Compare with potentially conflicting criteria
   - The outranking situation
   - Taking into account the performances' imprecision
   - Considering large performance differences
2. Theoretical foundation of the outranking approach
   - Overall preference concordance
   - Taking into account vetoes
   - The bipolar-valued outranking relation
3. The Rubis best-choice recommender system
   - Best-choice recommender system design
   - Resolving a best-choice problem
   - The RUBIS best-choice recommendation

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The outranking situation

Definition

- We say that "a decision alternative a outranks a decision alternative b" if and only:
  1. There is a significant majority of criteria (or objectives) who warrant that a is perceived at least as good as b and,
  2. No considerable negative performance difference is observed between a and b on any criterion (or objective).
- We say that "a decision alternative a does not outrank a decision alternative b" if and only if:
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Best office choice

- Let us reconsider the best office choice problem from lecture 5.
- Below the performances of the seven potential office sites with respect to the three objectives:

Site	Costs (in €)	Turnover (0-81%)	Work Cond. (0-19%)
A	-35 000	70.6	10.2
B	-17 800	29.5	9.9
C	-6 700	43.8	3.6
D	-14 100	42.3	10.0
E	-34.800	49.1	15.7
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Significant preferential judgment

Example

- The CEO of the SME judges the "*Costs*" and the cumulated "*Benefits*" objectives ("*Turnover*" and "*Working Conditions*") to be equi-significant for selecting the best office site.
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Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
G	-12 000	49.1	10.4
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- Site G certainly outranks site F as G is at least as well performing than F on all three objectives (unanimous concordance = Pareto dominance).

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Indeterminate outranking situation

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Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
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- As site *F* is less expensive than site *E* (-18 600 against -34 800), but also, at the same time less advantageous on objective "*Turnover*" (16.1 against 49.1) and objective "*Work Cond.*" (4.8 against 15.7), one can neither confirm, nor reject this outranking situation.

This indeterminate situation is similar to a voting result where the number of votes in favour perfectly balance the number of votes in disfavour.

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- Yet, are the grades 9.9 and 10.2 on the "*Work. Cond*" really different ?

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Definition (Discrimination thresholds)

The concept of discrimination threshold allows to take into account on each criterion (or objective) the:

- imprecision of our knowledge about present or past facts,
- uncertainty which necessarily affects our knowledge of the future,
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- Let us reconsider the performance table of our best office choice problem:

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- Hence, site *B* outranks site *A*, as the former is clearly less expensive (-17 800 against -35 000) and also more or less at least as good as *A* on objective "*Work Cond.*" (9.9 against 10.2, difference smaller than the supposed indifference threshold).

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- The concept of veto situation allows us to take into account on each criterion (or objective):

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- Consider the performances of alternatives A and F with respect to the three objectives:

Site	Costs	Turnover	Work. Cond.
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  The outranking situation between A and F is indeterminate.

- The CEO of the SME considers that a performance difference of 50 points on the "Turnover" objective attests a veto situation.

  Hence, the out-performance on objective "*Turnover*" of site A over site F ($70.6 - 16.1 = 54.6 > 50.0$ pts) resolves this indterminateness in favour of site A.

  Similarly, site F does certainly not outrank site A, as the counter-performance on objective "*Turnover*" is so high that it renders insignificant the fact that F is less expensive (-18600 against -35000).

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  Similarly, site F does certainly not outrank site A, as the counter-performance on objective "*Turnover*" is so high that it renders insignificant the fact that F is less expensive (-18600 against -35000).

1. Compare with potentially conflicting criteria
   - The outranking situation
   - Taking into account the performances' imprecision
   - Considering large performance differences
2. Theoretical foundation of the outranking approach
   - Overall preference concordance
   - Taking into account vetoes
   - The bipolar-valued outranking relation
3. The Rubis best-choice recommender system
   - Best-choice recommender system design
   - Resolving a best-choice problem
   - The RUBIS best-choice recommendation

Notation

- Let X be a finite set of p decision alternatives.
- Let F be a finite set of n criteria supporting an increasing real performance scale from 0 to M_j ($j = 1, ...n$).
- Let $0 \leqslant ind_j < pr_j < v_j \leqslant M_j + \epsilon$ represent resp. the indifference, the preference, and the veto discrimination threshold observed on criterion j.
- Let w_j be the significance of criterion j.
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Performing marginally *at least as good as*

Each criterion j is characterizing a double threshold order $\geqslant_j$ on A in the following way:

$$r(x \geqslant_j y) = \begin{cases} +1 & \text{if} \quad x_j - y_j \geqslant -ind_j \\ -1 & \text{if} \quad x_j - y_j \leqslant -pr_j \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

+1 signifies x is *performing at least as good as* y on criterion j,

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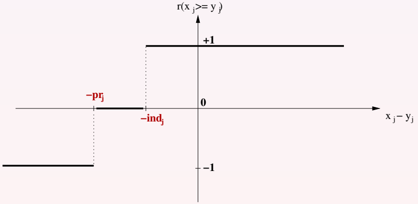

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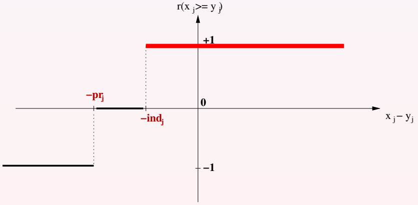

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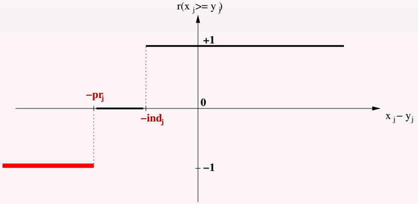

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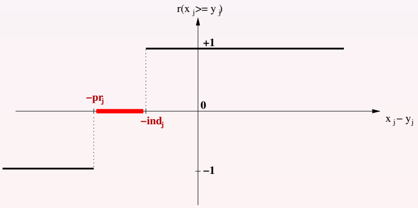

Performing globally *at least as good as*

Each criterion j contributes the significance w_j of his "*at least as good as*" characterisation $r(\succeq_j)$ to the characterisation of a global "*at least as good as*" relation $r(\succeq)$ in the following way:

$$\boxed{r(x \succeq y) \;=\; \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \succeq_j y) \right]} \qquad (2)$$

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Performing marginally and globally *less than*

Each criterion j is characterising a double threshold order $<_j$ (*less than*) on A in the following way:

$$r(x \prec_j y) \quad = \quad \begin{cases} +1 & \text{if} \quad x_j + pr_j \leqslant y_i \\ -1 & \text{if} \quad x_j + ind_j \geqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

And, the *global less than* relation ($\prec$) is defined as follows:

$$\boxed{r(x \prec y) = \textstyle\sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \prec_j y) \right]} \tag{4}$$

Property

The global "less than" relation $\prec$ is the dual ($\not\succsim$) of the global "at least as good as" relation $\succsim$.

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Marginal *considerably better* or *worse performing* situations

We define a single threshold order, denoted $\ll_j$ which represents *considerably less performing* situations as follows:

$$r(x \ll_j y) = \begin{cases} +1 & \text{if} \quad x_j + v_j \leqslant y_j \\ -1 & \text{if} \quad x_j - v_j \geqslant y_j \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

And a corresponding dual *considerably better performing* situation $\gg_j$ characterised as:

$$r(x \gg_j y) = \begin{cases} +1 & \text{if} \quad x_j - v_j \geqslant y_i \\ -1 & \text{if} \quad x_j + v_j \leqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

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Global *considerably better* or *considerably worse performing* situations

A global *veto*, or *counter-veto* situation is defined as follows:

$$r(x \ll y) = \bigcirc\!\!\!\!\vee_{j \in F}\, r(x \ll_j y) \quad (7)$$

$$r(x \gg y) = \bigcirc\!\!\!\!\vee_{j \in F}\, r(x \gg_j y) \quad (8)$$

where $\bigcirc\!\!\!\!\vee$ represents the epistemic polarising (Bisdorff 1997) or symmetric maximum (Grabisch et al. 2009) operator:

$$r \,\bigcirc\!\!\!\!\vee\, r' = \begin{cases} \max(r, r') & \text{if} \quad r \geqslant 0 \,\wedge\, r' \geqslant 0, \\ \min(r, r') & \text{if} \quad r \leqslant 0 \,\wedge\, r' \leqslant 0, \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

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Characterising veto and counter-veto situations

1. $r(x \ll y) = 1$ iff there exists a criterion i such that $r(x \ll_i y) = 1$ and there does not exist otherwise any criteria j such that $r(x \gg_j y) = 1$.
2. Conversely, $r(x \gg y) = 1$ iff there exists a criterion i such that $r(x \gg_i y) = 1$ and there does not exist otherwise any criteria j such that $r(x \ll_j y) = 1$.
3. $r(x \gg y) = 0$ if either we observe no considerable performance differences or we observe at the same time, both a considerable positive and a considerable negative performance difference.

Lemma

$r(\not\ll)^{-1}$ is identical to $r(\gg)$.

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The bipolar outranking relation $\succsim$

From an epistemic point of view, we say that:

1. alternative x *outranks* alternative y, denoted ($x \succsim y$), if
   1.1 a significant majority of criteria validates a global outranking situation between x and y, and
   1.2 no considerable counter-performance is observed on a discordant criterion,
2. alternative x *does not outrank* alternative y, denoted ($x \not\succsim y$), if
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Polarising the global "*at least as good as*" characteristic

The bipolar-valued characteristic $r(\succsim)$ is defined as follows:

$$\boxed{r({\color{red}x \succsim y}) = r(x \succeq y) \circledvee r(x \not\lll_1 y) \circledvee \ldots \circledvee r(x \not\lll_n y)}$$

Properties:

1. $r(x \succsim y) = r(x \succeq y)$ if no considerable positive or negative performance differences between x and y are observed,
2. $r(x \succsim y) = 1.0$ if $r(x \succeq y) \geqslant 0$ and $r(x \ggg y) = 1.0$,
3. $r(x \succsim y) = -1.0$ if $r(x \succeq y) \leqslant 0$ and $r(x \lll y) = 1.0$,
4. The bipolar outranking relation $\succsim$ is weakly complete: either $r(x \succsim y) \geqslant 0$ or $r(y \succsim x) \geqslant 0$.

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1. $r(x \succsim y) = r(x \succeq y)$ if no considerable positive or negative performance differences between x and y are observed,
2. $r(x \succsim y) = 1.0$ if $r(x \succeq y) \geqslant 0$ and $r(x \gg y) = 1.0$,
3. $r(x \succsim y) = -1.0$ if $r(x \succeq y) \leqslant 0$ and $r(x \ll y) = 1.0$,
4. The bipolar outranking relation $\succsim$ is weakly complete: either $r(x \succsim y) \geq 0$ or $r(y \succsim x) \geq 0$.

Polarising the global "*at least as good as*" characteristic

The bipolar-valued characteristic $r(\succsim)$ is defined as follows:

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Coherence of the bipolar-valued outranking concept

Property

The dual ($\not\gtrsim$) of the bipolar outranking relation $\gtrsim$ is identical to the strict converse outranking $\precnsim$ relation.

Proof: We only have to check the case where $r(x \ll_i y) \neq 0.0$ for all $i \in F$. If $r(x \ll y) \neq 0.0$:

$$\begin{aligned} r(x \not\gtrsim y) &= -r(x \gtrsim y) \;=\; -\big[r(x \succeq y) \circledvee -r(x \ll y)\big] \\ &= \big[-r(x \succeq y) \circledvee r(x \ll y)\big] \\ &= \big[r(x \not\succeq y) \circledvee -r(x \gg y)\big] \\ &= \big[r(x \prec y) \circledvee r(x \not\gg y)\big] \;=\; r(x \precnsim y). \end{aligned}$$

Else, there exist conjointly two criteria i and j such that $r(x \ll_i y) = 1.0$ and $r(x \gg_i y) = 1.0$ such that $r(x \gtrsim y) = r(x \not\gtrsim y) = r(x \precnsim y) = 0.0$. □

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Semantics of the bipolar valuation

The valuation $r(\succsim)$ has following interpretation:

- $r(x \succsim y) = +1.0$ signifies that the statement $x \succsim y$ is certainly valid.
- $r(x \succsim y) = -1.0$ signifies that the statement $x \succsim y$ is certainly invalid.
- $r(x \succsim y) > 0$ signifies that the statement $x \succsim y$ is more valid than invalid.
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The bipolar outranking (Condorcet) digraph

Definition

- We denote $\widetilde{G}(X, r(\succsim))$ the bipolar-valued digraph modelled by $r(\succsim)$ on the set of potential decision alternatives X.
- We denote $G(X, \succsim)$, the crisp digraph associated with $\widetilde{G}$ where we retain all arcs such that $r(x \succsim y) > 0$.
- $G(X, \succsim)$ is called the Condorcet or median cut digraph associated with $\widetilde{G}(X, r(\succsim))$.

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The office site choice problem revisited

If we consider:

1. a preference threshold of 5 pts on objective "*Turnover*",
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$r(\succsim)$	A	C	G
A	200	0	-19
C	0	200	0
G	38	0	200

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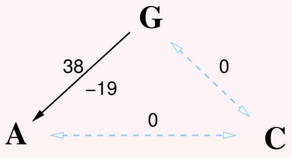

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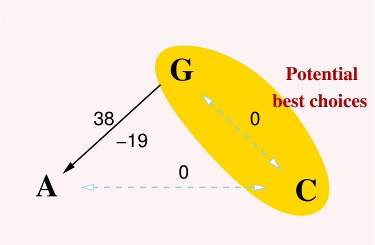

The office site choice problem – continue

Comment

- *The bipolar outranking characteristics show that:*
  1. *Site G is significantly at least as well performing as site A ($r(G \succsim A) = 38$)*
  2. *A is not significantly performing as well as site G ($r(A \succsim G) = -19$),*

     between sites G and C nor, between sites A and C.
- *Hence G and C may be recommended as potential best choices.*

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
C	-6 700	43.8	3.6
G	-12 000	49.1	10.4

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Designing a best-choice recommender system

- Traditionally, solving a best-choice problem consists in finding the unique best decision alternative.
- In RUBIS , we adopt a modern recommender system's approach which shows a subset of alternatives which contains by construction the potential best alternative(s).
- If not reduced to a singleton, the actual "best choice", the recommendation has to be refined in a later decision process phase.

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Pragmatic principles for a best-choice recommendation (BCR)

$\mathcal{P}_1$: Elimination for well motivated reasons.
Each eliminated alternative has to be outranked by at least one alternive in the BCR.

$\mathcal{P}_2$: Minimal size.
The BCR must be as limited in cardinality as possible.

$\mathcal{P}_3$: Efficient and informative.
The BCR must not contain a self-contained sub-recommendation.

$\mathcal{P}_4$: Effectively better.
The BCR must not be ambiguous in the sense that it is both a best choice as well as a worst choice recommendation.

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The BCR is, of all potential best-choice recommendations, the most determined one in the sense of the characteristics of the bipolar-valued outranking relation $\succsim$.

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Qualification of a BCR in an outranking digraph

Let Y be a non empty subset of X, called a choice in an outranking digraph $\widetilde{G}(X, r(\succsim)$.

- Y is called outranking (resp. outranked) iff for all non retained alternative x there exists an alternative y retained such that $r(y \succsim x) > 0.0$ (resp. $r(x \succsim y) > 0.0$).
- Y is called independent iff for all $x \neq y$ in Y, we observe $r(x \succsim y) \leqslant 0.0$.
- Y is an outranking kernel (resp. outranked kernel) iff Y is an outranking (resp. outranked) and independent choice.
- An outranking digraph $\widetilde{G}$ may not admit an outranking (resp. outranked) kernel if it contains a chordless odd circuit. The case given, we recursively break up all circuits at their weakest link.

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Translating the pragmatic principles in terms of choice qualification

$\mathcal{P}_1$: Elimination for well motivated reasons.
The BCR is an outranking choice.

$\mathcal{P}_{2+3}$: Minimal and stable recommendation.
The BCR is a kernel.

$\mathcal{P}_4$: Effectivity.
The BCR is a choice which is strictly more outranking than outranked.

$\mathcal{P}_5$: Maximal determination.
The BCR is the most determined one in the set of potential outranking kernels observed in a given bipolar outranking digraph $\widetilde{G}(X, r(\succsim))$.

Property (BCR Decisiveness)

Every bipolar strict outranking digraph $\widetilde{G}(X, r(\succsim))$ without chordless odd circuit admits at least one outranking and one outranked kernel.

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$\mathcal{P}_5$: Maximal determination.
The BCR is the most determined one in the set of potential outranking kernels observed in a given bipolar outranking digraph $\widetilde{G}(X, r(\succsim))$.

Property (BCR Decisiveness)

Every bipolar strict outranking digraph $\widetilde{G}(X, r(\succnsim))$ without chordless odd circuit admits at least one outranking and one outranked kernel.

The RUBIS best-choice recommendation (RBCR)

- A (strict) outranking kernel of maximal determination renders a RBCR. By default, we compute the RBCR on the strict (codual) outranking digraph where we previously break all chordless odd circuits.
- A RBCR verifies the five pragmatic principles.
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- A RBCR must not be confused with the actual best choice retained by the decision maker.
- Being only a best-choice recommendation, the RUBIS decision aid approach is only convenient in a progressive decision process.
- Mind that enumerating and breaking chordless circuits, the case given, are operationally difficult problems. The same is true for enumerating (strict) outranking kernels. Hence, a RBCR may in general only be computed for decision problems involving less than 50 decision alternatives.

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The performance table of the office choice problem

Criterion	w_i	Alternatives A	B	C	D	E	F	G
Costs	45	-35000	-17800	-6700	-14100	-34800	-18600	-12000
Proximity	32	100	20	80	70	40	0	60
Visibility	26	60	80	70	50	60	0	100
Standing	23	100	10	0	30	90	70	20
Work. Space	10	75	30	0	55	100	0	50
Comfort	6	0	100	10	30	60	80	50
Parking	3	90	30	100	90	70	0	80
W	145							

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Default performance discrimination thresholds

Criterion	Thresholds (in points or €) indiff.	pref.	veto
Costs	1000 €	2500 €	35 000 €
Proximity	10 pts	20	80
Visibility	10	20	80
Standing	10	20	80
Work. Space	10	20	80
Comfort	10	20	80
Parking	10	20	80

The bipolar outranking digraph

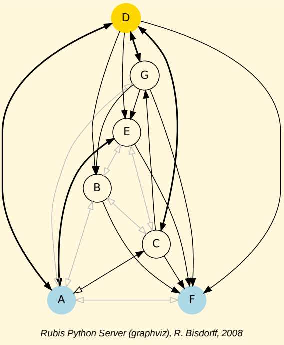
```
>>> from outrankingDigraphs import *
>>> t = PerformanceTableau('officeChoice')
>>> g = BipolarOutrankingDigraph(t)
>>> g.recodeValuation(-145,+145)
>>> g.showHTMLRelationTable(ndigits=0)
```

Characteristics multiplied by $W = 145$.

$r(\succsim)$	'A'	'B'	'C'	'D'	'E'	'F'	'G'
'A'	145	0	+145	+43	+113	0	0
'B'	0	145	0	-81	0	+145	-87
'C'	0	+0	145	+67	0	+145	+15
'D'	+15	+81	+3	145	+67	+145	+36
'E'	+75	0	0	-15	145	+145	-61
'F'	0	-145	-145	-145	-145	145	-145
'G'	0	+133	-15	+145	+79	+145	145

The Rubis best-choice recommendation

```
>>> g.computeChordlessCircuits()
 [] # no chordless outranking circuits detected
>>> g.showBestChoiceRecommendation(CoDual=False)
>>> g.exportGraphViz(bestChoice=['D'],worstChoice=['A','F'])
```



Choice	Determ. (%)	Qualification as $\succsim$	$\not\succsim$	indep.
$\{D\}$	51.0	3	145	145
$\{A, G\}$	50.0	113	0	0
$\{C, B, E\}$	50.0	15	145	0
$\{A, F\}$	50.0	0	145	0

The Rubis best-choice recommendation – continue

Comment

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- *A second and third potential BCR, but without a majority support, recommend the pair* $\{A, G\}$ *and the triplet* $\{C, B, E\}$*.*
- *A potential worst choice recommends the pair* $\{A, F\}$*. Alternative* A *(a weak Condorcet winner and looser) appears hence conjointly in a potential best, as well as worst, recommendation: a consequence of its weak comparability (high benefits combined with highest costs).*
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Is alternative G outranking alternative A?

```
>>> g.showPairwiseComparison('G','A')
```

Criterion	w_i	G	A	$G - A$	sign.	veto
Costs	45	-12000	-35000	+23000	+45	-1
Proximity	32	60	100	-40	-32	-1
Visibility	26	100	60	+40	+26	-1
Standing	23	20	100	-80	-23	+1
Work Space	10	50	75	-25	-10	-1
Comfort	6	50	0	+50	+6	-1
Parking	3	80	90	-10	+3	-1
W	145			$r(G \succsim A) =$		

Is alternative C outranking alternative A?

```
>>> g.showPairwiseComparison('C','A')
```

Criterion	w_i	C	A	$C-A$	sign.	veto
Costs	45	-6700	-35000	+28300	+45	-1
Proximity	32	80	100	-20	-32	-1
Visibility	26	70	60	+10	+26	-1
Standing	23	0	100	-100	-23	+1
Work Space	10	0	75	-75	-10	-1
Comfort	6	10	0	+10	+6	-1
Parking	3	100	90	+10	+3	-1
W	145			$r(C \succsim G) =$		

Is alternative D a significant Condorcet winner ?

Exercise(s)

Alternative D is outranking all the other office site alternatives.

1. *Analyse in detail the outranking situation between alternatives D and C.*
2. *What happens to the previous outranking situation, if a performance difference of 10 pts on the benefits criteria may not be anymore disregarded ?*
3. *Under what hypothesis may alternative C become a better alternative than D ?*
4. *What becomes the BCR if the CEO would consider all his three decision objectives as equally important ?*

Conclusions

- Similarly to the MAVT, the outranking approach stresses the necessicity to follow a consistent and systematic approach for evaluating the performances of the potential decision alternatives.
- Similarly to the MAVT, the outranking approach allows to model costs and benefits with the help of multiple qualitative and/or quantitative performance criteria.
- Contrary to the MAVT, the outranking approach does not make the assumption that the evaluations on all the criteria must be commensurable in order to model global preferences.
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- Taking into account performance discrimination thresholds allows to efficiently model imprecision, uncertainties and even considerable positive and negative differences in the performance data.
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