Identifying fibre material parameter distributions with little experimental efforts

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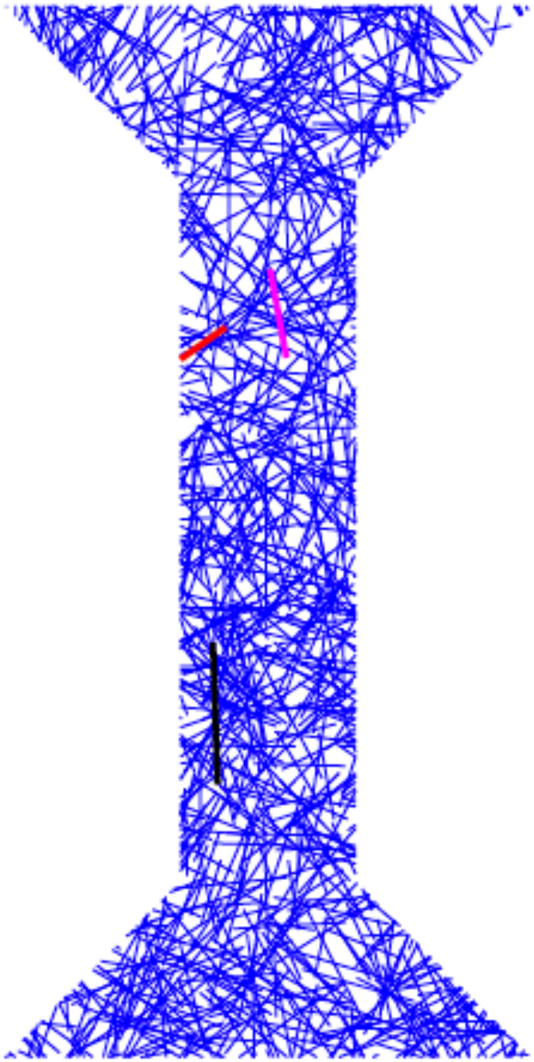
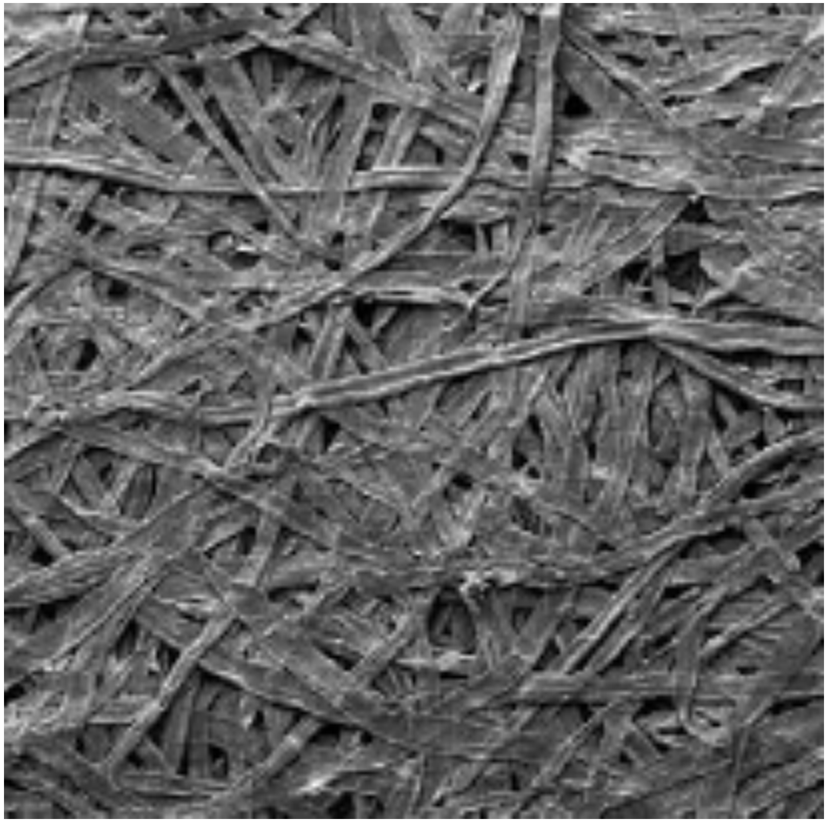
Beam models





1. Geometrical randomness
2. Base material randomness

Identification scheme for 1 parameter

1. Harvest and test fibres

result: 20 stress-strain curves

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1. Harvest and test fibres

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3. Identify parameters of the material parameter PDF

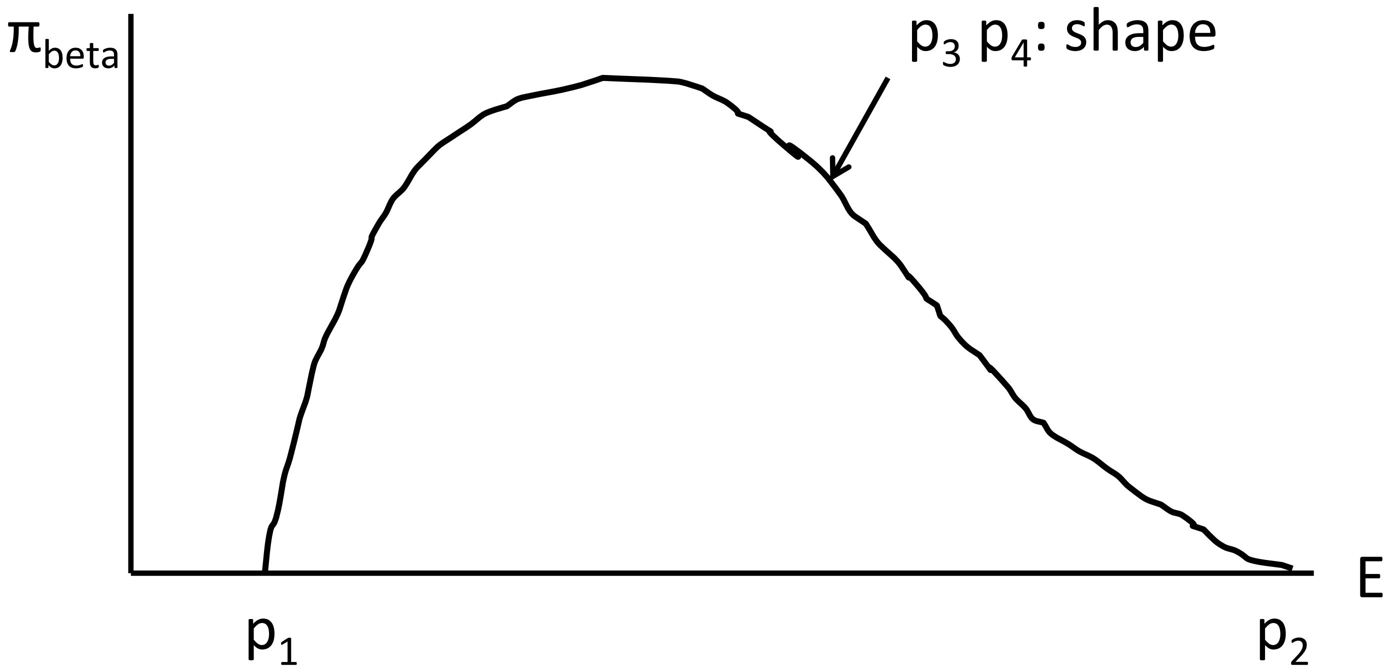
HOW?? Bayesian inference

Bayesian inference

$\underline{E}=[E_1\ E_2\ \ldots\ E_{20}]$: identified Young's moduli

$\underline{p}=[p_1\ p_2\ p_3\ p_4]$: PDF's parameters to be identified

$\pi_{beta}(E\,|\,\underline{p})$



Bayesian inference

$\underline{E}=[E_1\ E_2\ \dots\ E_{20}]$: identified Young's moduli

$\underline{p}=[p_1\ p_2\ p_3\ p_4]$: PDF's parameters to be identified

Bayes' theorem:

$$\pi(\underline{p}\,|\,\underline{E}) \propto \pi(\underline{E}\,|\,\underline{p})\,\pi(\underline{p})$$

$\pi(\underline{p})$: Assumed PDF for PDF's parameters

Prior

Bayesian inference

$\underline{E}=[E_1\ E_2\ \ldots\ E_{20}]$: identified Young's moduli

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Bayes' theorem:

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$\pi(\underline{p})$: Assumed PDF for PDF's parameters
Prior

$\pi(\underline{E}|\underline{p})$: Possibility to obtain $\underline{E}$, for given $\underline{p}$
Likelihood

Bayesian inference

Likelihood

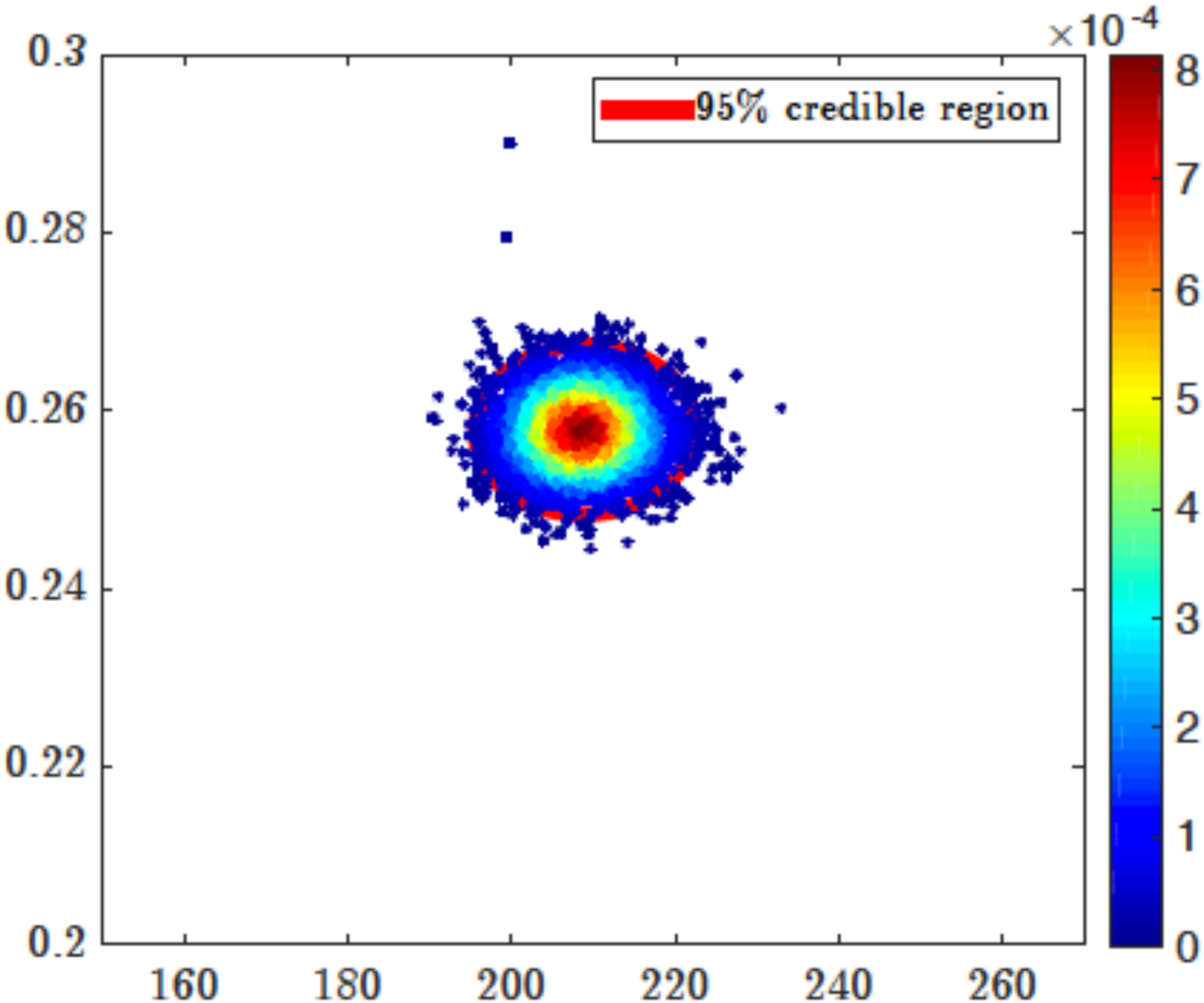
$$\pi(\underline{E}\,|\,\underline{p}) = \prod_{i=1}^{20} \pi_{beta}(E_i\,|\,\underline{p})$$

Posterior

$$\pi(\underline{p}\,|\,\underline{E}) \propto \pi(\underline{E}\,|\,\underline{p})\,\pi(\underline{p})$$

Sample with Monte Carlo Markov chain

Monte Carlo Markov chain



Identification scheme for 1 parameter

1. Harvest and test fibres

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HOW?? Bayesian inference

Identification scheme for several parameters

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3. Identify parameters of *<u>each</u>* material parameter PDF

HOW?? Bayesian inference

Identification scheme for several parameters

1. Harvest and test fibres

result: 20 stress-strain curves

2. Identify material parameters of each fibre

result: 20 Young's moduli, stored in $\underline{E}$

3. Identify parameters of *each* material parameter PDF

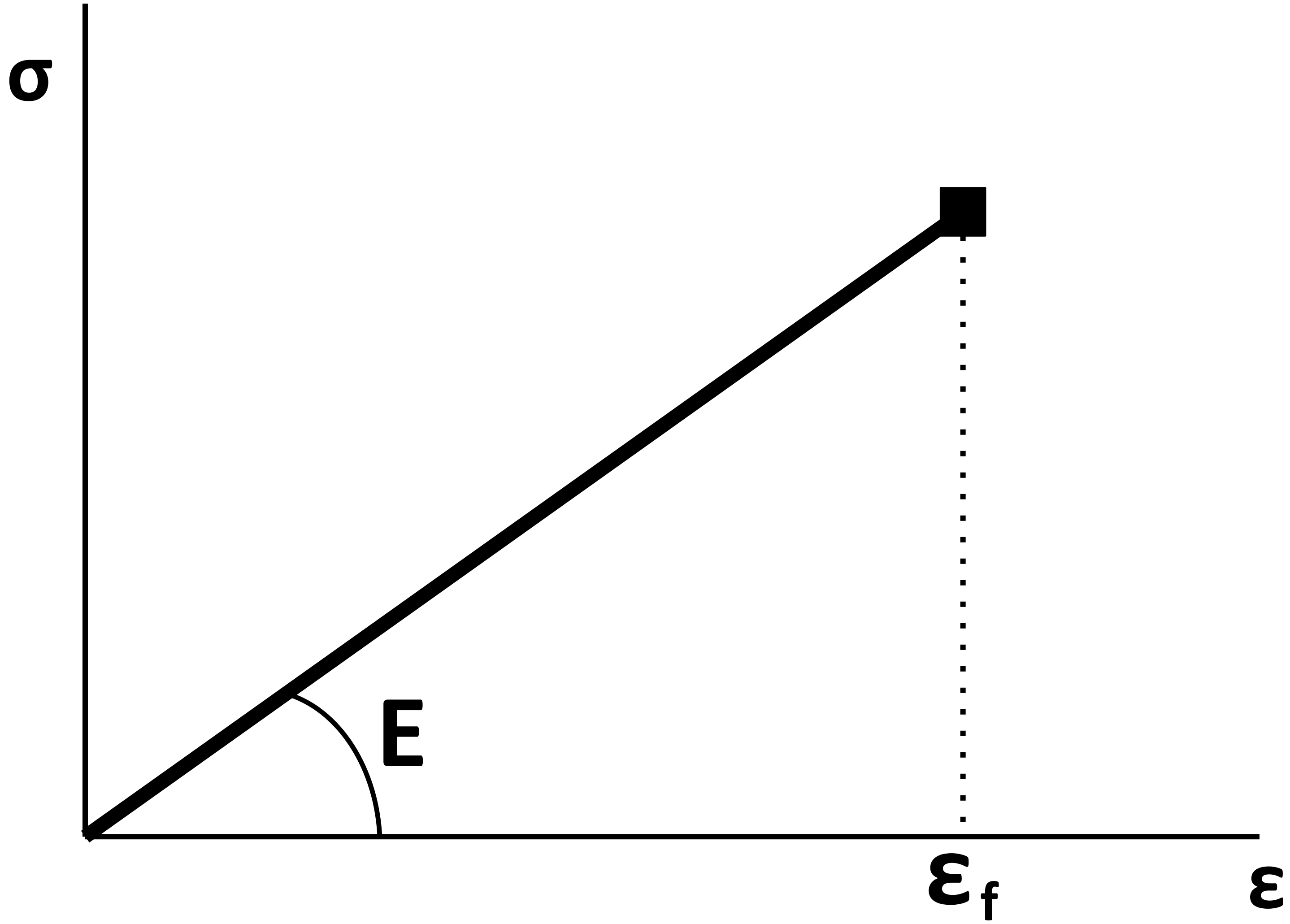
HOW?? Bayesian inference

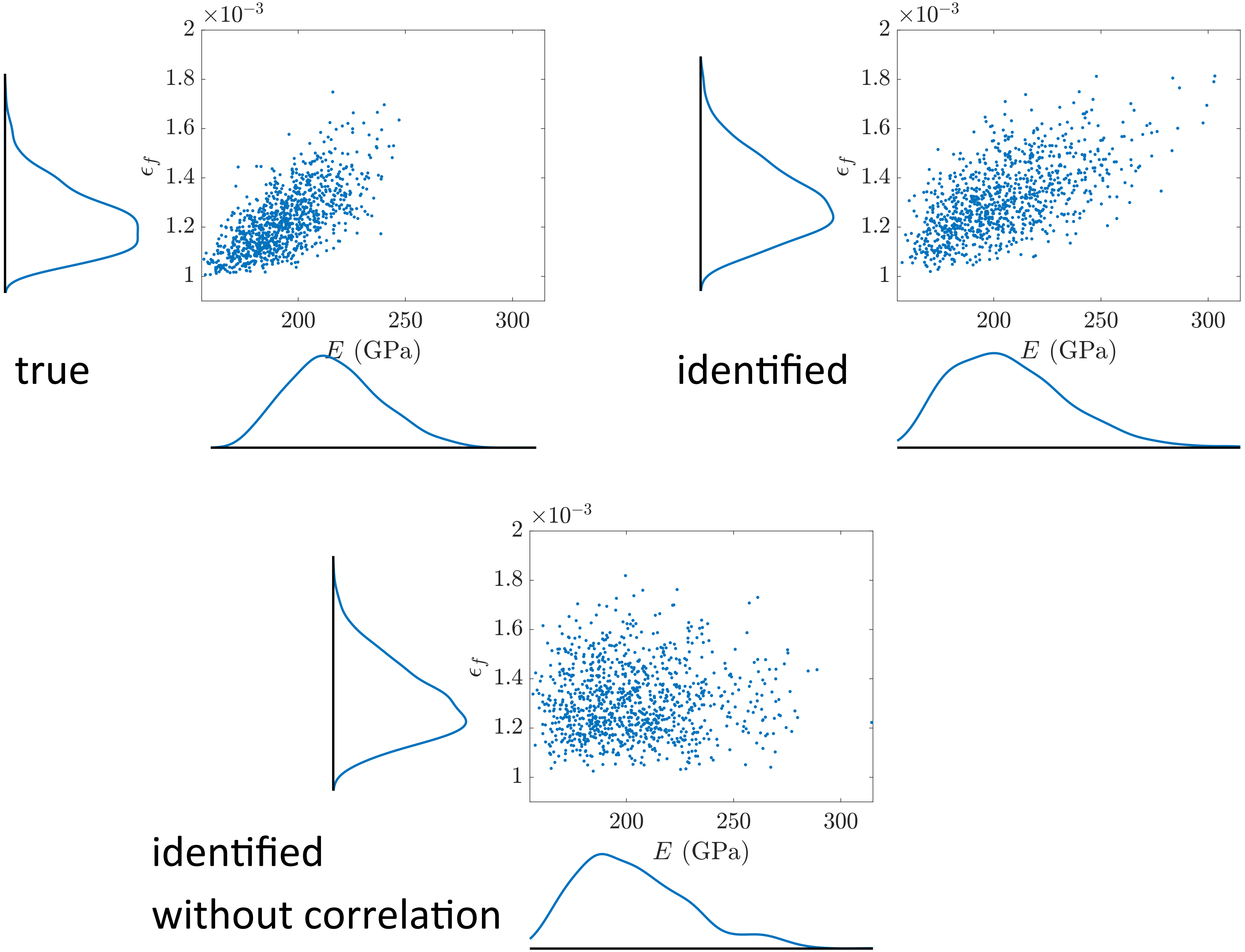
4. Identify the parameters of the copula that *couples* all material parameter PDFs in one joint PDF.

HOW?? Bayesian inference

Identification results: 2 parameter model

Brittle damage

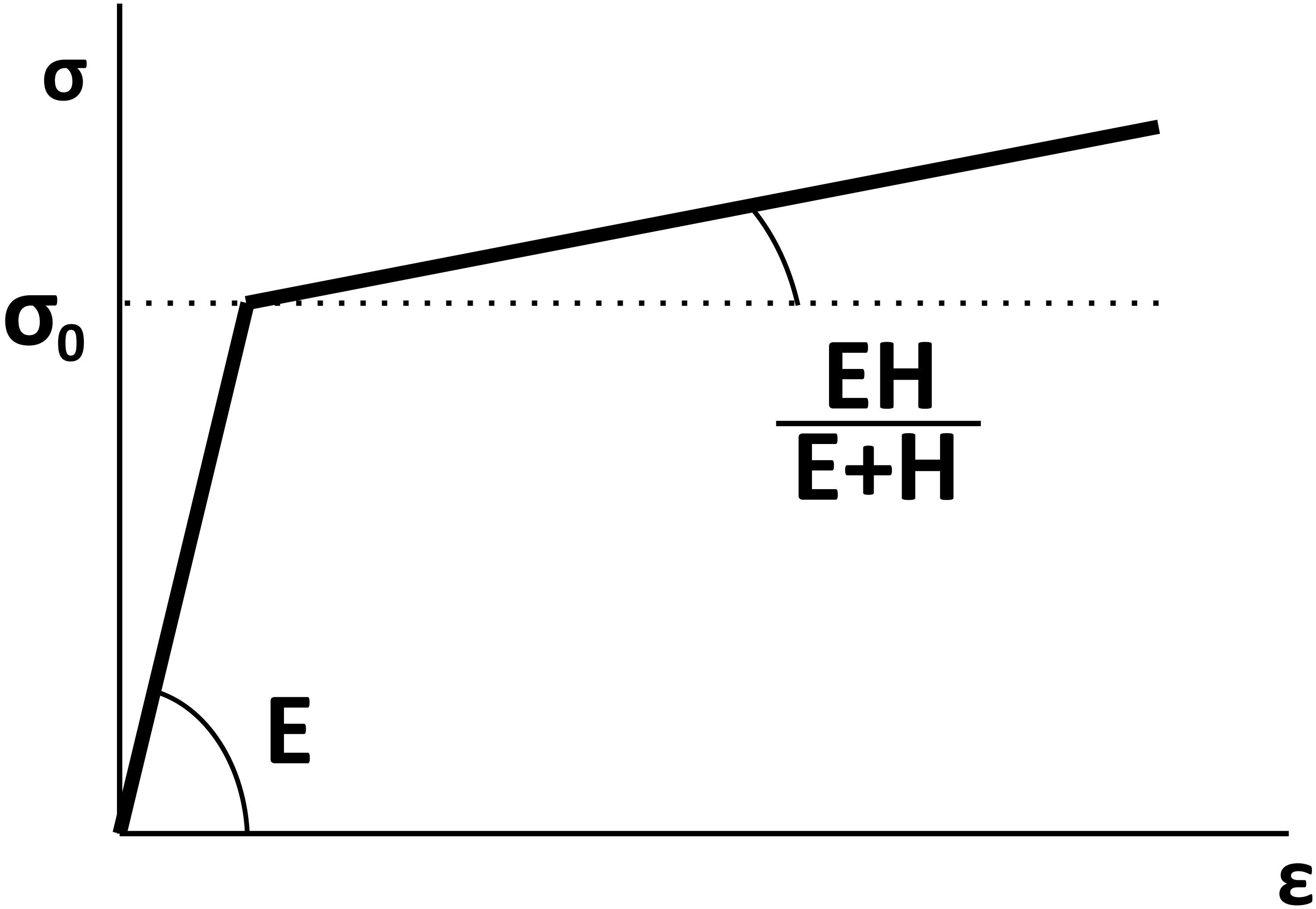


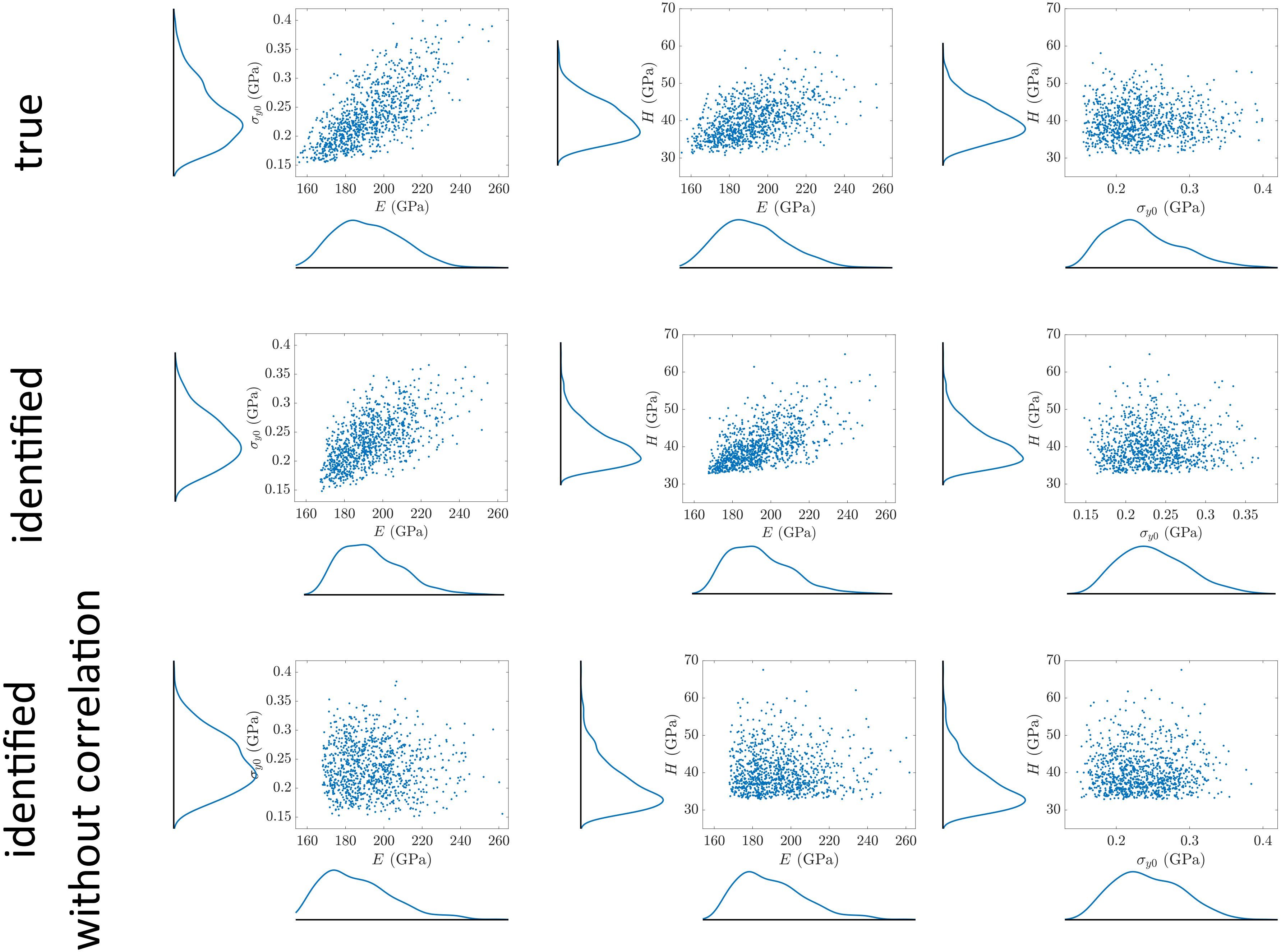


×10^-3
2
1.8
1.6
1.4
1.2
1
ε_f
200
250
300
E (GPa)
true
identified
identified
without correlation

Identification results: 3 parameter model

Elastoplasticity

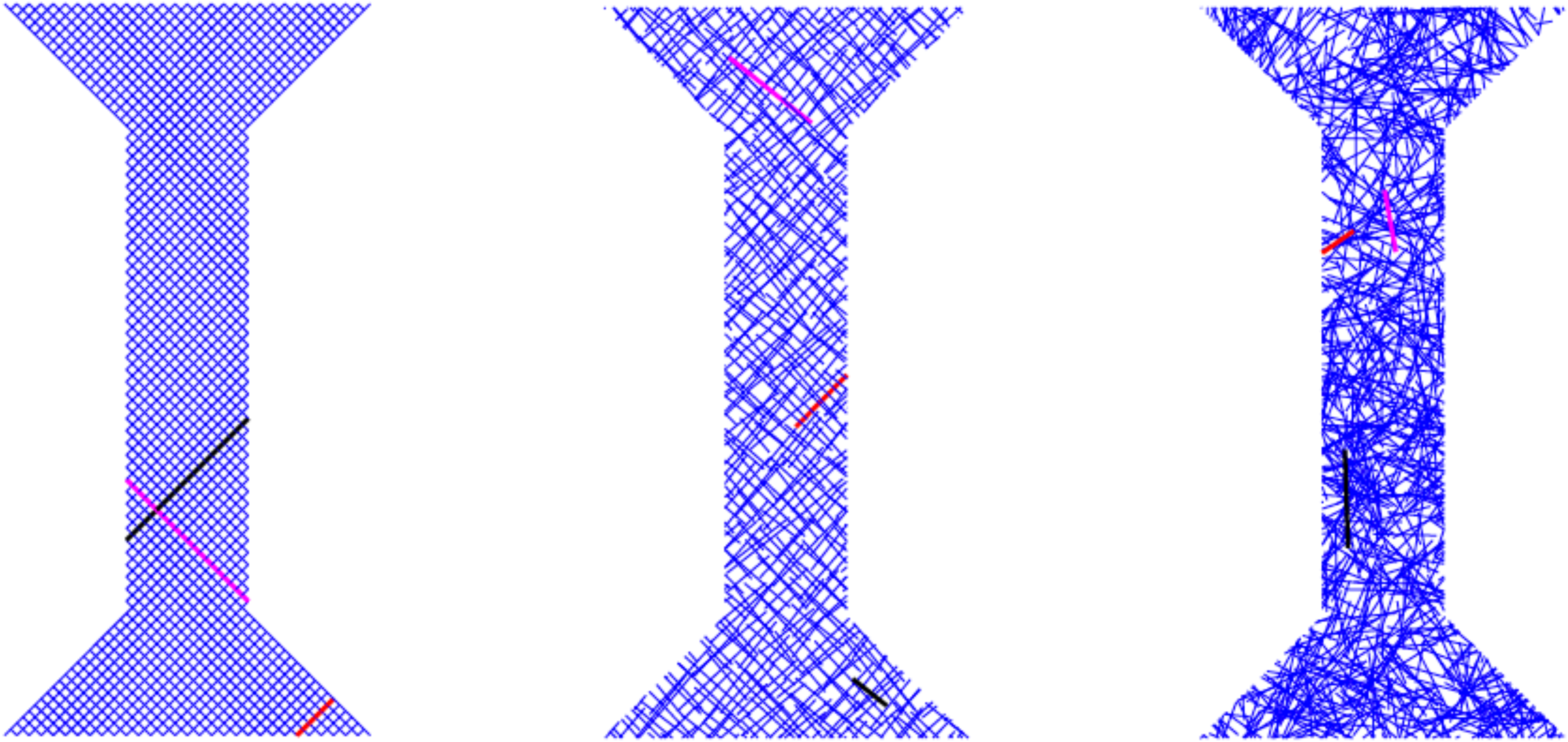




true
identified
identified
without correlation
σ_{y0} (GPa)
E (GPa)
H (GPa)

Influence of geometrical randomness

Geometrically Euler Bernoulli beam networks

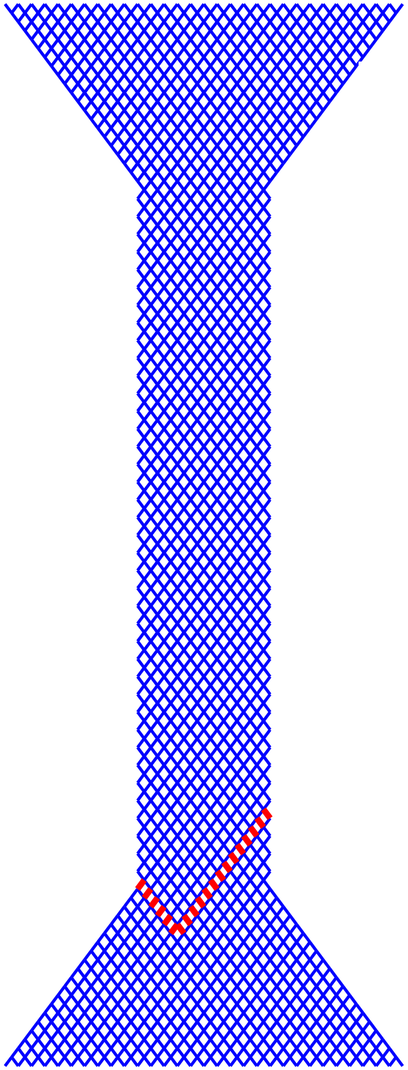


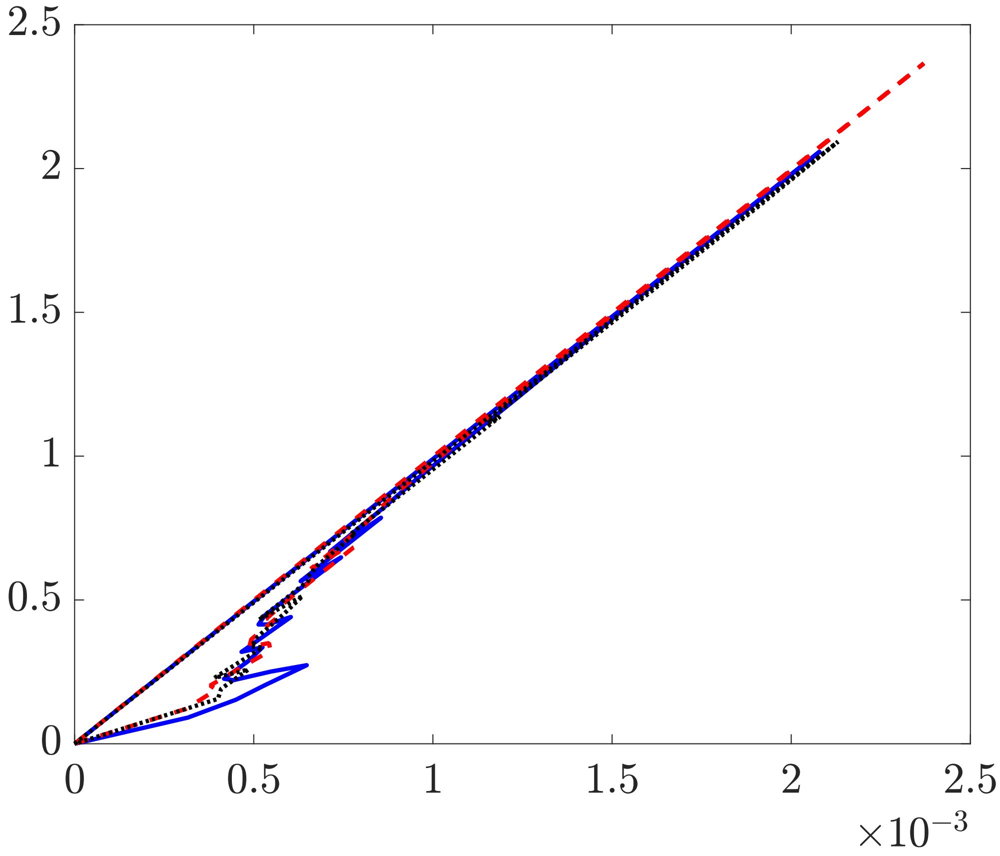
A: random fibre assignments

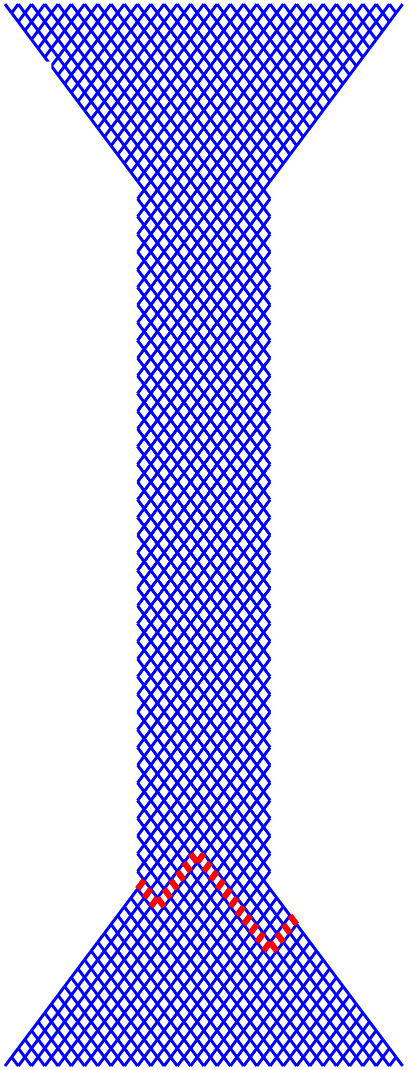
B: partially random

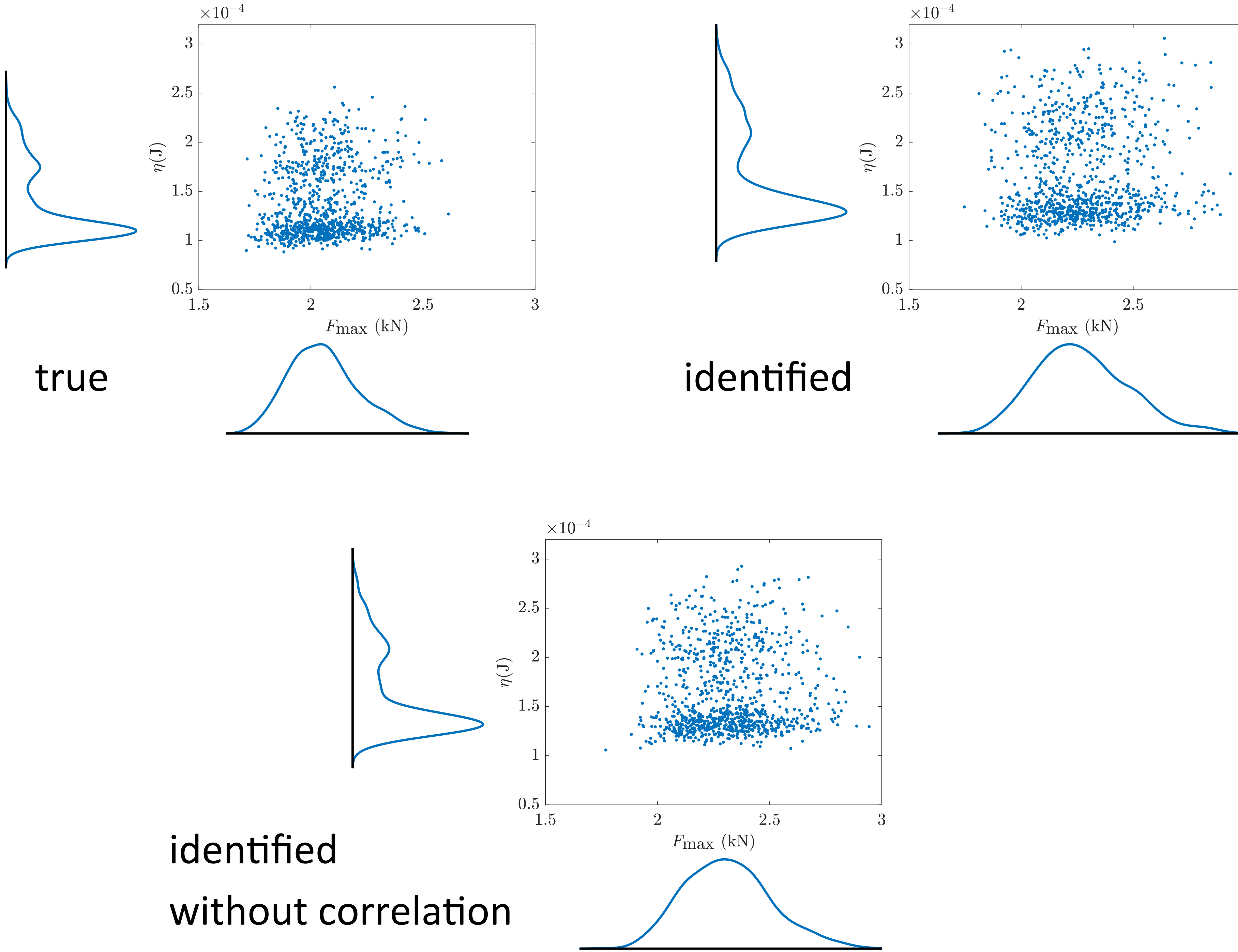
C: random

Results: Damage A



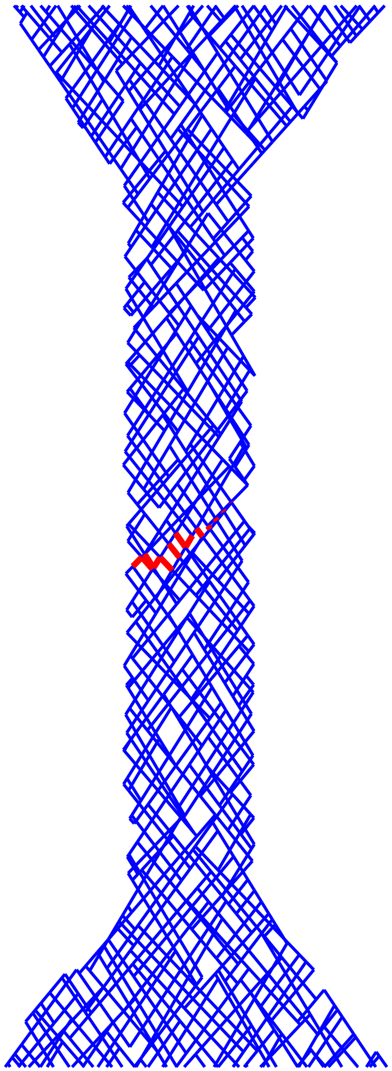


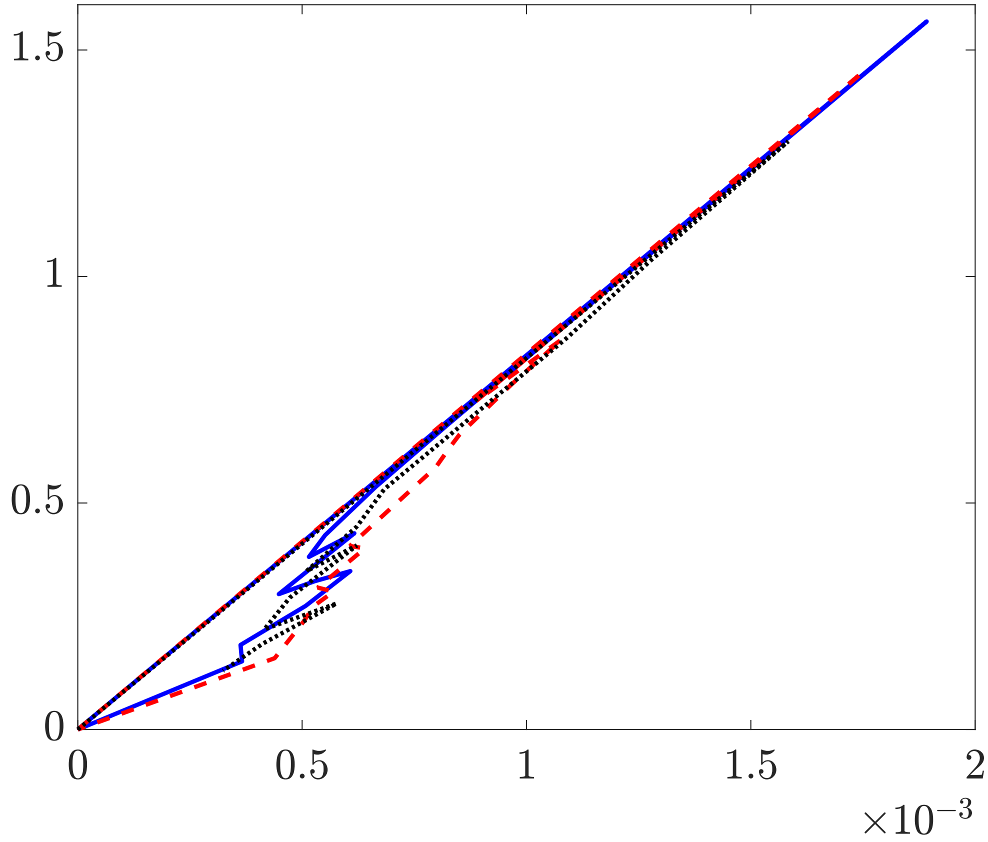


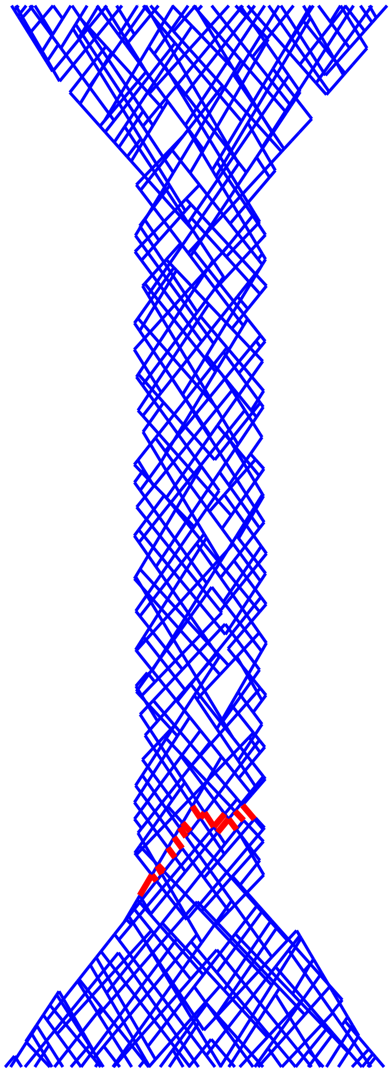


true
$\times 10^{-4}$
η(J)
$F_{\max}$ (kN)
identified
$\times 10^{-4}$
η(J)
$F_{\max}$ (kN)
identified
without correlation
$\times 10^{-4}$
η(J)
$F_{\max}$ (kN)

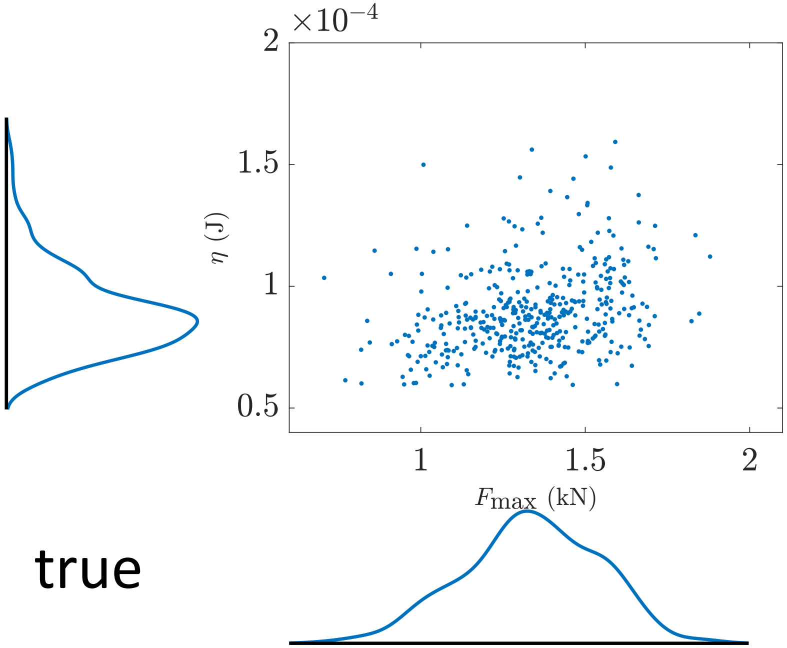
Results: Damage B (long fibres)



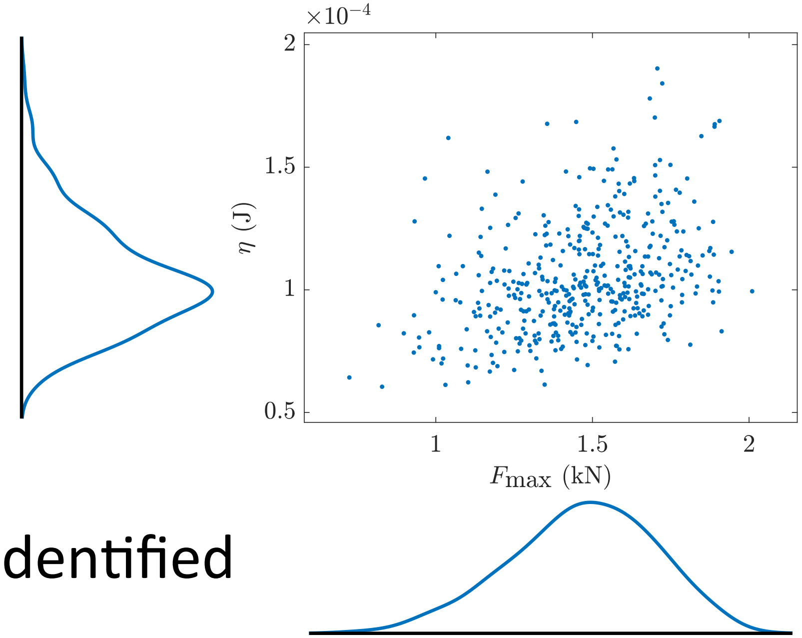


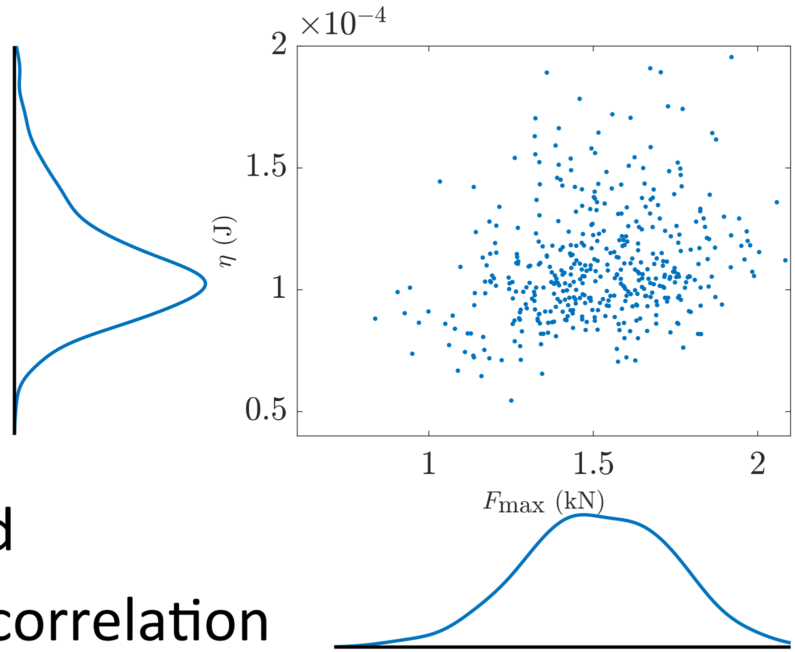


true



identified

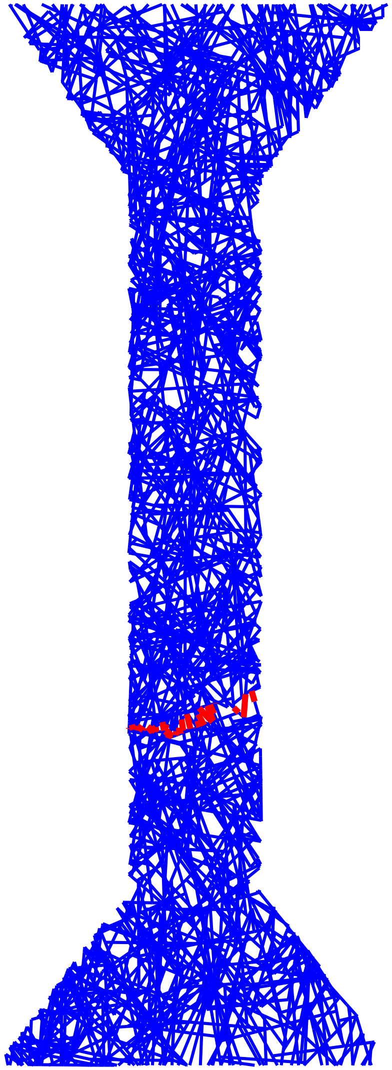


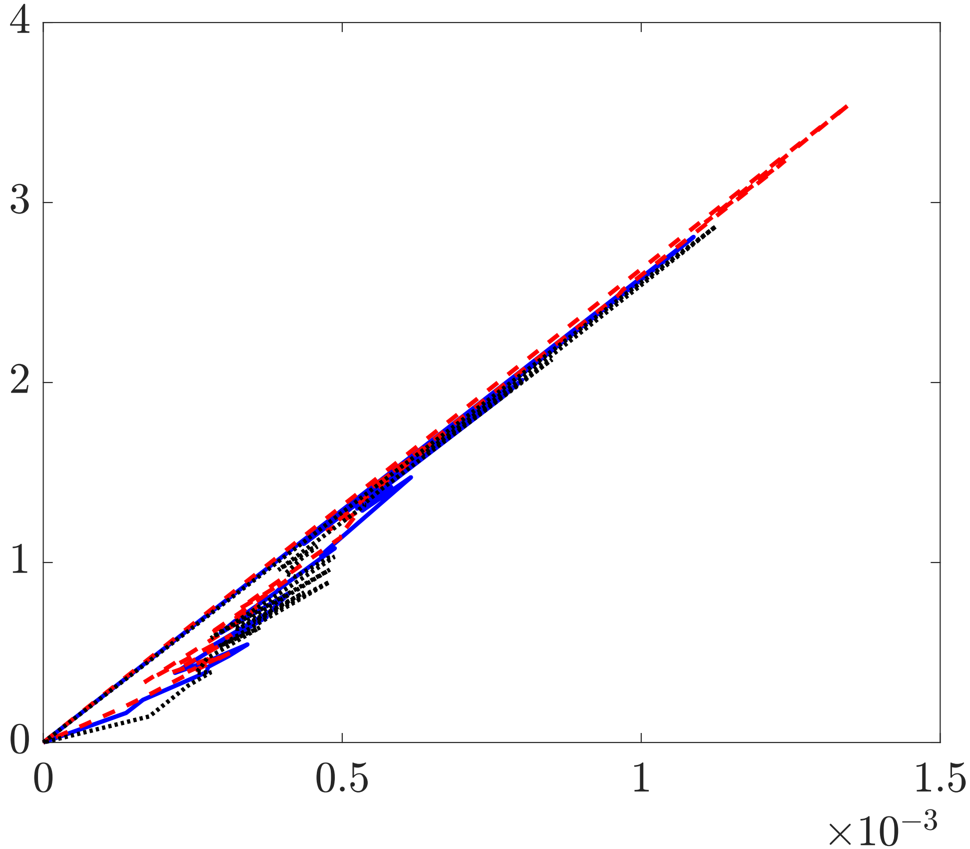


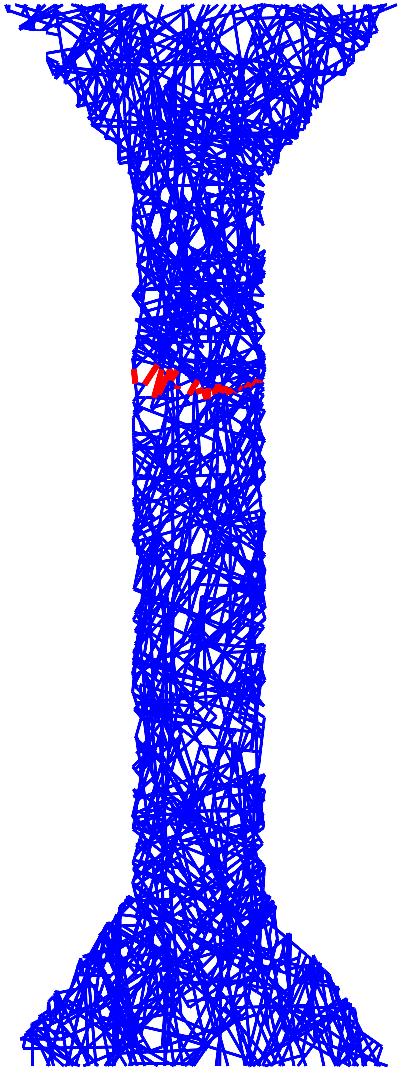
identified

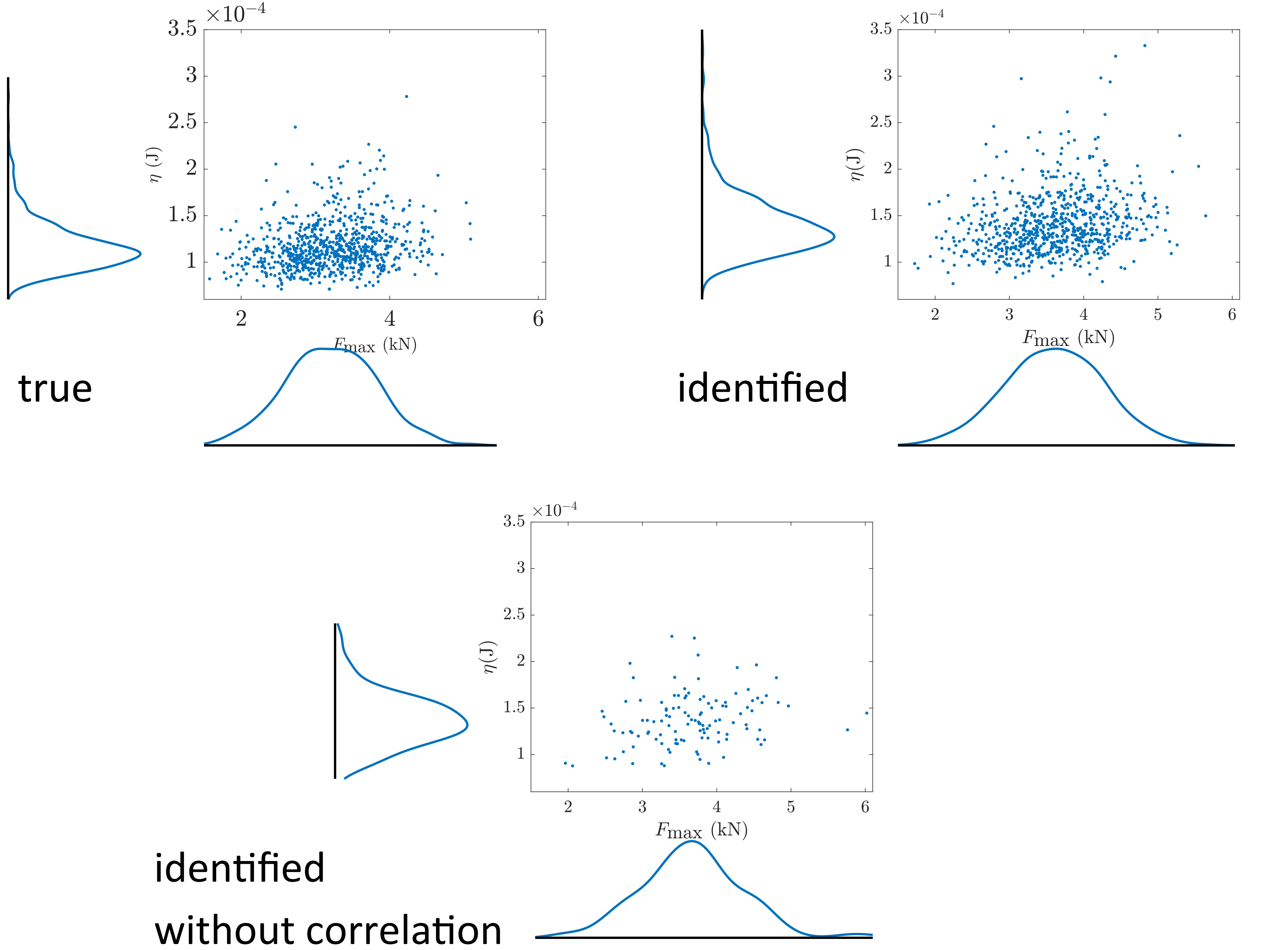
without correlation

Results: Damage C

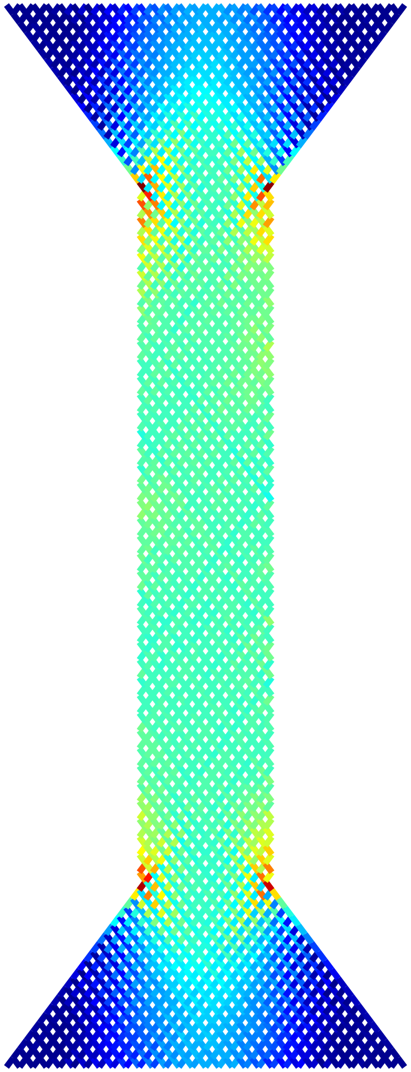


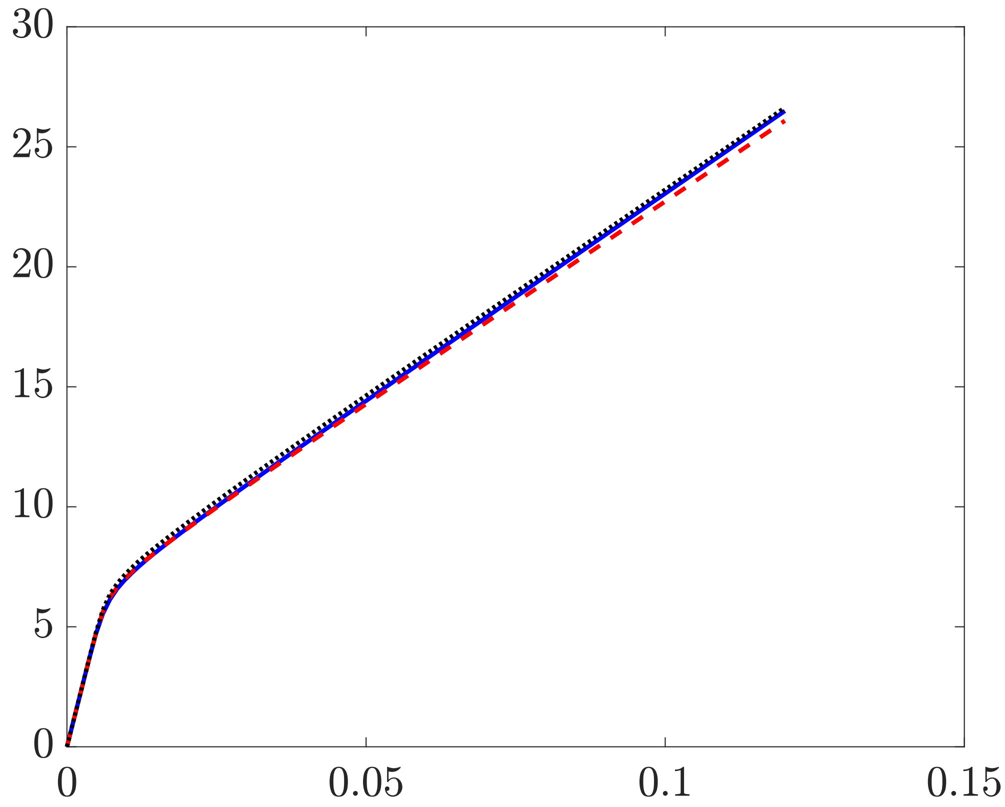


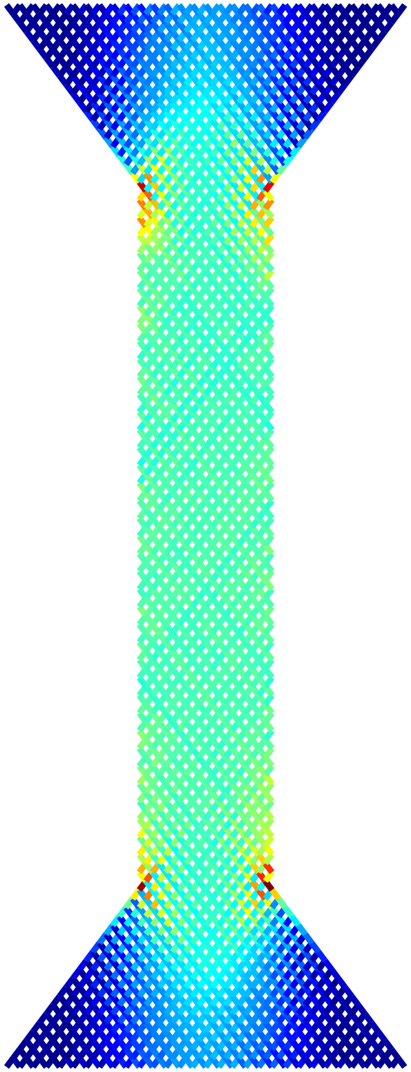


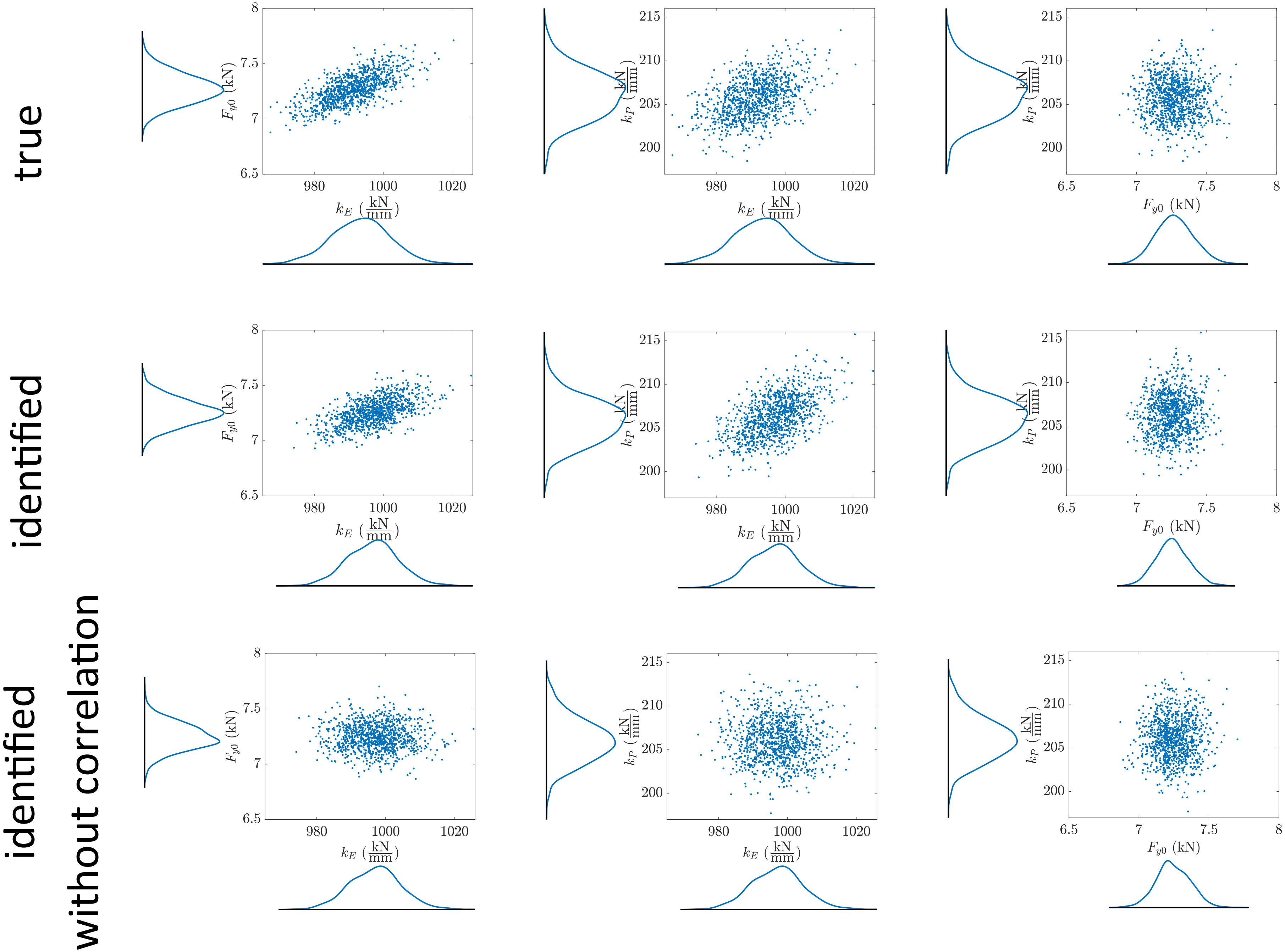

$\times 10^{-4}$
3.5
3
2.5
2
1.5
1
η (J)
2
4
6
$F_{\max}$ (kN)
true
$\times 10^{-4}$
3.5
3
2.5
2
1.5
1
η(J)
2
3
4
5
6
$F_{\max}$ (kN)
identified
$\times 10^{-4}$
3.5
3
2.5
2
1.5
1
η(J)
2
3
4
5
6
$F_{\max}$ (kN)
identified
without correlation

Results: Elastoplasticity A

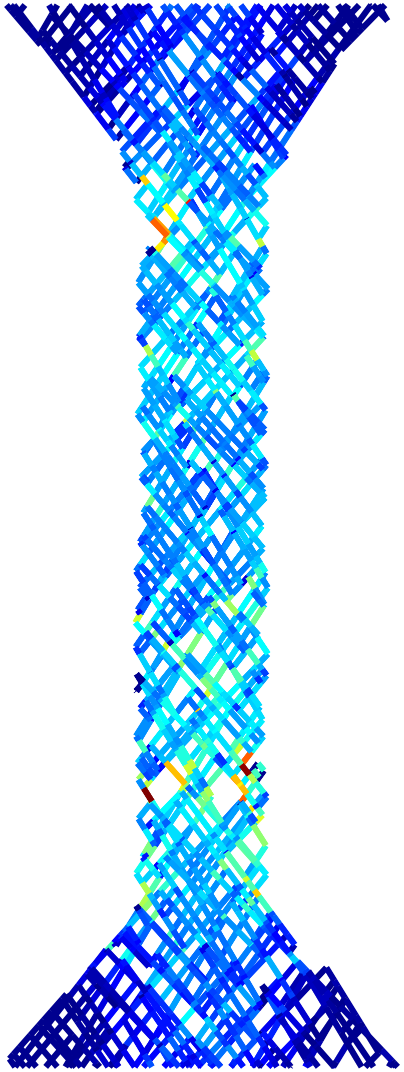


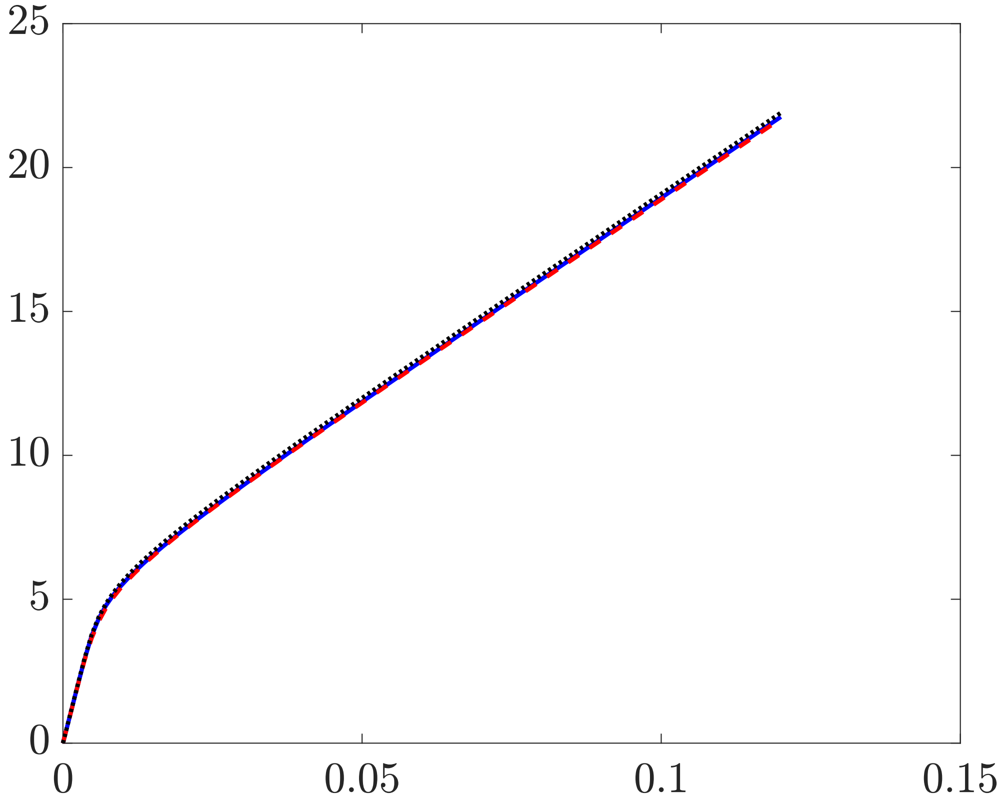


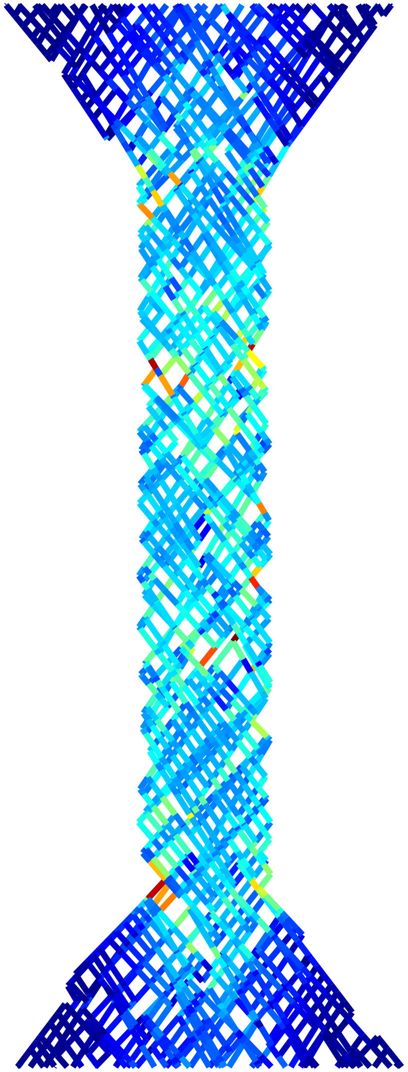


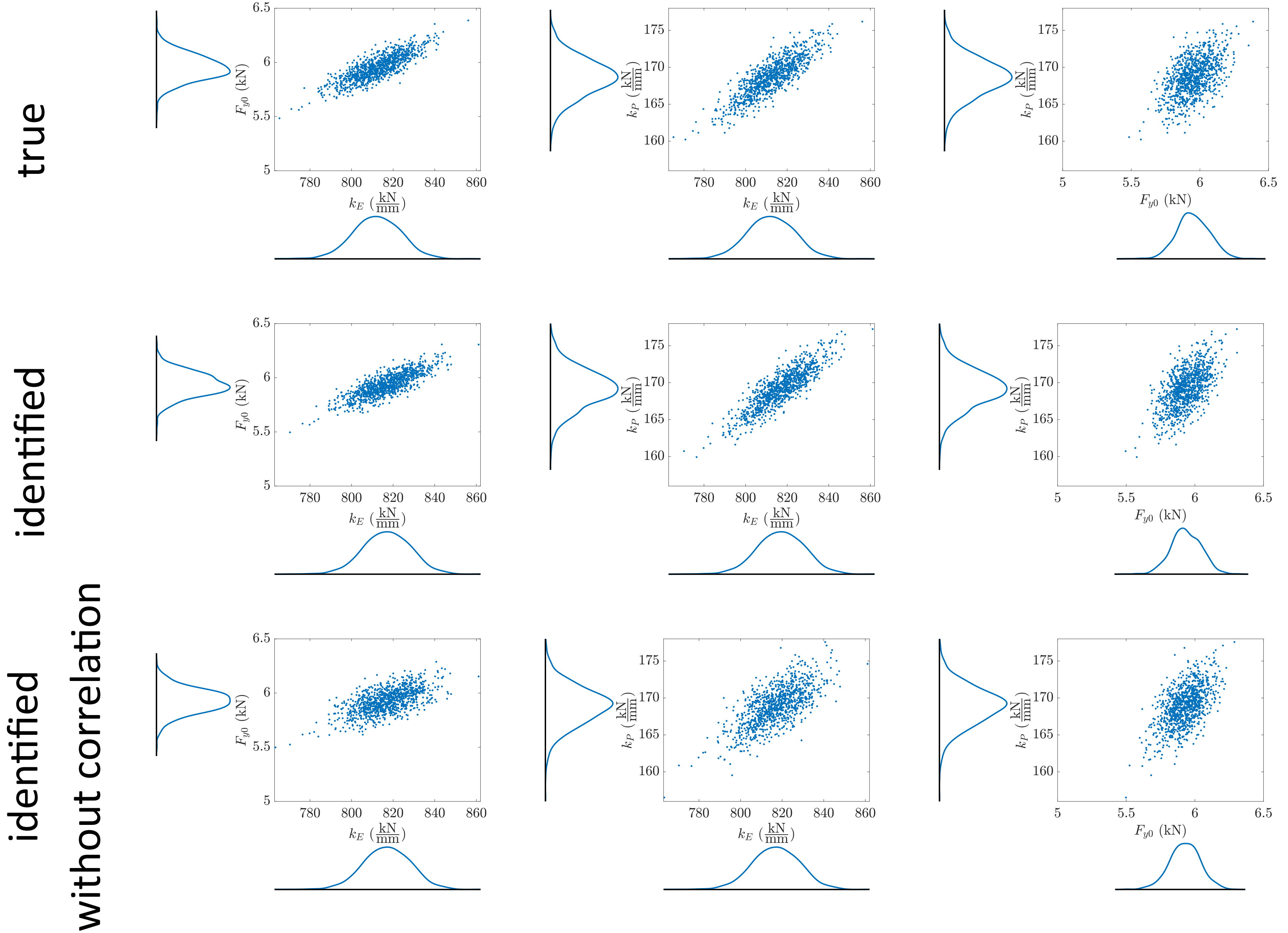

true
identified
identified
without correlation
F_{y0} (kN)
k_E ($\frac{\mathrm{kN}}{\mathrm{mm}}$)
k_P ($\frac{\mathrm{kN}}{\mathrm{mm}}$)
F_{y0} (kN)
6.5
7
7.5
8
980
1000
1020
200
205
210
215

Results: Elastoplasticity B



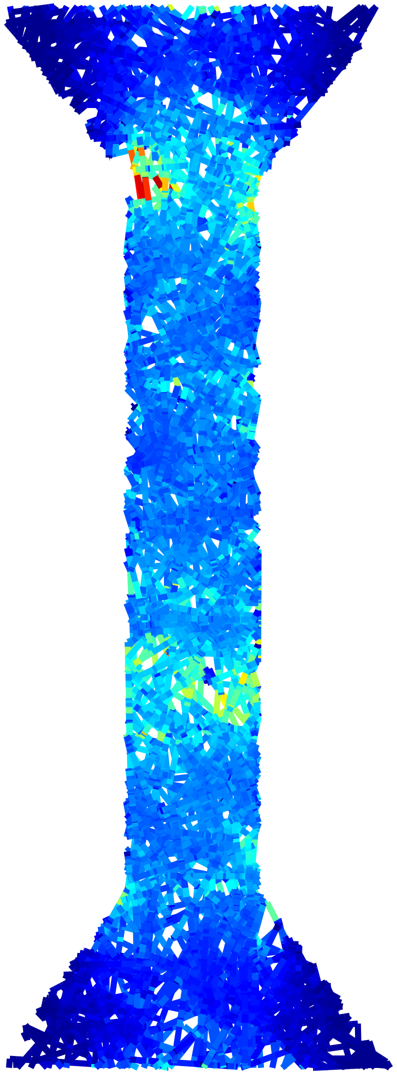


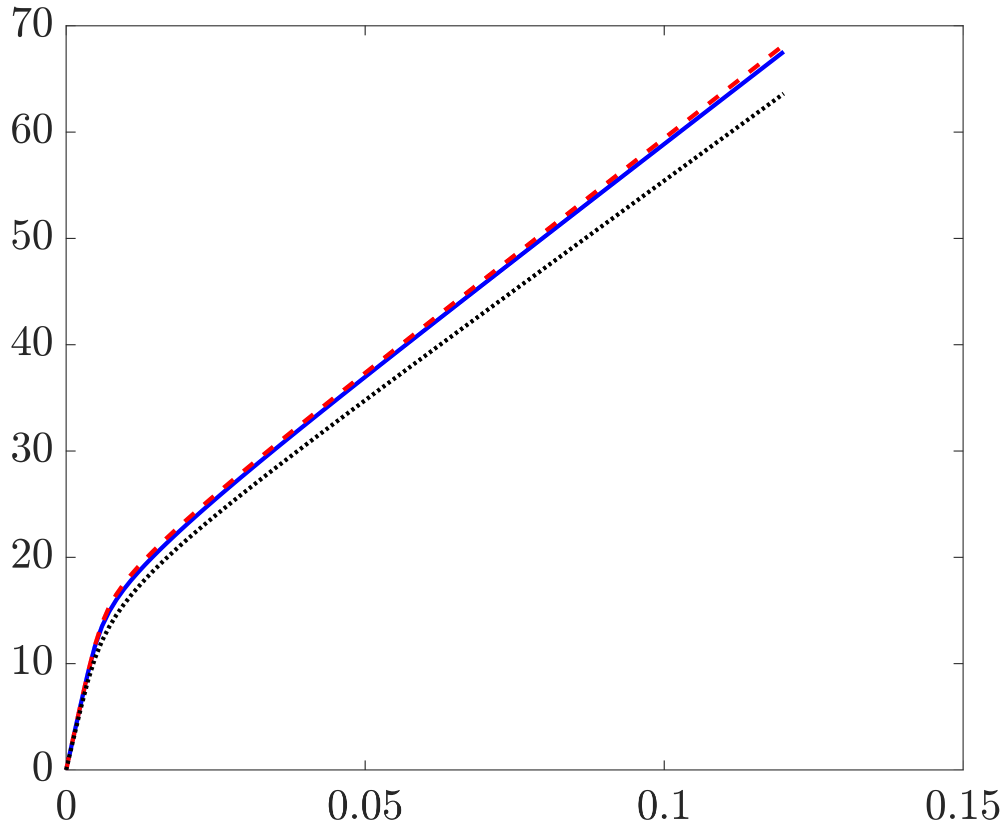


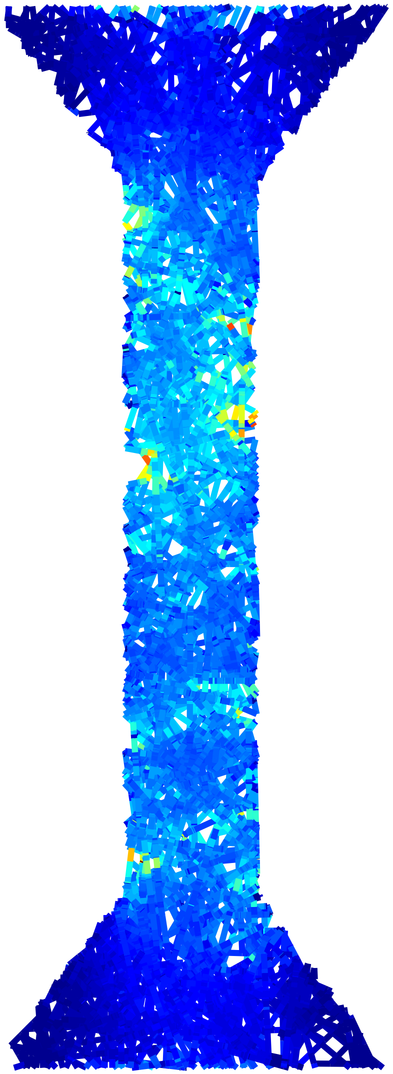


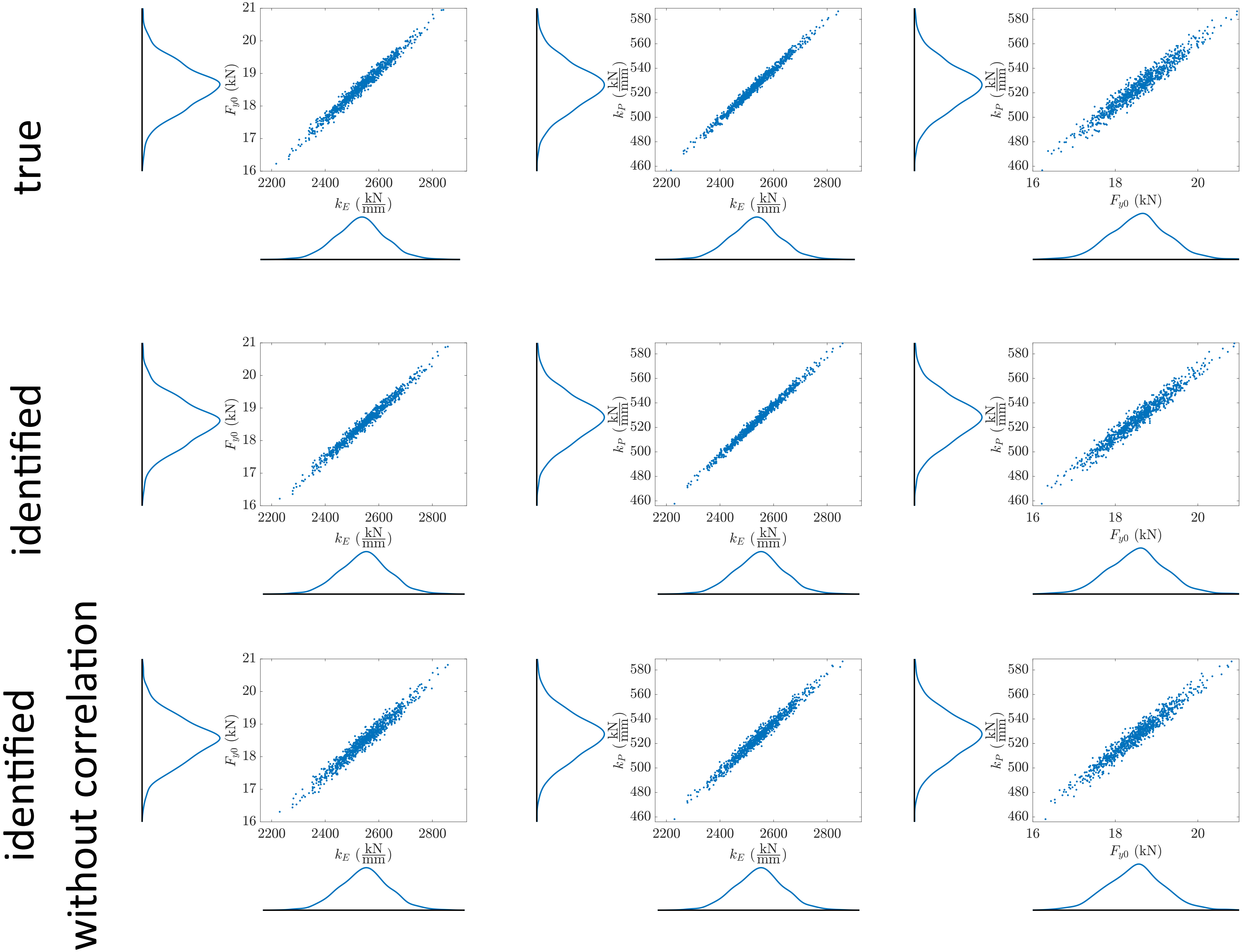
true
identified
identified
without correlation
F_{y0} (kN)
k_E $(\frac{kN}{mm})$
k_P $(\frac{kN}{mm})$

Results: Elastoplasticity C









true
identified
identified without correlation
F_{y0} (kN)
k_P $(\frac{\mathrm{kN}}{\mathrm{mm}})$
k_E $(\frac{\mathrm{kN}}{\mathrm{mm}})$

Summary

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1. Harvest and test fibres

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2. Identify material parameters of each fibre

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Bayesian inference

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Including the correlation between material parameters improves accuracy

Conclusion

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Damage: including correlation has no influence

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Elastoplasticity

Correlation is less important for more geometrical randomness

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Elastoplasticity

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Correlation is more important for long fibres (not shown)

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Elastoplasticity

Correlation is less important for more geometrical randomness

Correlation is more important for long fibres (not shown)

Correlation is more important for large fibre densities (not shown)