# Clones of pivotally decomposable operations

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Several classes of operations $f\colon A^n \to A$ admit a decomposition scheme

$$f(\mathbf{x}) \;=\; \mathit{\Pi}(\mathbf{x}_k, f(\mathbf{x}_k^1), f(\mathbf{x}_k^0)), \qquad k \leq n, \tag{1}$$

where $\mathit{\Pi}\colon A^3 \to A$ and $\{0,1\} \subseteq A$, and where $\mathbf{x}_k^c$ denotes the $n$-tuple obtained from $\mathbf{x}$ by substituting its $k$-th component by $c \in A$. Shannon decomposition (or Shannon expansion) [2,5] is an instance of (1) for Boolean functions, where $\mathit{\Pi}(x,y,z) = xy + (1-x)z$. More recent examples include the class of polynomial operations over a bounded distributive lattice. They are decomposable [3] as in (1) for $\mathit{\Pi}(x,y,z) = (x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$.

More generally, if $\mathit{\Pi}\colon A^3 \to A$, we say that a class of operations on $A$ is *$\mathit{\Pi}$-decomposable* [4] if it is the class of those operations that satisfy (1). In this talk, we investigate those $\mathit{\Pi}$-decomposable classes of operations that constitute clones. The results presented in this talk are published in [1].

## References

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