

$$\mathcal{L} = \frac{1}{2} \left( \frac{R}{r} - \frac{r}{R} \right)$$