

Associative and Quasitrivial Operations on Finite Sets: Characterizations and Enumeration

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We investigate the class of binary associative and quasitrivial operations on a given finite set. Here the quasitriviality property (also known as conservative-ness) means that the operation always outputs one of its input values. We also examine the special situations where the operations are commutative and nondecreasing, in which cases the operations reduce to discrete uninorms (which are discrete fuzzy connectives playing an important role in fuzzy logic).

Interestingly, associative and quasitrivial operations that are nondecreasing are characterized in terms of total and weak orderings through the so-called single-peakedness property introduced in social choice theory by Duncan Black.

We also address and solve a number of enumeration issues: we count the number of binary associative and quasitrivial operations on a given finite set as well as the number of those operations that are commutative and/or nondecreasing.

References

1. Couceiro, M., Devillet, J., Marichal, J.-L.: Quasitrivial semigroups: characterizations and enumerations. *Semigroup Forum*, <https://doi.org/10.1007/s00233-018-9928-3> (2018).