## On associative, idempotent, symmetric, and nondecreasing operations

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## Abstract

The study of aggregation functions defined on finite ordinal scales (i.e., finite chains) encounters an increasing interest since the last decades (see, e.g., [1-3, 5, 7, 9-16]). Among these functions, discrete t-norms, t-conorms, uninorms, and nullnorms are binary operations that play an important role in fuzzy logic. In particular, these operations share the properties of being associative, symmetric, and nondecreasing (in each variable).

It is known that the class of associative, idempotent, and symmetric binary operations is in one-to-one correspondence with the class of partial orders of semilattices (see, e.g., [6]). We provide a full description of the class of associative, idempotent, symmetric, and nondecreasing binary operations defined on a finite chain in terms of properties of the Hasse diagram of the corresponding semilattice. In particular, given an operation belonging to the latter class, we provide a recursive construction of the corresponding semilattice. We also provide an associativity test for idempotent, symmetric, and nondecreasing operations. Moreover, the enumeration of the class of associative, idempotent, symmetric, and nondecreasing operations leads to a new occurrence of the Catalan numbers and provides another construction of this sequence.

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