Workshop on Supergeometry and Applications

University of Luxembourg – December 14-15, 2017

Invited lecturers

Andrew Bruce (University of Luxembourg)
Steven Duplij (University of Münster)
Rita Fioresi (University of Bologna)
Janusz Grabowski (Polish Academy of Sciences, Warsaw)
Florian Hanisch (University of Potsdam)
Hovhannes Khudaverdian (University of Manchester)
Norbert Poncin (University of Luxembourg)
Zoran Skoda (University of Zagreb)

Topics

The aim of the meeting is to bring together mathematicians and physicists working in supergeometry. We plan a more informal event consisting of invited talks on current research and new trends in the field, including the recently developed colored supergeometry, its integration theory and applications. There will be enough time available for discussions between the participants.

Organizers

Andrew Bruce and Norbert Poncin

Sponsors

The workshop was supported by the University of Luxembourg and its Mathematics Research Unit.

Talks

Andrew Bruce (University of Luxembourg)

On a Z_2^n- graded version of Minkowski superspace

We show one possible way of extending the notion of Minkowski superspace to the setting of Z_2^n - Geometry. The resulting Z_2^n - manifold formally resembles N-extended superspace (with central charges) but there are subtle differences due to the exotic nature of the grading employed.

Steven Duplij (University of Münster)

Developing new supermanifolds by revitalizing old ideas

First, we apply the regularity concept for supermanifolds and obtain so called obstructed (or regular) supermanifolds (semi-supermanifolds), generalizing von Neumann regularity to n-regularity. In the patch definition, the transition functions become noninvertible in some sense and satisfy special cocycle conditions described by semi-commutative diagrams. The obstruction, which measures the distinction from the standard supermanifold, is proportional to the difference of some idempotent self-maps from identical morphisms. The same procedure of regularization is applied for categories and functors. Second, we introduce another noninvertible analog of superconformal transformations which arise from the alternative possibility of supermatrix reduction in the simplest one-grade case. They are always noninvertible, having nilpotent Berezinian and twist parity of the tangent space together with the so called mixed cocycle condition. For N=2 the role of superjacobian is played by permanent, also the remarkable formula connected Berezinian, permanent and determinant is obtained. Third, we consider some aspects of ternary supersymmetry. Fourth, we introduce the ring of polyadic integer numbers, which leads to unusual polyadic fields. The ways of generalization for the above constructions to the multigraded supersymmetry is outlined.

Rita Fioresi (University of Bologna)

Admissible systems and Harish-Chandra representations for semisimple real Lie supergroups

Real forms of classical Lie superalgebras are in one to one correspondence with Cartan automorphisms and Cartan decompositions. We introduce the notion of admissible root system with respect to a given Cartan automorphism and we show that the condition of admissibility is equivalent to the existence of infinite dimensional holomorphic representations of the real Lie supergroups underlying the given real form. We briefly discuss generalization to Lie superalgebras infinite dimensional and to Kostant root systems.

Janusz Grabowski (Institute of Mathematics, Polish Academy of Sciences)

Splitting theorem for colored supermanifolds

The classical Batchelor-Gawedzki theorem says that any smooth supermanifold is (non-canonically) diffeomorphic to the `superization' ΠE of a vector bundle E. It is also known that this result fails in the complex analytic category. Hence, it is natural to ask whether an analogous statement goes through in the category of colored supermanifolds (Z_2^n - supermanifolds) with its local model made of formal power series with Z_2^n - gradation and Z_2^n - commutation rules. We show that any smooth colored supermanifold is (non-canonically) diffeomorphic to the `superization' ΠE of an n-fold vector bundle E. The latter can be chosen split.

Florian Hanisch (University of Potsdam)

Berezin integration on super loop space and applications to index theory

The super loop space associated to an ordinary (spin) manifold is the space of ``maps'' from the super circle $S^{1|1}$ to M. We will briefly describe its infinite-dimensional supermanifold structure and show that the resulting algebra functions is closely related to the algebra of differential forms on ordinary loop space. In particular, it is possible to relate the action of $S^{1|1}$ on superfunctions to the equivariant differential on forms. Based on the Wiener integral, it is possible to define an analogue of Berezin integration. Similar to the finite-dimensional case, it turns out that the odd part of this integral may be expressed by an infinite-dimensional Pfaffian, the latter being defined using zeta-regularization. If time allows, we will eventually describe how these concepts can be used to obtain ``path integral-type'' proofs of index theorems by integrating Bismut-Chern characters. The argument indicates that supersymmetry enters these calculations in a crucial way.

This talk is based on joint work (partly in progress) with Matthias Ludewig.

Hovhannes Khudaverdian (University of Manchester)

Thick morphisms and higher Koszul brackets

It is a classical result in Poisson geometry that the cotangent bundle of a Poisson manifold has the structure of a Lie algebroid. One of the manifestations of this structure is the ``Koszul bracket'' of differential forms. There is a natural homomorphism from the resulting differential Lie superalgebra into the superalgebra of multivector fields with respect to the canonical Schouten bracket and the Lichnerowicz differential.

In the talk, we shall present a homotopy analog of the above results. When an ordinary Poisson structure is replaced by a homotopy one, instead of a single Koszul bracket there arises an infinite sequence of ``higher Koszul brackets'' introducing an L_∞-algebra structure in the space of differential forms (Khudaverdian-Voronov, 2008). We shall show how to use thick morphisms

of supermanifolds to construct a non-linear transformation, which is an L_∞-morphism, from this L_∞-algebra of differential forms to the Lie superalgebra of multivector fields with the canonical Schouten bracket. (A thick morphism of manifolds is a new notion recently introduced, see e.g. http://arxiv.org/abs/1411.6720v5.)

The talk is based on joint work with Th. Voronov.

Norbert Poncin (University of Luxembourg)

Higher supergeometry revisited

The aim of the talk is to present a generalization of superalgebra and supergeometry to Z_2^n - gradings, Z_2^n = Z_2 x … x Z_2, n>1. The corresponding sign rule is not given by the product of the parities, but by the scalar product of the involved Z_2^n - degrees. This Z_2^n - supergeometry exhibits interesting differences with classical supergeometry, provides a sharpened viewpoint, and has better categorical properties. Further, it is closely related to Clifford calculus: Clifford algebras have numerous applications in Physics, but the use of Z_2^n - gradings has never been investigated. In particular, the Z_2^n - Berezinian determinant and the corresponding integration theory will be discussed.

Zoran Škoda (University of Zadar)

Symmetry objects for geometries based on monoidal categories

As the basic organizing principle of supersymmetry, as well as of its Z_2^n refinement, one often emphasizes that all algebraic constructions live in an appropriate symmetric monoidal category. In the context of monoidal categories one studies the symmetry objects like supergroups and their anyonic and braided analogues. Using actions of these symmetry objects, one comes to important examples of geometries, the superhomogeneous and some quantum homogeneous spaces. After presenting some generalities on symmetries in the context of monoidal categories, I will focus on attempts at combining the infinitesimal symmetries or differential operators with the coordinate superalgebra of the space into the analogues of groupoids as novel symmetries in this context. Main examples comprise super-Hopf algebroids. I will also explain their relevance in the deformation quantization.

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Zoran Škoda (Zareb)

VENUE

The workshop will take place at the University of Luxembourg's Belval campus.

KEY TOPICS

Supergeometry
$\mathbb{Z}_2^n$-geometry
Applications

ORGANISERS

Norbert Poncin (University of Luxembourg)
Andrew Bruce (University of Luxembourg)

CONTACT

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WEBPAGE

http://math.uni.lu/SuperWork/Supergeometry.html

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Mathematics Research Unit (RMATH)