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volatility: Evidence from
Germany and the US**

Louis Chauvel, Anne Hartung, Flavia Palmisano

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German Socio-Economic Panel (SOEP)
DIW Berlin
Mohrenstrasse 58
10117 Berlin, Germany

Contact: soeppapers@diw.de



Dynamics of income rank volatility: Evidence from Germany and the US

Louis Chauvel

University of Luxembourg

Anne Hartung

University of Luxembourg

Flaviana Palmisano

University of Rome

Abstract

This paper presents a methodology for comparing income rank volatility profiles over time and across distributions. While most of the existing measures are affected by changes in marginal distributions, this paper proposes a framework that is based on individuals' relative positions in the distribution, and is neutral in relation to structural changes that occur in the economy. Applying this approach to investigate rank volatility in Germany and the US over three decades, we show that while poorer individuals within both countries are the most volatile, the volatility trend for the middle class in each of these countries differs.

Keywords: rank volatility, income, risk, inequality, middle class, Germany, US.

JEL codes: D31, J6, I30.

I. INTRODUCTION

In recent decades, rising inequality in many industrialized countries has been a disquieting fact that has prompted a renewed interest in understanding distributional dynamics. At the same time, a large number of studies have shown that instability of individual earnings or incomes is also quite high (see, for example, Dahl et al. 2011) and has significantly increased, especially in the US, contributing to the exacerbation of disparities between individuals (Bania and Leete 2009; Comin and Rabin 2009; Dynan et al. 2012; Gottschalk and Moffit 2002; Nichols 2010; Shin and Solon 2011). However, most of these studies are based on earnings or incomes and use aggregate measures of instability for the reference distribution¹ such as alternative estimators of transitory variance and volatility. Consequently, they end up being affected by structural variations within a distribution, for example, by the persistent increase in inequality that has characterized many countries. Moreover, the use of aggregate measures implies that these analyses end up hiding the possible countervailing effects of volatility over the entire distribution. This paper addresses these issues by proposing a framework that is not affected by structural changes in the economy, because it is based on the relative positions of individuals within the distribution of income. Moreover, being based on profiles rather than on aggregate measures, it enables the identification of sections of the population that are most affected by instability. These two key features distinguish our approach from those in the existing literature.

Specifically, this paper focuses on instability in relation to the *ranks* of individuals within a society rather than on the instability of their incomes or labor earnings. Individuals' ranks are consequently defined by their relative positions within the overall distribution of household incomes as opposed to the distribution of labor earnings. We thus subscribe to the view that while changes in the earning distribution are exclusively related to labor market

¹ By "aggregate measure" we mean the use of a representative aggregate to express instability within a given society as opposed to the use of individual level data on instability.

dynamics, a broader concept of income also incorporates the role of the welfare state and family dynamics in absorbing negative events and income shocks. Instability can be costly for individuals. However, the extent of its undesirability depends on individual risk preferences, risk pooling possibilities, and levels of insurance available against risks of income loss. Social protection as well as income transfers and the labor supply within households provide such income-smoothing insurance. Therefore, in the context of our analysis, the use of household income distribution to identify an individual's rank appears to be a more appropriate methodological choice compared with approaches that are based on individual earnings (see Jenkins 2011). We then measure instability of individual income ranks through the concept of volatility.

There are many appealing features that are characteristic of such a rank-based approach.² Disregarding household level income details for each individual, we can focus instead on comparisons that are independent from the marginal distributions since these are transformed into a common uniform one. This is of paramount importance because, for instance, in periods of sustained variations in inequality, income changes can be seriously affected by these kinds of structural transformations (see, for example, Jenkins and Van Kerm 2006; Van Kerm 2004). Because our framework is based on income ranks, it allows for an assessment of volatility that is generated by the real movements of individuals within the distribution and not by structural changes in the distribution.³ In addition, ranks represent a stable benchmark in contrast to incomes that are not stable during times of structural change.

In this study, we do not use income ranks *per se*. Instead, we use the logit transformation of these ranks (hereafter, logitrans). This transformation permits the establishment of

² See D'Agostino and Dardanoni (2009) for an alternative application of the rank-based approach in the context of mobility measurement.

³ Note that the use of rank in volatility analyses may also be motivated on the base of the positional goods framework or on Easterlin's theory of relative utility (Easterlin 1974), the relevance of which has been confirmed by recent research. Higher positions within an income distribution, rather than absolute income or an individual's position, have been demonstrated to lead to utility gains compared with reference wages (Clark et al. 2008; 2009; Alpizar et al. 2005).

proportionality between rank volatility and volatility of a specifically defined relative income that we refer to as medianized income, thereby enabling estimation problems at the boundaries to be solved.

Lastly, our framework does not require the estimation of a formal model of income dynamic. In this paper, we measure volatility by applying the magnitude of the change in income ranks rather than by isolating the transitory components of those changes. In this respect, our methodology can be considered a complementary approach to the Gottschalk-Moffitt procedure. The use of a “descriptive approach,” such as the one applied here does not enable transitory shocks to be disentangled from permanent ones. However, the results obtained using this approach do not depend on underlying assumptions about the income generating process, as is the case with results obtained using the Gottschalk-Moffitt procedure (see Cameron and Tracy 1998; Congressional Budget Office 2007; Dynan et al. 2008; Dynarski and Gruber 1997; Moffit and Gottschalk 2011; Shin and Solon 2011).

This approach also differs from previous ones through its focus on disaggregate as opposed to aggregate measures of volatility. The latter may lead to unsatisfactory results, because aggregate volatility may hide countervailing volatility and volatility trends across the distribution. Using data from the US, Jensen and Shore (2008) have shown that a systematic rise in the volatility of incomes over time within the population at large does not accompany a decomposition of changes in the average volatility. They argued that an increase in the average volatility was largely driven by a sharp rise in highly volatile incomes.

We concur with these views and argue for the need to develop an alternative methodology to evaluate volatility from a microeconomic perspective based on volatility profiles rather than on aggregate measures. In line with the above arguments, and complementing previous studies, we argue that such profiles and their trends can be asymmetric, that is, they may have differential impacts on different sections of the distribution. One reason for this is that

institutions affect the stability of income careers, and job flexibility usually hits those individuals occupying the lowest positions in the income distribution harder than those positioned higher up.

Some studies have begun to comprehensively explore individual heterogeneity within this trend. For instance, Dynan et al. (2012) have found that volatility in the US rose in the early 1970s as well as in the late 2000s, and that this widening of the income distribution was a phenomenon that was especially related to changes in the tails of the distribution. In view of the comparably high volatility of household incomes at the extreme ends of the income distribution, Hardy and Ziliak (2013) have alluded to a “wild ride” experienced at the top and the bottom of the distribution. Such heterogeneity also finds support in the work of Bania and Leete (2009), who demonstrated that US volatility was highest for lower income households and that their instability showed a sharper increase than that of other groups during the 1990s. Gottschalk (1997) found that in the US, the probability of remaining within the lowest quintile was lower than that of remaining within the top quintile. However, most of these studies have either been based on income levels, as opposed to ranks, or they have considered large-scale units such as quintiles that ignore intra-group volatility. Therefore, in contrast to previous studies, here we investigate the heterogeneity of volatility across a distribution, adopting a volatility profile.⁴

We then apply this framework to evaluate income rank volatility and its trends in Germany and the US between 1983 and 2009. We demonstrate the relevance of our framework by revisiting an ongoing debate on comparisons of income volatility in these two countries (e.g., Maasoumi and Trede 2001; Gottschalk and Spolaore 2002; Schluter and Van de gaer 2011). The point of departure for our study, in relation to this debate, is the observation that, when using a volatility measure based on income ranks, the US is not ranked

⁴ Note that the use of “heterogeneity” here does not refer to different sources of volatility, but rather to variations in volatility and volatility trends among different sections of the distribution.

as being more volatile than Germany, contrary to received wisdom. Furthermore, a comparison between these two countries is appropriate, because in addition to being characterized by intrinsically different economic and political structures, they have exhibited different levels of inequality and inequality dynamics in recent decades.

Although a few other studies have examined the volatility of household incomes in Germany and the US, this is the first contribution to explore volatility using a rank-based approach for these countries. To accomplish this aim, we use the Cross National Equivalent File (CNEF), a dataset containing harmonized data on these countries. Our study reveals the following findings. First, those who were relatively poor experienced much higher volatility than those who were richer in both countries. Second, the poor were found to be less volatile in Germany compared with the poor in the US. Third, the volatility gap between the poor and the rich tended to decline in Germany but not in the US. Last, while volatility increased consistently in the US for the lower middle class, it decreased consistently in Germany for the upper middle class.

Thus, the contribution of our study to the existing literature is twofold. The first contribution is methodological, as we introduce a measurement framework for evaluating distributional profiles of rank volatility. In doing so, we are able to observe volatility trends that are net of structural changes. The second contribution is empirical. Specifically, through the application of our methodological framework, we provide new insights on volatility trends that have prevailed in Germany and the US during the last few decades.

The rest of the paper is organized as follows. Section two introduces the methodological framework. Section three presents the results of our empirical analysis. Section four offers our conclusions.

II. THE METHODOLOGICAL FRAMEWORK

Let a society's income distribution at time t be represented by the cumulative distribution function (cdf) $F: R_+ \rightarrow [0,1]$. Hence, $F(y_t) = P(\tilde{y}_t \in R_+ : \tilde{y}_t \leq y_t)$, that is, the cdf returns the probability $p_t \in [0,1]$ of observing income that is less or equal to \tilde{y}_t in that society at time t . The ranks of individuals in this society can then be defined by $p_t = F(y_t)$.

The logit transformation of the income rank (the logitransformation) can then be expressed as follows:

$$\text{logit}(p_t) = \ln\left(\frac{p_t}{(1-p_t)}\right) \quad (1)$$

This transformation of the concerned variable allows for a more accurate estimation of the volatility experienced by individuals situated at the tails of the distribution. It should be noted that the use of ranks *per se* implies that while those individuals positioned in the center of the distribution can move in two directions (up or down), those positioned at the bottom (or top) of the distribution can only move in one direction, up (or down). The logit transformation enables us to overcome this drawback. Thus, in contrast to the use of percentile ranks, our framework does not accord greater importance to movements at the center. When an individual with rank $p = .99$ gets richer, substantial income movement has weak effects in terms of rank variation but not in terms of logitransformation variation.

In addition, the logitransformation approximates the Champernowne-Fisk distribution (CF) as follows:

$$\ln(\bar{y}_t) = \alpha * \text{logit}(p_t) \quad (2)$$

where α measures the degree of inequality, understood as the “stretching out” of the distribution. In particular, if the income distribution is log-logistic, α will be equivalent to the

Gini coefficient. \bar{y}_t denotes the medianized income; that is, $Me(y_t)$ is the median income of the distribution (Chauvel 2015; Dagum 2006; Fisk 1961).⁵

$$\bar{y}_t = \frac{y_t}{Me(y_t)}. \quad (3)$$

In other words, $logit(p_t)$ is proportional (with coefficient α depending on the country and year), and thus an equivalent measure to the log of the medianized income. Table 1 shows the conversion between logitranks and percentile ranks, which is useful for interpreting the results. For instance, a magnitude of -2 relates to quantile .119, close to the first decile, while a magnitude of 2 relates to an income 2.7 times higher than the median.

TABLE 1
Magnitudes of Logitranks and Percentile Ranks

Logitranks	-5	-4	-3	-2	-1	0	1	2	3	4	5
Rank	0.007	0.018	0.047	0.119	0.269	0.500	0.731	0.881	0.953	0.982	0.993

It should be noted that the CF, on which the proposed methodological framework is based, is one of several statistical laws applied to model incomes. There are at least three key reasons motivating the use of CF as an approximation to income distributions. First, because they are characterized by two parameters ($Me(y_t)$ and α), CF is highly parsimonious, with appropriate Pareto-type power-tails found at both extremes. This parsimony is notable, and the coefficient α plays a significant role in the measurement of inequality, because its value corresponds to the Gini coefficient. Second, CF entails a simplified generalized beta distribution of the second kind (GB2).⁶ While CF is much less flexible than GB2, it does share some important features with the latter such as power-tails. Third, CF produces income distributions that are solidly grounded in mathematical expressions. Last, while CF is applied

⁵ In a comparison of 212 samples, Chauvel (2015) has shown that this relation approximates quite well the empirical distributions in terms of the living standard (post-tax and income transfer) per consumption unit.

⁶ Although GB2 provides a better fit for income distribution, CF provides a simple framework that can be used to capture changes in local inequality.

at the intersection of different theoretical traditions, its formula nevertheless remains very simple.⁷

We subsequently use logitranks to describe and visualize income rank volatility and its changes over time. Let us denote the average logitrank of individuals between two periods as follows:

$$\overline{\text{logit}(p)} = \frac{\text{logit}(p_t) + \text{logit}(p_{t+1})}{2}. \quad (4)$$

For the sake of simplicity, we refer to equation (4) as an intertemporal logitrank. Let $\delta(p_t) = \text{logit}(p_t) - \text{logit}(p_{t+1})$ be the change in the logitrank between the initial and final periods.

We measure individual volatility as the standard deviation of $\text{logit}(p_t)$ and $\text{logit}(p_{t+1})$, which reflects the intensity/magnitude of moves, or the instability of a position. Individual income rank volatility can then be defined as follows:

$$v(\overline{\text{logit}(p)}) = \sqrt{\frac{1}{2} \sum_{t=1}^2 (\text{logit}(p_t) - \overline{\text{logit}(p)})^2}. \quad (5)$$

Plotting equation (5) against each $\overline{\text{logit}(p)}$, we obtain volatility profiles as shown in in Figure A1 in the Appendix. This graphical tool provides intuitive information on the extent of income rank volatility across the distribution.

The advantage of this logitrank-based measure of volatility is that it is not affected by changes in inequality over space or time. When there is a rise in inequality, and if the intrinsic volatility regime remains constant, the log of the medianized income-based measure of volatility correspondingly increases as a trivial consequence. By contrast, the logitrank-based volatility is not affected by distributional changes. This is because its construction is net of any inequality transformation.

⁷ In microeconomics, GB2 and, as a consequence, CF can be considered to be derived from Parker's neoclassic model of firm behavior (Parker, 1999). A number of other theoretical constructions such as stochastic processes of income attainment yield the same distribution. In a study conducted in the field of finance, Gabaix (2009) considered stochastic models based on geometric Brownian motion that could generate this type of distribution. Thus, studies from a number of different research fields have confirmed the importance of the CF model.

It is important here to emphasize the appropriateness of using logitranks for measuring volatility. Applying this procedure, we can simply transform the empirical quantile function of any distribution in its vertical projection. In the case of panel analysis for two or more years, the logitrunk transformation consists in the reshaping of the empirical distributions on an invariable reference distribution of shape defined in equation (2). This implies that logitrunk-based volatility absorbs all structural transformations, retaining the sole exchange mobility.⁸ Given our focus on pure volatility, the application of this approach enabled us to conduct meaningful comparisons over time and across countries.

Second, the use of descriptive statistics and more elaborate models based on logitrunk as a dependent variable is appropriate, because, according to the CF model, logitranks are simple linear transformations of the log of incomes. Therefore, apart from the difference between these two kinds of models, entailed in the fact that logitrunk variations are depurated from structural changes, working with the former is equivalent to working with the latter. Furthermore, as for any other ranking strategy, we dispose of a fixed point - relative position with respect to the rest of the distribution - that cannot be obtained when using incomes.

Our approach also entails other appealing technical features. First, existing rank-based studies on the volatility of incomes or earnings have often examined quintile or decile transitions over varying time periods (e.g., Gottschalk 1997). However, this method is unable to differentiate between the magnitudes of changes in income ranks. Thus, for example, it treats changes from the 19th to the 21st percentile in the same way as it treats transitions from the 1st to the 39th percentile. Our framework, based on a continuum of ranks, is able to account for distance. A further advantage of this framework is that it is not dominated by small changes in income levels near zero that usually lead to huge or infinite changes when a methodology entailing the log of income is applied.

⁸ See Jenkins and Jantti (2015) on the relevance of focusing on rank rather than on income in the measurement of mobility.

III. INCOME RANK VOLATILITY PROFILES FOR GERMANY AND THE US.

Data

Our empirical analysis is based on the panel component of the Cross National Equivalent File (CNEF, Frick et al. 2007 and 2008). The CNEF was designed at Cornell University to provide harmonized data for a set of eight country-specific surveys representative of the respective resident populations. For the present study, we consider all of the waves between 1983 and 2009 for Germany (SOEP, Socio-Economic Panel 2016) and the US (PSID).

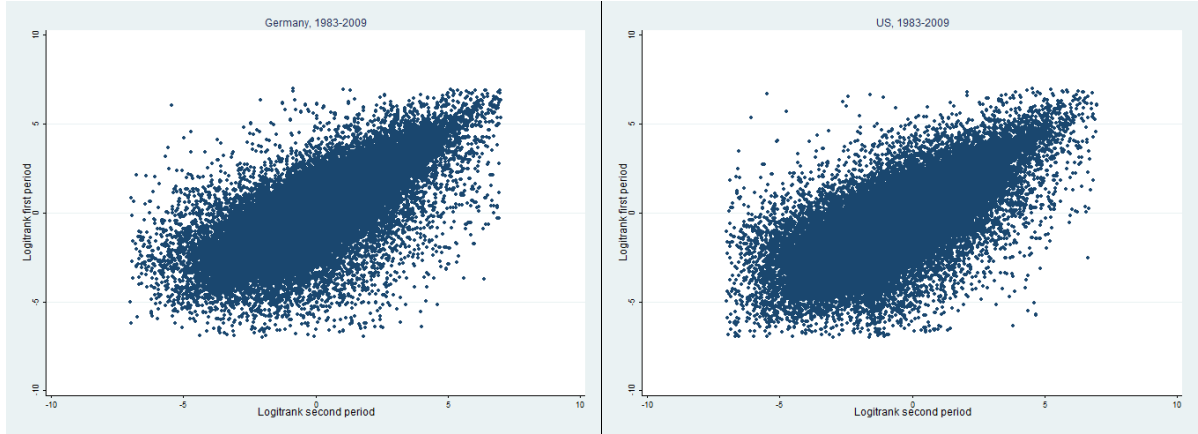
The unit of observation is the individual. Our data cover all individuals aged between 20 and 65 years. We restrict our US sample to African Americans and Caucasian Americans, excluding other racial groups such as Asians and Hispanics. We exclude East Germany from the German sample because of its late start. The measure of living standards is disposable household income, which includes income after transfers and the deduction of income tax and social security contributions. Incomes are expressed in constant 2005 prices and are adjusted for differences in household size, using the square root of the household size. Individual volatility is measured over a 2-year period. We use sample weights to compute all estimates with standard errors obtained through 500 bootstrap replications.

Before discussing the details of our analysis, it is worth pointing out that some of the statistical properties relating to the use of the logitrans can be useful in the context of volatility analysis. By plotting the logitrans of all individuals at two different points in time for both Germany and the US over the entire study period, we are able to detect cases of complete stability or absence of volatility that are apparent when every individual has the same rank in t and $t + 1$. In such cases, each individual would be placed on the diagonal. Figure 1 shows that the observation cloud is anisotropic rather than bi-normally distributed. It suggests that there is less variation in ranks across the two time points at the top of the income

distribution (upper right corner), as observations are clustered closer to the diagonal than in other parts of the income distribution.

FIGURE 1

Distribution of Logitrans in Germany and the US, 1983–2009



Note: For the conversion between logitrans and percentile ranks see Table 1. Source: Authors' computations based on CNEF.

The particular distribution of the logitrans change is better captured in Figure A2 in the Appendix, which shows the density of the change in the logitrans of individuals over the two periods. Indeed, this change indicates a significant deviation from a normal distribution. Logitrans variations entail more extreme values than those found in the normal hypothesis. We detect a typical Lévy alpha-stable distribution belonging to the general family of stable distributions. A general stable distribution can be described by four parameters. These are: an index of stability or characteristic exponent $\theta \in (0; 2]$ ($\theta = 2$ for a normal distribution and R (Nolan 2009; Umarov et al. 2010). Here, the value of θ is close to 1.3. Leptokurtic non-normal stable distributions are also known as stable Paretian distributions. These heavy-tailed distributions are common in finance statistics and assets volatility analysis.

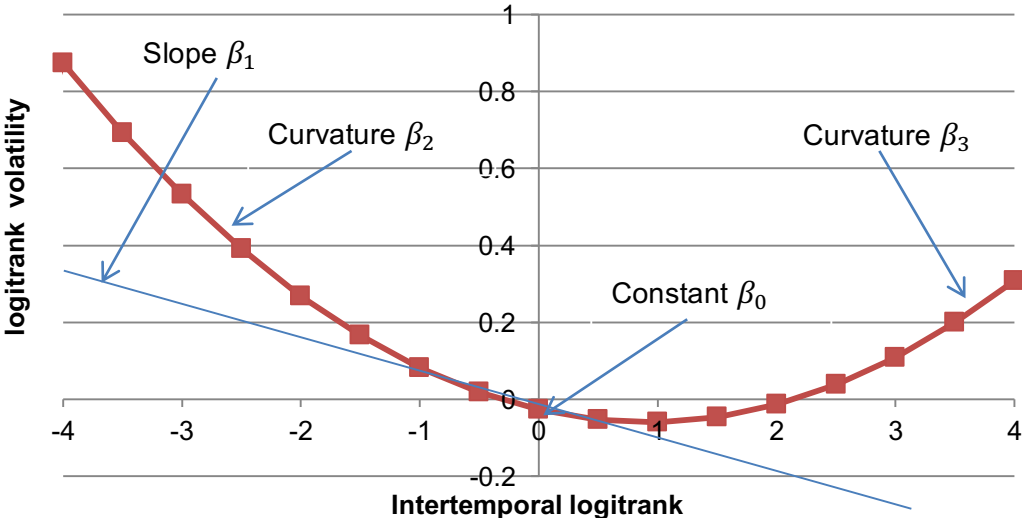
The above properties and the particular shapes of the volatility profiles shown in Figure A2 (see Appendix) enable us to estimate a volatility profile as follows:

$$\ln \left(v(\overline{\logit(p)}) \right) = \beta_0 + \beta_1 \overline{\logit(p)} + \beta_2 \overline{\logit(p)}^2_{|\logit(p)<0} + \beta_3 \overline{\logit(p)}^2_{|\logit(p)>0} + \varepsilon. \quad (6)$$

Volatility profiles are estimated with a polynomial in which the log of the logitrank volatility is expressed as a function of the intertemporal logitrank and its square. To account for dissymmetry, we collapse the curvatures (β_2 and β_3) in two parameters located, respectively, below and above the median (logitrank = 0). Figure 2 illustrates our conceptualization of estimated volatility profiles developed using equation (6).

FIGURE 2

Profile of Volatility



Note: For the conversion between logitranks and percentile ranks see Table 1.

As shown in Figure 2, estimating profiles of volatility through equation (6) is very useful as it allows capturing relevant information related to the income dynamics under analysis. These are the constant β_0 that catches volatility near the median, the slope β_1 that denotes the degree to which volatility is higher (or lower if $\beta_1 < 0$) at the top than at the bottom, and β_2 that expresses the degree of increase of volatility at the extremes of the distribution ($\beta_2 > 0$). Therefore, variations in the three parameters provide interpretable information on the increase or decrease of volatility in specific parts of the distribution. A positive change in β_0 implies that volatility increases for all parts. A rise in the value of β_1 indicates more volatility for the richest individuals. Finally, higher values of β_2 and β_3 indicates more volatility for extreme

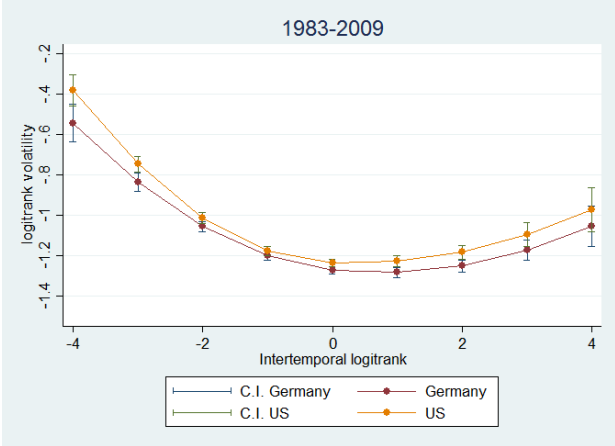
values (and relatively higher stability at the median level). Consequently, we are able to better understand in which part of the distribution income rank volatility increases or decreases more steeply.

Results

Figure 3 shows the estimated profiles of income rank volatility for Germany and the US over the entire study period. The volatility profiles for both countries appear to be U-shaped. This particular shape is neither a natural nor a trivial outcome of our methodology. It denotes greater changes, on average, at the bottom and at the top of the distribution compared with the change occurring at the center of the distribution.

FIGURE 3

A Comparison of Logitransk Volatility in Germany and the US, 1983–2009



Notes: Volatility depicted on the y-axis is estimated using equation (6). Intertemporal logitransk refers to equation (4). For the conversion between logitransks and percentile ranks see Table 1. C.I. are the confidence intervals. Source: Authors’ computations based on the CNEF.

An important feature of the U-shaped curvature characterizing all the volatility profiles depicted in Figures 3, 4, and 5 is its asymmetry. This indicates that in both countries, the middle-income classes have been and continue to be more stable than the top-income classes. Whereas the top-income classes are still more stable than those at the bottom of the income

hierarchy, but more fluid (or unstable) than the middle. Over time, changes in this asymmetry reveal different trends for incomes situated near the median, the top, and the bottom of the distribution.

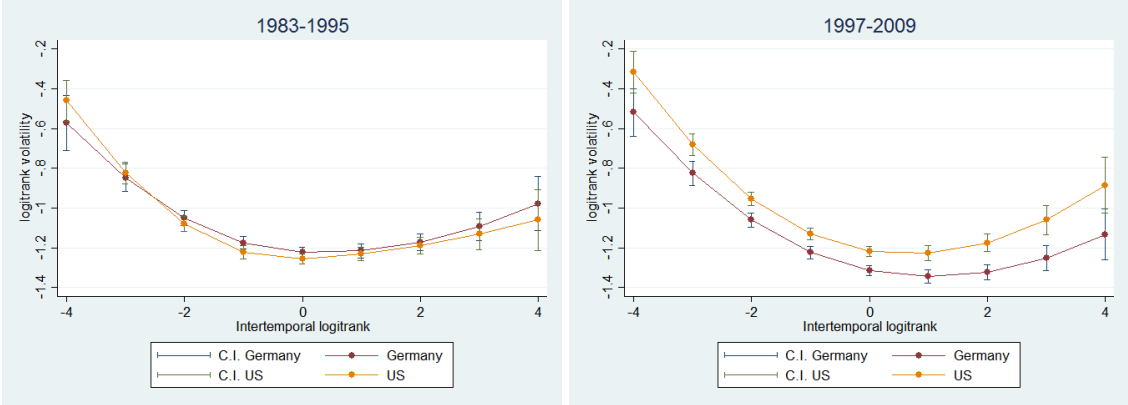
Considering 95% confidence intervals, the overall regimes of logitrunk-based volatility for Germany and the US do not differ substantially during the entire period from 1983 to 2009 (Figure 3). In fact, their profiles showed similar patterns. Although this may appear to be a disappointing result, it is consistent with the findings of previous studies showing that prior to reunification, West Germany was a relatively mobile society, and was even more mobile than the US (e.g., Bayaz-Ozturk et al. 2014; Maasoumi and Trede 2001). Conversely, this result does not corroborate the findings of previous studies focusing on household income volatility (log-income) that incomes in the US tended to fluctuate more during a similar period than those in Germany and were thus more “risky” (e.g., Gottschalk and Spolaore 2002; Van Kerm 2004). However, these findings were not net of distributional differences between Germany and the US (it is obvious to experience higher income volatility in more unequal countries). The added value of our results (in terms of the logitrunk of income) is the insight that the inner systems of volatility (net of distributional changes) in Germany and the US do not greatly differ during this period (1983–2009).

When we compare Germany and the US over shorter time intervals, the dynamics seem to be more complicated. During the earlier period (1983–1995), there is a considerable overlap in the confidence intervals of the volatility curves for the two countries (Figure 4, left panel). Conversely, a significant divergence is observed during the subsequent period (1997–2009), with volatility being intrinsically higher in the US than in Germany (Figure 4, right panel). The difference between these two countries is always statistically significant in all cases except those of individuals ranked at the very bottom and top of the distribution. The general patterns in these two countries evidenced during this latter period are in line with the findings

reported in other studies. These indicate that European labor markets are more regulated than the American market, which has become increasingly flexible, unstable, and risky for employees. Consequently, volatility is intrinsically higher in the US than in Germany.

FIGURE 4

A Comparison of Logitrunk Volatility in Germany and the US, 1983–1995 (left) and 1997–2009 (right)



Notes: Volatility on the y-axis is estimated using equation (6). Intertemporal logitrunk refers to equation (4). See Table 1 for the conversion between logitranks and percentile ranks. C.I. denotes the confidence intervals. Source: Authors’ computations based on the CNEF.

However, more insights on volatility can be gained by exploring if and how these profiles have changed over time in these two countries (Figure 5). In fact, in the US, the entire distribution and, most importantly, the lower middle class have become more volatile. By contrast, less volatility in relation to the overall distribution is currently being experienced in Germany than previously, with the exception (though not significant) of the very poor. In particular, volatility has significantly decreased for the upper middle class between the two periods.

The estimated coefficients of equation (6), shown in Table 2, are graphically plotted in Figures 3, 4, and 5, thereby providing a precise and insightful depiction of the nature of changes in volatility. As the dependent variable is volatility, higher coefficients indicate greater positive impacts on volatility, while negative coefficients indicate negative impacts.

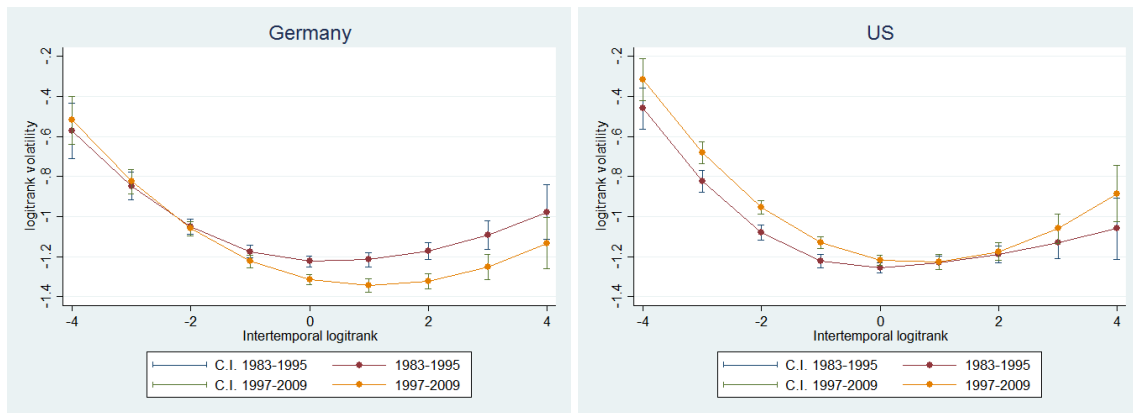
With respect to β_0 , which refers to the extent of volatility at the median of the income distribution, the positions of Germany and the US are reversed during the two study periods. While Germany appears to be more volatile than the US near the median during the period prior to 1996, the US becomes more volatile and Germany much less volatile during the period after 1996 compared with the previous situation (a decrease in β_0).

Regarding β_1 , the general slope or the first order gradient of income volatility differences between the bottom and the top, Table 2 shows in each case a negative coefficient – with the exception of the US in the earlier period (.0198). In other words, income rank volatility is, on average, higher among the poor than among the rich. An examination of the size of the coefficients suggests that an extensive change appears to have occurred at this level over time in terms of the balance between the rich and the poor. Both countries evidence a reduction of this coefficient, which is greater in the US.

Lastly, β_2 and β_3 indicate the weight of volatility at the tails of the distribution. These second order coefficients describe the (quadratically shaped) curvature at the bottom and top of the distribution, respectively. That is, they depict the deviation of volatility at the extreme ends compared with the median. The first coefficient, β_2 , appears to be consistently higher in the US than in Germany, implying that higher volatility occurred at the bottom compared to the median of the distribution in the US as opposed to Germany (.0548 and .0464 in the US compared with .0180 and .0250 in Germany). By contrast, the second coefficient, β_3 , is higher in Germany (.0180) than in the US (.0074) during the first period, whereas it is higher in the US (.0306) than in Germany (.0250) during the latter period. However, if the entire period is considered, it is lower in the US than in Germany. This indicates that volatility at the top, as compared to that at the bottom, increased in both countries, but more so in the US.

FIGURE 5

A Comparison of Logitrunk Volatility During the Two Periods, 1983–1995 and 1997–2009 in Germany (left) and the US (right)



Notes: Volatility on the y-axis is estimated using equation (6). Intertemporal logitrunk refers to equation (4). See Table 1 for the conversion between logitrunks and percentile ranks. C.I. denotes the confidence intervals. Source: Authors' computations based on the CNEF.

Overall, these results confirm that the volatility gap between the poor and the rich tends to decline in Germany, in contrast to the US, and that the difference between the volatility experiences of the two countries is clearly related to the volatility trend of the middle class. In a nutshell, the German winners in terms of income stability belong to the upper middle class (between the median and the 95th percentile), significantly above the median. While income risks decline for this income group, lower class Americans (between percentile 5 and 27) face increasing instability. This has occurred in a context wherein the relative incomes of this income group have declined compared with the American median, because of increasing inequality within the US. The German upper-middle class has somehow experienced a double gain, whereas the same class in the US has experienced a double loss in terms of both poverty and economic insecurity.

An in depth analysis of determinants of these trends is beyond the scope of this paper. However, some hypotheses clearly apply. First, the increase in inequality relating to earnings in the US during the 1980s has been attributed to accelerated skill-biased technical changes

favoring skilled workers, a decline in unionization, and decreasing real wages. By contrast, the rise in inequality during the 1990s was most pronounced at the top of the income distribution. Second, increasing volatility in the US has traditionally been attributed to increasing work incentives and labor market flexibility, leading to a reduction in welfare and job security. The fact that the increased volatility was concentrated in the lower-middle class section of the population should raise concerns about whether consumption and well-being in that section of the population has been adversely affected. Such concerns are particularly salient given the high likelihood of liquidity constraints for this income group and the imperfect public social insurance available to them. Conversely, the better educated upper-middle class has been better able to defend its economic status even if this defense has been at a cost in terms of labor intensity (Schor 1992; Gershuny 2000). Similarly, volatility in earnings has been found to differ across educational groups, with the least skilled individuals and high school dropouts showing the highest levels of volatility compared with highly skilled individuals between 1973 and 1984. However, this trend was reversed during the period between 1986 and 2008 (Ziliak et al. 2011).

Changes in the structure of earnings in West Germany after reunification can explain the particular changes in volatility found here. These changes, together with a decrease in the mobility of labor earnings, may be the key reasons for reduced volatility in the upper-middle section of the German distribution. In this respect, the German upper-middle class has evidenced a strong wage-earning pattern in which income security and stabilization over time is still (or even more so than previously) a valuable economic resource, defended as such by the skilled population. Conversely, the specificity of the American system has been to develop a large group of unstable poor workers who increasingly contrast with the relatively more stable upper-middle class skilled workers.

The differences in the US and German income rank volatility trends can also be explained by differences in the effectiveness of government taxes and transfers and in government policies and the welfare system, in general, aimed at leveling income volatility. Indeed, the US government's redistribution efforts seem to have been less effective than those of the German government, particularly in relation to the abovementioned hyperflexible low-wage income group.

A useful way to illustrate the difference between the logitrunk-based approach and the traditional log-income-based approach is through a simulation, in which we assume that the only change affecting income distribution is the variation in inequality over time.⁹ Figure A3 in the Appendix shows estimates of the logitrunk and log-medianized income volatilities in this hypothetical situation. This clearly indicates that while volatility based on the traditional approach (log-medianized income volatility) increases with an increase in inequality, logitrunk volatility remains constant.¹⁰

⁹ The STATA dofile for replicating the simulation is available as supplementary material.

¹⁰ This evidently follows from equation (2).

TABLE 2

Estimated Coefficients of the Logitrunk Volatility Profiles

	Figure 6: Overall (1983–2009)		Figure 8: First period (1983–1995)		Figure 8: Second period (1997–2009)	
	US	Germany	US	Germany	US	Germany
β_1	-.0112	-.0332	.0198	-.0103	-.0395	-.0546
(s.e.)	(.0135)	(.0126)	(.0186)	(.0174)	(.0172)	(.0173)
[c.i.]	[-.0375, .0152]	[-.0580, -.0084]	[-.0166, .0562]	[-.0444, .0237]	[-.0733, -.0057]	[-.0885, -.0206]
β_2	.0507	.0372	.0548	.0382	.0464	.0360
(s.e.)	(.0048)	(.0052)	(.0066)	(.0075)	(.0063)	(.0069)
[c.i.]	[.0412, .0601]	[.0271, .0474]	[.0418, .0678]	[.0235, .0529]	[.0340, .0587]	[.0225, .0496]
β_3	.0193	.0219	.0074	.0180	.0306	.0250
(s.e.)	(.0057)	(.0054)	(.0082)	(.0075)	(.0073)	(.0074)
[c.i.]	[.0081, .0305]	[.0113, .0326]	[-.0087, .0234]	[.0032, .0327]	[.0163, .0450]	[.0105, .0394]
β_0	-1.2359	-1.2705	-1.2577	-1.2229	-1.2178	-1.3137
(s.e.)	(.0098)	(.0098)	(.0130)	(.0137)	(.0133)	(.0131)
[c.i.]	[-1.2551, -1.2167]	[-1.2896, -1.2514]	[-1.2831, -1.2323]	[-1.2499, -1.1960]	[-1.2438, -1.1918]	[-1.3395, -1.2880]

Source: Authors' computations based on the CNEF.

The value of applying our framework can be further grasped by comparing the results obtained for logitrunk-based volatility with those obtained using the standard income-based volatility measure (hereafter referred to as income volatility). These results are shown in: Figure 6 (a comparison of volatility in Germany and the US over the entire study period from 1983–2009), Figure 7 (a comparison of volatility in Germany and in the US during the first and second periods, that is, 1983–1995 and 1997–2009, respectively), and Figure 8 (showing the separate evolution of volatility in Germany and the US over time).

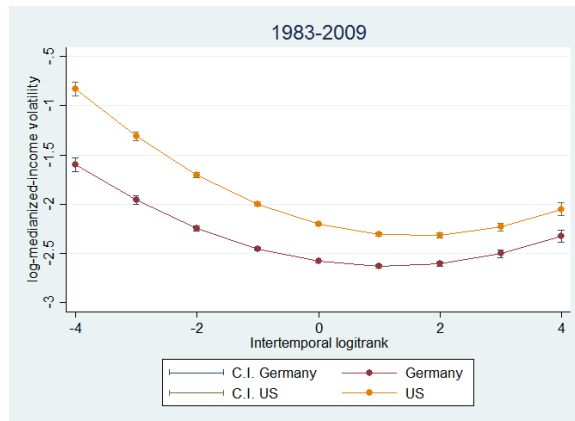
The U-shaped volatility profile shown in Figures 3, 4, and 5 is confirmed with the income volatility profile in which mobility is of a higher amplitude further away from the median. However, volatility profiles based on income may, in general, prove to be an inconvenient tool for conducting comparisons, because income volatility is trivially higher in more unequal countries. In fact, apart from the U-shape of the profiles, patterns of income volatility clearly differ in relation to several features from the patterns obtained for logitrunk volatility. For instance, income volatility is observed to be consistently lower than logitrunk volatility, independently of the country or the period considered. By contrast, the gap between the profiles of the two countries shows a considerable increase when we focus on income volatility, both for the entire time span (1983–2009) and over the two shorter periods (1983–1995 and 1997–2009). The wider gap between Germany and US obtained using income volatility, in contrast to that obtained using logitrunk volatility, reflects the higher level of inequality in the US compared to Germany (see Table 3). This can be explained as the impact of inequality on the measurement of income volatility as opposed to the neutrality of logitrunk volatility in relation to inequality. It is also interesting to note that income volatility in the US consistently outplay income volatility in Germany during the first period. As we previously observed, logitrunk volatility during this same period appeared, conversely, to be higher in Germany than in the US (Figure 4), with the exception of the lowest section of the

distribution, that is, the section for which the gap between income volatility in Germany and the US appeared to be higher.

Moreover, it should be noted that the time trend of the volatility profile for each country changes with a focus on income rather than on logitrans. This difference again reflects the sensitivity of income-based measures of volatility to inequality in contrast to the neutrality that characterizes logitrans-based measures of volatility. This is particularly evident in the German case. Figure 5 (the panel on the left), which shows logitrans volatility, indicates that a decline in volatility was experienced across almost the entire distribution, with the exception of the poorest segments of the population. By contrast, Figure 8 (the panel on the left) shows that volatility measured on the basis of incomes has remained quite stable over time, especially for the middle section of the distribution. As for the US, Figure 8 (panel on the right) is consistent with Figure 5 (panel on the right), indicating that the second period was the most volatile. However, the difference between the first and second periods is noticeably greater in the case of income volatility than it is in the case of logitrans volatility (Figure 5, panel on the right). Divergent features related to the use of income volatility as opposed to logitrans volatility can once again be attributed to the sensitivity of income-based measures of volatility to differences in the stretching out of the distribution in the two countries, evidenced by the differential increases in their Gini coefficients (see Table 3). The stability of the German income volatility profile over time, in contrast to the dramatic increase observed in the US profile, is mostly attributable to differences in the variation of the Gini coefficients in these countries: the increase of this index in the US was almost double that of its increase in Germany. As previously discussed, our methodology, based on logitrans, enables an assessment of volatility net of the changes generated by variations in income inequality.

FIGURE 6

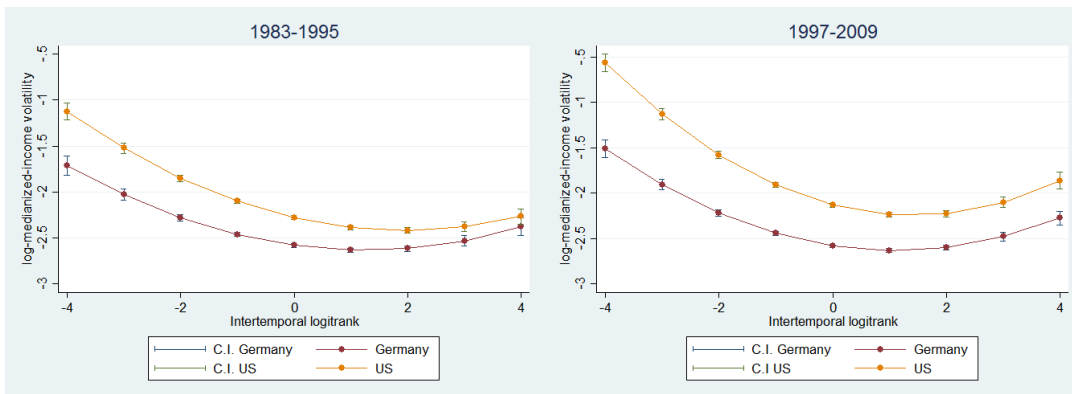
A Comparison of Log-Medianized Income Volatility in Germany and the US, 1983–2009



Notes: Volatility on the y-axis is estimated using equation (6), but the log of the log-medianized income volatility is used as the dependent variable. Intertemporal logitrans refers to equation (4). For the conversion between logitrans and percentile ranks see Table 1. C.I. denotes the confidence intervals. Source: Authors' computations based on the CNEF.

FIGURE 7

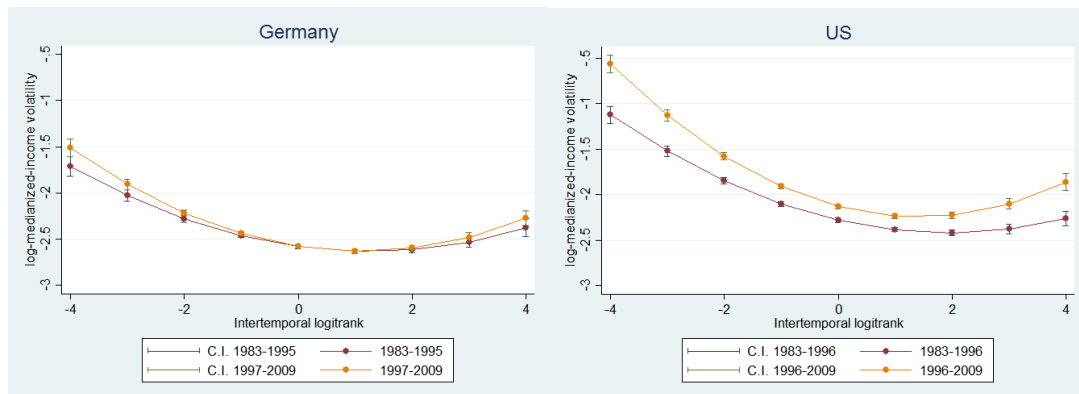
A Comparison of Log-Medianized Income Volatility in Germany and the US during the Periods 1983–1995 and 1997–2009



Notes: Volatility on the y-axis is estimated based on equation (6), but the log of the medianized-income volatility is used as the dependent variable. Intertemporal logitrans refers to equation (4). For the conversion between logitrans and percentile ranks see Table 1. C.I. denotes the confidence intervals. Source: Authors' computations based on the CNEF.

FIGURE 8

A Comparison of Log-Medianized Income Volatility During the Periods 1983–1995 and 1997–2009 in Germany (left) and the US (right)



Notes: Volatility on the y-axis is estimated using equation (6), but the log of the medianized-income volatility is used in this case as the dependent variable. Intertemporal logitranks refers to equation (4), for the conversion between logitranks and percentile ranks see Table 1. C.I. denotes the confidence intervals. Source: Authors' computations based on the CNEF.

TABLE 3

Period-Specific Gini Coefficients

Years (first period)	Gini coefficients		Average income	
	Germany	US	Germany	US
1983	0.250	0.305	21769.93	26809.69
1985	0.245	0.324	22096.55	29034.3
1987	0.242	0.325	22915.62	30164.49
1989	0.247	0.355	24189.96	33680.41
1991	0.251	0.343	25793.79	33001.94
1993	0.258	0.353	25589.26	33968.36
1995	0.270	0.366	24280.53	33059.54
Average first period	0.252	0.339	23805.09	31388,39
Years (second period)	Gini coefficients		Average income	
	Germany	US	Germany	US
1997	0.260	0.362	24332.32	36460.88
1999	0.258	0.394	25925.71	38319.85
2001	0.270	0.375	26325.57	41942.97
2003	0.283	0.372	27036.98	40676.45
2005	0.302	0.410	27478.54	43186.96
2007	0.298	0.429	28463.79	42563.5
2009	0.292	0.412	29042.36	42299.12
Average second period	0.280	0.393	26943.61	40778.53

Source: Authors' computations based on CNEF.

IV CONCLUSIONS

In this paper, we have proposed a methodological framework for evaluating and comparing rank volatilities and their trends over time. This framework has departed from the prevailing view that a better understanding of volatility can be obtained by examining the extent of this phenomenon within different parts of the distribution. Moreover, it has endorsed the view that income rank volatility may provide additional information that would usefully complement the rooted consensus among scientists on the extent of income instability. Although there is an extensive literature across different knowledge domains that emphasizes the significance of individuals' relative positions in the income distribution, rank volatility has not yet been explored. To better capture movements occurring at the tails of the distribution, we have used the logit transformation of rank. This transformation satisfies a number of additional statistical properties making our methodology an appealing tool for application in the domain of volatility analysis.

We have applied our framework to evaluate and compare the dynamics of income rank volatility in Germany and the US over the last three decades. Using the CNEF, we have demonstrated that, in general over the entire period, the rank volatility profiles of these two countries do not differ substantially and that the poorer sections of these populations experienced more volatility than the richer sections. This volatility gap appears to be, however, higher in the US than in Germany. The poorer sections also experienced increasing volatility over time, especially in the US. Conversely, upper middle class households have steadily become relatively more stable with respect to their income rank, especially in Germany. These transformations are consistent with transformations in labor regulations and welfare regimes specific to the countries we have studied.

This work can be extended in a number of directions. From a methodological perspective, it would be interesting to extend this framework to an analysis of structural versus exchange

mobility. From a more empirical viewpoint, a potential area of application of our work would be the European context in which countries exhibit very different systems of labor and income regulations. When more current data become available, relevant questions to investigate would be whether the U-shape of the income rank volatility profiles has been affected by the recent financial crisis and whether national specificities have been evident in this process. Such an analysis would also allow understanding whether and how the austerity policies introduced in some countries have affected individual instability.

ACKNOWLEDGEMENTS

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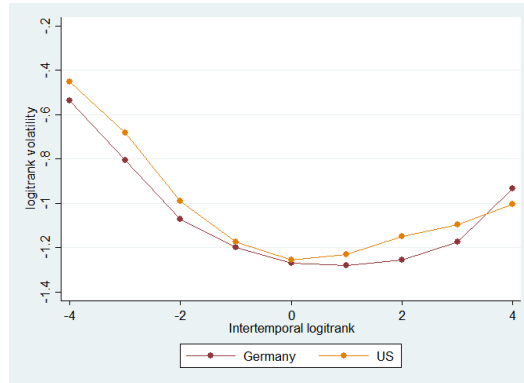
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APPENDIX

FIGURE A1

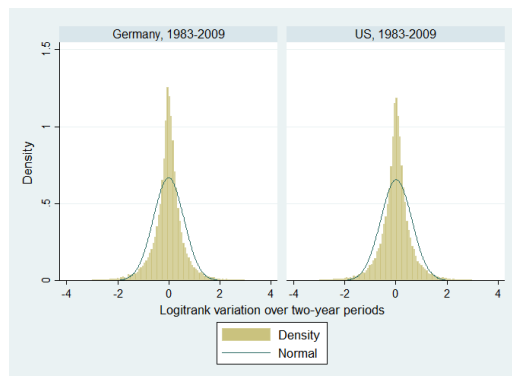
Observed Average Two-Year Volatility in Germany and US, 1983–2009



Notes: Volatility on the y-axis refers to the log of equation (5). Intertemporal logitrans refers to equation (4). Table 1 shows the conversion between logitrans and percentile ranks. C.I. denotes the confidence intervals. Source: Authors' computations based on the CNEF.

FIGURE A2

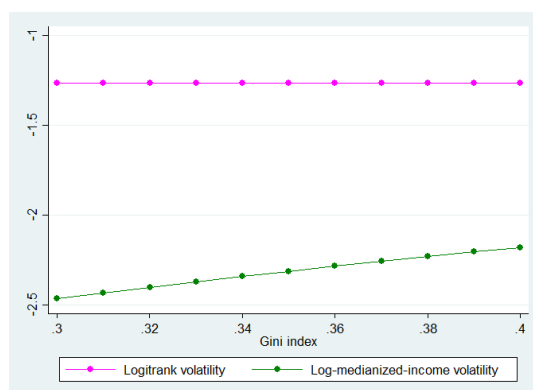
Logitrans Variations During the Two Periods in Germany and the US, 1983–2009



Source: Authors' computations based on CNEF.

FIGURE A3

A Comparison of Logitrans and Logincome Volatility with Increasing Inequality



Note: Logitrans volatility and log-medianized income volatility refer to individuals whose logitrans is 0, that is, those situated at the median of the distribution. Source: Authors' simulation based on CNEF (US).