

A performance evaluation of weight-constrained conditioned portfolio optimization

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Outline

1 Portfolio optimization with conditioning information

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- 2 General formulation of the problem

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- 3 Empirical study
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Problem context

- Discrete-time optimization
- Minimize portfolio variance for a given expected portfolio mean
- Postulate that there exists some relationship $\mu(s)$ between a signal s and each asset return r observed at the end of the investment interval:

$$r_t = \mu(s_{t-1}) + \epsilon_t,$$

with $E[\epsilon_t | s_{t-1}] = 0$.

- How do we optimally use this information in an otherwise classical (unconditional mean / unconditional variance) portfolio optimization process?

Problem history

- Ferson and Siegel (2001): closed-form solution of unconstrained mean-variance problem using unconditional moments
- Chiang (2008): closed-form solutions to the benchmark tracking variant of the Ferson-Siegel problem
- Basu et al. (2006), Luo et al. (2008): empirical studies covering conditioned optima of portfolios of trading strategies

Possible signals

Taken from a continuous scale ranging from purely macroeconomic indices to investor sentiment indicators. Indicators taking into account investor attitude may be based on some model or calculated in an ad-hoc fashion.

Examples include

- short-term treasury bill rates (Fama and Schwert 1977);
- CBOE Market Volatility Index (VIX) (Whaley 1993) or its European equivalents (VDAX etc.);
- risk aversion indices using averaging and normalisation (UBS Investor Sentiment Index 2003) or PCA reduction (Coudert and Gex 2007) of several macroeconomic indicators;
- global risk aversion indices (GRAI) (Kumar and Persaud 2004) based on a measure of rank correlation between current returns and previous risks;
- option-based risk aversion indices (Tarashev et al. 2003);
- sentiment indicators directly obtained from surveys (e.g. University of Michigan Consumer Sentiment Index)

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Unconditioned expected return and variance given conditioning information

These are obtained as expectation integrals over the signal domain. If a risk-free asset with return r_t is available,

$$E(P) = E[u'(s)(\mu(s) - r_t 1)] = E[l_1(u, s)]$$

and

$$\begin{aligned}\sigma^2(P) &= E\left[u'(s)[(\mu(s) - r_t 1)(\mu(s) - r_t 1)' + \sigma_\epsilon^2]u(s)\right] - \mu_P^2 \\ &= E[l_2(u, s)] - \mu_P^2\end{aligned}$$

for an expected unconditional return of μ_P and a conditional covariance matrix σ_ϵ^2 .

Optimal control formulation

Minimize $J_{[s^-, s^+]}(x, u) = \int_{s^-}^{s^+} l_2(u, s) p_s(s) ds$ as $s^- \rightarrow -\infty, s^+ \rightarrow +\infty$

subject to $\dot{x}(s) = l_1(u, s) p_s(s) \forall s \in [s^-, s^+]$, with

$$\lim_{s \rightarrow -\infty} x(s) = x_-, \quad \lim_{s \rightarrow +\infty} x(s) = x_+,$$

and $u(s) \in U, \forall s \in [s^-, s^+]$

where $U \subseteq \mathbb{R}^n$, $x(s) \in \mathbb{R}^m$ and L as well as f are continuous and differentiable in both x and u .

Since the signal s is not necessarily bounded, the resulting control problem involves expectation integrals with infinite boundaries in the general case.

Necessity and sufficiency results

- The Pontryagin Minimum Principle (PMP) and Mangasarian sufficiency theorem are shown to continue holding if the control problem domain corresponds to the full real axis: the corresponding optimal control problems are well-posed.
- The PMP is then used to show that the given optimal control formulation of the conditioned mean-variance problem generalizes classical (Ferson and Siegel; Markowitz) problem expressions

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Aim of the study

Carry out backtests executing constrained-weight conditioned optimization strategies with different settings.

Data set

- 11 years of daily data, from January 1999 to February 2010 (2891 samples)
- Risky assets: 10 different EUR-based funds commercialized in Luxembourg chosen across asset categories (equity, fixed income) and across Morningstar style criteria
- Risk-free proxy: EURIBOR with 1 week tenor
- Signals: VDAX, volatility of bond index, PCA-based indices built using both 2 and 4 factors and estimation window sizes of 50, 100 and 200 points, Kumar and Persaud currency-based GRAI obtained using 1 month and 3 month forward rates

Benchmark problem

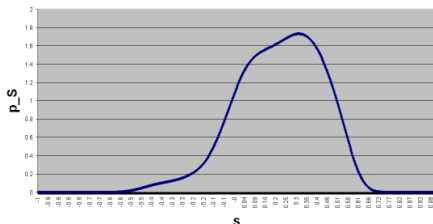
- Take VDAX index as signal, with 60 point estimation window and weights constrained to allow for long investment only
- Rebalance Markowitz-optimal portfolio alongside conditioned optimal portfolio, both with and without the availability of a risk-free proxy asset, over the 11-year period
- Assume lagged relationship $\mu(s)$ between signal and return can be represented by a linear regression
- Use kernel density estimates for signal densities
- Estimate the above using a given rolling window size (15 to 120 points)
- Use direct collocation discretisation method for numerical problem solutions

Benchmark problem (2)

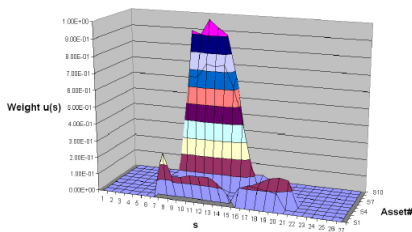
- Vary the parameters to check both for robustness of strategy results and whether results can be further improved while staying with a linear regression model for the relationship between signals and returns
- Obtain efficient frontier for every date and choose portfolio based on quadratic utility functions with risk aversion coefficients between 0 and 10
- Compare sharp ratios (ex ante), additive observed returns (ex post), observed standard deviations (ex post) of both strategies
- Try different window sizes, different signal lags, weight averages over different signal points, different signals

Typical kernel density estimate for signal and resulting optimal weight functionals

Kernel density estimate for GRAI 3M signal, 26 Mar 99

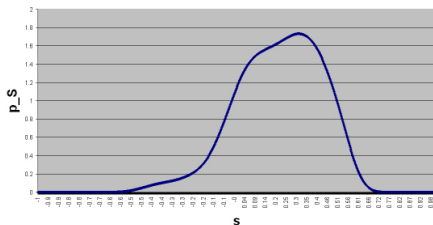


Optimal weight functionals using GRAI 3M signal, positive weights only, 26 Mar 99

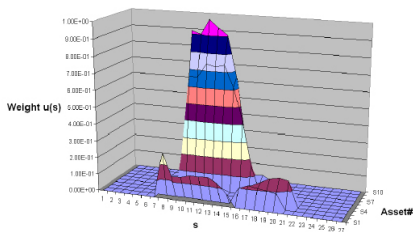


Typical kernel density estimate for signal and resulting optimal weight functionals

Kernel density estimate for GRAI 3M signal, 26 Mar 99

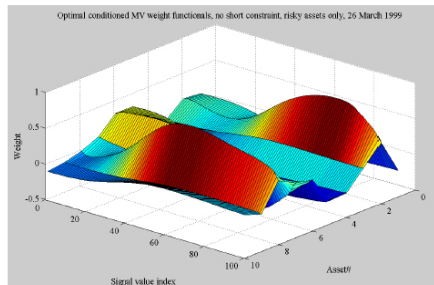


Optimal weight functionals using GRAI 3M signal, positive weights only, 26 Mar 99

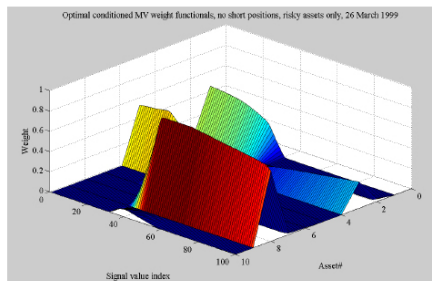


As would be expected, the constrained optimal weights are not simply a truncated version of the unconstrained optimal (Ferson-Siegel) weights.

Weights in the constrained and unconstrained case



(a) Unconstrained weights.



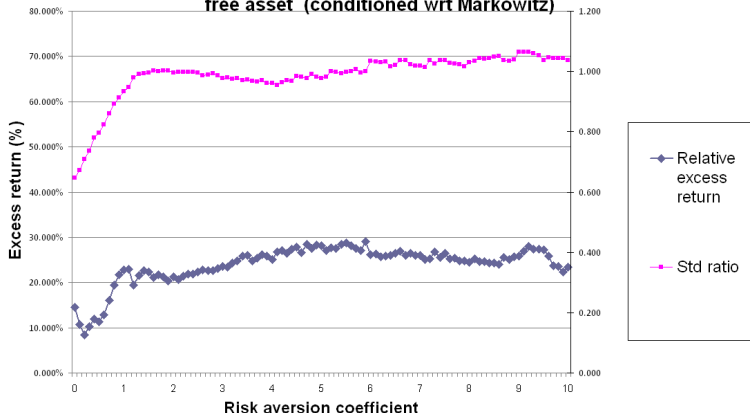
(b) Constrained weights.

FIGURE 4.6: Optimal weight functionals for conditioned problem, no short constraint (left) / weights constrained to be positive(right), 26 March 1999, 60 day estimation windows, VDAX index.

With risk-free asset

Ex post observed relative excess additive returns, standard deviation ratios

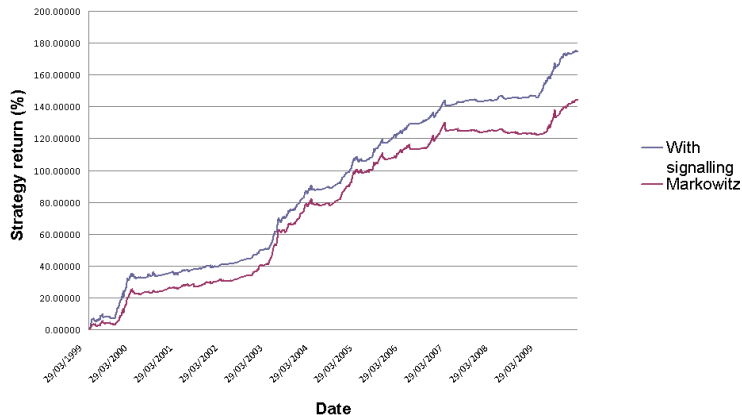
Observed excess returns and std ratios for base case with risk-free asset (conditioned wrt Markowitz)



With risk-free asset

Time path of additive strategy returns for $\lambda = 2$

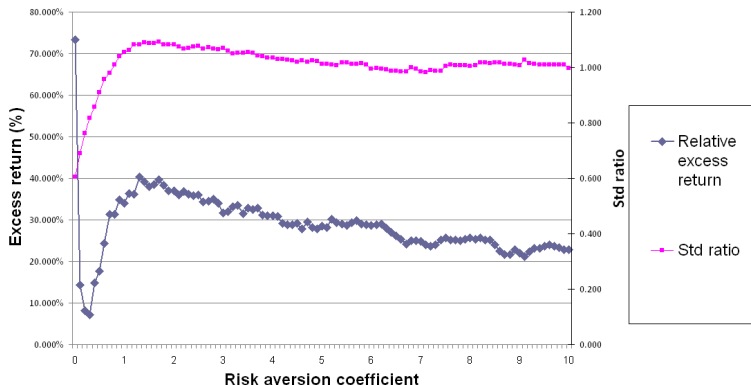
Cumulative strategy returns (risk aversion = 2, positive weights only)



With risky assets only

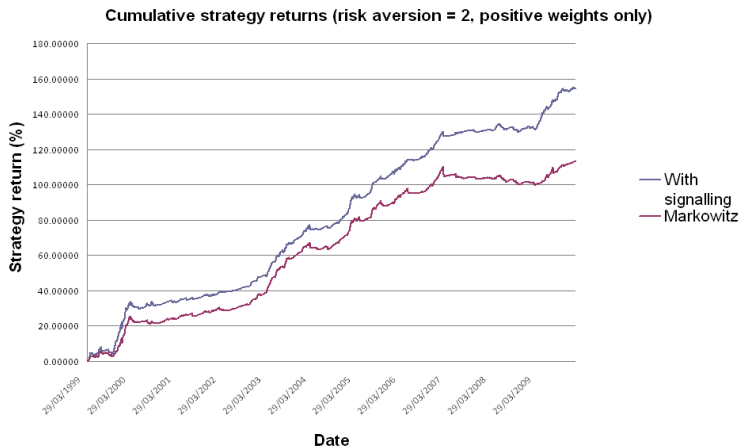
Ex post observed relative excess additive returns, standard deviation ratios

Observed excess returns and std ratios for risky asset only base case
(conditioned wrt Markowitz)



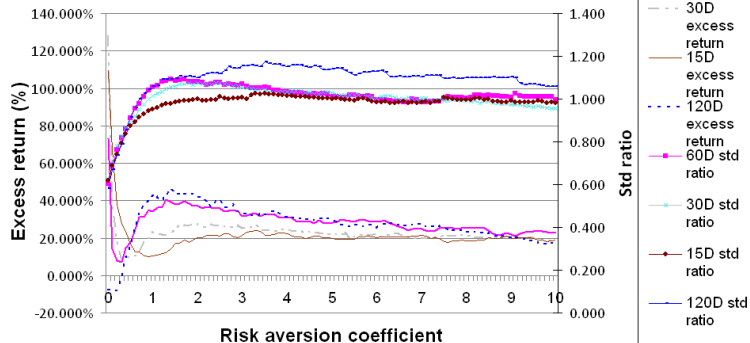
With risky assets only

Time path of additive strategy returns for $\lambda = 2$



Ex post results for different estimation window sizes

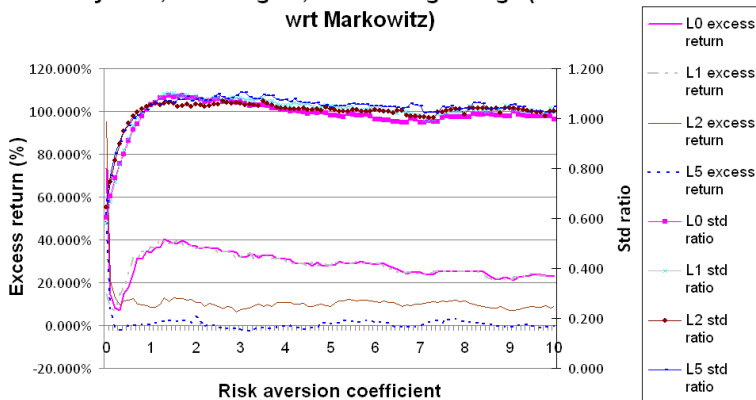
Observed excess returns and std ratios for risky asset
only case, VDAX signal, different estimation window sizes
(conditioned wrt Markowitz)



- Excess returns (and standard deviations) larger as window sizes increase
- Trade-off between statistical quality of estimates and impact of conditional nonstationarities

Ex post results for different signal lags

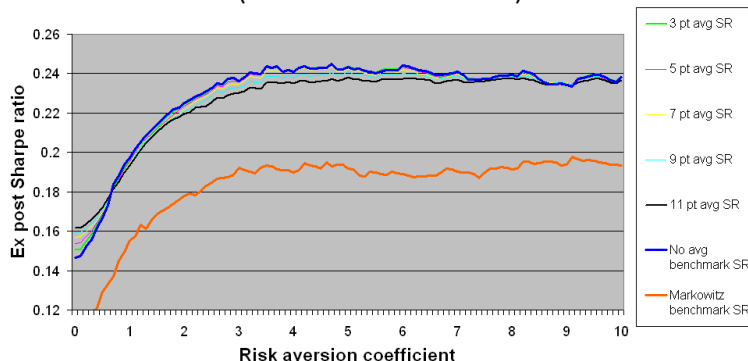
**Observed excess returns and std ratios for risky asset
only case, VDAX signal, different signal lags (conditioned
wrt Markowitz)**



- Excess returns larger and standard deviations smaller as lag size increases
- Trade-off between statistical quality of estimators and easier modelling

Ex post results for weight averages over different number of signal points

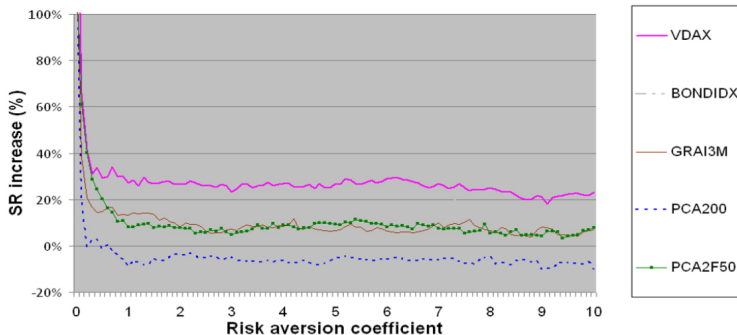
Observed Sharpe ratios for risky asset only case, VDAX signal, averaging weights over different intervals (conditioned wrt Markowitz)



- Negligible changes in excess returns, slight changes in standard deviations: little risk attached to signal observations

Ex post results for different signals

**Observed Sharpe ratio increases for risky asset only case,
VDAX signal, different conditioning signals (conditioned
wrt Markowitz)**



- Best results seen for baseline VDAX signal, averaging seems to distract from signal power