Quantum States and Disjunctive Attacks in Talmudic Logic

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This paper provides logical modelling for the results contained in the twelfth monograph on Talmudic logic entitled *Fuzzy Logic and Quantum States in Talmudic Reasoning* [2].[1]

This paper directly impacts on abstract argumentation theory, temporal and fuzzy arguments and disjunctive collapse. It deals with attacks on a target set of arguments which results in the target to be considered in a quantum like superposition state. The attack is not crisp enough and so cannot be said to be focussed on any individual member or any clear subset of the target. As a result the target set needs to be treated like a quantum superposition of its members.

1 Background and orientation

We begin our discussion with several examples.

We thank the referees for most valuable comments

[1]As we have indicated in our first paper and in our book [1] on Talmudic Logic, the aim of this series (of possibly 25-30 books) is twofold:

1. Import logical tools to the service of modelling and explaining Talmudic reasoning and debate.
2. Export ideas and logical constructions from Talmudic debate for the application and use in general logical theory, artificial intelligence and agency and norms.

Example 1.1 (Disjunctive attacks: Story 1)**.** *Mr. Smith is a rich old man who wants to donate a very rare classic painting to one of two national museums. He committed the donation in a letter to the two museums and copied and approved by Charity Commission, so the donation to one of the two museums was legally done, accepted and in force, except that the choice as to which of the two museums the painting will be given has not been made yet. Mr. Smith said that he would inform the Charity Commission and the museums which museum he would choose in a few days. The donation is in force, however, regardless the status of the choice. Mr. Smith unfortunately died before he made that choice. We are now left with an unclear legal situation regarding ownership. Let a, b and x denoted as follows:*

$b =$ the painting does belong to museum b

$a =$ the painting does belong to museum a

$x =$ body of laws regarding ownership.

We have, of course, that a and b are mutually exclusive. Therefore we have that x disjunctively attacks (see [6]) the set $\{a, b\}$. The attack says one of $\{a, b\}$ must be false. We must be clear here.

Suppose we are dealing with n museums . The options are then

$$a_i = \textit{the painting belongs to the } i\textit{-th museum, } i = 1, \ldots n.$$

Then we have that x implies *that exactly on of a_i holds.*

Put differently, x implies the set $\{a_i | i = 1, .., n\}$, where the meaning of imply a set of formulas *is that exactly one of a_i is true, or equivalently that exactly one $\neg a_i$ is false.*

Then again reformulating we can say that x attacks the set $\{\neg a_i | i = 1, \ldots, n\}$, *where the meaning of* attacking a set *is that exactly one member $\neg a_i$ is false, namely exactly one a_i is true.*

In case $n = 2$, we have $a = \neg b$ and $b = \neg a$ and so we have that x attacks $\{\neg a, \neg b\}$ *is the same as x attacks $\{b, a\} = \{a, b\}$. Talmudic logic debate distinguishes several views on this scenario. The facts on the ground are that the museum's claim that there was a legally binding donation and as for the question of who is beneficiary, a or b, a reasonable deal can be worked out, such as an agreed arrangement of co-ownership, or sharing, or we can let the estate of Mr. Smith continue and choose a museum or we can flip a coin, or ... whatever other symmetrically reasonable solution.*

Talmudic logic debate offers two main views on this:

View 1. Quantum like view. *This view is that , since Mr. Smith died before making a choice of a museum, ownership is superimposed evenly on both museums,* *in the same sense as, nowadays, modern quantum mechanics treats the two slits experiment [5]. Recall that in the two slit experiment a single electron is sent towards two slits a and b and the electron passes through both slits as a wave and interferes with itself. So even though logically in classical mechanics the electron is expected to pass only through one slit, it is also a wave according to quantum mechanics and so it passes through both.*

The Talmudic debaters holding this view are divided in their verdict:

Option 1. *Since a and b are mutually exclusive, there is no longer a donation. The superposition of ownership cancels the donation. The actual Talmudic debate is in connection with marriages but we have adapted the story to Mr Smith and his donation of paintings . See Talmud Bavli, Kidushin, Page 51a and after.*

Option 2. *The superposition of ownership does not cancel the donation. There is a donation the superposition holds but the fact is that the superposition causes lack of clarity of what to do and it should be undone by court order. The museums should waive their "ownership" back to the estate for otherwise the normal flow of life would be disrupted. After that, the estate can re-donate the painting if they want to.*

Of course, those options have implications towards estate tax duties, etc.

Note that both options agree that ownership is super-imposed on both $\{a, b\}$*. They differ in their verdict.*

View 2. Fuzzy probabilistic view. *There is no superposition. There was a donation and we view the scenario as if there was a choice of a museum, except that we do not know what it was, i.e. we treat the case as if Mr. Smith did choose a museum, wrote a letter but died and the letter was lost). So we have a case of purely epistemic uncertainty here and we are expected to provide some mechanism to divide/allocate the painting. For example:*

1. *Share ownership 50/50.*

2. *Make a case for one museum over the other, for example, if the painting was in the special area of museum a, then we can argue and reasonably claim that there is high probability that a was chosen.*

3. *Recommend other arrangements, such as decision by lottery, or time sharing, etc.*

Note that there are further implications to View 2. For example if the number of museums involved is very large, we could on probabilistic grounds, agree that the painting remains with the estate, as the probability for each museum to be the owner is very low. This is an interesting Talmudic view. We might think that it is possible for all the museums to form a coalition and ask for the painting. The Talmud will not allow this. To explain this aspect of this view, think of a different scenarioes.

Scenario 2.1:(Compare with Example 1.2). *There is one painting which was donated to one museum, from among many paintings and we do not know which one it is. Say both the donator and the museum curator die suddenly. The problem is whether we forbid the estate owners of these paintings to sell any of their paintings for fear that it is the one belonging/donated to the museum. The Talmud view in this case is that since the majority of paintings was not donated we allow the sale.*

Compare this scenario with the following variation:

Scenario 2.2. *This scenario is the case of donating one painting but not yet deciding which one, and before a decision is made, the owner dies. In this case, the Talmud says that each of the paintings could have been chosen, and so the museum is part owner/potential owner in each painting and so none of them can be sold! This is like modern quantum superposition view.*

There are other contexts where this practical probabilistic reasoning makes sense. If one Ebola infected person passed through an airport around the time when there were 2000 others present, we can assume about each of the others that he/she is not infected but cannot treat them as such.

We now conclude our discussion of View 1 and View 2 of the story of Mr. Smith donation of one painting to one of two museums. The main thrust of the story is that there is an attack on the set $\{a, b\}$ *without there being any specific attacks on* a *or on* b*. The story can continue as follows:*

Suppose each of a *and of* b*, independently attacks* c*, the details of the attack are not important (maybe* c *is an art critic claiming the painting is a forgery). What is important are the formal options for handling the situation. We have several options for reasoning here*

1. *c must be out (i.e. false), since either a or b is in (i.e. true) and both attack c.*

2. *c must be in, since the disjunctive attack is super-imposed on both a and b, so neither is safely to be considered in (true).*

3. *c is undecided since we do not know exactly what is going on with $\{a, b\}$.*

4. *c joins $\{a, b\}$ in the status of being a member of the superposition set. In other words, we have that if x disjunctively attacks $\{a, b\}$ and a attacks c, and the attack of x on $\{a, b\}$ is perceived as a superposition on $\{a, b\}$, then the constellation of [x disjunctively attacks $\{a, b\}$ and a attacks c, and the attack of x on $\{a, b\}$ is perceived as a superposition on $\{a, b\}$] is taken to be equivalent to the constellation [x attacks $\{a, b, c\}$ and the attack is perceived as a superposition of $\{a, b, c\}$].*

Example 1.2 (Disjunctive attack Story 2)**.** *Mr. Smith is a rich man owning 2 original masterpieces. He decides to donate one of these paintings to the museum (a charity). There are several steps to be taken to accomplish this properly. Select the painting, transfer ownership, put conditions on its use and exhibition, get tax relief on the donation, etc., etc.*

These steps are persistent in time. Once accomplished they remain so. So the temporal flow is to execute each step properly and then legally end up with the result. The Talmudic scenario is to study, debate and rule in cases where the steps become fuzzy. The question is then to determine what final result we have in this case. The logic behind the Talmudic debate of the various scenarios is the Talmudic fuzzy logic and Talmudic disjunctive attacks.

Scenario 3. *Mr. Smith commits a painting to the museum. The museum sends Mr. Jones to go with Mr. Smith to the "storage vault" and choose a painting.*

Storyline 1. *On the way both die (tragic traffic accident).*

Question 1. *What does the museum get/own? What would the heirs/estate of Mr. Smith do?*

Storyline 2. *Mr. Smith and Mr. Jones get to the storage and choose a painting. On the say out of the storage they both die. So we know a painting was chosen but we do not know which one and we have no way of knowing.*

Question. *Same as before.*

Storyline 3. *Mr. Smith authorises Mr. Jones to go to the storage and choose a painting. Mr. Jones does that and telephones Mr. Smith and tells him what he chose. A few minutes later Mr. Smith dies of a heart attack and Mr. Jones dies in a tragic accident. The museum knows the government was secretly and unlawfully* *recording all telephone conversations of prominent citizens. They could try and get the recording of which painting was chosen. This is very difficult because the Government will never admit that it is listening to its citizens. In this scenario, we could find out what painting was chosen, but for all practical purposes, we find ourselves in Storyline 2.*

We note that once we are in a state of superposition, like when a painting was donated to one of two museums but not decided which one or one of two paintings was donated to a single museum but not decided which one, we can collapse the superposition retrospectively by, for example, flipping a coin. This is parallel to quantum superposition which can collapse when we do measurements.

Let us now analyse these stories. Let

$$S = \{\pi, \pi'\}$$

be the set of paintings and let $G(x)$ be the predicate that x was given by Mr. Smith to the museum. First we ask: do we know for sure that $\exists x G(x)$ must hold? The problem is that if no painting was chosen, was there a donation?

If we decide that there was a donation, then which painting? Can the museum sell something? Can the museum transfer to another legal entity whatever it has?

Storyline 1. *This is the case where a painting was donated but none was chosen. Compare with Example 1.1.*

Rava opinion. *There is no deal. The museum gets nothing. (Compare with View 1 of Example 1.1.)*

Abeyei opinion. *There was a valid deal. $\exists x G(x)$ is true but we are in doubt as to which painting was given to the museum. We have a case of superposition here.*

According to this view, we can recommend some options.

A1: *The museum is to give up voluntarily the donation. This is what the law forces them to do.*

A2: *Alternatively, in practice, they may reach some deal.*

1. *The estate of Mr. Smith can donate all the paintings to the museum.*
2. *Choose a paining now.*
3. *Rotate the donation, rotate every season a different painting.*
4. *etc.*

This may be OK for paintings and museums, but there are other scenarios which are less flexible. Mr. Smith may have two beautiful daughters and he has agreed to give one of his daughters in marriage to Mr. Jones' son. According to Abayei's approach, only option A1 can be taken. No sharing or rotation or anything is possible, only divorce from each of them. According to the law one cannot be married to two sisters at the same time. One cannot even choose one later, because the new choice may not be the correct one and if married to one you cannot have a marital relationship with her sister. If we look at Storyline 2, here there was a choice of painting or daughter, but we do not know which one. So we can apply a different logical machinery to this case. Maybe we can argue that the museum has all the paintings of Van Gogh except the one which Mr. Smith owns and so it is most likely that the last van Gogh was chosen or in the case of marriage, one can argue that perhaps one of the daughters already knows Mr. Jones' son and the process was most likely aimed at choosing her?

To sharpen the difference between Storyline 1 Abayei and Storyline 2 Abayei, we note the following:

Our storyline 1, Abayei, we accept that $\exists x G(x)$ holds but we do not accept for any $x \in S$ that $G(x)$ holds in a clear cut way, as opposed to some fuzzy way. So Rava says there is no engagement and Abayei says that there is, but it is fuzzy. In Storyline 2, we also accept that for one of $x \in S, G(x)$ holds, but we do not know which one.

We need a logic which can model such distinctions!

2 Argumentation networks

We need to model the above examples. We shall use a version of disjunctive argumentation networks [6, 3].

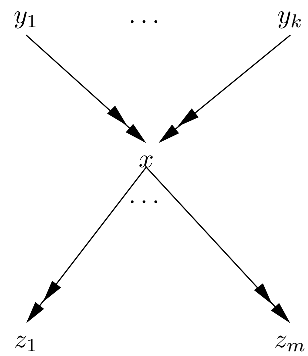
Definition 2.1. *A finite argumentation network has the form (S, R), where S is a finite non-empty set of arguments, and $R \subseteq S \times S$ is an attack relation. We also write $x \twoheadrightarrow y$ in diagrams to express xRy, x attacks y.*

Example 2.2. *Imagine two pairs of parents planning a joint wedding for their children. They need to compose a list of guests of several types.*

1. *Relatives from each family*

2. *Neighbours and friends of parents*

3. *Friends of the bride and bridegroom*



$a \twoheadrightarrow b$ means aRb

Figure 1

4. Colleagues and co-workers

Inviting family can be a problem!

Auntie Bertha might say "I am not coming if that bastard ex-husband of mine is invited". I.e., Bertha $\twoheadrightarrow$ ex-husband.

Grandma Teresa might say "I don't want these kids inviting too many of these hippy crazy friends of theirs, espeically not the drummers". I.e., Teresa $\twoheadrightarrow$ {set of hippies}.

Figure 1 can describe the problematic map which exists:

x is one possible invitee, say Grandma Teresa. She is 109 years old and $y_1, \ldots, y_k$ object to inviting her. Possibly because she is too old and they are worried about her health The reason does not matter. The important fact here is the double arrow $y_i \twoheadrightarrow x$. This means y_i wants x out. So if y_i is invited, x cannot be invited. Similarly, x objects to $z_1, \ldots, z_m$. So the Figure 1 describes the entire configuration around x. We want to define a maximal set E of invited guests such that the following holds:

1. *$x, y \in E \Rightarrow x$ does not attack y, (i.e., not xRy). I.e., E is conflict free. No member x of E says "I object" to another member of E.*

2. *If any x says "why did you invite $z \in$ and you did not invite me? How could you invite this terrible person z"? (i.e., we have $z \twoheadrightarrow z$), then we can say, "we had to invite $y \in E$ and unfortunately, y was against you x" (i.e. for some $y \in E, y \twoheadrightarrow x$).*

Such a set E which is also maximal, is called in the argumentation community "a preferred extension". These always exist.

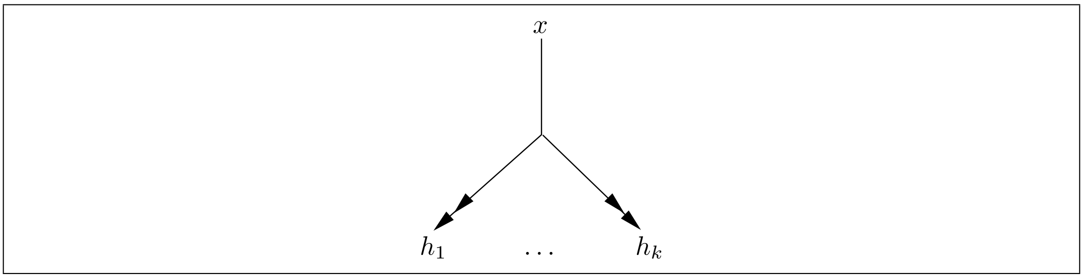


Figure 2

A disjunctive attack has the form $x \twoheadrightarrow H$ where $H \subseteq S$. Its meaning is

- if $x \in E$ then for some $y \in H(y \notin E)$.

This means if you invite x then one of H must not be invited. For example x may be having an affair with both (h_1, h_2). So it is bad taste to invite both. We know about it, but h_1 and h_2 do not know about each other, so it is better not to have them both, says x. We use the notation of Figure 2

Definition 2.3 (See [6], Definition 3.3)**.**

1. *A finite disjunctive argumentation network has the form $\mathcal{A} = (S, \rho)$, where S is a finite set of arguments and $\rho \subseteq S \times (2^S - \varnothing)$, i.e. ρ is a relation of (disjunctive attacks) between elements $x \in S$ and non-empty subsets $H \subseteq S$ denoted as $(x\rho H)$.*

   Let (S, ρ) be a network and let $E \subseteq S$:

   (a) *We say E is conflict free iff for no $x \in E$ and $H \subseteq E$ do we have $x\rho H$.*

   (b) *We say that E protects α iff for any $z\rho H \cup \{\alpha\}$ there exists a $\beta \in E$ and $E_3 \subseteq E$ and $H_3 \subseteq H$ such that $\beta\rho H \cup H_3 \cup E_3 \cup \{z\}$.*

   (c) *We say E protects itself if it protects each of its members.*

   (d) *We say E is a complete extension if E is conflict free, protects itself and contains all the elements it protects.*

Talmudic attack $x\rho H$ wants exactly one *$y \in H$ to be out. Talmudic logic thinks of it as a collapse of $x\rho H$ to xRy.*

The next definition, 3.1 will explain what we mean by collapse, and give a more correct way to obtain the complete extensions according to Talmudic logic.

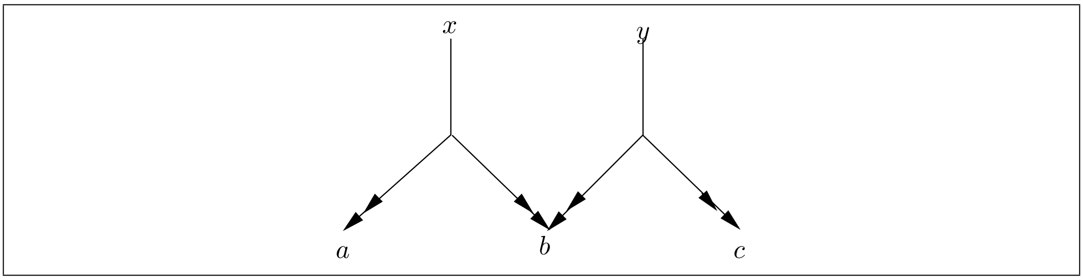


Figure 3

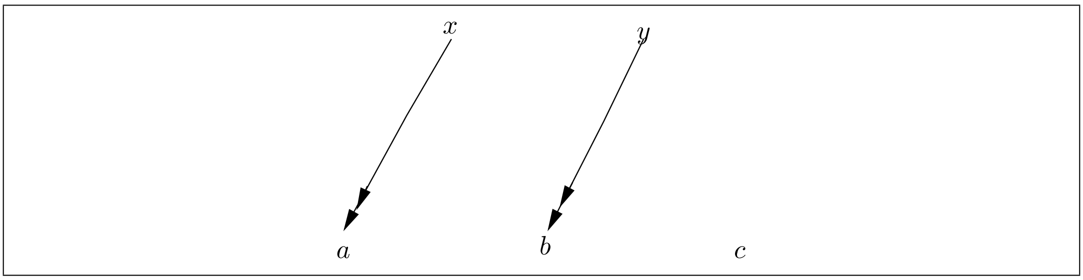


Figure 4

3 Talmudic argumentation systems

Definition 3.1. *Let $\mathcal{A}$ be a finite disjunctive network and let $x\rho H$ be one of its attacks. We say that a set $\mathbb{F}((x,H))$ is a collapse set for (x,H) if it is the set of all $\mathcal{A}_y, y \in H$ of the form $\mathcal{A}_y = (S, \rho^x_y)$, where $\rho^x_y = (\rho - \{x,H)\}) \cup \{(x,\{y\})\}$. In other words, (S, ρ^x_y) is the network where $x\rho H$ is replaced by $x\rho\{y\}$, i.e. $x\rho H$ collapses to $x\rho\{y\}$.*

For each $x\rho H$, let $\mathbf{f}(x,H)$ choose one pair $(x,y), y \in H$. Let $\mathcal{A}_\mathbf{f}$ be the total collapse of $\mathcal{A}$ according to $\mathbf{f}$, defined as $(S, R_\mathbf{f})$, where $R_\mathbf{f} = \{\mathbf{f}(x,H) | x\rho H\}$.

Example 3.2.

1. **Complete collapse**. *Consider the network of Figure 3.*

   Here we have $x\rho\{a,b\}$ and $y\rho\{b,c\}$. The total collapses are the networks in Figures 4, 5, 6 and 7.

2. **Partial collapse.** *We may have that say $x\rho\{a,b\}$ collapses while $y\rho\{b,c\}$ does not collapse. So we have in this case the possible Figures 8 and 9.*

Remark 3.3. *We have to decide what the Talmud would say about attacks emanating from non-collapesed nodes. Consider Figure 10*

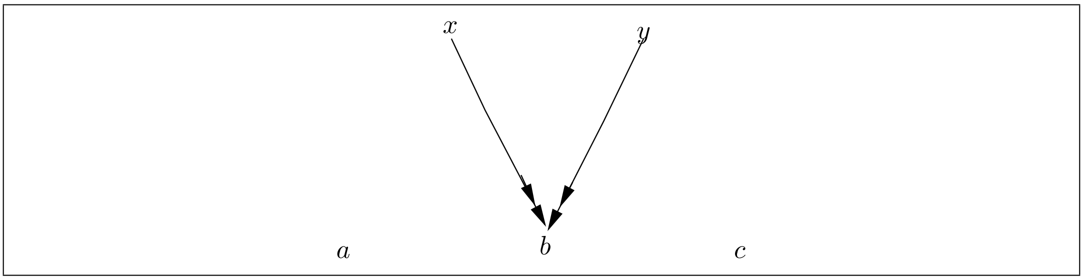


Figure 5

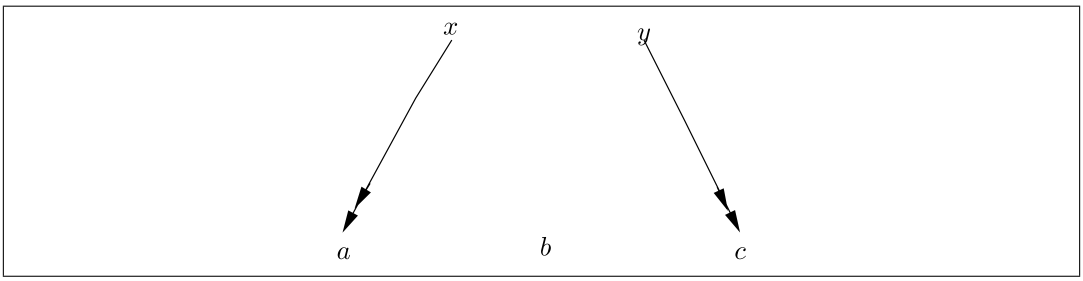


Figure 6

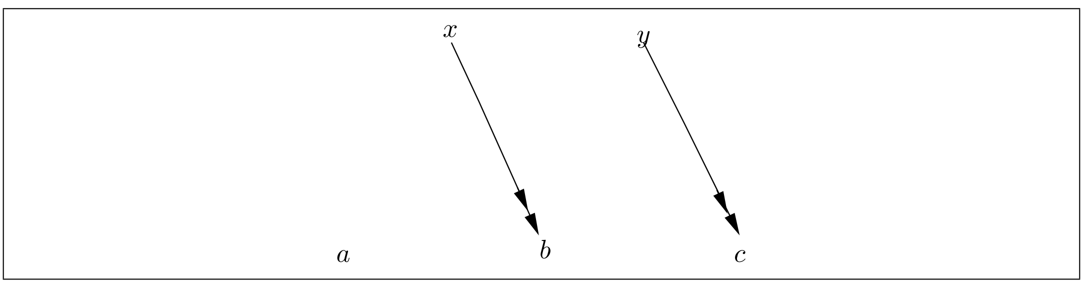


Figure 7

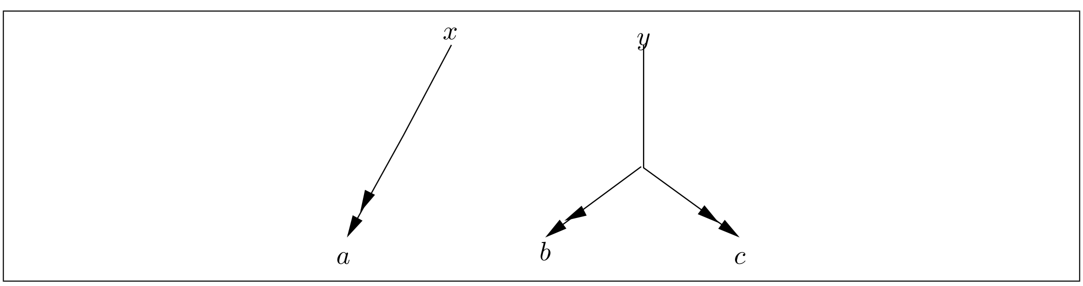


Figure 8

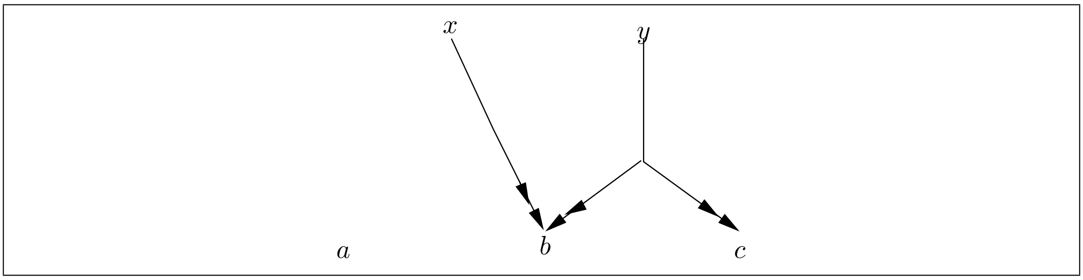


Figure 9

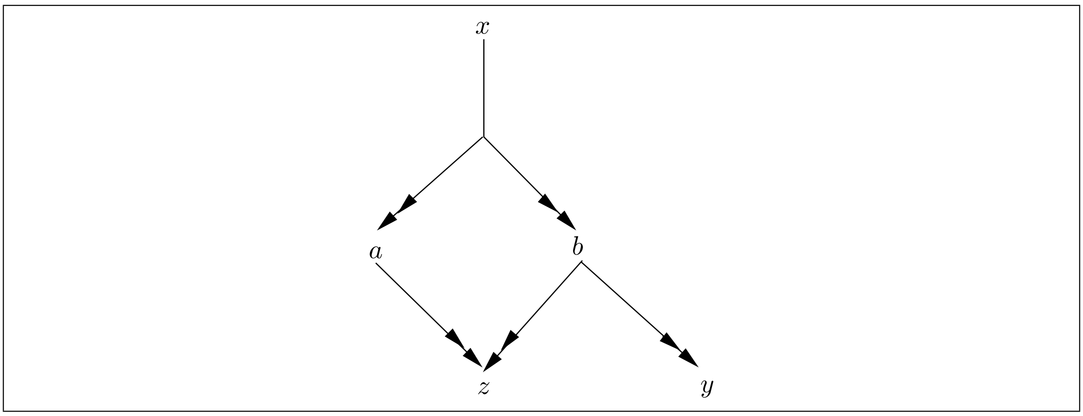


Figure 10

In this figure the attack of x on $\{a, b\}$ remains uncollapsed. So this is the final fixed figure. What is our view of $\{a, b\}$? Do we consider them as both in/true (since there is no collapse) for the purpose of the attacks $b \twoheadrightarrow y$, $b \twoheadrightarrow z$ and $a \twoheadrightarrow z$? Or do we regard then as undecided? Do we give them fuzzy values?

The Talmud approach can be modelled by four values {in, out, undecided, wave}. So we use labelling $x \in \{in, out, und, wave\}$.

So, in Figure 10 we may have that $\{a, b\}$ does not collapse, so we give a, b value "wave" each. This value is passed on to y and z.

If y or z further attack some nodes, they will pass on the value "wave" to their targets.

Remark 3.4. *Compare with the traditional Caminada labellings and other approaches in [4]. Let us look again at Figure 1 where $y_1, \dots, y_k$ are all the attackers of x and let us write the conditions on any $\lambda : S \mapsto$ {in, out, und, wave} to be a legitimate Talmudic labelling for a traditional network (S, R) without disjunctive attacks.*

(TC1) $\lambda(x) =$ *out, if for some* $y_i, \lambda(y_i) =$ *in.*

(TC2) $\lambda(x) =$ *in, if for all* $y_i, \lambda(y_i) =$ *out.*

(TC3) $\lambda(x) =$ *und, if none of* y_i *has value* $\lambda(y_i) =$ *in and some of* $\lambda(y_i) =$ *und.*

(TC4) $\lambda(x) =$ *wave, if none of* $\lambda(y_i) =$ *in and none of* $\lambda(y_i)$ *is und and some of* $\lambda(y_i) =$ *wave.*

Remark 3.5. *We now have to define what is a legitimate* λ *for a network* (S, ρ) *with disjunctive attacks* $\rho \subseteq S \times (2^S - \varnothing)$*. We shall reduce this concept by induction to the traditional case with four values as defined in Remark 3.4. The reduction is by induction on the number of disjunctive attacks in* (S, ρ)*. We first need a concept of constraints on* λ*.*

1. *Let* (S, R) *be an argumentation network of any kind (traditional or Talmudic) with* $R \subseteq S \times S$*. Let* λ_1 *be a partial function* λ_1 *: Subset* E *of* $S \mapsto$ *values. We say* λ *is a legitimate extension under the constraint* λ_1 *if* λ *is legitimate and* λ *agrees with* λ_1 *on its values.*

2. *For example in the configuration of Figure 1 we may have the constraint* $\lambda_1(y_1) =$ *wave. However, if the figure is part of a larger network and* y_1 *is attacked by a node which needs to be in, then* λ *cannot overrule* λ_1 *on the value of* y_1*.*

   When we have a constraint λ_1 *it may be the case that no legitimate* λ *exists with such a constraint.*

3. *We now define what it means to be a legitimate Talmudic extension for* (S, ρ)*.*

   This is done by induction on the number of disjunctive attacks in (S, ρ)*. We choose a disjunctive attack and do a case analysis of "imaginary" options, (being option (a), (b,i), (b,ii) and (b,iii) below). With each such option we associate a family* $\mathbb{F}$ *(option) of networks with a lesser number of disjunctive attacks. Each member of each family will yield some legitimate* λ *by the induction hypothesis, and the totality of these* λ *are the legitimate extensions for* (S, ρ)*.*

   So let us begin:

   Base Case. *There are no disjunctive attacks, but there are constraints* λ_1*, requiring values from {in, out, und, wave} . Use principles (TC1)–(TC4) of Remark 3.4 to get the extensions, if possible.*

Inductive Case. *There are disjunctive attacks and there are constraints λ_i. In this case we choose one disjunctive attack. Define the case analysis below and define the sets $\mathbb{F}$ (case number). Any λ found by the inductive hypothesis for any element of these sets will do for our (S, ρ).*

So let us begin the inductive case: Let $x\rho\{h_1, \ldots, h_k\}$ as in Figure 2.

We distinguish two cases for the Talmudic complete extension λ.

(a) **case of collapse** *In this case the attack of x on $\{h_1, \ldots, h_k\}$ does collapse to one of the attacks $x \twoheadrightarrow h_i$, for some i.*

Therefore we define the legitimate λ for (S, ρ) as any legitimate λ for $\mathbb{F}$ (case (a)) $= \{(S, \rho_i)|$ where $\rho_i = (\rho - \{(x, \{h_1, \ldots, h_k\}))\cup\{(x, \{h_i\})\}\}$ respecting the constraints λ_1.

(b) **case of no collapse** *In this case we distinguish three cases.*

i. *x is out. In this case there is no attack and we let the legitimate λ for (S, ρ) to be any one of the legitimate λ of $\mathbb{F}$ (case (b,i)) $= \{(S, \rho_i)$ of case (a) but with the additional constraint to λ_1 being the constraint $x =$ out$\}$.*

ii. *x is in or $x =$ wave. In this case there is no collapse and so we have the additional constraints for λ_1 being $h_1 = h_2 = \ldots = h_k =$ wave. So we let the legitimate λ for this case for (S, ρ) to be any legitimate λ for the network $\mathbb{F}$ (case (b,ii)) $= \{(S, \rho')$ where $\rho' = \rho - \{(x, \{h_i, \ldots, h_k\})\}$ under the constraint λ_1 augmented by the additional constraints $x =$ in or $x =$ wave ,respectively and $h_i =$ wave for $i = 1, \ldots, k\}$.*

iii. *x is und. In this case we look at (S, ρ') as in case (ii), with the additional constraints to λ_1 being the constraint $x = h_1 = \ldots = h_k =$ und.*

Example 3.6. *Let us see what the Talmud would do with Figure 10.*

Here we have only one disjunctive attack $x\rho\{a, b\}$ for which we know $x =$ in because x is not attacked. So there are two possibilities for this attack.

1. *The attack collapses and so $x \twoheadrightarrow \{a, b\}$ is to be replaced either by $x \twoheadrightarrow a$ or by $x \twoheadrightarrow b$, giving rise to Figure 11 or Figure 12.*

2. *The attack does not collapse, giving rise to Figure 13 with the constraints shown.*

   So the possible extensions according to Remark 3.4 are:

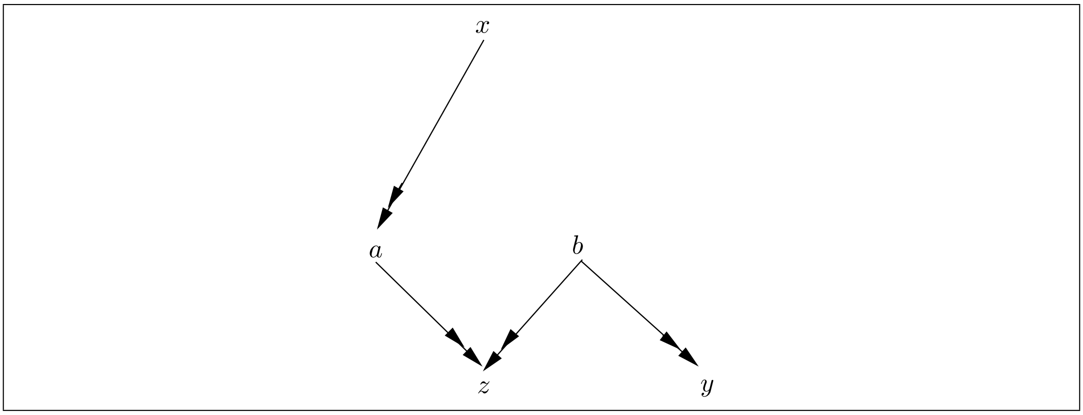


Figure 11

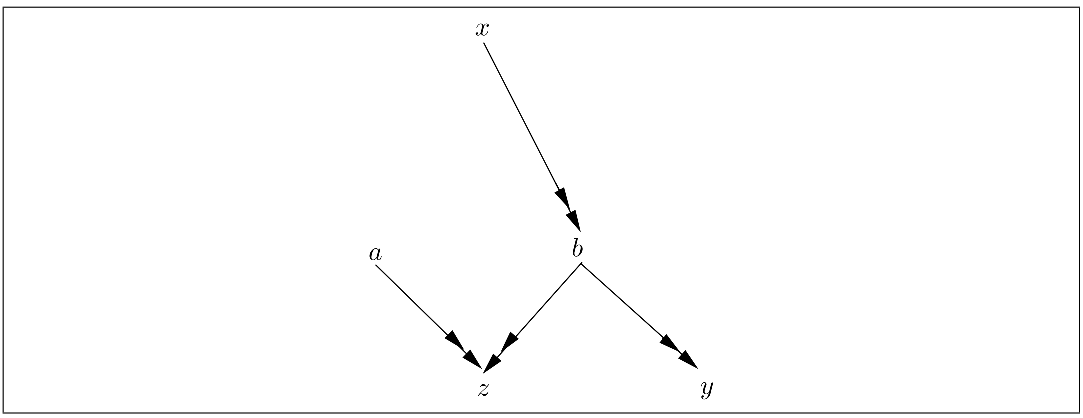


Figure 12

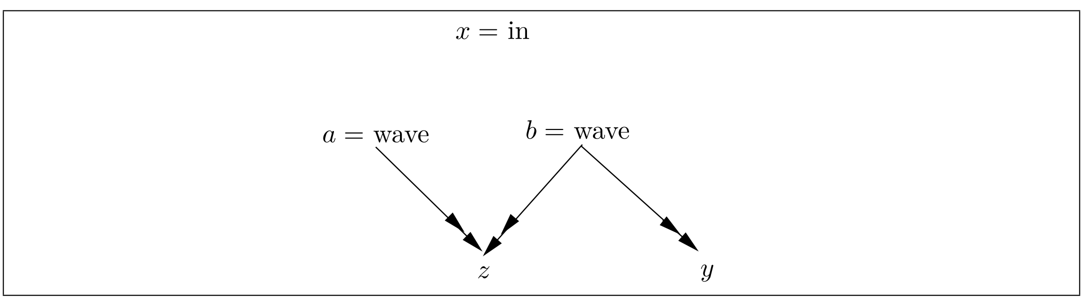


Figure 13

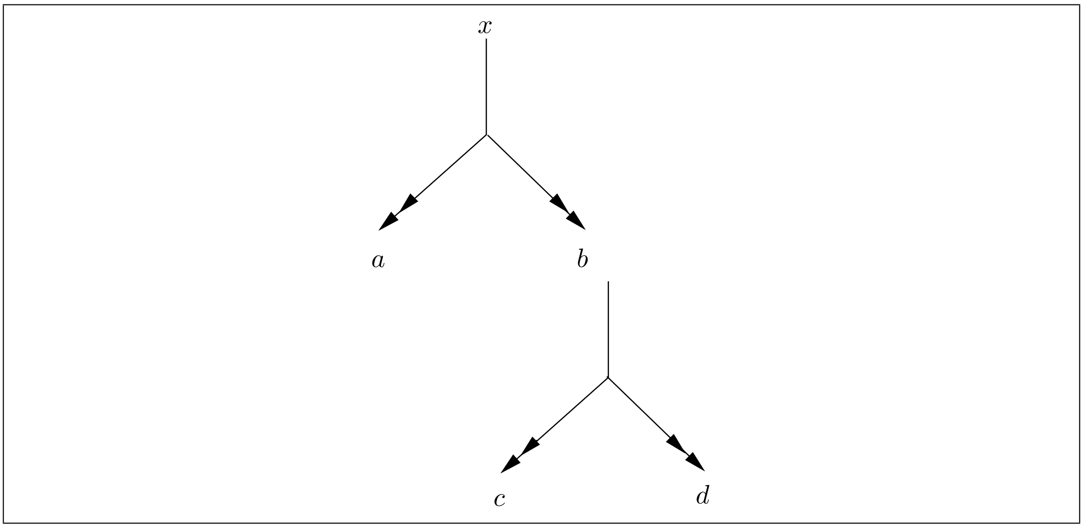


Figure 14

$$\begin{aligned}
\lambda_1 &: x = \textit{ in}, \ a = \textit{ out}, b = \textit{ in}, z = y = \textit{ out} \\
\lambda_2 &: x = \textit{ in}, \ a = \textit{ in}, b = \textit{ out}, z = \textit{ out}, y = \textit{ in} \\
\lambda_3 &: x = \textit{ in}, \ a = b = z = y = \textit{ wave.}
\end{aligned}$$

Example 3.7. *Consider the network of Figure 14.*

Let us agree that $x\rho\{a, b\}$ does collapse while $b\rho\{c, d\}$ does not collapse.

The extensions are the following, calculated intuitively.

$$\begin{aligned}
\lambda_1 &: x = in, \ a = \textit{ out}, \ b = \textit{ in}, \ c = d = \textit{ wave} \\
\lambda_2 &: x = in, \ a = \textit{ in}, \ b = \textit{ out}, \ c = d = \textit{ in}
\end{aligned}$$

Let us now follow our inductive procedure of Remark 3.5 and let us start inductively from $b\rho\{c, d\}$. We get four options, as seen in Figures 15, 16, 17 and 18. The constraints are written in the figures.

For each of the Figures 15–18 we deal with the attack $x\rho\{a, b\}$. These split into two figures each. One with the attack of x on a and one with the attack of x on b.

Some of these will not be possible.

Here are the Figures:

We see that the inductive procedure gave us λ_1 and λ_2 as we expected.

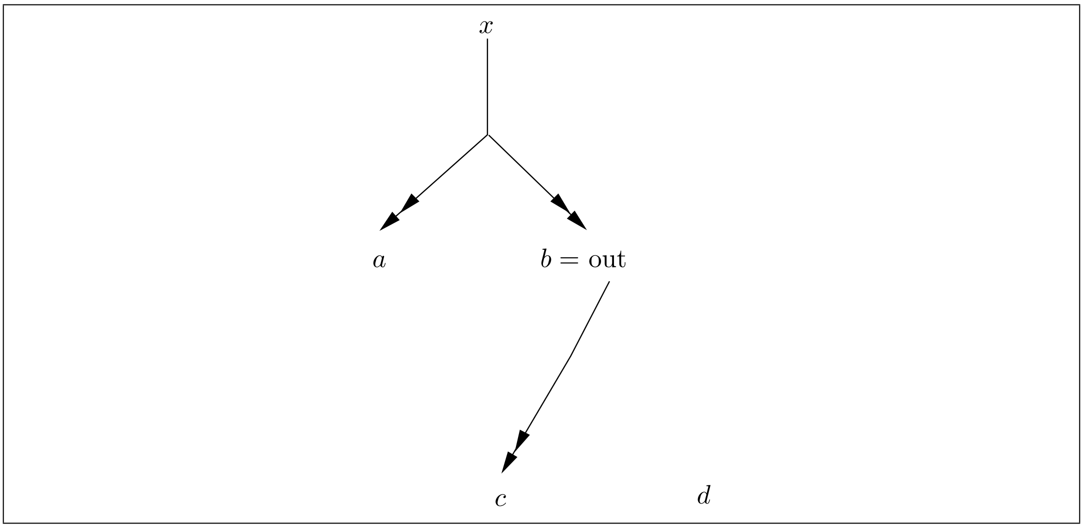


Figure 15

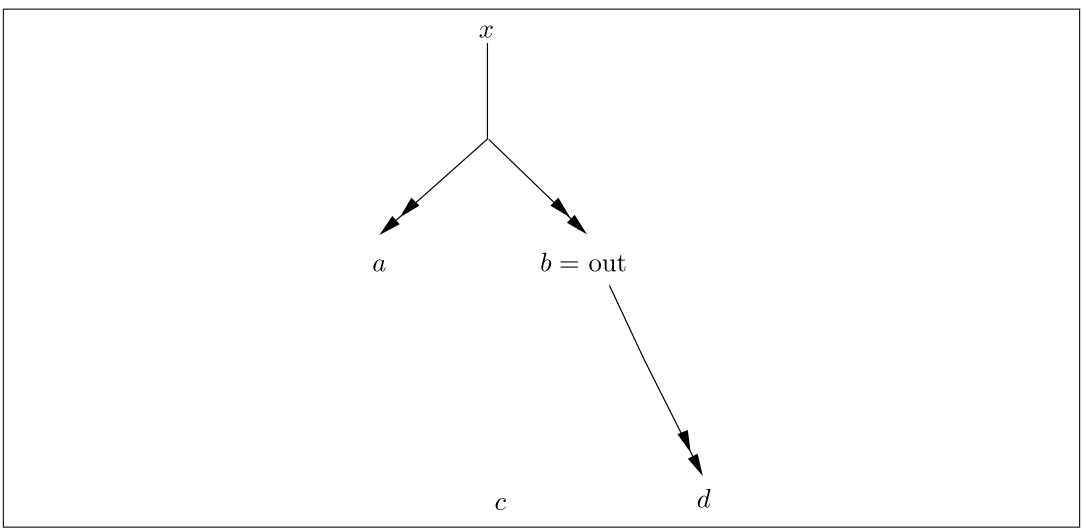


Figure 16

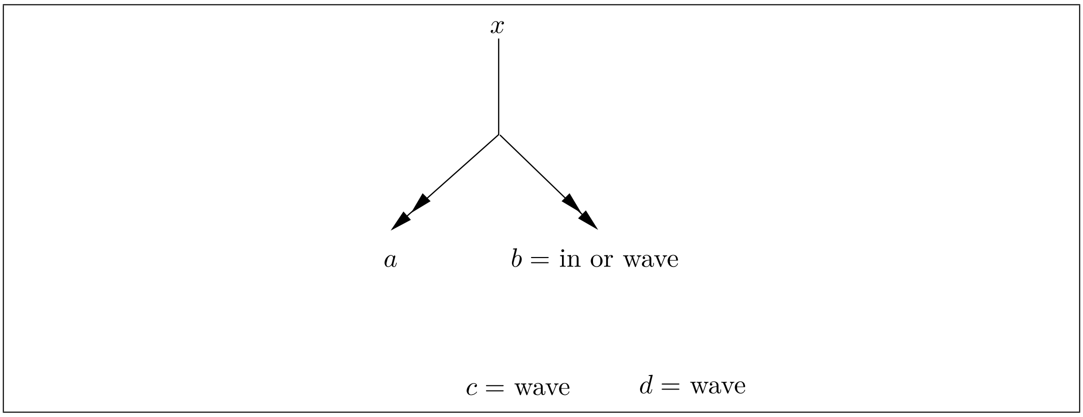


Figure 17

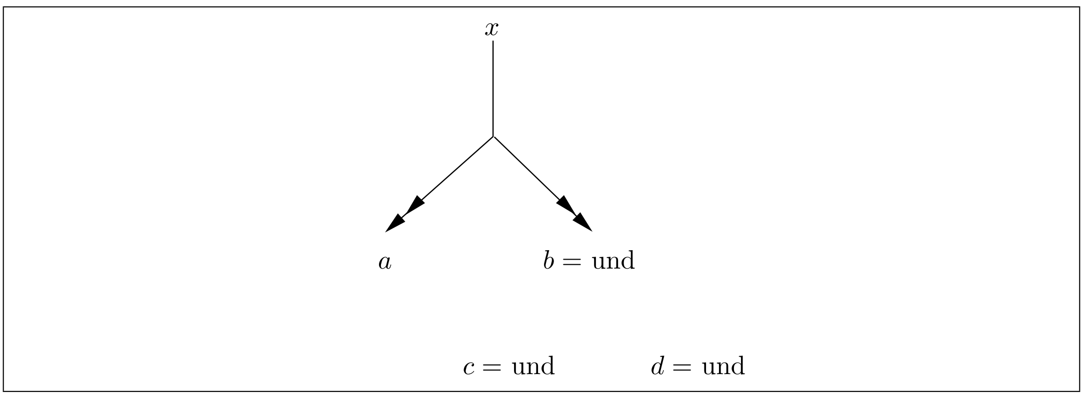


Figure 18

4 Using Bochman's collective argumentation

In his paper [3], Bochman considered conjunctive disjunctive attacks of the form $G \twoheadrightarrow H$, where both G and H are subsets of S. The intended meaning of $G \twoheadrightarrow H$ is that if all embers of G are in, then at least one member of H is out. The treatment of this notion is straightforward (see [6] and Bochman [3]) using axiomatic properties on $G \twoheadrightarrow H$ to characterise various types of semantics. For example, he considered the following axioms:

Montonicity. If $G \twoheadrightarrow H$ then $G \cup G' \twoheadrightarrow H \cup H'$.

Symmetry. If $G \twoheadrightarrow H_1 \cup H_2$ then $G \cup H_1 \twoheadrightarrow H_2$.

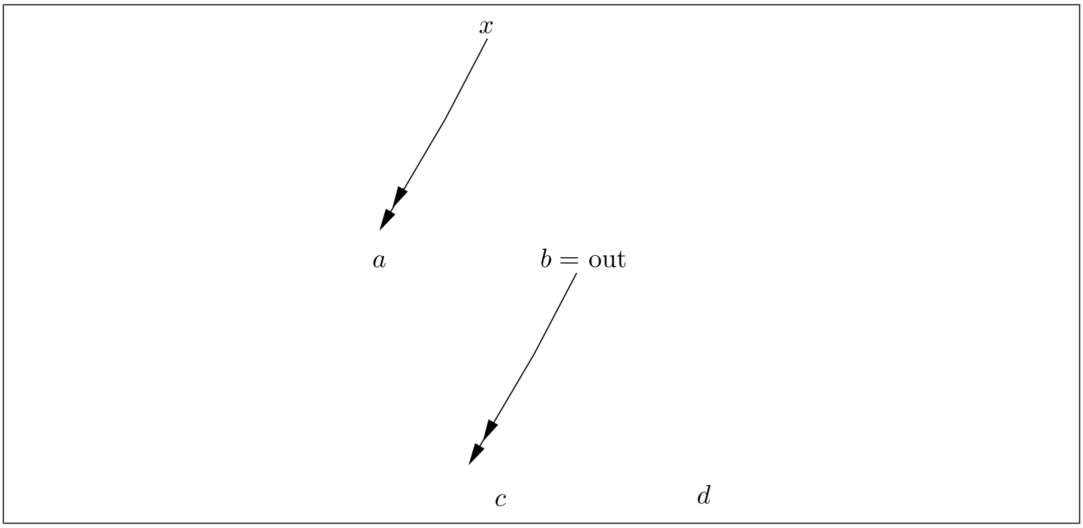


Figure 19: Not possible.

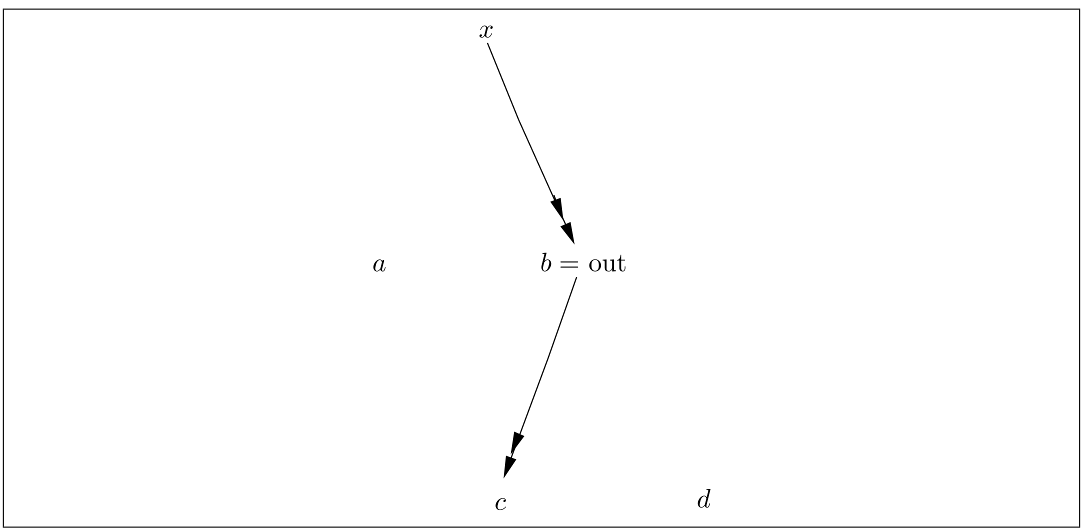


Figure 20: Possible, gives λ_2.

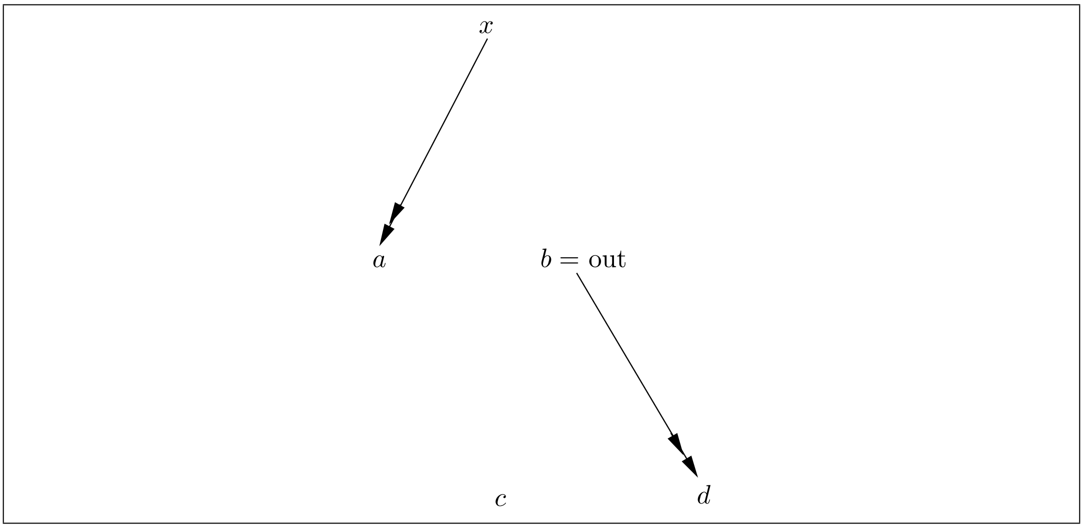


Figure 21: Not possible.

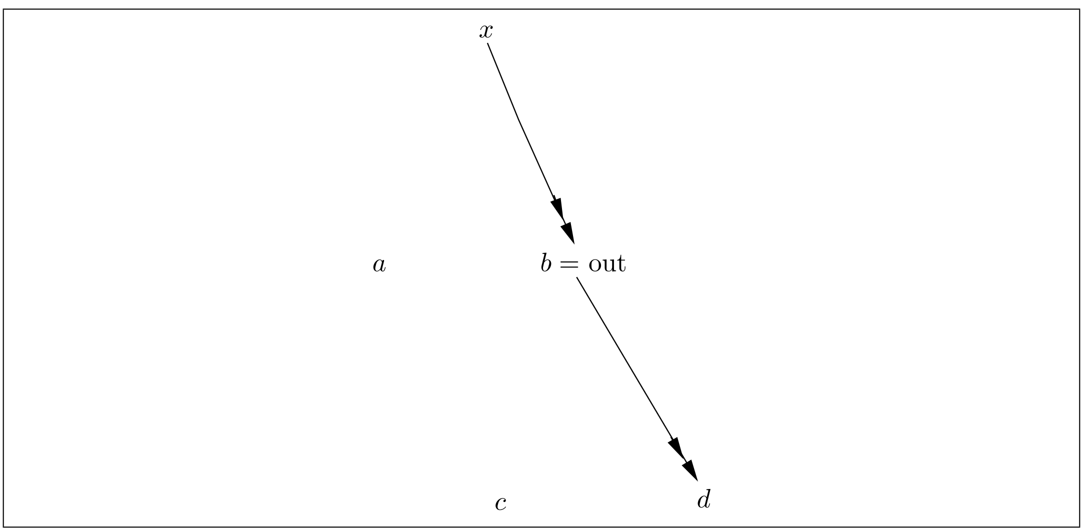


Figure 22: Possible, gives λ_2.

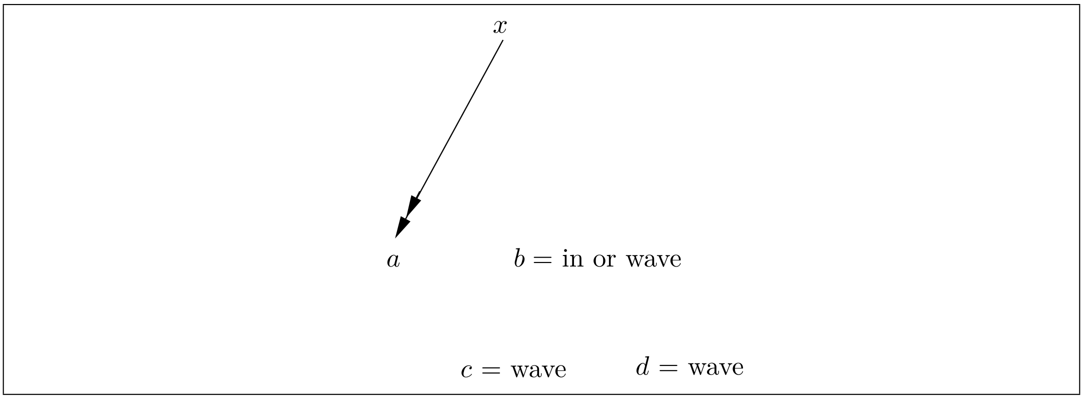


Figure 23: Possible only with $x =$ in. Gives λ_1.

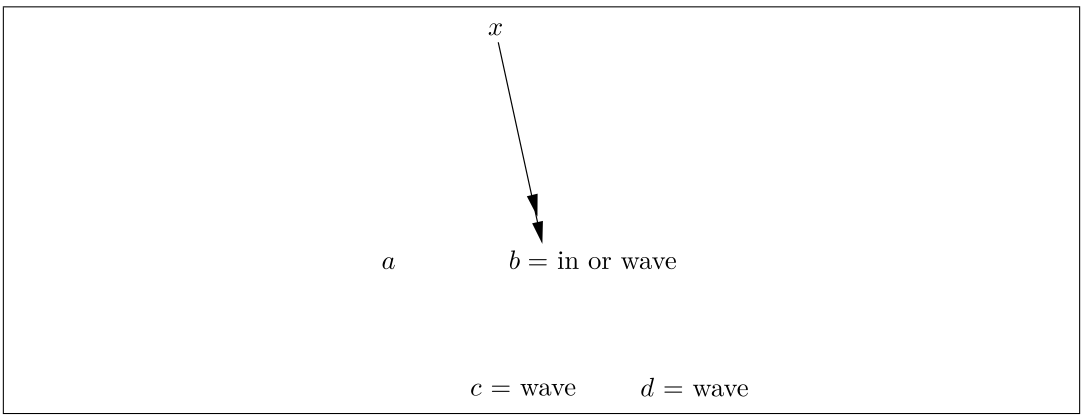


Figure 24: Not possible

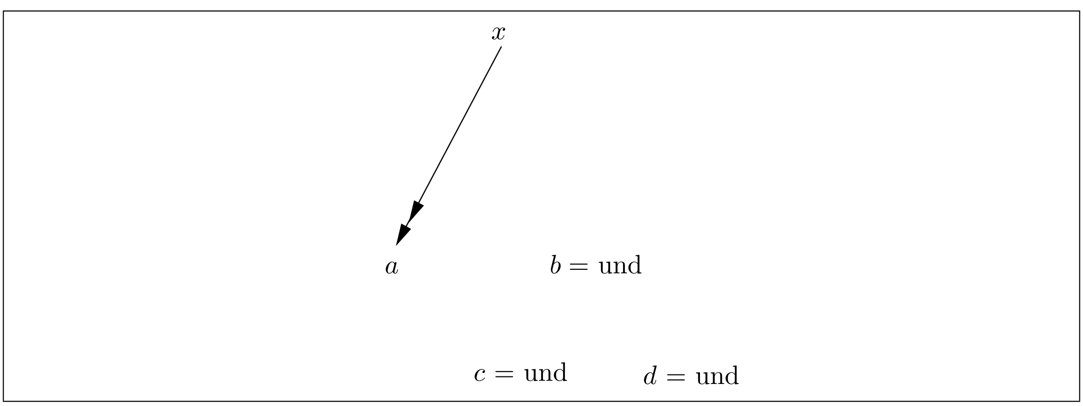


Figure 25: Not possible.

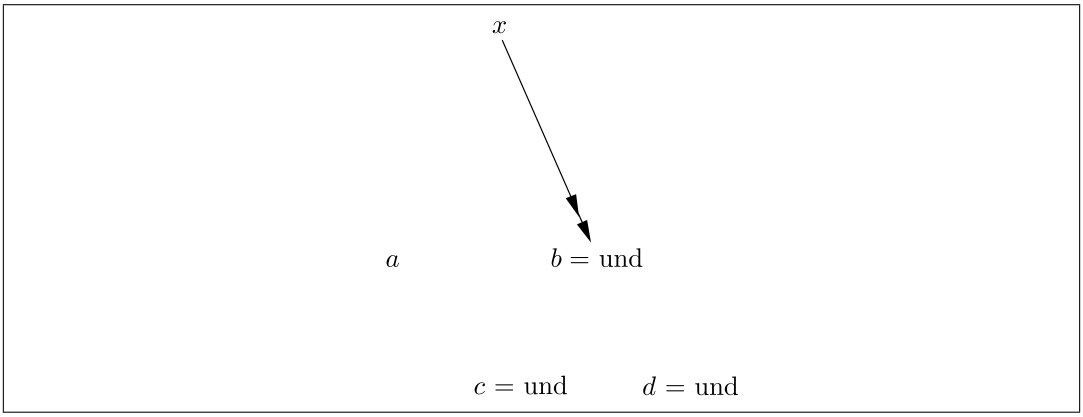


Figure 26: Not possible

Affirmativity. The empty set is not attacked.

Locality. If $G \twoheadrightarrow H_1 \cup H_2$ then $G \twoheadrightarrow H_1$ or $G \twoheadrightarrow H_2$.

Using this axiomatic approach, we want to characterise Talmudic disjunctive attacks. Let us list the properties we need to characterise:

(P1) If $G \twoheadrightarrow H$ then either (a) or (b) holds

(a) For exactly one $y \in H$ we have $G \twoheadrightarrow \{y\}$. (This is collapse.)

(b) For none of $H' \subsetneq H$ do we have $G \twoheadrightarrow H'$. This is quantum superposition.)

(P2) If $G \twoheadrightarrow H$ is a quantum superposition attack on H and for some $H' \neq \varnothing$ such that $H' \subseteq H$ we have that $H' \twoheadrightarrow K$ then $H' \twoheadrightarrow K$ is also a quantum superposition attack and furthermore $G \twoheadrightarrow (H - H') \cup K$ also holds as a quantum superpsoition attack.

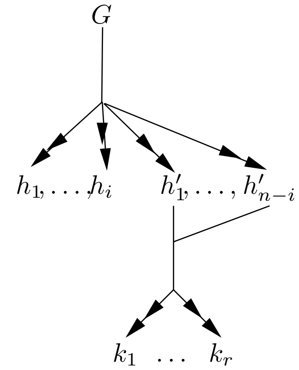
Figure 27 explains the idea in terms of labels. Any attacker with a wave label propagates this label to its targets.

Note that (P2) can be better understood in positive terms. If $G \rightarrow H$ and $H' \rightarrow K$ and $H' \subseteq H$, then $G \rightarrow (H - H') \cup K$.

The following Bochman-style axioms can characterise (P1) and (P2).

(AP1) If $G \twoheadrightarrow H$ then $H \neq \varnothing$ and

$$\bigvee_{y \in H} \{[G \twoheadrightarrow \{y\}] \wedge \bigwedge_{z \neq y} \neg G \twoheadrightarrow \{z\}\} \vee (\bigwedge_{H' \subsetneq H} (\neg(G \twoheadrightarrow H')))$$



If the value of h'_j is "wave" because of G then the values of $k_1, \ldots, k_r$ is also "wave".

Figure 27

(AP2) If $G \twoheadrightarrow H$ and $\varnothing \neq H' \subsetneq H$ and $H' \twoheadrightarrow K$ then $G \twoheadrightarrow (H - H') \cup K$ and $\bigwedge_{K' \subsetneq K}(\neg(H' \twoheadrightarrow K'))$ and $\bigwedge_{K' \subsetneq (H-H') \cup K}(\neg(G \twoheadrightarrow K'))$.

5 Conclusion and discussion

We saw that Talmudic disjunctive attacks require four values, {in, out, und, wave} and differs from [6] in two senses:

1. The attack on a target set H can turn the target set into having the value "wave" for quantum like superposition. This value is then propagated further by the members of the target set when they attack further targets.

2. When attacking a target set H the attack can collapse to attacking a single $y \in H$. Note and compare that the disjunctive attacks in general can collapse to attacking a subset $\varnothing \neq G \subseteq H$.

   In our case the options for the attack on a set H is either an attack on a single member of H or rendering H into a wave quantum superposition state.

3. We note here, that in view of references [6] and [3], one of our referees commented as follows:

   > "Two things should be distinguished here. First, the quantum superposition idea, taken by itself, is more or less comprehensible and internally coherent. The only question is whether it is relevant to

argumentation, and it is here that I have my reservations. An argument may collectively attack a set of arguments H just because it disproves one of the joint conclusions of H. This is not a quantum phenomenon, and different arguments in H can still be separated by other arguments, and by different attacks they (separately) create against further arguments. So, if your interpretation insists on existence of a special 'superposition' of a set of arguments, it ought to be represented as an entirely new connective for combining arguments, over and above the existing argumentation machinery."

We comment in return that adding a connective on a set H which turns it into a quantum state is too strong a move. In our system a set H may turn into a quantum state only when attacked. It is the nature of the attack that causes the quantum state. We can certainly investigate a connective, say $\mathbb{Q}(H)$, which turn H into a quantum state but it will be different

(a) A disjunctive attack cannot turn a single point set into a quantum state, but the connective $\mathbb{Q}$ can do that (and make this point propagate the "wave" value).

(b) Adding $\mathbb{Q}$ yields a different logic. (Certainly worthy of investigation.)

4. Another referee brought to our attention the works of Andrew Schumann, [7, 8], connecting Talmudic Logic to parallel computation. The referee spent a lot of effort going through our papers commenting how the quantum view can, and maybe should, be replaced by a parallel computation view. Let us respond to the referee's proposal, i.e. comment on the connection between the phenomena that we describe and parallel computation. Parallel computation describes computational processes that are carried out in parallel. It focuses on processes that cannot be done serially. The focus of the logical problems in our case (a man that marries one of two women) is not connected to the serial question. If the problem was that one cannot decide the state of the one without previously knowing the state of the other or vice versa, the question of serialism would have been relevant. But our problem is totally different. The state cannot be fully decided, even if we do the computation serially. The fact that one of the women is married prevents the marriage of the other, without any connection to the order of computation. It is therefore a problem of Quantum Logic (intertwining of states) and not Parallel Computation Logic. Said another way, in our case there is a complex interaction between the two channels of the problem (like the interaction between distant particles in an ERP experiment). This is the focus of our investigations, and not just the

existence of two parallel channels. The logic of the created state is what we discuss, and this logic is Quantum Logic. We have no interest in how to do the computation that helps us reach the conclusion that this is indeed the state.

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