Prioritized Norms and Defaults in Formal Argumentation

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Abstract

Deontic logic sentences define what an agent ought to do when faced with a set of norms. These norms may come into conflict such that a priority ordering over them is necessary to resolve these conflicts. Dung's seminal paper raised the — so far open — challenge of how to use formal argumentation to represent non monotonic logics, highlighting argumentation's value in exchanging, communicating and resolving possibly conflicting viewpoints in distributed scenarios. In this paper, we propose a formal framework to study various properties of prioritized non monotonic reasoning in formal argumentation, in line with this idea. More precisely, we show how a version of prioritized default logic and Brewka-Eiter's construction in answer set programming can be obtained in argumentation via the weakest and last link principles. We also show how to represent Hansen's recent construction for prioritized normative reasoning by adding arguments using weak contraposition via permissive norms, and their relationship to Caminada's "hang yourself" arguments.

Keywords: Abstract argumentation theory, prioritized normative reasoning.

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1 Introduction

Since the work of Alchourrón and Makinson [1] on hierarchical normative systems, in which a priority or strength is associated with the authority which promulgated a norm, reasoning with priorities of norms has been a central challenge in deontic logic. This has led to a variety of non-monotonic formalisms for prioritized reasoning in deontic logic, including a well known approach from prioritized default logic (PDL) and answer set programming — recently given argumentation semantics [13] (and to which we refer as the *greedy* approach); an approach by Brewka and Eiter [3] (which we refer to as the Brewka-Eiter construction); and a recent approach in hierarchical normative reasoning by Hansen [9], which we refer to as the Hansen construction. Given as input a set of norms with priorities, these approaches may produce different outputs. Consider the following benchmark example introduced by Hansen [9], and which results in the *prioritized triangle*.

Example 1.1 [Prioritized triangle – Hansen [9]]

Imagine you have been invited to a party. Before the event, you receive several imperatives, which we consider as the following set of norms.
- Your mother says: if you drink (p), then don't drive ($\neg x$).
- Your best friend says: if you go to the party (a), then you'll drive (x) us.
- An acquaintance says: if you go to the party (a), then have a drink with me (p).

We assign numerical priorities to these norms, namely '3', '2' and '1' corresponding to the sources 'your mother', 'your best friend' and 'your acquaintance', respectively. Whereas default and answer set programming-based approaches derive p, Hansen [9] argues convincingly that in normative reasoning p should not be derived. Meanwhile, the greedy approach and the Hansen construction return x, but the Brewka-Eiter construction returns $\neg x$.

Given that these different non-monotonic approaches yield different results, and further given Young and colleagues [13] representation result for prioritized default logic in argumentation, we wish to investigate the representation of such prioritized normative systems in formal argumentation. Therefore, the research question we answer in this paper is: *how can Brewka-Eiter's and Hansen's approaches for prioritized non monotonic reasoning be represented in formal argumentation?*

In this paper, we aim to make as few commitments as possible to specific argumentation systems. We therefore build on Tosatto et al. [11]'s abstract normative systems, and a relatively basic structured argumentation framework which admits undercuts and rebuts between arguments, and allows for priorities between rules making up arguments. We show that different approaches to lifting priorities from rules to arguments (based on the weakest and last link principles) allow us to capture the greedy and Brewka-Eiter approaches, while the introduction of additional arguments through the principle of weak contraposition, or through so called *hang yourself arguments*, allows us to obtain the Hansen construction.

A key point of our formal framework is that it addresses the challenge raised by Dung [6] aiming at representing non-monotonic logics through formal argumentation. In particular, argumentation is a way to exchange and communicate viewpoints, thus having an argumentation theory representing a non-monotonic logic is desirable for such a logic, in particular when the argumentation theory is simple and efficient. Note that it is not helpful for the development of non-monotonic logics themselves, but it helps when we want to apply such logics in distributed and multiagent scenarios.

The layout of the paper is as follows. First, we introduce our formal framework, and the three constructions. Second, we present our representation results, and demonstrate the relation between weak contraposition and hang yourself arguments. Finally, in concluding remarks, we discuss the main contributions of our approach, and highlight the future directions to be investigated.

2 Prioritised abstract normative system

In this section, we introduce the notion of prioritized abstract normative system (PANS) and three different approaches to compute what normative conclusions hold (referred to as an *extension*). A PANS captures the context of a system and the normative rules in force in such a system, together with a set of permissive norms which identify exceptions under which the normative rules should not apply. There is an element in the universe called $\top$, contained in every context, and in this paper we consider only a finite universe. A PANS also encodes a ranking function over the normative rules to allow for the resolution of conflicts.

Tosatto et al. [11] introduce a graph based reasoning framework to classify and organize theories of normative reasoning. Roughly, an *abstract normative system* (ANS) is a directed graph, and a context is the set of nodes of the graph containing the universe. In a context, an abstract normative system generates or produces an obligation set, a subset of the universe, reflecting the obligatory elements of the universe.

Based on the notion of abstract normative system defined by Tosatto and colleagues [11], a PANS is defined as follows.

Definition 2.1 [Prioritized abstract normative system] A prioritized abstract normative system PANS is a tuple $\mathcal{P} = \langle L, N, P, A, r \rangle$, where

- $L = E \cup \{\neg e \mid e \in E\} \cup \{\top\}$ is the universe, a set of literals based on some finite set E of atomic elements;
- $N \subseteq L \times L$ is a set of ordinary norms;
- $P \subseteq L \times L$ is a set of permissive norms;
- $A \subseteq L$ is a subset of the universe, called a context, such that for all a in E, $\{a, \neg a\} \not\subseteq A$;
- $r : N \cup P \rightarrow \mathbb{N}$ is a function from the norms to the natural numbers;

and where $N \cap P = \emptyset$.

Ordinary norms are of the kind "if you go to the party, then you should have a drink with me", whilst permissive norms take the form of statements such as "if you go to the party, then you don't have to have a drink with me". Both ordinary norms and permissive norms are *conditional norms*, requiring some condition to hold (e.g., going to the party) before their conclusion can be drawn. To distinguish the ordinary norms of N from the permissive norms of P, we write (a, x) for the former and $\langle a, x\rangle$ for the latter, where $a, x \in L$ are the antecedent and conclusion of the norm respectively. When no confusion can arise, a permissive norm is also represented as (a, x). Let $u, v \in N \cup P$ be two norms, we say that v is at least as preferred as u (denoted $u \leq v$) if and only if $r(u)$ is no more than $r(v)$ (denoted $r(u) \leq r(v)$), where $r(u)$ is also called a rank of u. We write $u < v$ or $v > u$ iff $u \leq v$ and $v \not\leq u$. Given a norm $u = (a, x)$ or $\langle a, x\rangle$, we write *ant*(u) for a to represent the antecedent of the norm, and *con*(u) for x to represent the conclusion of the norm. We say that a PANS is totally ordered if and only if the ordering $\leq$ over $N \cup P$ is antisymmetric, transitive and total. We assume that the set of norms is finite. For $a \in L$, we write $\overline{a} = \neg a$ if and only if $a \in E$, and $\overline{a} = e$ for $e \in E$ if and only if $a = \neg e$. Given a set S, we use $S \nvdash \perp$ to denote that $\nexists a, b \in S$ s.t. $a = \overline{b}$, i.e., a and b are not contradictory.

Example 2.2 [Prioritized triangle [9]] In terms of Def. 2.1, the prioritized triangle can be represented as a PANS $\mathcal{P}_1 = \langle L, N, P, A, r\rangle$, where

- $L = \{a, p, x, \neg a, \neg p, \neg x\}$,
- $N = \{(a, p), (p, \neg x), (a, x)\}$,
- $P = \emptyset$, $A = \{a, \top\}$,
- $r((a, p)) = 1$, $r((p, \neg x)) = 3$, and $r((a, x)) = 2$.

Figure 1 visualizes the prioritized triangle, with the crossed line between a and $\neg x$ denoting the norm (a, x).

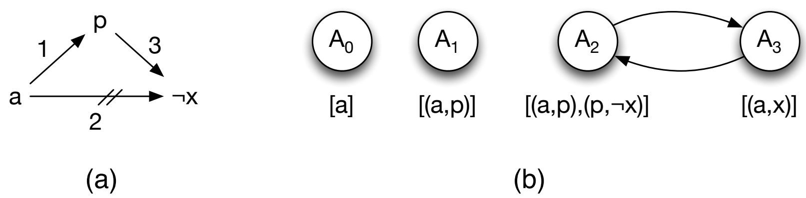

Fig. 1. The prioritized triangle (a), with the related arguments and the attacks among them visualized as directed arrows (b).

Given a totally ordered PANS, existing approaches of reasoning with prioritized norms may give different consequences. We consider three approaches (among others) that give three distinct consequences to the prioritized triangle example: the greedy approach of PDL, the Brewka-Eiter construction and the Hansen construction. Existing approaches consider only PANSs without permissive norms (i.e., $P = \emptyset$). In this paper, we extend these approaches to PANSs with permissive norms. So, the following definitions are applicable for both cases when $P = \emptyset$ and $P \neq \emptyset$.

First, a greedy approach (as used in PDL) always applies the norm with the highest priority among those which can be applied if this does not make the extension inconsistent.

Definition 2.3 [Greedy approach] Given a totally ordered PANS $\mathcal{P} = \langle L, N, P, A, r\rangle$, a norm $u \in N \cup P$ and a set $S \subseteq L$:

- We say that u is acceptable with respect to S, if and only if the following conditions holds:
  · $ant(u) \in S$,
  · $S \cup \{con(u)\} \nvdash \bot$, and
  · $\nexists v \in N \cup P$ such that $v > u$, v has not been previously applied, $ant(v) \in S$, and $S \cup \{con(v)\} \nvdash \bot$.
- Let $G_{\mathcal{P}} : 2^L \to 2^L$ be a function, such that $G_{\mathcal{P}}(S) = S \cup \{con(u)\}$ if $u \in N \cup P$ is acceptable with respect to S; otherwise, $G_{\mathcal{P}}(S) = S$.
- Given A, $G_{\mathcal{P}}$ has a fixed point (denoted as $G^{\infty}_{\mathcal{P}}(A)$, such that the extension of $\mathcal{P}$ by using the Greedy approach (denoted as *Greedy*($\mathcal{P}$)) is equal to:

$$\{a \in G^{\infty}_{\mathcal{P}}(A) \mid \exists\{b_1, \ldots, b_k\} \subseteq G^{\infty}_{\mathcal{P}}(A) : b_1 \in A, \\ \forall i \in \{1, \ldots, k-1\}, (b_i, b_{i+1}) \in N, \\ \text{and } (b_k, a) \in N\}$$

Note that since $\mathcal{P}$ is totally ordered, using the Greedy approach guarantees that there is a unique extension.

Building on the Greedy approach, Brewka and Eiter [3] defined the following construction.

Definition 2.4 [Brewka-Eiter construction] Given a totally ordered PANS $\mathcal{P} = \langle L, N, P, A, r\rangle$, and a set $X \supseteq A$:

- Let $\mathcal{P}^X = \langle L, N', P', A, r'\rangle$, where
  · $N' = \{(\top, l_2) \mid (l_1, l_2) \in N, l_1 \in X\}$ is the set of ordinary norms,
  · $P' = \{\langle\top, l_2\rangle \mid \langle l_1, l_2\rangle \in P, l_1 \in X\}$ is the set of permissive norms,
  · and $r'((\top, l_2)) = r((l_1, l_2))$ for all $(l_1, l_2) \in N \cup P$ are priorities over norms.
- If $X = Greedy(\mathcal{P}^X)$, then X is an extension of $\mathcal{P}$ by using the Brewka-Eiter construction, denoted as $X \in BnE(\mathcal{P})$.

Our definition (Def. 2.4) and the original formalism of Brewka and Eiter [3] are different, in the sense that in our definition we do not make use of default negation to represent the exceptions, i.e., the defeasibility of a (strict) rule, but we use defeasible rules and the notion of applicability of such rules. This means that the correct translation of the prioritized triangle of Example 2.2 ends up as the following logic program [1]:

[1] Note that in [3] $\mathtt{r}_0 < \mathtt{r}_3$ means that $\mathtt{r}_0$ has higher priority than $\mathtt{r}_3$.

$\mathtt{r_0 : a.}$
$\mathtt{r_1 : p\ :\text{-}\ not\ \neg p, a.}$
$\mathtt{r_2 : x\ :\text{-}\ not\ \neg x, a.}$
$\mathtt{r_3 : \neg x\ :\text{-}\ not\ x, p.}$
$\mathtt{r_0 < r_3 < r_2 < r_1}$

If priorities are disregarded, then this logic program has two answer sets: $\{a, p, x\}$ and $\{a, p, \neg x\}$. Thus, considering priorities, the former is the unique preferred answered set, as pointed out in Example 2.6 below.

Similarly, Hansen [9] defined the following construction by building on the Greedy approach.

Definition 2.5 [Hansen construction] Given a totally ordered PANS $\mathcal{P} = \langle L, N, P, A, r\rangle$:

- Let $T = \{u_1, u_2, \ldots, u_n\}$ be a linear order on $N \cup P$ such that $u_1 > u_2 > \cdots > u_n$.
- For all $R \subseteq N \cup P$, let $R(A) = \{x \mid x$ can be derived from A with respect to $R\}$.
- We define a set Φ as $\Phi = \Phi_n$ such that
  - $\Phi_0 = \emptyset$,
  - $\Phi_{i+1} = \Phi_i \cup \{u_i\}$, if $A \cup R(A) \nvdash \perp$ where $R = \Phi_i \cup \{u_i\}$; otherwise, $\Phi_{i+1} = \Phi_i$.
- The extension of $\mathcal{P}$ by using Hansen construction (denoted as *Hansen*($\mathcal{P}$)) is equal to *Greedy*($\mathcal{P}'$), where $\mathcal{P}' = \langle L, N', P', A, r\rangle$, in which $N' = N \cap \Phi$ and $P' = P \cap \Phi$.

Example 2.6 [Prioritised triangle: extensions] Regarding $\mathcal{P}_1$ in Example 2.2, we get three different extensions when using these approaches. For the greedy approach we obtain $S_1 = \{a\}$, $G^1_{\mathcal{P}_1}(S_1) = \{a, x\}$, $G^\infty_{\mathcal{P}_1}(S_1) = G^2_{\mathcal{P}_1}(S_1) = \{a, p, x\}$. For the Brewka-Eiter construction, given $X = \{a, p, \neg x\}$, we have $\mathcal{P}_1^X = \langle L, N', P', A, r'\rangle$, where $N' = \{(\top, p), (\top, x), (\top, \neg x)\}$, $P' = \emptyset$, $r'((\top, p)) = 1$, $r'((\top, \neg x)) = 3$ and $r'((\top, x)) = 2$; *Greedy*($\mathcal{P}_1^X$) $= X$. Since no other set could be an extension, *BnE*($\mathcal{P}$) $= \{\{a, p, \neg x\}\}$. Finally, for the Hansen construction, let $u_1 = (p, \neg x)$, $u_2 = (a,\ x)$, and $u_3 = (a, p)$, and $T = \{u_1, u_2, u_3\}$. Then $\Phi_0 = \emptyset$, $\Phi_1 = \{u_1\}$, $\Phi_2 = \{u_1, u_2\}$, and $\Phi = \Phi_3 = \Phi_2 = \{u_1, u_2\}$. So, $\mathcal{P}_1' = \langle L, N', P', A,\ r\rangle$, where $N' = \{u_1, u_2\}$, $P' = \emptyset$. Since *Greedy*($\mathcal{P}_1'$) $= \{a, x\}$, *Hansen*($\mathcal{P}_1$) $= \{a, x\}$.

3 Argumentation theory for a PANS

In this section, we introduce an argumentation theory on prioritised norms. This theory builds on ideas from *ASPIC*$^+$ [10]. Given a PANS, we first define arguments and defeats between them, then compute extensions of arguments in terms of Dung's theory [6], and from these, obtain conclusions.

In a PANS, an argument is an acyclic path in the graph starting in an element of the context. We assume minimal arguments — no norm can be applied twice in an argument and no redundant norm is included in an argument. Permissions are undercutting arguments containing at least one permissive norm. We use $concl(\alpha)$ to denote the conclusion of an argument α, and $concl(E) = \{concl(\alpha) \mid \alpha \in E\}$ for the conclusions of a set of arguments E.

Definition 3.1 [Arguments and sub-arguments] Let $\mathcal{P} = \langle L, N, P, A, r\rangle$ be a PANS.

A context argument in $\mathcal{P}$ is an element $a \in A$, and its conclusion is $concl(a) = a$.

An ordinary argument in $\mathcal{P}$ is an acyclic path $\alpha = [u_1, \ldots, u_n]$, $n \geq 1$, such that:
(i) $\forall i \in \{1, \ldots, n\}$, $u_i \in N$;
(ii) $ant(u_1) \in A$;
(iii) $con(u_i) = ant(u_{i+1})$, $1 \leq i \leq n-1$;
(iv) $\{ant(u_1), \ldots, ant(u_n)\} \nvdash \bot$; and
(v) $\nexists i, j \in \{1, \ldots, n\}$ such that $i \neq j$ and $u_i = u_j$.
Moreover, we have that $concl(\alpha) = con(u_n)$.

An undercutting argument in $\mathcal{P}$ is defined in terms of an ordinary argument, by replacing the first condition with (1') $\exists i \in \{1, \ldots, n\}$ such that $u_i \in P$.

The sub-arguments of argument $[u_1, \ldots, u_n]$ are, for $1 \leq i \leq n$, $[u_1, \ldots, u_i]$. Note that context arguments do not have sub-arguments.

The set of all arguments constructed from $\mathcal{P}$ is denoted as $Arg(\mathcal{P})$. For readability, $[(a_1, a_2), \ldots, (a_{n-1}, a_n)]$ may be written as $(a_1, a_2, \ldots, a_{n-1}, a_n)$. The set of sub-arguments of an argument α is denoted as $sub(\alpha)$.

We follow the tradition in much of preference-based argumentation [2,10], and use *defeat* as the relation among arguments on which the semantics is based, whereas *attack* is used for a relation among arguments which does not take the priorities among arguments into account. To define the defeat relation among prioritized arguments, we assume that *only* the priorities of the norms are used to compare arguments. In other words, we assume a lifting of the ordering on norms to a binary relation on sequences of norms, written as $\alpha \succeq \beta$, where α and β are two arguments, indicating that α is at least as preferred as β.

There is no common agreement about the best way to lift $\geq$ to $\succeq$. In argumentation, two common approaches are the weakest and last link principles, combined with the elitist and democratic ordering [10]. However, Young and colleagues [13] show that elitist weakest link cannot be used to calculate $\succeq$, and proposes a *disjoint elitist order* which ignores shared rules. Based on these ideas we define the orderings between arguments according to the weakest link and last link principles (denoted as $\succeq_w$ and $\succeq_l$ respectively) as follows.

Definition 3.2 [Weakest link and last link] Let $\mathcal{P} = \langle L, N, P, A, r\rangle$ be a PANS, and $\alpha = [u_1, \ldots, u_n]$ and $\beta = [v_1, \ldots, v_m]$ be two arguments in $Arg(\mathcal{P})$. Let $\Phi_1 = \{u_1, \ldots, u_n\}$ and $\Phi_2 = \{v_1, \ldots, v_m\}$. By the weakest link principle,

$\alpha \succeq_w \beta$ iff $\exists v \in \Phi_2 \setminus \Phi_1$ s.t. $\forall u \in \Phi_1 \setminus \Phi_2, v \leq u$. By the last link principle, $\alpha \succeq_l \beta$ iff $u_n \geq v_m$.

When the context is clear, we write $\succeq$ for $\succeq_w$ or $\succeq_l$. We write $\alpha \succ \beta$ for $\alpha \succeq \beta$ without $\beta \succeq \alpha$.

Given a way to lift the ordering on norms to an ordering on arguments, the notion of defeat can be defined.

Definition 3.3 [Defeat among arguments] Let $\mathcal{P} = \langle L, N, P, A, r\rangle$ be a PANS. For all $\alpha, \beta \in \mathit{Arg}(\mathcal{P})$,

α **attacks** β iff β has a sub-argument β' such that
(i) $\mathit{concl}(\alpha) = \overline{\mathit{concl}(\beta')}$

α **defeats** β iff β has a sub-argument β' such that
(i) $\mathit{concl}(\alpha) = \overline{\mathit{concl}(\beta')}$ and
(ii) α is a context argument, or $\beta' \not\succ \alpha$.

The set of defeats between the arguments in $\mathit{Arg}(\mathcal{P})$ is denoted as $\mathit{Def}(\mathcal{P}, \succeq)$.

In what follows, an argument $\alpha = [u_1, \ldots, u_n]$ with ranking on norms is denoted as $u_1 \ldots u_n : r(\alpha)$, where $r(\alpha) = (r(u_1), \ldots, r(u_n))$.

Example 3.4 [Prioritised triangle, continued] Consider the prioritised triangle in Example 2.2. We have the following arguments, visualized in Figure 1.b:

A_0 a (context argument)

A_1 $(a, p) : (1)$ (ordinary argument)

A_2 $(a, p)(p, \neg x) : (1, 3)$ (ordinary argument)

A_3 $(a, x) : (2)$ (ordinary argument)

We have that A_2 attacks A_3 and vice versa, and there are no other attacks among the arguments. Moreover, A_2 defeats A_3 if $(2) \not\succ (1, 3)$ (last link), and A_3 defeats A_2 if $(1, 3) \not\succ (2)$ (weakest link).

It is worth mentioning that Dung [7] proposes the notion of a *normal attack relation*, which satisfies some desirable properties that cannot be satisfied by the ASPIC^+ semantics, i.e., the semantics of structured argumentation with respect to a given ordering of structured arguments (elitist or democratic preorder) in ASPIC^+. In the context of the current paper, this notion could be defined as follows. Let $\alpha = (a_1, \ldots, a_n)$ and $\beta = (b_1, \ldots, b_m)$ be arguments constructed from a PANS. Since we have no Pollock style undercutting argument (as in ASPIC^+) and each norm is assumed to be defeasible, it says that α normally attacks argument β iff β has a sub-argument β' s.t. $\mathit{concl}(\alpha) = \overline{\mathit{concl}(\beta')}$, and $r((a_{n-1}, a_n)) \geq r((b_{m-1}, b_m))$. According to Def. 3.2 and 3.3, the normal defeat relation is equivalent to the defeat relation using the last link principle in this paper.

Given a set of arguments $\mathcal{A} = \mathit{Arg}(\mathcal{P})$ and a set of defeats $\mathcal{R} = \mathit{Def}(\mathcal{P}, \succeq)$, we get an argumentation framework (AF) $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. For a set $B \subseteq \mathcal{A}$, B is conflict-free iff $\nexists \alpha, \beta \in B$ s.t. $(\alpha, \beta) \in \mathcal{R}$. B defends an argument α iff $\forall (\beta, \alpha) \in \mathcal{R}$, $\exists \gamma \in B$ s.t. $(\gamma, \beta) \in \mathcal{R}$. The set of arguments defended by B

in $\mathcal{F}$ is denoted as $\mathcal{D}_{\mathcal{F}}(B)$. A set of B is a complete extension of $\mathcal{F}$, iff B is conflict-free and $B = \mathcal{D}_{\mathcal{F}}(B)$. B is a preferred (grounded) extension iff B is a maximal (resp. minimal) complete extension. B is a stable extension, iff B is conflict-free, and $\forall \alpha \in \mathcal{A} \setminus B$, $\exists \beta \in B$ s.t. $(\beta, \alpha) \in \mathcal{R}$. We use $sem \in \{cmp, prf, grd, stb\}$ to denote complete, preferred, grounded, or stable semantics. A set of argument extensions of $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ is denoted as $sem(\mathcal{F})$. Then, we write *Outfamily* for the set of conclusions from the extensions of the argumentation theory, as in [12].

Definition 3.5 [Conclusion extensions] Given a prioritised abstract normative system $\mathcal{P} = \langle L, N, P, A, r\rangle$, let $\mathcal{F} = (Arg(\mathcal{P}), Def(\mathcal{P}, \succeq))$ be the AF constructed from $\mathcal{P}$. The conclusion extensions, written as $Outfamily(\mathcal{P}, \succeq, sem)$, are the conclusions of the ordinary and context arguments in argument extensions.

$$\{\{concl(\alpha) \mid \alpha \in S, \alpha \text{ is an ordinary or context argument}\} \mid S \in sem(\mathcal{F})\}$$

Multi-extension semantics can yield different conclusions when norms may yield multiple most preferred results. Additionally, it is important to note that conclusions of a PANS are drawn only from ordinary and context arguments.

Example 3.6 [Prioritized triangle, continued] According to Example 3.4, let $\mathcal{A} = \{A_0, \ldots, A_3\}$. We have $\mathcal{F}_1 = (\mathcal{A}, \{(A_2, A_3)\})$ where $A_2 \succeq_l A_3$, and $\mathcal{F}_2 = (\mathcal{A}, \{(A_3, A_2)\})$ where $A_3 \succeq_w A_2$. For all $sem \in \{cmp, prf, grd, stb\}$, $Outfamily(\mathcal{P}, \succeq_l, sem) = \{\{a, p, \neg x\}\}$, and $Outfamily(\mathcal{P}, \succeq_w, sem) = \{\{a, p, x\}\}$.

We now turn our attention to the properties of the argumentation theory for a PANS. Since all norms in a PANS are defeasible, it is obvious that our theory maps to the framework of $ASPIC^+$. According to the corresponding properties in [10], the following three propositions follow directly.

Proposition 3.7 *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AF constructed from a PANS. For all $\alpha, \beta \in \mathcal{A}$: if α attacks β, then α attacks arguments that have β as a sub-argument; if α defeats β, then α defeats arguments that have β as a sub-argument.*

Proposition 3.8 (Closure under sub-arguments) *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AF constructed from a PANS. For all $sem \in \{cmp, prf, grd, stb\}$, $\forall E \in sem(\mathcal{F})$, if an argument $\alpha \in E$, then $sub(\alpha) \subseteq E$.*

Proposition 3.9 (Consistency) *Elements of Outfamily are conflict free.*

The following two properties formulate the relations between non-argument-based and argument-based approaches for reasoning with a totally ordered PANS without permissive norms.

Proposition 3.10 (Greedy is weakest link) *Given a totally ordered PANS $\mathcal{P} = \langle L, N, P, A, r\rangle$ where $P = \emptyset$, and $\mathcal{F} = (Arg(\mathcal{P}), Def(\mathcal{P}, \succeq_w))$. It holds that $\mathcal{F}$ is acyclic, and $Greedy(\mathcal{P}) = concl(E)$ where E is the unique complete extension of $\mathcal{F}$.*

Proof. First, since $\mathcal{P}$ is totally ordered, under $\succeq_w$, the relation $\succeq_w$ among arguments is acyclic. Hence, $\mathcal{F}$ is acyclic, and therefore has a unique extension under all argumentation semantics mentioned above.

Second, given *Greedy*($\mathcal{P}$), let $E = \{(a_1, \ldots, a_n) \in Arg(\mathcal{P}) \mid \{a_1, \ldots, a_n\} \subseteq Greedy(\mathcal{P})\}$. According to Def. 2.3, it holds that $concl(E) = Greedy(\mathcal{P})$. Now, we verify that E is a stable extension of $\mathcal{F}$:

(1) Since all premises and the conclusion of each argument of E are contained in *Greedy*($\mathcal{P}$) which is conflict-free, it holds that E is conflict-free.

(2) $\forall \beta = (b_1, \ldots, b_m) \in Arg(\mathcal{P}) \setminus E$, $b_m \notin Greedy(\mathcal{P})$ (otherwise, if $b_m \in Greedy(\mathcal{P})$, then $(b_1, \ldots, b_{m-1}) \subseteq Greedy(\mathcal{P})$, and thus $\beta \in E$, contradicting to $\beta \notin E$). Then $\exists \alpha = (a_1, \ldots, a_n) \in E$, s.t. $a_n = \overline{b_j}$, $2 \leq j < m$. Then, we have the following two possible cases:

- (a_{n-1}, a_n) and (b_{j-1}, b_j) are applicable at the same time: in this case, since $a_n \in Greedy(\mathcal{P})$, $r((a_{n-1}, a_n)) \geq r((b_{j-1}, b_j))$. It follows that $(a_1, \ldots, a_n) \succeq_w (b_1, \ldots, b_j)$. So, β is defeated by α.
- (a_{n-1}, a_n) is applicable, (b_{j-1}, b_j) is not applicable: in this case, there are in turn two possibilities:
  · $(a_1, \ldots, a_n) \succeq_w (b_1, \ldots, b_j)$: β is defeated by α.
  · $(b_1, \ldots, b_j) \succ_w (a_1, \ldots, a_n)$: in this case, $\exists \gamma = (c_1, \ldots, c_k) \in E$ s.t.: $c_k = \overline{b_i}$, $(c_1, \ldots, c_k) \succeq_w (b_1, \ldots, b_i)$, $2 \leq i < j$. Then, β is defeated by γ.

Since E is conflict-free and for all $\beta \in Arg(\mathcal{P}) \setminus E$, β is defeated by an argument in E, E is a stable extension. Since $\mathcal{F}$ is acyclic, E is the unique complete extension of $\mathcal{F}$. □

Proposition 3.11 (Brewka-Eiter is last link) *Given a totally ordered PANS* $\mathcal{P} = \langle L, N, P, A, r\rangle$ *where* $P = \emptyset$, *and* $\mathcal{F} = (Arg(\mathcal{P}), Def(\mathcal{P}, \succeq_l))$. *It holds that* $BnE(\mathcal{P}) = \{concl(E) \mid E \in stb(\mathcal{F})\}$.

Proof. ($\Rightarrow$:) $\forall H \in BnE(\mathcal{P})$, let $E = \{(a_1, \ldots, a_n) \in Arg(\mathcal{P}) \mid \{a_1, \ldots, a_n\} \subseteq H\}$. According to the Brewka-Eiter construction [3], $H = concl(E)$, because $\forall a \in H$, there exists at least one argument $(a_1, \ldots, a_n)$ s.t. $a_n = a$ and $\{a_1, \ldots, a_{n-1}\} \subseteq H$, which is in turn because if $a_n \in H$, then (a_{n-1}, a_n) is applicable w.r.t. H, and hence $a_{n-1} \in H$; recursively, we have $a_i \in H$ for all $i \in \{1, \ldots, n-1\}$.

Let $(Args_0, Defeats_0)$ be an AF, in which $Args_0 = \{\alpha \mid sub(\alpha) \subseteq E\}$, $Defeats_0 \subseteq Args_0 \times Args_0$ that is constructed in terms of the last link principle. It holds that $Defeats_0 \subseteq Def(\mathcal{P}, \succeq_l)$. For all $\alpha \in Args_0 \setminus E$, $concl(\alpha) \notin H$. Then, $\exists \beta \in E$ s.t. $concl(\alpha) = \overline{concl(\beta)}$ and β defeats α by using the last link principle. It follows that E is a stable extension of $(Args_0, defeats_0)$. Now, let us prove that E is a stable extension of $\mathcal{F}$.

We need only to verify that for all $\alpha \in Arg(\mathcal{P}) \setminus Args_0$, α is defeated by E. It follows that α has at least one sub-argument (otherwise, it should be included in E, contradicting $\alpha \notin Args_0$). Let β be a sub-argument of α such that β has no sub-argument. It follows that β is in $Args_0$. Then we have the following two possible cases:

- β is defeated by E: In this case, α is defeated by E.
- β is not defeated by E: In this case, β is in E (since E is a stable extension). Then, according to the definition of $Args_0$, the direct super argument of β (say β') is in $Args_0$. We in turn have two possible cases similar to the cases with respect to β. Recursively, we may conclude that α is defeated by E or, α is in E (this case does not exist).

($\Leftarrow$:) For all $E \in \mathit{stb}(\mathcal{F})$, let $\mathcal{P}' = \langle L, N', P', A, r' \rangle$ where $P' = \emptyset$, $N' = \{(\top, b) \mid (a, b) \in N \text{ and } a \in \mathit{concl}(E)\}$, and $r'(\top, b) = r(a, b)$ for all $(a, b) \in N$ and $a \in \mathit{concl}(E)$.

Let $E' = \{(\top, a_n) \mid (a_1, \ldots, a_n) \in E\}$.

In order to prove that $\mathit{concl}(E)$ is an extension of $\mathcal{P}$ in terms of the Brewka-Eiter construction, according to Proposition 3.10, we only need to verify that E' is a stable extension of $(\mathit{Arg}(\mathcal{P}'), \mathit{Def}(\mathcal{P}', \succeq'_w))$ which is an AF of $\mathcal{P}'$ by using the weakest link principle. This is true because:

- Since E is conflict-free, E' is conflict-free.
- For all $\beta' \in \mathit{Arg}(\mathcal{P}') \setminus E'$, let β be a corresponding argument in $\mathit{Arg}(\mathcal{P}) \setminus E$ s.t. $\beta = (b_1, \ldots, b_n)$, $\beta' = (\top, b_n)$, and all sub-arguments of β are in E. Since β is not in E, it is defeated by E. Since all sub-arguments of β are not defeated by E, there exists an argument in E whose conclusion is in conflict with $\mathit{concl}(\beta) = \mathit{concl}(\beta')$. So, β' is defeated by E'.

□

4 Weak contraposition

Geffner and Pearl [8] introduces conditional entailment, combining extensional and conditional approaches to default reasoning. Conditional entailment determines a prioritization of default knowledge bases. A distinguishing property of conditional entailment is what we can call *weak contraposition*, which inspires our weak contraposition property.

Output under weak contraposition is obtained by adding the contrapositives of the norms to the permissive norms. The priorities of the permissive norms are the same as the priorities of the original norms.

Definition 4.1 [Weak contraposition] Let $\mathit{wcp}(N) = \{\langle \overline{x}, \overline{a} \rangle \mid (a, x) \in N\}$. $\mathit{Outfamily}_{wcp}(\langle L, N, P, A, r \rangle, \succeq, \mathit{sem}) = \mathit{Outfamily}(\langle L, N, P \cup \mathit{wcp}(N), A, r' \rangle, \succeq, \mathit{sem})$, where $r'(\langle \overline{x}, \overline{a} \rangle) = r((a, x))$, and $r'((a, x)) = r((a, x))$ otherwise.

In the running example we add three contrapositives. Given a contextual argument a, the undercutting arguments for $\neg a$ do not affect the result, as they are always defeated by the contextual argument. So the only additional argument to be considered is the undercutting argument for $\neg p$. This can block the argument for p, as required.

Example 4.2 [Prioritized triangle, continued] Consider $\mathcal{P}_1$ in Example 2.2, visualized in Figure 2.a. We have $\mathit{wcp}(N) = \{\langle \neg p, \neg a \rangle, \langle x, \neg p \rangle, \langle \neg x, \neg a \rangle\}$, and assume that contrapositives have the same priority as the original norms, i.e., $r(\mathit{wcp}(N)) = (1, 3, 2)$. We have the following arguments:

A_0 a (context argument)

A_1 $(a,p):(1)$ (ordinary argument)

A_2 $(a,p)(p,\neg x):(1,3)$ (ordinary argument)

A_3 $(a,x):(2)$ (ordinary argument)

A_4 $(a,x)\langle x,\neg p\rangle:(2,3)$ (undercutting argument)

A_5 $(a,x)\langle x,\neg p\rangle\langle \neg p,\neg a\rangle:(2,3,1)$ (undercutting arg.)

A_6 $(a,p)(p,\neg x)\langle \neg x,\neg a\rangle:(1,3,2)$ (undercutting arg.)

Argument A_0 is not defeated by any argument, and defeats A_5 and A_6. We therefore consider only arguments A_1 to A_4.

As before, A_2 attacks A_3 and vice versa. In addition, A_4 attacks both A_1 and A_2, A_1 attacks A_4, and A_2 attacks A_4.

By using the last link principle, we have that A_4 defeats A_1 and thus A_2; and that A_2 defeats A_3 and thus A_4. In this case, under the stable and preferred semantics, there are two extensions $\{A_0, A_1, A_2\}$ and $\{A_0, A_3, A_4\}$. So, $Outfamily_{wcp}(\mathcal{P}_1, \succeq_l, sem) = \{\{a, p, \neg x\}, \{a, x\}\}$, where $sem \in \{prf, stb\}$.

By using the weakest link principle, we have that A_4 defeats A_1 and thus A_2; and that A_3 defeats A_2. In this case, for all $sem \in \{cmp, grd, prf, stb\}$, $\{A_0, A_3, A_4\}$ is the only extension. So, $Outfamily_{wcp}(\mathcal{P}_1, \succeq_w, sem) = \{\{a, x\}\}$.

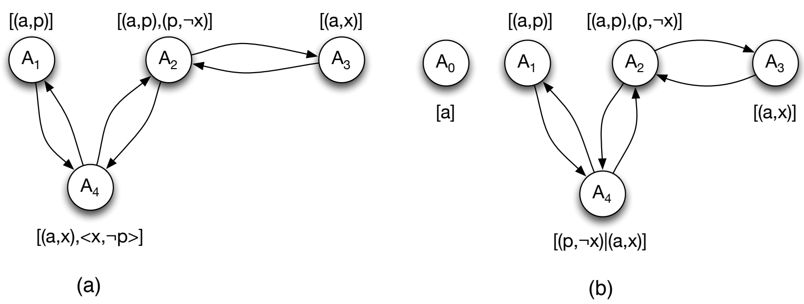

Fig. 2. The prioritized triangle of Example 4.2 (a) and of Example 5.3 (b).

The following proposition shows that the Hansen construction can be represented in formal argumentation by weakest link, if the set of permissive norms is extended with the contrapositions of the norms in N. Note that to capture Hansen's reading of the prioritized triangle, we need to add more structure to the example. The proof is along the lines of the proof of Proposition 3.10.

Proposition 4.3 (Hansen is weakest link plus wcp) *Given a totally ordered PANS* $\mathcal{P} = \langle L, N, P, A, r\rangle$ *where* $P = \emptyset$, $\mathcal{P}' = \langle L, N, P', A, r'\rangle$ *with* $P' = wcp(N)$ *and* $r'(\langle \overline{x}, \overline{a}\rangle) = r((a, x))$, *and* $r'((a, x)) = r((a, x))$ *otherwise, and* $\mathcal{F} = (Arg(\mathcal{P}'), Def(\mathcal{P}', \succeq_w))$. *It holds that* $Hansen(\mathcal{P}) = concl(E)$ *where* E *is the set of ordinary arguments of the unique complete extension of* $\mathcal{F}$.

Proof. [Sketch] First, closure under sub-arguments and consistency follow from Proposition 2 and 3, because we reuse the definitions of weakest link.

Second, $\mathcal{P}$ does not have to be totally ordered, as there may be permissive norms with the same rank as one of the ordinary norms. Thus, it no longer holds that $Arg(\mathcal{P})$ must be totally ordered under $\succeq_w$, and thus $\mathcal{F}$ is not necessarily acyclic. Nevertheless, thanks to the properties we imposed on arguments, there is still only one unique extension under all the argumentation semantics mentioned above.

Third, let $E = \{(a_1, \ldots, a_n) \in Arg(\mathcal{P}) \mid \{a_1, \ldots, a_n\} \subseteq Hansen(\mathcal{P})$ or $\exists i < n$ such that $a_i \notin Hansen(\mathcal{P})\}$. According to Definition 4, it holds that $concl(E) = Hansen(\mathcal{P})$. Now, we first prove that E is a stable extension of $\mathcal{F}$:

(1) Since all premises and the conclusion of each argument of E are contained in $Hansen(\mathcal{P})$ which is conflict-free, or one of the premises is not in $Hansen(\mathcal{P})$, it holds that E is conflict-free.

(2) $\forall \beta = (b_1, \ldots, b_m) \in Arg(\mathcal{P}) \setminus E$, $b_m \notin Hansen(\mathcal{P})$ (otherwise, if $b_m \in Hansen(\mathcal{P})$, then $(b_1, \ldots, b_{m-1}) \subseteq Hansen(\mathcal{P})$, and thus $\beta \in E$, contradicting the requirement that $\beta \notin E$). Then $\exists \alpha = (a_1, \ldots, a_n) \in E$, such that $a_n = \overline{b_j}$, $2 \leq j < m$. The two cases are analogous to the two cases in the proof of Proposition 3.10.

Since E is conflict-free and for all $\beta \in Arg(\mathcal{P}) \setminus E$, β is defeated by an argument in E, E is a stable extension. E is thus the unique complete extension of $\mathcal{F}$. □

5 Hang Yourself Arguments

We now introduce another type of argument, the *hang yourself argument* (abbreviated HYA) for prioritized normative systems. HYAs were introduced in a non-prioritized setting by [4,5] [2]. A HYA is made up of a *hypothetical argument* α, and an ordinary argument β, with contradictory conclusions. A third argument, γ, serves as the premise for α. If argument $\gamma;\alpha$ (where ; denotes concatenation of arguments to obtain a super-argument) is an ordinary argument which conflicts with β, then a contradiction exists, meaning that either γ or the HYA is invalid.

Definition 5.1 [Hang yourself arguments] Given a prioritized abstract normative system PANS $\mathcal{P} = \langle L, N, P, A, r\rangle$.

A hypothetical argument in $\mathcal{P}$ is similar to an ordinary argument in Definition 3.1. The only difference is that in a hypothetical argument, $ant(u_1) \notin A$.

A hang yourself argument in $\mathcal{P}$, written $\alpha|\beta$ consists of a hypothetical argument α and an ordinary argument β with opposite conclusions, such that for sub-arguments α', β' of α and β respectively, we have that if α' and β' have opposite conclusions, then $\alpha = \alpha'$ and $\beta = \beta'$.

[2] They are also called Socratic-style arguments due to their connection with Socratic style argumentation.

For convenience, given an argument $\beta;\alpha$ where $\beta = [(a_1, a_2), \ldots, (a_{i-1}, a_i)]$ and $\alpha = [(a_i, a_{i+1}), \ldots, (a_{n-1}, a_n)]$, where $(a_j, a_j + 1)$ has rank r_j, we write $r(\beta;\alpha^{-1})$ to denote the priority obtained from the sequence of ranks $r_1, \ldots, r_i, r_{n-1}, \ldots, r_{i+1}$.

Definition 5.2 [Defeat for HYAs] A HYA $\alpha|\beta$ defeats an argument γ iff there is a sub argument γ' of γ such that $\gamma';\alpha$ is an argument, and $r(\beta;\alpha^{-1}) \not\prec r(\gamma')$. A HYA $\alpha|\beta$ is defeated by an argument γ if and only if

(i) γ defeats β; or

(ii) there is a sub argument γ' of γ such that $\gamma';\alpha$ is an argument, and $r(\gamma') \not\prec r(\beta;\alpha^{-1})$;

Example 5.3 [Prioritized triangle, continued] Consider $\mathcal{P}_1$ in Example 2.2, visualized in Figure 2.b. The only relevant hang yourself argument is $(p, x)|(a, \neg x)$ which defeats (a, p) depending on the ranking of $(p, x), (a, \neg x)$. We thus have the following arguments:

A_0	a	(context argument)
A_1	$(a, p) : (1)$	(ordinary argument)
A_2	$(a, p)(p, \neg x) : (1, 3)$	(ordinary argument)
A_3	$(a, x) : (2)$	(ordinary argument)
A_4	$(p, \neg x)\|(a, x) : (3), (2)$	(hang yourself arg.)

A_1 and A_2 each defeats A_4 if $(2, 3) \not\succ (1)$. A_3 defeats A_2 if $(1, 3) \not\succ (2)$. A_4 defeats A_1 and A_2 if (1) and $(1, 3) \not\succ (2, 3)$ respectively.

For weakest link, A_4 defeats A_1 and A_2, and A_3 defeats A_2. We therefore have $\mathit{Outfamily}(\mathcal{P}_1, \succeq_w, sem) = \{\{a, x\}\}$ for all complete semantics.

An argument containing weak contrapositives may be seen as a kind of HYA. More precisely, consider an argument $A = [(a_1, a_2), \ldots, (a_{n-1}, a_n)]$, and another argument $B = [(b_1, b_2), \ldots, (b_m, \overline{a_n})]$. These two arguments result in a sequence of weak contrapositive arguments:

$B; [\langle\overline{a_n}, \overline{a_{n-1}}\rangle]$
$B; [\langle\overline{a_n}, \overline{a_{n-1}}\rangle, \langle\overline{a_{n-1}}, \overline{a_{n-2}}\rangle]$
...
$B; [\ldots; \langle\overline{a_2}, \overline{a_1}\rangle]$

Note that the last argument in the sequence is always defeated by the context argument. The remaining arguments attack (and may defeat) the different sub-arguments of A.

We now prove that the hang yourself argument is equivalent to the weak contrapositive argument.

Proposition 5.4 *The HYA $\delta = [(a_i, a_{i+1}), \ldots, (a_{n-1}, a_n)]|\beta$ is equivalent to the weak contrapositive argument $\omega = \beta; \ldots; (\overline{a_{i+1}}, \overline{a_i})$ in the sense that δ defeats a subargument α' of α if and only if ω defeats α'.*

Proof. Without loss of generality, assume that ω attacks α' on its last argument. Then the rank of ω is $r(\omega) = r(\beta), r((a_{n-1}, a_n)), \ldots, r((a_i, a_{i+1}))$. From Definition 5.2, the HYA defeats α if $r(\alpha') \not\succ r(\omega)$. Similarly, α defeats the HYA if $r(\omega) \not\succ r(\alpha')$. The final situation in which the weak contraposition is defeated holds if α defeats β. In such a situation, the HYA is also defeated. Thus, the situation where the weak contraposition defeats (is defeated by) α is identical to when the HYA defeats (is defeated by) α. □

6 Conclusions

In this paper, we provide a step towards studying non-monotonic logics through formal argumentation theory. Here, we begin addressing this challenge by considering three distinct systems for prioritized nonmonotonic reasoning, showing that they are different forms of our theory of argumentation. In particular, we showed how the Greedy approach of prioritized default logic can be represented by the weakest link principle; the Brewka-Eiter approach of answer set programming by the last link principle; and the Hansen approach of deontic logic using the weakest link principle extended with weak contraposition. We also showed that for weakest link, weak contraposition is a special case of hang yourself arguments.

While most work in formal argumentation uses very general frameworks to study argumentation systems, we use a very simple argument system to study the links between argumentation and prioritized norms. In particular, we utilised prioritized abstract normative systems, where norms are represented by a binary relation on literals, priorities are represented by natural numbers, and all norms have a distinct priority.

The main lessons that can be learned from our results are as follows. The weakest link principle corresponds to the greedy approach which is computationally attractive, but conceptually flawed. It should be adopted only when computational efficiency is the most important property. Thus, to get a more balanced result, the last link approach seems to be better for a wide number of potential applications, e.g., multiagent systems. This means that the pros and cons of both solutions have to be considered, and the decision regarding which to use depends on the application scenario of interest. Finally, Hansen's approach is a sophisticated way to deal with prioritized rules, and can be modeled using weakest link to handle conflicts, as we have shown. Our results are relevant not only when modelling normative systems, but also potentially when a developer must make a choice regarding which link principles to use when developing an argumentation system.

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