The Combined Effect of Periodic Signals and Noise on the Dilution of Precision of GNSS Station Velocity Uncertainties

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INTRODUCTION

Station velocity uncertainties determined from a series of Global Navigation Satellite System (GNSS) position estimates depend on both the deterministic and stochastic models applied to the time series. While the deterministic model generally includes parameters for a linear and several periodic terms, the stochastic model is a representation of the noise character of the time series in form of a power-law process. For both of these models the optimal model may vary from one time series to another while the models also depend, to some degree, on each other. In the past various power-law processes have been shown to fit the time series and the sources for the apparent temporally-correlated noise were attributed to, for example, mismodelling of satellites orbits, antenna phase centre variations, troposphere, Earth Orientation Parameters, mass loading effects and monument instabilities.

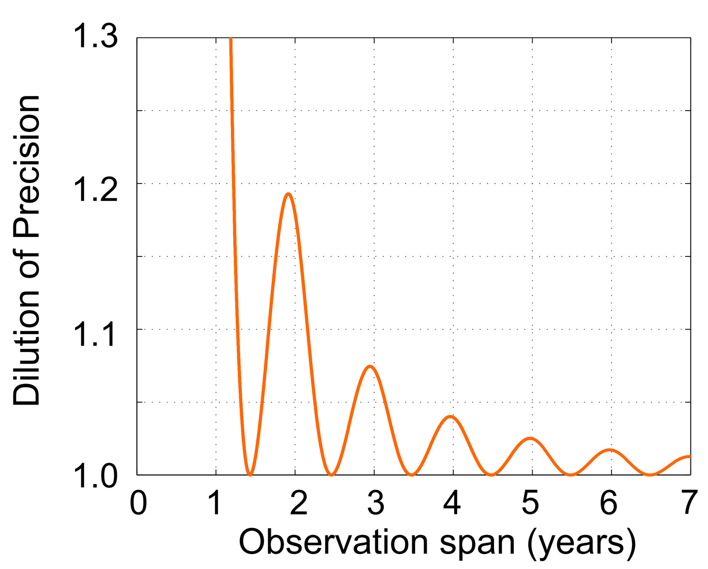
Blewitt and Lavallée (2002) developed a model to calculate the bias level while one may not account for annual signals or so-called **DILUTION OF PRECISION (DP)**:

$$DP = \frac{\sigma_{v_1}}{\sigma_{v_2}} = \left[1 - \frac{6}{(\pi \cdot f \cdot \tau)^2} \cdot \frac{\cos(\pi \cdot f \cdot \tau) - \frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau}}{1 - \frac{\cos(\pi \cdot f \cdot \tau) \cdot \sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau}}\right]^{-\frac{1}{2}}$$

where τ is the time span, v_1 and v_2 denote the velocities determined without and with accounting for the periodic terms of frequency f_i, respectively.

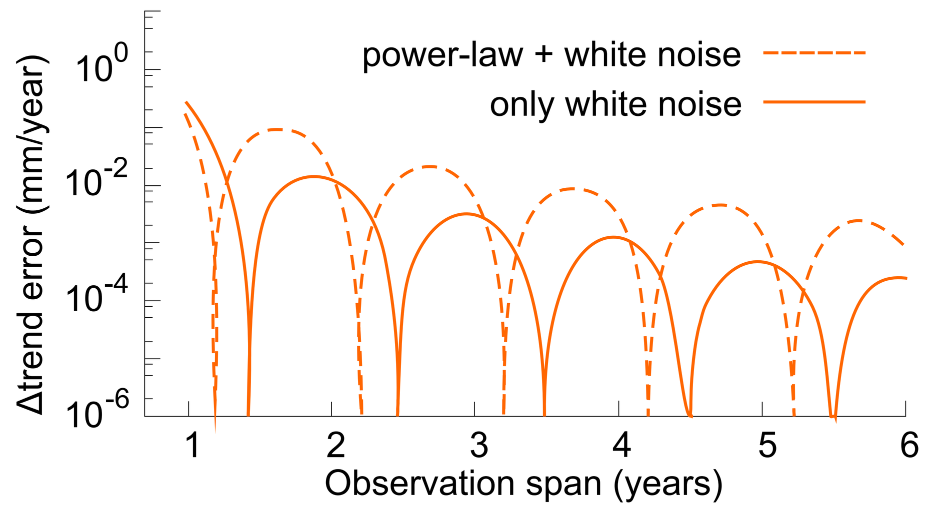
Blewitt and Lavallée (2002) demonstrated how improperly modelled seasonal signals affected the estimates of station velocity uncertainties. However, in their study they assumed that the time series followed a white noise process with no consideration of additional temporally-correlated noise.

Figure 1: Dilution of precision for models: with linear velocity and linear velocity plus periodic terms (reproduced from Blewitt and Lavallée, 2002):



Bos et al. (2010) empirically showed for a small number of stations that the noise character was much more important for the reliable estimation of station velocity uncertainties than the seasonal signals.

Figure 2: Dilution of precision for models: with linear velocity and linear velocity plus periodic signals with certain noise process applied (reproduced from Bos et al., 2010):



In this presentation we pick up from Blewitt and Lavallée (2002) and Bos et al. (2010), and have derived formulas for the computation of the General Dilution of Precision (GDP) under presence of periodic signals and temporally-correlated noise in the time series.
We show, based on simulated and real time series from globally distributed IGS (International GNSS Service) stations processed by the Jet Propulsion Laboratory (JPL), that periodic signals dominate the effect on the velocity uncertainties at short time scales while for those beyond four years, the type of noise becomes much more important. In other words, for time series long enough, the assumed periodic signals do not affect the velocity uncertainties as much as the assumed noise model.

GENERAL DILUTION OF PRECISION (GDP)

A difference between two uncertainties estimated with and without periodic terms can be better understood by computing a ratio between them. We called it the **General Dilution of Precision** (GDP) to be consistent with Blewitt and Lavallée (2002), but taking into consideration power-law noise in the stochastic part. We calculated the GDP to be the ratio between two errors of velocity: without and with inclusion of seasonal terms of periods equal to one year and its overtones. To all these cases power-law processes of white, flicker and random-walk noise were added separately.
We have adopted two approaches: widely used annual and semi-annual terms to be subtracted (n=2), annual plus its 3 overtones: semi-annual, four and three months (n=4) along with the approach consistent with Bogusz and Klos (2015): tropical and draconitic up to their 9th harmonics plus 1st and 2nd Chandlerian (n=20) in:

$$x(t) = x_0 + v_x \cdot t + \sum_{i=1}^{n}\left[AC^i \cdot \cos(\omega_i^T \cdot t) + AS^i \cdot \sin(\omega_i^T \cdot t)\right] + \varepsilon(t)$$

The parameters of the model are determined by means of Maximum Likelihood Estimation (MLE) where $\mathbf{A}$ is the design matrix, $\boldsymbol{\theta}$ is the vector with the parameters of the model and $\boldsymbol{\varepsilon}$ is the vector of residuals:

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{A}^T \cdot \mathbf{C}_{\varepsilon\varepsilon}^{-1} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \cdot \mathbf{C}_{\varepsilon\varepsilon}^{-1} \cdot \mathbf{x}$$

The covariance matrix of the determined parameters with the covariance matrix $\mathbf{E}$ resulting from power-law process, and σ_{pl} and σ_{wn} being the standard deviations of power-law and white noise:

$$\mathbf{C}_{\hat{\theta}\hat{\theta}} = \sigma_{pl}^2 \cdot \mathbf{E}(\kappa) + \sigma_{wn}^2 \left[\mathbf{A}^T \cdot \mathbf{A}\right]^{-1}$$

GNSS DATA

Blewitt and Lavallée (2002) theoretically predicted the minimum velocity bias at integer-plus-half years, while Bos et al. (2010) found minimal influence closer to integer-plus-a-quarter year by considering coloured noise. Following those two conclusions we started a simulation with a minimum length of 2 years and extended it with an increment of 0.25 years. We used real data from selected (115) IGS stations.

Figure 3: The General Dilution of Precision as a function of the observation span for MATE (Matera, Italy), North (yellow dots) and East (red triangles) components:

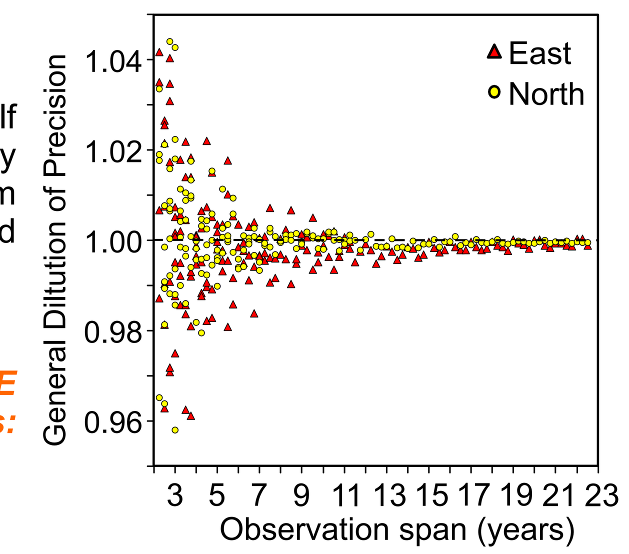
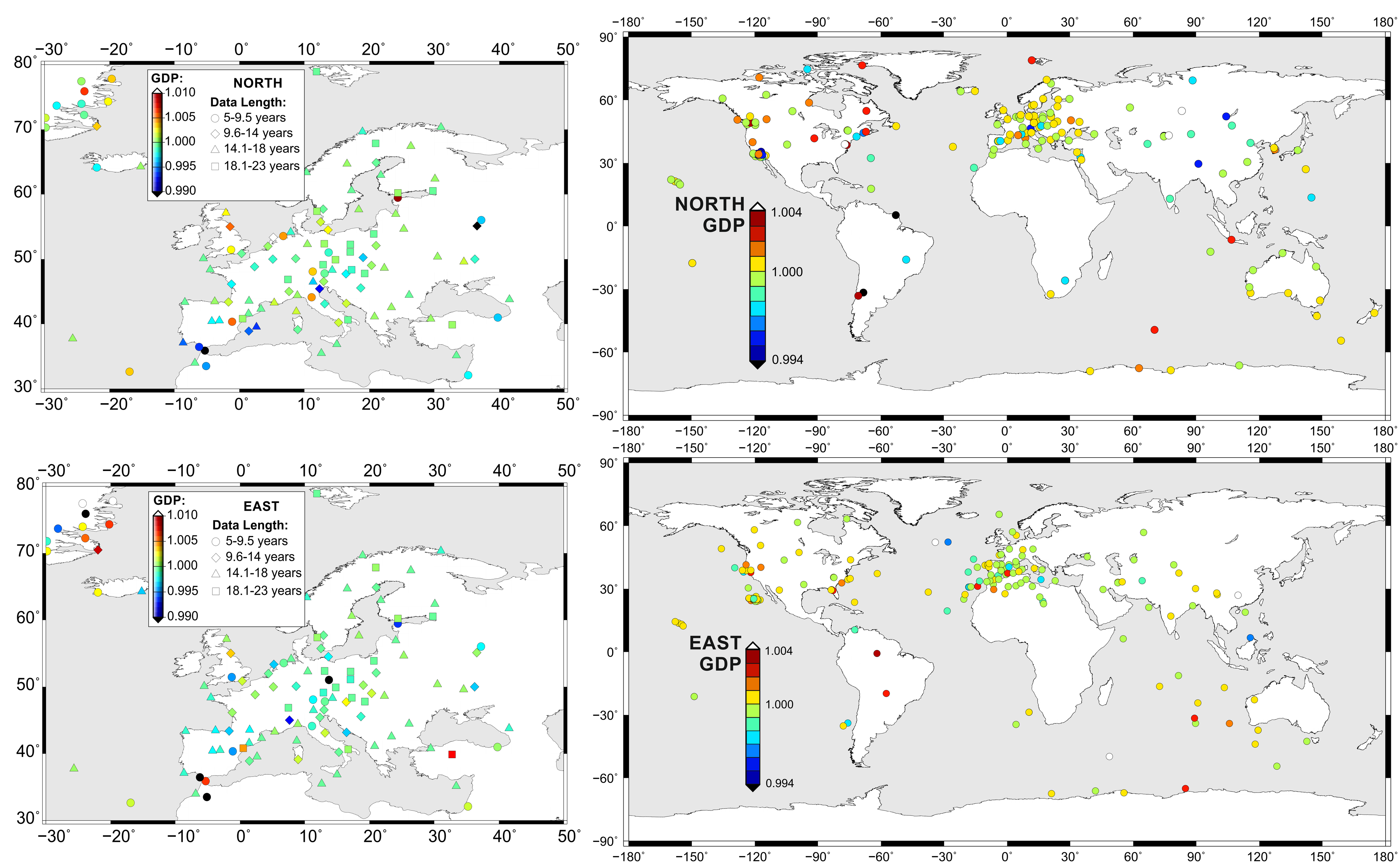


Figure 4: The General Dilution of Precision for set of IGS stations when two approaches were adopted: widely used annual and semi-annual terms to be subtracted (n=2), and the approach consistent with Bogusz and Klos (2015):



SIMULATED TIME SERIES

The simulations were performed up to a maximum length of time series equal to 26 years. We estimated the relative difference in the variance of trend between two deterministic models as:

$$\Delta\sigma_v^2 = \frac{\sigma_{v1}^2 - \sigma_{v2}^2}{\sigma_{v1}^2}$$

σ_{v1}^2 - variance of trend for model with linear velocity,
σ_{v2}^2 - variance of trend for model with linear velocity and periodic terms.

Figure 5: Variance of the slope $\Delta\sigma_v^2$ (in %) for different lengths of time series. The integer spectral indices of white (blue), flicker (red) and random walk (green) processes are examined. Two deterministic models are considered: with linear velocity (σ_{v1}^2) and with linear velocity plus periodic terms (σ_{v2}^2). We assumed that σ_{pl}^2=1 and σ_{wn}^2=1:

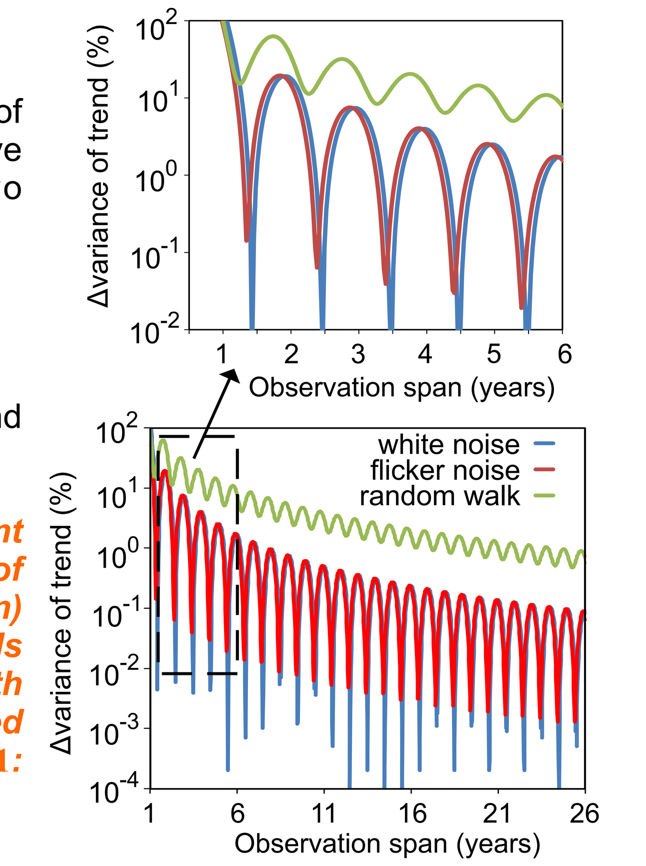
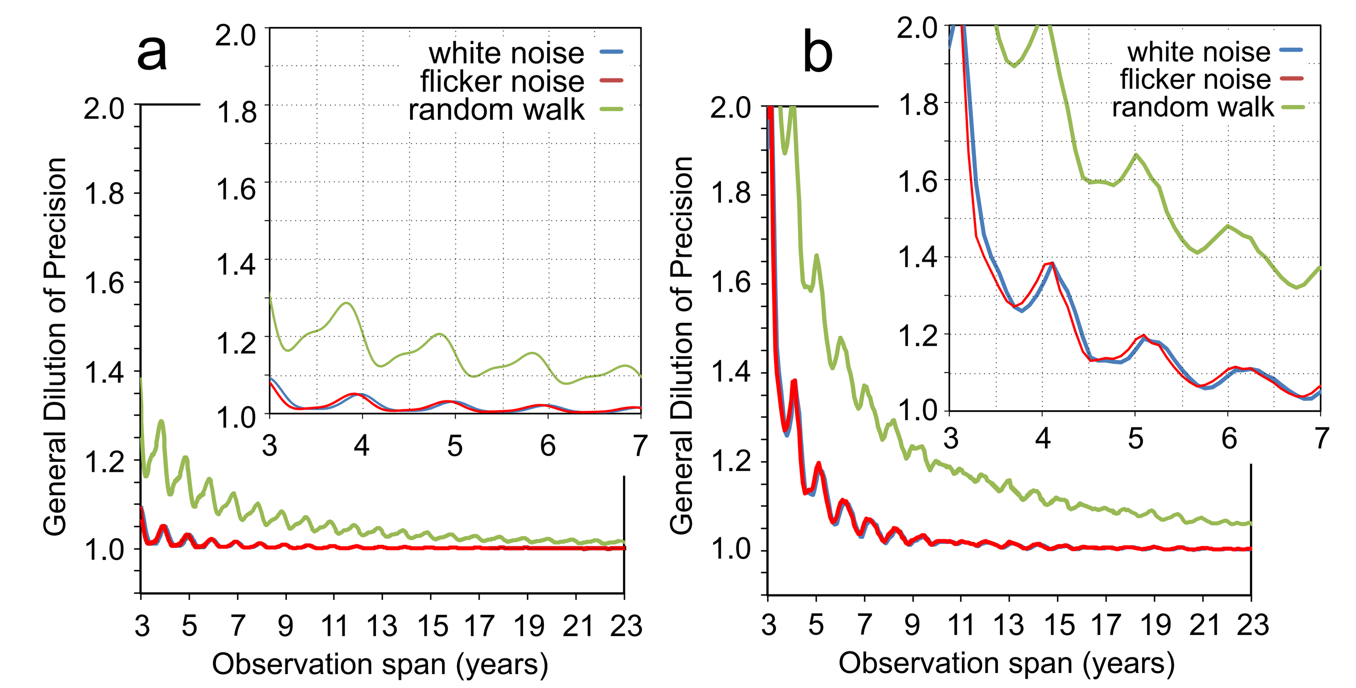


Figure 6: A General Dilution of Precision (GDP) for white noise (blue), flicker noise (red) and random walk (green) plotted for deterministic model containing a linear trend plus: (a) annual and semiannual, (b) annual and draconitics up to their 9th harmonics plus 1st and 2nd Chandlerian (Bogusz and Klos, 2015):



DISCUSSION

1. Along with the increasing spectral index, the amplitudes of oscillations also increase. This arises from the fact that any power-law process with κ<0 brings a correlation between amplitudes of seasonal terms and velocity. In this way, the GDP value is much higher for any time series length considered.
2. Strong peaks in the oscillations are indicated for short time scales, especially for the random walk case. On the other hand, the oscillations play a significant role and are even more important than the assumed noise character. The noise character starts to become important for data longer than 9 years.
3. The local minima and maxima are also being enlarged together with a change towards random walk. This shows, that the GDP differs from integer-plus-half years by Blewitt and Lavallée (2002), who considered only white noise. This is clearly noticed for random-walk noise.
4. With increasing spectral index, the General Dilution of Precision decreases more slowly.
5. Blewitt and Lavallée (2002) used the value of 5% to calculate the minimum velocity bias. However, this value is disputable. With the ever increasing demands on velocities, we argue that even a change of 2% in GDP could be considered as significant. The value of 2% results from the median ratio of the horizontal velocity error to the velocity itself as derived from real GNSS data. This means, that 13 years of continuous observations are enough to reduce the GDP below 2% when white and flicker noise were assumed. However, when random walk is present, this time can be as long as 48 years, which indicates that it is possible to omit periodic oscillations in the deterministic part and that only the appropriate noise model needs to be considered.

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