Asymmetric Competition among Nation States: A Differential Game Approach*

Yutao Han, Patrice Pieretti, Skerdilajda Zanaj†, Benteng Zou
CREA, University of Luxembourg

Abstract

This paper analyzes the impact of foreign investments on a small country's economy in the context of international competition. To that end, we model tax and public input competition within a differential game framework between two unequally sized countries. The model accounts for the widely recognized characteristic that small states are more flexible in their political decision making than larger countries. However, we also acknowledge that small size is associated with limited institutional capacity in the provision of public services. The model shows that the long-term outcome of international competition crucially depends on the degree of capital mobility. In particular, we show that flexibility mitigates against - but does not eliminate - the likelihood of collapse in a small economy. Finally, we note that the beneficial effect of flexibility in a small state increases with its inefficiency in providing public services and with the degree of international openness.

Keywords: Tax/public input competition, Open-loop/Markovian strategies, Differential games.

JEL classification: H25, H73, O30, O43.

*We appreciate the valuable discussion and comments with Arnaud Bourgain, Raouf Boucekkine, Robin Boadway, Herbert Dawid, David de la Croix, Pierre Picard and Myrna Wooders in the early stage of this work and we are indebted to Deborah Schwartz for her research assistance. All eventual mistakes and errors are ours.

†Corresponding author. E-mail: skerdilajda.zanaj@uni.lu. Address: 162a, avenue de la Faiencerie, L-1511, Luxembourg.

1 Introduction

Small states generally suffer from limited access to capital and labor resources, both in amount and in variety. Then, foreign direct investments (in short, FDI, hereafter) can contribute significantly to the development of small states (Read, 2008). In fact, small economies tend to have high level of access to private foreign capital as a ratio of total capital formation (Streeten, 1993). Using data from the World Bank, Figure 1 suggests that the ratio of FDI flows to the gross fixed capital formation is higher in small countries (i.e., population less than two million)[1] than in large countries (i.e., population in excess of 30 million)[2]. Moreover, the economic well-being of small countries is positively correlated with the ratio of FDIs. The data in Figure 1 indicate that small countries above the average line, such as Luxembourg, Malta, Cyprus or Estonia, exhibit a high level of per capita GDP, whereas small countries below this threshold have a lower level of per capita GDP. This is confirmed in Figure 2, which suggests that a direct relationship exists between the level of GDP per capita and foreign investments in small economies. In the cluster of larger countries, however, this relationship is hardly apparent.[3] Countries, such as Poland, Italy, Turkey, India and Spain appear above the threshold in Figure 1, whereas the USA, Ukraine, Nepal, Greece among others, are situated below it.[4]

Given these facts, this paper analyzes the impact of foreign investment flows on the economic performance of a small country competing internationally for mobile production factors. In this context, we investigate the conditions undder which the economies of such countries can be viable, or even expand, in the long term. To that end, we develop a dynamic framework to study how a small country attracts foreign capital through two policy instruments, namely taxes and public services.[5]

[1] Our data set contains 51 countries with population less than 2 million. This represents 72% of all the existing "small" countries. An exhaustive description cannot be provided due to a lack of relevant information.

[2] Our data set of countries with population in excess of 30 million is exhaustive. It contains 41 countries.

[3] Note that, we have not controlled for other determinants of per capita GDP; for example, the availability of natural resources. Taking into account oil reserves and the recent increase in oil prices would explain the position of Qatar or Brunei in our figures.

[4] The ambiguous role of FDIs on the economic performance of countries is documented in the literature (see, for example Alfaro *et al.* 2004).

[5] These public services contribute to the domestic attractiveness of private capital, as they are supposed to enhance private productivity. Examples of this are spending for the operation and maintenance of

For the sake of simplicity, we focus on two competing countries of uneven size. In this study, size is defined as number of capital-owners in a respective country and these capital owners are simultaneously entrepreneurs and workers. By adopting this approach, our model focuses on the economic size of a country.

The dynamic aspect of international competition is addressed by a differential game framework in which the strategic behavior of the small country differs from that of its larger rival. We account for the widely recognized characteristic that small states are more flexible in their political decision making than much larger countries (see, in particular, Streeten, 1993).

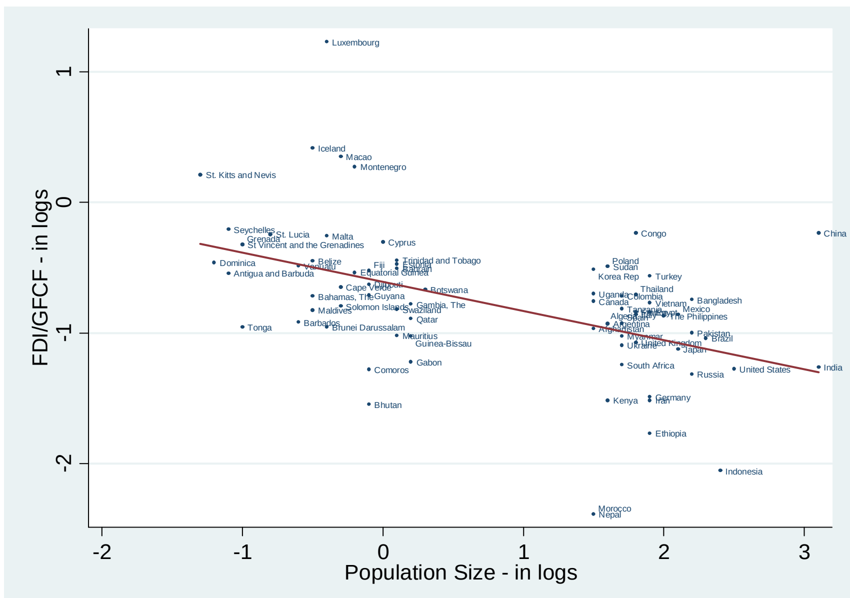

Figure 1: Relationship between the ratio of FDIs to Gross Fixed Capital Formation and population from 2000 to 2010. Source: World Bank

power and transportation infrastructures, operating costs of universities, but also the enforcement of property rights and the provision of capital market, labor and environmental regulations. It follows that countries' attractiveness may also be due to the quality of their institutions. In the Oxford Handbook of Entrepreneurship (2007), it is argued that the abundance of entrepreneurs in a country depends on the existence of regulations, property rights, accounting standards and disclosure requirements, among other factors. Furthermore, in recent years, there has been a surge of national and cross-country studies relating economic development to institutions, especially institutions affecting capital market development and functionality (see, for example, La Porta et al.,1997).

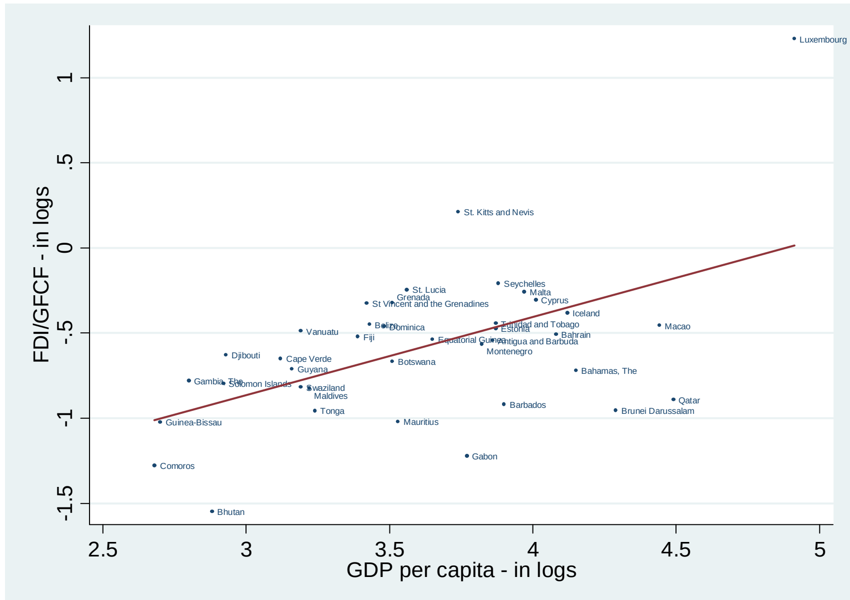

Figure 2 : Relationship between GDP per capita of small countries and the ratio of FDIs to Gross Fixed Capital Formation from 2000 to 2010. Source: World Bank

We thus assume that the small country adopts a Markovian feed-back behavior (i.e., the policy variables are continuously reset in response to the dynamics of the states of the world), whereas the larger country chooses an open-loop rule (i.e., the policy variables are set only once at the initial time). We also acknowledge that small size is associated with handicaps, as, small economies are generally characterized by limited institutional capacity in the provision of public goods (Commonwealth Secretariat, 2000) relative to large countries. Finally, we assume that the capital owners living in both countries have heterogeneous attitudes toward their attachment to home. Thus, they incur costs related to moving abroad. The extend of these costs depends on their attitudes toward their countries. Additionally, their decision to relocate their capital is affected by capital taxation and by productivity-enhancing public services.

The main results of the paper can be summarized as follows. First, the model shows that GDP, in particular the GDP per capita, of the small country increases with the flow of FDIs, which is consistent with the facts presented above. Moreover, the long-run solutions show that the economy of the small country can expand, shrink or even collapse. In this context, two cases can be distinguished: one exhibits high international openness and another exhibits low international openness. The fundamental difference between these cases is that the small country will only experience economic collapse if capital mobility is high (i.e., high international openness). However, higher efficiency in the provision of public services can partially countervail this effect by decreasing the likelihood of collapse. In the second case, when capital mobility is low, international competition for capital can eventually reduce the size of the small economy without provoking its collapse. If capital mobility is very low, the model shows that international competition tends to expand the economy of the small country. We also assess the extent to which flexibility is beneficial to the small country, given that it suffers from limited institutional capacity. By comparing the Markovian and open-loop outcomes, we find that flexibility mitigates against - but does not eliminate- the likelihood of a small economy collapse. Finally, we show that the benefit of flexibility increases in tandem with the inefficiency of public service provision and with the degree of international openness in the small country.

Our paper contributes to the existing literature in the following ways. First, we provide a dynamic counterpart to previous static papers in which countries compete with two instruments. Following Zodrow-Mieszkowski (1986) model, there has been a growing body of literature on the joint role of taxes and public inputs in attracting mobile production factors. For example, Zissimos and Wooders (2008) analyze how the provision of public goods designed to reduce the production cost of private firms is able to relax international tax competition between governments of equal size. Benassy-Quéré et al. (2007) provide an empirical analysis of the impact of taxes and public goods on the allocation of private capital. They find that both corporate taxes and public capital contribute significantly to inward FDIs. Pieretti and Zanaj (2011) propose a two-stage game in which both a small and large jurisdiction compete for capital using taxes and public goods as policy variables. These contributions are, however, static and thus unable to provide insights into dynamic outcomes. Differential games have already been applied to oligopolistic competition (Dockner and Jorgensen, 1984, Karp and Perloff, 1993, Cellini and Lambertini, 2004); however, few studies have applied differential games to tax competition. For example, Coates (1993) deals with the issue of property tax competition and partially analyzes the open-loop equilibrium of a dynamic game.[6] Secondly, by assuming that small countries are more flexible in taking decisions than their larger rivals but at a higher institutional cost as explained

[6]As mentioned by Cardarelli (2002).

above, we account for behavioral and institutional asymmetries which, to the best of our knowledge, are not considered in the traditional tax competition literature.

We assume the economic magnitude expressed in terms of productive resources can vary endogenously as a consequence of public policy and international competition, while the political size is fixed. Similar to our model, the contribution of de la Croix and Dottori (2008) is also concerned with the collapse of a community. To explain the tragedy of Easter Island, these authors show how a closed system can collapse as a result of non-cooperative bargaining between clans. The context and the methodology of their paper is, however, different from ours, given that they use an overlapping generations model in which people live for two periods and compete in fertility rates.

The paper is organized as follows. The next section models the dynamic competition between two countries of asymmetric size. In Section 3, we derive long-run solutions and Section 4 analyzes the long-run conditions of a small country. The importance of flexibility in small economies is assessed in Section 5 and Section 6 presents the conclusion.

2 The model

Suppose that the world is composed of two countries (regions) with unequal populations. Country size may be defined by population, area, or national income (Streeten, 1993). In this study, population, rather than area, is used to define country size. More precisely, size is defined with respect to the number of capital owners who populate the country and these capital owners are simultaneously entrepreneurs and workers. By adopting this approach, our model identifies a country by the size of its economy. Furthermore, capital owners (and their associated activites) are free to relocate to the neighbor country at any point in time. At time $t = 0$, capital flows have not yet taken place, so the population size in each country coincides with its native population.

At $t = 0$, the population of jurisdictions is evenly distributed with unit density on the interval $[-S_1(0), S_2(0)]$. The small country extends from $-S_1(0)$ to the origin 0, and the rest of the world extends from 0 to $S_2(0)$. It follows that the small economy has a size of $S_1(0)$, and the rest of the world has a size of $S_2(0)$, with $S_1(0) < S_2(0)$. We assume that the total number of firms is constant over time and is normalized to one. Thus, for any future time $t \geq 0$, $S_1(t) = S(t)$ and $S_2(t) = 1 - S(t)$.

Entrepreneurs Each citizen is endowed with one unit of capital which is combined with her labor to establish a firm. Therefore, all citizens are self-employed entrepreneurs. Throughout the rest of the paper, we thus use firms and entrepreneurs interchangeably. The firms are distributed at their respective sub-interval according to their disposition to establish a firm outside of their home location. As in Ogura (2006), we assume that this population of entrepreneurs is heterogeneous in the degree of their attachment to the home country.[7] Within the model, we dictate that the closer entrepreneurs are located to extremes of the interval, the more they are attached to their current location. Conversely, the closer that firms are to the border 0, the less they are attached to their territory, and the easier it will be for them to relocate abroad.[8] This means that a firm of type $\alpha \in [-S_1(0), 0]$ located in the home country incurs a disutility of relocating abroad equal to kx, where x is the distance between 0 and α. The coefficient k represents the unit cost of moving capital abroad and can also be interpreted as the degree of international openness.

As in Pieretti and Zanaj (2011), we assume that each firm produces $q+a_i$ $(i = 1, 2)$ units of a final good, where q is the private component of (gross) productivity. The fraction a_i of the produced good depends on the public input supplied by the home (foreign) jurisdiction.[9] Note that the product $S_i \cdot (q + a_i)$ represents the total output or GDP produced in country $i = 1, 2$. This implies that $q + a_i$ is the per capita output in a respective country. The total output is sold in a competitive (world) market at a given price normalized to one. Thus, we suppose that both countries have equal access to a common market. This also implies that the smaller jurisdiction does not suffer from a reduced home market. We further consider that the unit production cost is constant and equal to zero without loss of

[7] This characteristic was first considered in the fiscal competition research of Mansoorian and Myers (1993).

[8] For reasons of simplicity, we assume that firms can only relocate to their neighboring jurisdiction.

[9] A public input satisfies the local public good characteristics; that is, it is jointly used without rivalry by firms located within the same jurisdiction. It follows that the benefits and costs of these goods only accrue at the jurisdictional level. As in Zissimoss and Wooders (2008), we abstract congestion costs . Incorporating congestion into the model would complicate our framework without qualitatively improving the results. Moreover, if public input represents immaterial goods as laws and regulations (e.g., protecting intellectual property and, specifying accurate rules for dispute resolution), the lack of congestion in our model is justified by the particular nature of these goods.

generality. Each entrepreneur pays a tax on capital which is denoted by T_i $(i = 1, 2)$ and levied in the country $i = 1, 2$.[10]

The temporal perspective of the setting described above is as follows. For each period $t \in [\Delta t, +\infty)$ and for any $\Delta t > 0$, governments update their choices in terms of the public services and taxes offered.

Suppose that an entrepreneur of type α is initially located in the small country and considers staying at home or investing her physical capital abroad. If she decides not to move, her profit is given by[11]

$$\pi_1(t) = q(t) + a_1(t) - T_1(t). \tag{1}$$

If she invests abroad, her profit becomes

$$\pi_2(t) = q(t) + a_2(t) - T_2(t) - kx(t).$$

It follows that the marginal entrepreneur x who is indifferent between investing abroad and staying at home verifies the condition

$$q(t) + a_1(t) - T_1(t) = q(t) + a_2(t) - T_2(t) - kx(t).$$

Consequently, we obtain

$$x(t, a_1, a_2, T_1, T_2) = \frac{a_2(t) - T_2(t)}{k} - \frac{a_1(t) - T_1(t)}{k}. \tag{2}$$

In other words, the large country attracts capital $(x > 0)$ from the smaller jurisdiction if the net gain of investing abroad, $a_2(t) - T_2(t)$, is higher than the net gain of staying at home, $a_1(t) - T_1(t)$ after taking into account the mobility cost kx. If $x < 0$, capital moves from the large jurisdiction to the smaller one.

The motion equation of the size of the small country's economy $S(t)$ is given by

$$\dot{S}(t) = -x = \frac{a_1(t) - T_1(t)}{k} - \frac{a_2(t) - T_2(t)}{k}. \tag{3}$$

[10] Given that each entrepreneur invests exactly one unit of capital in our model, the total tax will be T_i $(i = 1, 2)$.

[11] For the sake of simplicity, we consider that q is such that the profit of each firm is positive for all equilibrium levels of public goods and taxes.

We further assume that the preferences for the home location will change in the following way.[12] For the firms that do not move, attachment to home will increase by x if the small economy is attractive to foreign investors ($x < 0$), and it will decrease if the foreign location attracts capital from the small country ($x > 0$). For the capital owners who move abroad, the higher their attachment to the country they leave, the lower the attachment to the new location will be.

Governments Adopting a public-choice perspective, we posit that the governments maximize tax revenue.[13] To this end, countries compete simultaneously by using taxes and public services to attract entrepreneurs, and firms decide where to locate based on these government policies. We suppose that the effective (net) tax revenue collected by the governments does not coincide with the gross amount of tax revenue collected. Following Vaillancourt (1989) and Blumenthal and Slemrod (1992), tax collection is costly due to the administration, monitoring and enforcing procedures associated with it (Kenny and Winer, 2006). If the marginal cost of collecting taxes rises, then the net tax revenue $R(t)$ at time t is a convex function of the collected taxes. For tractability reasons, the net tax revenue will be given by $R_i = \sqrt{S_i T_i}$.

The instantaneous objective function of government $i (i = 1, 2)$ is thus given by the following:

$$w_i(T_i, a_i) = \sqrt{S_i T_i} - \frac{\beta_i}{2} a_i^2, \tag{4}$$

where the second term is the cost of providing public inputs, which is assumed quadratic, whereas β_i is a country specific efficiency parameter. Indeed, the higher the value of β_i, the higher the unit and marginal costs of providing public servive.

The key focus of this paper is the long-run behavior of small states. To this end, we

[12] After relocation of a subset of firms, the attachment to home will change according to the following rule. For all $\widetilde{\alpha}(t) \in [-S(t), S^*(t)]$, we define $\widetilde{\alpha}(t) = \widetilde{\alpha}(t - \Delta t) + x$, where $\widetilde{\alpha}(t) = \begin{cases} \alpha(t) \in [-S(t), O(t)] \\ \alpha^*(t) \in [O(t), S^*(t)] \end{cases}$ and $O(t)$ stands for the origin at period t.

[13] This assumption should not be interpreted in the classical sense given by Brennan and Buchanan (1980) and applied to Leviathan governments. We do not consider here that regulators are self-interested governments. We simply assume that collected taxes are used to finance productive public services but also public goods that do not directly affect the productivity of firms, such as green spaces, swimming pools, and security bodies.

highlight two opposing features of small open economies.

First, according to the Commonwealth Secretariat (2000), the public sector of mini-states generally suffers from limited institutional capacity.[14] Moreover, it may be difficult for small states to recruit high-quality civil servants given their limited pool of candidates(Streeten, 1993). These factors can reduce the efficiency and increase the unit costs for the provision of public services. To account for these facts, we assume that $\beta_1 \equiv \beta > \beta_2$. Normalizing β_2 to 1, we impose $\beta > 1$. It follows that β represents the inefficiency of the small country relatively to the large one.[15]

Secondly, small size can be considered an asset (Kuznets, 1960; Easterly and Kray, 2000) given the economic success of many micro-states. Streeten (1993) suggests that problems related to collective action can be solved more easily in small countries,[16] whereas the larger jurisdiction is not able or not willing to attain this degree of flexibility in its decision making.[17] To capture this difference, we assume that the large jurisdiction commits to a policy path that was adopted at the beginning of the game (i.e., open-loop strategy), whereas policy-makers in the small jurisdiction adopt a Markovian feed-back strategy.

This mixed representation offers a convenient way of modeling differences in flexibility of decision making (Dockner *et al.*, 2000). Although small in a political sense, the mini-state can grow larger as a result of sustained capital inflows. The small country's size could thus exceed a critical threshold that would cause the large country to react more aggressively by also adopting a Markovian strategy. To rule out such a behavioral change,

[14] In small states, the median wage bill of the public sector as a proportion of GDP is 31 percent, whereas the ratio is 21 percent in large developing countries (Commonwealth Secretariat and World Bank, 2000).

[15] To be consistent, the parameter β should be inversely correlated with the size of the small country. Taking into account this feature would however complicate the analysis without important additional insight. Therefore, we shall assume that the small country is tiny enough to consider β as given. For that reason we assume that the size $S_1$is bounded from above by $\overline{S}$ where $\overline{S} < \frac{1}{2}$.

[16] These attributes facilitate greater single-mindedness and focus on economic policy-making and a more rapid and effective response to exogenous change (Armstrong and Read, 1995). Hence, in the present paper, we assume that the small economy updates its decision variables at each period t and is thus able to condition its actions based on current observations.

[17] This could result from the higher costs of social and political heterogeneity. Indeed, after having reached a policy consensus, changing this policy could be a very sensitive issue in a large country. Moreover, the extremely small size of the mini-state may influence thus, given that the large economy may consider it to be unimportant.

we assume that the size of the small economy will remain tiny enough. In other words, we assume that the size $S(t)$ is bounded from above and impose $S(t) \leq \overline{S} < \frac{1}{2}$, for any $t \geq 0$.

The dynamic objective-functions of the competing jurisdictions are respectively[18]

$$J_1 = \max_{a_1, T_1} \int_0^{+\infty} e^{-rt} w_1(T_1(S,t), a_1(S,t)) dt, \tag{5}$$

$$J_2 = \max_{a_2, T_2} \int_0^{+\infty} e^{-rt} w_2(T_2(t), a_2(t)) dt, \tag{6}$$

where r is the discount rate of the public decision-makers, which should reflect the degree of impatience of the population. Given that there is no evidence that this rate is dependent on the size of a population, we accept that r is common to both jurisdictions.

3 Steady states and the long-run policy mix

As explained above, we assume that the small jurisdiction adopts a Markovian strategy, and its larger rival chooses an open-loop approach when designing its optimal decision path. In the appendix we provide the full solution to this game. The steady state production potential of the small country is

$$\hat{S} = \frac{(kr)^{-\frac{3}{2}}}{6\sqrt{2}} \left(\frac{\sqrt{2}}{\beta} - 1 \right) + \frac{2}{3}.$$

Note that this steady state is saddle point stable and there is one monotonically convergent path leading to it.[19]

To guarantee that the production potential of the small country remains smaller than $\overline{S} < \frac{1}{2}$ in the long term, we impose that $k < k^* = \left(\frac{1}{2}\right)^{\frac{1}{3}} \frac{1}{r}$ and $\beta > \underline{\beta} = \frac{\sqrt{2}}{1-\sqrt{2}(kr)^{\frac{3}{2}}}$. The long

[18] Similar to Barro (1990), we consider that the government provides flows of public services. It follows that the public service provision will be treated as a control variable.

[19] We present the convergence path in the appendix.

run policy mix of countries related to taxes and public services is

$$\begin{aligned}
\widehat{a}_1 &= \frac{1}{2\beta}\left(\frac{1}{kr}\right)^{\frac{1}{2}}, \qquad \widehat{T}_1 = kr\widehat{S}, \\
\widehat{a}_2 &= \frac{1}{2}\left(\frac{1}{2kr}\right)^{\frac{1}{2}}, \qquad \widehat{T}_2 = 2kr(1-\widehat{S}).
\end{aligned}$$

These values allow us to define $\widehat{a}_i + q$ as the long-run per capita GDP of country $i = 1, 2$. According to the above solutions, it is possible to show that the variable $\widehat{a}_i$ [20] increases with the long-term size of the economy $\hat{S}$. Given that $\hat{S}$ is positively related to FDI inflows, our model is consistent with a stylized fact we highlighted in Figure 2, in which the per capita output of small economies improves with inward foreign investments.[21] This positive relationship results from spending on public services, which impacts the productivity of firms and, thus, affects the attractiveness of the location to foreign investments.

We also easily verify that $\widehat{a}_2 - \widehat{a}_1 = \frac{1}{4\beta}\left(\sqrt{2}\beta - 2\right)\sqrt{\frac{1}{kr}} > 0$ for $\beta > \sqrt{2}$ and $\widehat{T}_2 - \widehat{T}_1 = kr\left(2 - 3\widehat{S}\right) > 0$, given that $\widehat{S} < \frac{1}{2}$. In other words,

Proposition 1 *The small economy will always undercut tax rates but the public services it provides will never be attractive to investors.*

This result is reminiscent of the findings reported in the literature on tax competition among economies of uneven size (Bucovetsky, 1991, Wilson, 1991,Kanbur and Keen, 1993, Trandel, 1994), according to which the benefit of smallness translates into the ability to undercut the tax rates of larger countries. Contrary to research on inter-jurisdictional competition (based on taxes and public services), our model does not generate an equilibrium, which occurs when the small economy has higher taxe than its larger rival (Hindriks *et al.*, 2008, Pieretti and Zanaj, 2011). This does not occur because the small country is at a disadvantage in providing public services due to the limited capacity of its public sector.

Furthermore, the less efficient the small country is in providing public services, the more it will implement attractive tax policy. Indeed, the gaps $\widehat{a}_2 - \widehat{a}_1$ and $\widehat{T}_2 - \widehat{T}_1$ rise

[20] The steady-state value $\hat{a}_i$ written as a function of $\hat{S}$ is $\hat{a}_i = 3kr(\hat{S}_i - \frac{2}{3}) + (\frac{1}{2kr})^{\frac{1}{2}}$. It follows that $\frac{\partial \hat{a}_i}{\partial \hat{S}_i} > 0$ is always true.

[21] In other words, we see that the level of GDP, the GDP per capita and the production potential of the small country in particular increase with the flow of FDIs.

with β. It should be noted that increasing international openness (lower k) has the same effect as rising β on both gaps. Thus, the higher the capital mobility, the more the small country will be inclined to undercut the tax rates of its rival.

Finally, if the long-run solutions have to guarantee non-negative net budget constraints of both economies, the following two conditions must hold. Either (a) $k^* > k \geq \overline{k}$ with $\overline{k} = (\frac{1}{32})^{\frac{1}{3}}\frac{1}{r}$, or (b) k verifies $\underline{k} < k \leq \overline{k}$, with $\underline{k} = \left(\frac{1}{50}\right)^{\frac{1}{3}}\frac{1}{r}$ and β satisfies $\underline{\beta} < \beta \leq \overline{\beta}$, with $\overline{\beta} = \frac{1}{2\sqrt{2}-16(kr)^{\frac{3}{2}}}$. The budget constraint of the large country will be satisfied if $\widehat{w}_1 \geq 0$, as there are less stringent conditions on the parameters of the large country than its smaller rival [22].

4 Will small states survive in the long run?

In this section, we focus our attention on the conditions under which the production potential of the small economy will expand ($\frac{1}{2} > \hat{S} > S(0)$), shrink ($\hat{S} < S(0)$) or even collapse ($\hat{S} = 0$).[23] Two cases can be considered according to the degree of capital mobility.

Case 1 *High degree of international openness*: $\underline{k} < k < \overline{k}$.

In this case, the survival of the small economy depends on its relative efficiency in providing public services. Two sub-cases can be distinguished: one in which capital mobility is very high, i.e., $\underline{k} < k < k^s$ with[24] $k^s = \dfrac{1}{2\left[2 + S(0)\right]^{\frac{2}{3}}}\dfrac{1}{r}$, and a second one in which capital mobility is moderately high, i.e., $k^s < k < \overline{k}$. In the first sub-case, it is readily verified that the small economy expands in the long run, $\widehat{S} > S(0)$, if $\beta < \overline{\beta}$. However, if the relative efficiency of provision of public services in the small economy is too low (i.e., if $\beta > \overline{\beta}$), it will collapse. Furthermore, as the mobility cost approaches its lower bound $\underline{k}$, the small country is more likely to collapse. This occurs because the small economy has to lower its taxes to such an extent that it can no longer sustain its public expenditures ($\widehat{w}_1 < 0$). There are two extreme outcomes in the long-run. Either the small economy expands, or collapses. Therefore, if it shrinks, it must collapse.

---

[22] It also appears that $\widehat{S} \in \left[0, \frac{1}{2}\right)$ in both (a) and (b).

[23] We impose (see proof in Appendix A.3) that $S(t) \leq \overline{S} < \frac{1}{2}$. If so, $\underline{\beta}$ would depend on the upper bound of $\overline{S}$. Thus, $\underline{\beta}(\overline{S}) = \frac{\sqrt{2}}{1+6\sqrt{2}(\overline{S}-\frac{2}{3})(kr)^{\frac{3}{2}}}$, in which $\overline{S}$ is decreasing.

[24] It is readily verified that $k^s < \overline{k}$ if $0 < S(0) < \frac{1}{2}$.

This extreme scenario changes in the second sub-case. According to the values taken by β, the small economy can expand, collapse and shrink without collapsing. If $\beta < \beta^s$ with $\beta^s = \frac{\sqrt{2}}{1-6\sqrt{2}\left[\frac{2}{3}-S(0)\right](kr)^{\frac{3}{2}}}$, it will expand, and if $\beta > \overline{\beta}$, it will collapse. For an intermediate efficiency value, i.e., $\beta^s < \beta < \overline{\beta}$, the small country will shrink but still survive.

The following proposition can then be stated:

Proposition 2 *Assume that international openness is high. The economy of the small country can expand if it is relatively efficient in providing public services. Otherwise, its economy will shrink or even collapse in the long run.*

In a world of mobile capital, a small economy may have difficulty surviving even if it is able to adapt to change more quickly than larger countries. This can occur because the efficient provision of public services and capital mobility are crucial to generating the resources necessary to afford further public amenities. In fact, the model shows that below a given threshold, rising capital mobility causes the small economy to cut its taxes to such an extent that its budgetary resources vanish. It follows that small states, but especially micro-states, can secure their status in a global economy if their public sectors provide public services with sufficient efficiency and if their tax rates are more favorable than those of larger countries. At best, this is a necessary condition for attracting foreign capital, or at least, surviving.

Case 2 *Low degree of international openness:* $k^* > k > \overline{k}$.

In this case, the relative inefficiency of the provision of public services can no longer lead to the collapse of an economy because budget resources are not constrained. Formally, the limit value $\overline{\beta}$ tends to ∞ if k approaches $\overline{k}$. This is in marked contrast with the first case, as - in this case- a low degree of financial openness makes capital more captive and provides sufficient tax revenues to cover public expenditures. At worst, the economy of the small country can contract ($0 < \widehat{S} < S(0)$). This occurs if $\widehat{k} > k > \overline{k}$ and $\beta > \beta^s$, with $\widehat{k} = (\frac{1}{8[2-3S(0)])^2})^{\frac{1}{3}}\frac{1}{r}$. However, if mobility is very low, i.e., $k^* > k > \widehat{k}$, the small economy will attract foreign capital and thus expand. Surprisingly, this scenario occurs independently of the level of inefficiency.

We conclude with the following proposition:

Proposition 3 *Assume that international openness is low. The small country's economic size never collapses but may shrink if the degree of international openness is not sufficiently low. In either case, the survival of the economy is independent of the efficiency of public service provision.*

We provide a summary illustration of the different cases with respect to the parameter values of k and β in Figure 3.

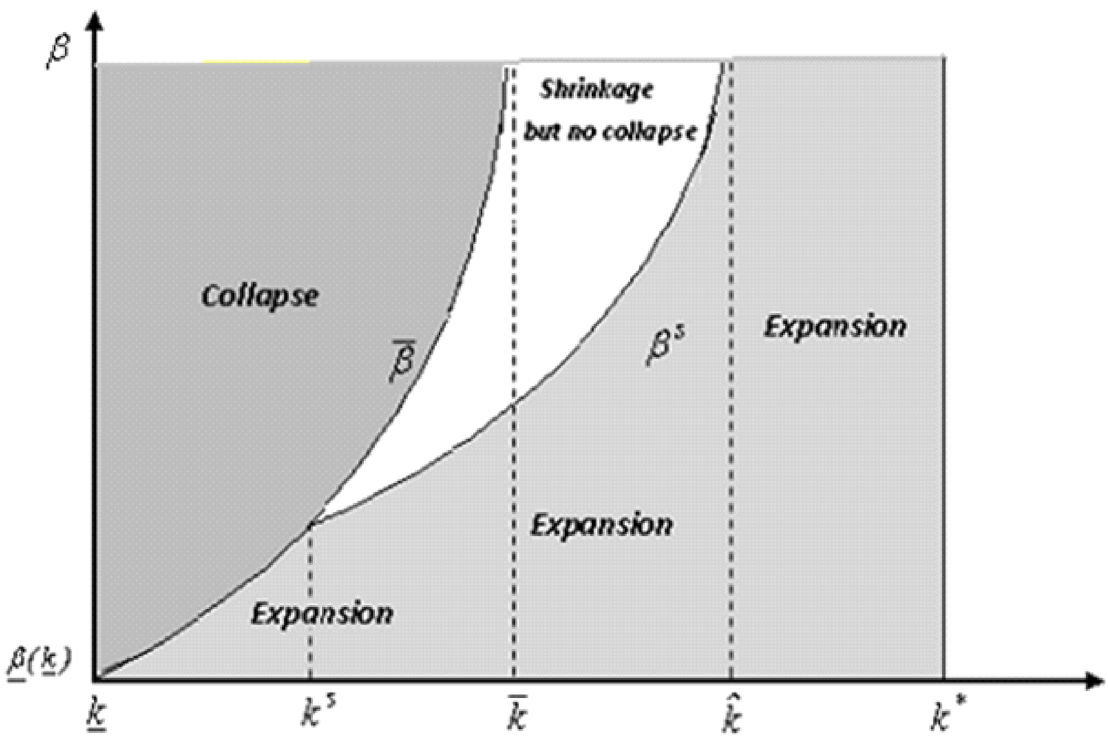

Figure 3 : The evolution of the small country's economic potential according to the mobility cost (k) and the degree of public inefficiency (β).

5 How important is flexibility to the small economy?

To assess how beneficial flexibility is to the small country, we first calculate the long-run production potential $\widetilde{S}$ of the small country if it chooses an open-loop behavior identical to its larger rival. We thus obtain

$$\tilde{S} = \frac{(kr)^{-\frac{3}{2}}}{4}(\frac{1}{\beta} - 1) + \frac{1}{2}.$$

The benefit of flexibility can be represented by the difference $\widehat{S} - \widetilde{S}$, which is obtained by comparing the Markovian and open-loop outcomes. It is easy to verify that this difference is always non-negative. Therefore, given the same parameters, the Markovian behavior adopted by the small country is preferable to the open-loop behavior. However, flexibility does not completely eliminate the potential for collapse; it only makes its occurrence less likely.

Given that $\frac{\partial(\widehat{S}-\widetilde{S})}{\partial\beta} > 0$, the advantage of the small country's flexibility increases with its inefficiency to provide public services. In other words, the economic size of the small country is more sensitive to an increase in efficiency (β decreases) in the Markovian scenario.[25] Consequently, flexibility counterbalances inefficiency, and the more inefficient a small country is in providing public inputs, the more valuable flexibility is to its long-run survival.

Furthermore, higher capital mobility increases the relative advantage of flexibility, given that $\frac{\partial(\widehat{S}-\widetilde{S})}{\partial k} < 0$. Note that increased capital mobility reduces (k increases) the long-term economic potential of the small economy; however, this occurs to a lesser extent in the Markovian scenario. It follows that flexibility countervails the negative effect of high capital mobility, and flexibility brings greater benefits to the small country when capital mobility is low. So, we can conclude by the following proposition.

Proposition 4 *The benefit of flexibility decreases with the small country's efficiency to provide public services and increases with capital mobility.*

We finally observe that similar to the Markovian scenario, the small country never collapses by adopting an open-loop behavior when capital mobility is sufficiently low. However, this condition becomes more restrictive in the open-loop scenario. Indeed, the absence

[25] In fact, it is convenient to verify that $\left|\frac{\partial\widehat{S}}{\partial\beta}\right| < \left|\frac{\partial\widetilde{S}}{\partial\beta}\right|$.

of flexibility in policy making requires now that the mobility cost is higher than $\overline{\overline{k}}$, which exceeds the threshold $\overline{k}$ corresponding to the Markovian case.[26]

6 Conclusion

In this paper, we investigate whether a small open economy can survive in the long-run when facing global competition. To this end, we model the dynamic competition between two unequally sized economies. The policy makers of these two countries compete simultaneously by taxing mobile capital and offering public services. Firms choose to locate their capital in the country where their profits are maximized. We characterize the heterogenous behaviors of the two governments within a differential game framework, in which the small state adopts Markovian (i.e., flexible) behavior, and its larger rival commits to a strategy developed at the initial time point (i.e., open-loop behavior).

The results show that under conditions of high capital mobility, the small economy will risk economic collapse if it provides public services inefficiently. When capital mobility is very low, the economy of the small state always expands despite its limited institutional capacity.

However, further research is needed. In the present study, countries are treated solely as maximizers of tax revenue, and this over-emphasizes the role of tax rates in the long-run outcomes. Therefore, it would be interesting to analyze a scenario in which governments are welfare maximizers and take into account the well-being of their populations. The present paper also models the private sector in an elementary way. Countries are undifferentiated in their ability to produce private goods and the production process is static. Future research should thus consider how international competition is able to impact the growth process of these competing economies when private productivity differs between jurisdictions.

[26] It is convenient to show that $\overline{\overline{k}} = (\frac{1}{4})^{\frac{1}{3}}\frac{1}{r}$.

References

[1] Alfaro L, A. Chanda, S. Kalemli-Ozcan, and S. Sayek, 2004. FDI and Economic Growth: The role of local financial markets. Journal of International Economics, 64(1): 89-112.

[2] Armstrong H.W. and R. Read, 1995. Western European Micro-states and EU autonomous Regions: the Advantages of Size and Sovereignty. World Development, 23(7):1229-1245.

[3] Barro, R.J. ,1990. Government Spending in a Simple Model of Endogenous Growth. Journal of Political Economy, 98:103-125.

[4] Benassy-Quere A., N. Golalraja and A. Tronnoy, 2007. Tax and public input competition. Economic Policy: 385-430.

[5] Blumenthal, M. and J. Slemrod, 1992. The compliance cost of the U.S. individual tax system: a second look after tax reform, National Tax Journal, 45(2): 186-202.

[6] Brennan, G. and J.M. Buchanan,1980. The Power to Tax: Analytical Foundations of a Fiscal Constitution. Cambridge: Cambridge University Press.

[7] Bucovetsky S., 1991. Asymmetric Tax Competition, Journal of Urban Economics, 30, 167–181.

[8] Cardarelli R., E. Taugourdeau, J.P. Vidal, 2002. A repeated interactions model of tax competition. Journal of Public Economic Theory 4, 19–38.

[9] Casson M., Y. Bernard, B. Anuradha, and W. Nigil, 2006. The Oxford Handbook of Entrepreneurship. Oxford: Oxford University Press.

[10] Cellini R. and L. Lambertini, 2004. Dynamic oligopoly with sticky prices: closed-loop, feedback, and open-loop solutions, Journal of Dynamical and Control Systems. Vol. 10, No. 3: 303-314.

[11] Cellini R. and L. Lambertini, 2007. A differntial oligopoly game with differentiated goods and sticky prices, European Journal of Operational Research, 176: 1131-1144.

[12] Coates D., 1993. Property tax competition in a repeated game. Regional Science and Urban Economics, 23:111—119.

[13] De la Croix D. and D. Dottori, 2008. Easter Island's collapse: a tale of a population race. Journal of Economic Growth, 13:27–55.

[14] Dockner, E. J., S. Jorgensen, N.V. Long, and G. Sorger, 2000. Differential Games in Economics and Management Science. Cambridge, Cambridge University Press.

[15] Dockner E. and S. Jorgensen S., 1984. Cooperative and non-cooperative differential game solutions to and investment and pricing problem. J. Opl Res. Soc. 35(8): 731-739.

[16] Easterly W. and A. Kray, 2000. Small states, small problems ? Income, growth, and volatility in small states. World Development, 28 (11): 2013-2027.

[17] Hindriks J, S. Peralta and S. Weber, 2008. Competing in taxes and investment under fiscal equalization. Journal of Public Economics, 92: 2392-2402.

[18] Kanbur, R., and M. Keen, 1993. Jeux Sans Frontières: Tax competition and tax coordination when countries differ in size. American Economic Review, 83(4): 877–893.

[19] Karp L.S. and J.M. Perloff , 1993. Open-loop and feedback models of dynamic oligopoly. International Journal of Industrial Organization, 11: 369-389.

[20] Kenny L. and S. Winer, 2006. Tax Systems in the World: An Empirical Investigation into the Importance of Tax Bases, Administration Costs, Scale and Political Regime. International Tax and Public Finance, 13(2): 181-215.

[21] Kuznets S.,1960. Economic Growth of Small Nations. In The Economic Consequences of the Size of Nations, ed. E.A.G. Robinson. Proceedings of a Conference Held by the International Economic Associations. MacMillan, Toronto.

[22] La Porta, R., F. Lopez-de-Silanes, A. Shleifer, and R. Vishny, 1997. Legal determinants of external finance. The Journal of Finance, 52, 1131-1150.

[23] Mansoorian A. , G.M. Myers,1993. Attachment to home and efficient purchases of population in a fiscal externality econom. Journal of Public Economics, 52: 117-132.

[24] Ogura L.M., 2006. A note on tax competition, attachment to home, and underprovision of public goods. Journal of Urban Economics, 59: 252-258.

[25] Pieretti P. and S. Zanaj, 2011. On tax competition, public goods provision and jurisdictions' size. Journal of International Economics, 84(1): 124-130.

[26] Read R., 2008. Foreign Direct Investment in Small Island Developing States. Journal of International Development, 20: 502–525.

[27] Report of the Commonwealth Secretariat/ World Bank Joint Task Force on Small States, 2000. Small States: Meeting Challeges in the Global Economy.

[28] Streeten P., 1993. The Special Problems of Small Countries. World Development, 21(2): 197-202.

[29] Trandel G., 1994. Interstate commodity tax differentials and the distribution of residents. Journal of Public Economics, 53: 435-457.

[30] Zodrow G.R. and P.M. Mieszkowsky, 1986. Pigou, property taxation and the underprovision of local public goods. Journal of urban economics 19, 356-370.

[31] Vaillancourt, François,1989. The Administrative and Compliance Costs of the Personal Income Tax and Payroll Tax System in Canada. Canadian Tax Papers, No. 86. Canadian Tax Foundation, c1989.

[32] Wilson J.D., 1991. Tax competition with interregional differences in factor endowments. Regional Science and Urban Economics, 21(3): 423-451.

[33] Zissimos B. and M. Wooders, 2008. Public Good Differentiation and the Intensity of Tax Competition. Journal of Public Economics, 92: 1105-1121.

A Appendix

A.1 Solution of the differential game in Section 3.

Wedefine as follows the notion of heterogenous strategic behavior that is used in Dockner *et al.*, 2000, pages 87–92.[27]

Definition 1 *A 2-tuple* (Ψ_1, Ψ_2) *of functions* $\Psi_1 : [0,1] \times [0,+\infty) \to \mathbb{R}^2_+$ *and* $\Psi_2 : [0,+\infty) \to \mathbb{R}^2_+$, *with* $\Psi_1 = (\Psi_{11}(S,t), \Psi_{12}(S,t)), \forall (S,t) \in [0,1] \times [0,+\infty)$ *and* $\Psi_2 = (\phi_{21}(t), \Psi_{22}(t))$, *is called a heterogenous Strategic Nash Equilibrium if, for each* $i = 1,2$, *an optimal control path* $(a_i(\cdot), T_i)$ *of player* i *exists and is given by the Markovian Strategy for player 1:* $(a_1(t), T_1(t)) = (\Psi_{11}(S(t),t), \Psi_{12}(S(t),t)) = \Psi_1(S(t),t)$, *and open-loop strategy for player 2:* $(a_2(t), T_2(t)) = (\Psi_{21}(t), \Psi_{22}(t)) = \Psi_2(t)$.

The small open economy (the Markovian strategic player) takes the large country's (open loop) strategy $\Psi_2(t)$ as given, and hence, faces the following optimization problem:

$$\begin{cases} \max\limits_{a_1, T_1} \displaystyle\int_0^\infty e^{-rt} \left[(S(t)T_1(S,t))^{\frac{1}{2}} - \frac{\beta}{2} a_1^2(S,t) \right], \\ \text{subject to } \dot{S}(t) = \dfrac{a_1(S,t) - T_1(S,t)}{k} - \dfrac{\Psi_{21}(t) - \Psi_{22}(t)}{k}. \end{cases} \tag{7}$$

The corresponding current-value Hamiltonian is

$$\mathcal{H}_1(T_1, S, a_1, \lambda_1) = \left[S^{\frac{1}{2}}(t) T_1^{\frac{1}{2}}(S,t) - \frac{\beta}{2} a_1^2(S,t) \right] + \lambda_1 \left(\frac{a_1(S,t) - T_1(S,t)}{k} - \frac{\Psi_{21}(t) - \Psi_{22}(t)}{k} \right)$$

where λ_1 denotes a costate variable.

The large economy faces the following problem:

$$\begin{cases} \max\limits_{a_2, T_2} \displaystyle\int_0^\infty e^{-rt} \left[((1 - S(t))T_2(t))^{\frac{1}{2}} - \frac{1}{2} a_2^2(t) \right], \\ \text{subject to } \dot{S}(t) = \dfrac{\Psi_{11}(S,t) - \Psi_{12}(S,t)}{k} - \dfrac{a_2(t) - T_2(t)}{k}. \end{cases} \tag{8}$$

[27] Different but similar idea of guessing *symmetric* strategies via Pontryagin maximum principle are also used in several studies by Cellini and Lambertini (2004 and 2007 and the references therein).

The large country *conjectures* that the small economy's strategies are $\Psi_{11}(S,t) = \frac{\rho}{\beta k}\lambda_1(t)$ and $\Psi_{12}(S,t) = \left(\frac{k}{2\rho\lambda_1(t)}\right)^2 S, \forall S \in [0,1]$ and $t \geq 0$.[28][29] Thus, the current-value Hamiltonian of the large economy is defined as

$$\mathcal{H}_2(T_2, S, a_2, \lambda_2) = \left[(1-S(t))^{\frac{1}{2}} T_2^{\frac{1}{2}}(t) - \frac{1}{2}a_2^2(t)\right] + \lambda_2 \left(\frac{\Psi_{11}(S,t) - \Psi_{12}(S,t)}{k} - \frac{a_2(t) - T_2(t)}{k}\right)$$

with λ_2 its costate variable.

The first order conditions yield the small economy's equilibrium choices $T_1(S,t) = \left(\frac{k}{2\lambda_1}\right)^2 S$, $a_1(S,t) = \frac{\lambda_1}{k\beta}$. The costate variable verifies the equation $\dot{\lambda}_1(t) = r\lambda_1 - \frac{k}{4\lambda_1}$ with the transversality condition $\lim_{t\to\infty} e^{-rt}\lambda_1(t)S(t) = 0$.

The optimal choices of the big economy are $a_2(t) = -\frac{\lambda_2(t)}{k}$, $T_2(t) = \left(\frac{k}{2\lambda_2(t)}\right)^2 (1-S(t))$ with the costate equation

$$\dot{\lambda}_2(t) = r\lambda_2 - \frac{k}{4\lambda_2} + \frac{k}{4}\frac{\lambda_2}{\lambda_1^2}. \tag{9}$$

The associated transversality condition is $\lim_{t\to\infty} e^{-rt}\lambda_2(t)S(t) = 0$.

Moreover, we can readily check that the maximized Hamiltonian $H_1^*(S,\lambda_1)$ and $H_2^*(S,\lambda_2)$ are given by

$$\mathcal{H}_1^*(S,\lambda_1,t) = \left[\frac{k}{2\rho\lambda_1}S - \frac{\beta}{2}\left(\frac{\lambda_1}{k\beta}\right)^2\right] + \lambda_1\left(\frac{\frac{\lambda_1}{k\beta} - (\frac{k}{2\lambda_1})^2 S}{k} - \frac{(-\frac{\lambda_2}{k}) - (\frac{k}{2\lambda_2})^2(1-S)}{k}\right)$$

and

$$\mathcal{H}_2^*(S,\lambda_2,t) = \left[-\frac{k}{2\lambda_2}(1-S) - \frac{1}{2}\left(-\frac{\lambda_2}{k}\right)^2\right] + \lambda_2\left(\frac{\frac{\lambda_1}{k\beta} - (\frac{k}{2\lambda_1})^2 S}{k} - \frac{(-\frac{\lambda_2}{k}) - (\frac{k}{2\lambda_2})^2(1-S)}{k}\right).$$

---

[28] There may be different strategies to choose, such as the one presented by Cellini and Lambertini(2007). Here, we only study one of its kind.

[29] The guessing of the others' strategies comes from open-loop strategy: Suppose both players play open-loop strategies, then their current value open-loop Hamiltonian functions can be easily written down and so do their first order conditions, respectively. The first order conditions offer the optimal open-loop strategies, provided some sufficient conditions are checked. Suppose those optimal open-loop strategies are: $\Psi_1(t) = \Psi_1(S(t),\lambda_1(t),t)$ and $\Psi_2(t) = \Psi_1(S(t),\lambda_2(t),t)$, for any t. In the guessing process, the open-loop strategy player(here, the large economy) takes the small economy's open-loop strategy as conjectured strategy with the following modification: large economy guesses that strategy $\Psi_1(t) = \Psi_1(S(t),t)$ will be $\Psi_1(S,t)$ with any state variable S, since the small economy plays Markovian strategy. Therefore, the large economy guess that the small economy's strategy is: $\Psi_1(S,t) = \Psi_1(S,\lambda(t),t)$, for any (S,t).

It is straightforward that the maximized Hamiltonian are concave with respect to the state variable S, hence, $a_i(t), T_i(t)$ $(i = 1,2)$ are optimal paths. Therefore, the large country's conjecture about the rival's strategy is optimal. Therefore, the solutions $\Psi_1(S,t) = (a_1(S,t), T_1(S,t))$ and $\Psi_2(t) = (a_1(t), T_2(t))$ for $S \in [0,1]$ and $t \geq 0$ is one pair of Nash Equilibria. QED.

A.2 Steady states

The long run solutions of the above dynamic system are given as follows:

Proposition 5 *At the Nash equilibrium, for any given parameters $\rho, k, r, \beta_i, i = 1,2$, there is a potential interior steady state*

$$\widehat{S} = \frac{1}{6\sqrt{2}} \left(\frac{1}{kr}\right)^{\frac{3}{2}} (\frac{\sqrt{2}}{\beta} - 1) + \frac{2}{3}, \tag{10}$$

$$\widehat{a}_1 = \frac{1}{2\beta} \left(\frac{1}{kr}\right)^{\frac{1}{2}}, \ \widehat{T}_1 = kr\widehat{S}, \ \widehat{a}_2 = \frac{1}{2} \left(\frac{1}{2kr}\right)^{\frac{1}{2}}, \widehat{T}_2 = 2kr(1-\widehat{S}), \tag{11}$$

with the costate variables $\widehat{\lambda}_1 = \frac{1}{2}\left(\frac{k}{r}\right)^{\frac{1}{2}}, \widehat{\lambda}_2 = -\frac{1}{2}\left(\frac{k}{2r}\right)^{\frac{1}{2}}$. Notice that he steady state is a saddle point of the canonical system and it is one dimensional locally asymptotically stable.

A.3 Trajectories

The above analysis shows that there is a stable trajectory associated with the dynamic system. In this subsection, we explore the convergence path to make clear how the steady state is attained. Taking into account of the initial and transversality conditions, the FOCs yield the explicit trajectories

$$\lambda_1(t) = \frac{1}{2} \left(\frac{k}{r}\right)^{\frac{1}{2}}, \ \lambda_2(t) = -\frac{1}{2} \left(\frac{k}{2r}\right)^{\frac{1}{2}}.$$

The state trajectory becomes

$$S(t) = (S(0) - \widehat{S})e^{-3rt} + \widehat{S} \tag{12}$$

which is the optimal convergence path leading to the steady state. The convergence speed is $3r$.

A.4 State constraint $S(t) \leq \overline{S} < \frac{1}{2}$

Recalling that the small country's size is constrained ($S(t) \leq \overline{S} < \frac{1}{2}$), we adapt the Lagrangian function as follows

$$\begin{aligned}\mathcal{L}_1(T_1, S, a_1, \lambda_1) &= \left[S^{\frac{1}{2}}(t)T_1^{\frac{1}{2}}(S,t) - \frac{\beta}{2}a_1^2(S,t)\right] + \lambda_1\left(\frac{a_1(S,t) - T_1(S,t)}{k} - \frac{a_2(t) - T_2(t)}{k}\right) \\ &+\mu(S - \overline{S}).\end{aligned}$$

The above first order conditions still hold, except the costate variable which now verifies the equation $\dot{\lambda}_1(t) = r\lambda_1 - \frac{k}{4\lambda_1} + \mu$. Furthermore, we introduce the Kuhn-Tucker condition

$$\mu\left(S - \overline{S}\right) = 0.$$

In other words, we have, either $S < \overline{S}$ with $\mu = 0$ or $S = \overline{S}$ with $\mu \geq 0$. However, since the small economy's size is constrained by the upper-bound $\overline{S}$, we impose that $\mu = 0$ whenever $S = \overline{S}$. QED.