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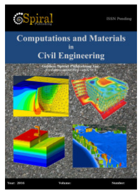

Research Paper

The influence of superposition and eccentric loading on the bearing capacity of shallow foundations

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Abstract. In 1920 Prandtl published an analytical solution for the bearing capacity of a centric loaded strip footing on a weightless in-finite half-space. Reissner (1924) extended this solution for a surrounding surcharge and Keverling Buisman (1940) for the soil weight. Terzaghi (1943) wrote this as a superposition of three separate bearing capacity components for the cohesion, surcharge and soil-weight. The first question is to what extent the currently used components are correct. The second question is to what extent the superposition is correct, because the failure mechanisms for these three components are not the same. A number of finite element calculations show that there is indeed an error, which is luckily not too large and leads to predictions on the safe side. Meyerhof (1953) extended the equation of Terzaghi with correction factors for the shape of the footing and the inclination of the load. For eccentric loading however, there are no correction factors. The common practice is to reduce the contact area of the foundation such that its centroid coincides with that of the load, which means that, the area of the foundation outside the effective area, is completely neglected. Therefore the third question is, if this reduction of the foundation area is an accurate method to describe the reduction of the bearing capacity due to eccentric loading. A number of finite element calculations show that this is indeed the case.

Keywords: Bearing capacity; Footings; Foundations; Finite Element Modelling.

1. Introduction

In 1920, Ludwig Prandtl published an analytical solution for the bearing capacity of a soil under a load, p, causing kinematic failure of the weightless infinite half-space underneath. The strength of the half-space is given by the angle of internal friction, ϕ, and the cohesion, c. The failure mechanism proposed by Prandtl (see the original drawing of Prandtl in Figure 1) has been validated by laboratory tests, for example those performed by Jumikis (1956), Selig and McKee (1961), and Muhs and Weiß (1972).

Prandtl subdivided the sliding soil part into three zones (Figure 2):

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1. Zone 1: A triangular zone below the strip load. Since there is no friction on the ground surface, the directions of the principal stresses are horizontal and vertical; the largest principal stress is in the vertical direction.
2. Zone 2: A wedge with the shape of a logarithmic spiral, in which the principal stresses rotate through $90°$, from Zone 1 to Zone 3. The pitch of the sliding surface equals the angle of internal friction; $\xi = \phi$, creating a smooth transition between Zone 1 and Zone 3.
3. Zone 3: A triangular zone adjacent to the strip load. Since there is no friction on the surface of the ground, the directions of principal stress are horizontal and vertical; the largest principal stress is in the horizontal direction.

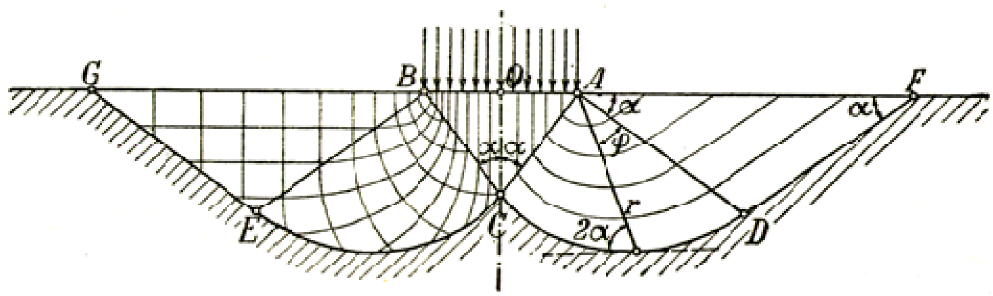

Fig. 1. The Prandtl-wedge failure mechanism (Original drawing of Prandtl, 1920).

The failure mechanism does not have to be symmetrical, with sliding in two directions, as proposed by Prandtl in Figure 1, but can also be unsymmetrical, with sliding in one direction only, as shown in Figure 2.

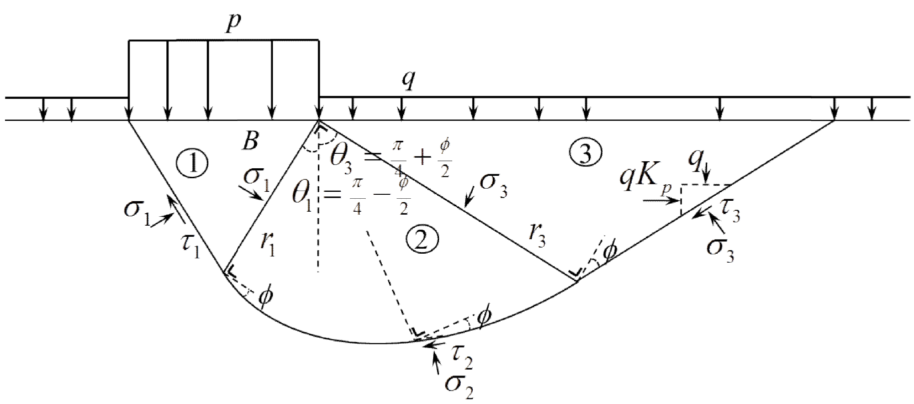

Fig. 2. The Prandtl-wedge with three zones.

The solution of Prandtl was extended by Hans J. Reissner (1924) for a surrounding surcharge, q, and was based on the same failure mechanism. Albert S. Keverling Buisman (1940) and Karl Terzaghi (1943) extended the Prandtl-Reissner formula for the soil weight, γ. It was Terzaghi (1943) who wrote the equation for the bearing capacity as a superposition of the three separate bearing capacity components for the cohesion, surcharge and soil-weight. George G. Meyerhof (1953) was the first to propose equations for inclined loads. He was also the first in 1963 to write the formula for the (vertical) bearing capacity p_v with dimensionless bearing capacity factors (*N*), inclination factors (*i*) and shape factors (*s*), for the three independent bearing components; cohesion (*c*), surcharge (*q*) and soil-weight (γ), in a way it was adopted by Jørgen A. Brinch Hansen (1970) and is still used nowadays:

$$p_v = i_c s_c c N_c + i_q s_q q N_q + i_\gamma s_\gamma \tfrac{1}{2} \gamma B N_\gamma . \tag{1}$$

where c is the cohesion (kPa), q is the surcharge (kPa), γ is the effective soil weight (kN/m^3), and B is the width of the strip load (m).

The purpose of this study is threefold: to check the correctness of the currently used bearing capacity factors for non-dilatant soil, to check the influence of the superposition on these factors, and to check the influence of eccentric loading on these factors.

2. The correctness of the bearing capacity factors for non-dilatant soil

2.1. *Theory*

The cohesion bearing capacity factor follows from the solution of Prandtl (1920):

$$N_c = \left(K_p \cdot e^{\pi \tan\phi} - 1 \right) \cot\phi \quad \text{with:} \quad K_p = \frac{1+\sin\phi}{1-\sin\phi} \tag{2}$$

where ϕ is the friction angle of the soil (°) and K_p is the passive earth pressure coefficient (-).

The surcharge bearing capacity factor follows from the solution of Reissner (1924):

$$N_q = K_p \cdot \left(\frac{r_3}{r_1} \right)^2 = K_p \cdot e^{\pi \tan\phi} \quad \text{with:} \quad K_p = \frac{1+\sin\phi}{1-\sin\phi} \tag{3}$$

Keverling Buisman (1940), Terzaghi (1943), Caquot and Kérisel (1953), Meyerhof (1951; 1953; 1963; 1965), Brinch Hansen (1970), Vesic (1973, 1975), and Chen (1975) subsequently proposed different equations for the soil-weight bearing capacity factor N_γ. Therefore the following equations for the soil-weight bearing capacity factor can be found in the literature:

$$\begin{aligned}
N_\gamma &= \left(K_p \cdot e^{\pi \tan\phi} - 1 \right) \tan(1.4\phi) \quad \text{(Meyerhof, 1963)}, \\
N_\gamma &= 1.5 \left(K_p \cdot e^{\pi \tan\phi} - 1 \right) \tan\phi \quad \text{(Brinch Hansen, 1970)}, \\
N_\gamma &= 2 \left(K_p \cdot e^{\pi \tan\phi} + 1 \right) \tan\phi \quad \text{(Vesic, 1973)}, \\
N_\gamma &= 2 \left(K_p \cdot e^{\pi \tan\phi} - 1 \right) \tan\phi \quad \text{(Chen, 1975)}.
\end{aligned} \tag{4}$$

The equation from Brinch Hansen is, as he writes, "*...based on calculations first from Lundgren and Mortensen (1953) and later from Odgaard and N. H. Christensen*". The equation of Vesic is almost identical to the solution of Caquot and Kérisel (1953) because it is, as he writes, based on "*...the numerical results of an analysis made by them*".

2.2. ***Results***

In order to check the correctness of the currently used bearing capacity factors for non-dilatant soil, both displacement and stress controlled finite element model calculations have been made with Plaxis 2D, with a Mohr-Coulomb soil model and a fine wide mesh with 15-node elements.

Figure 3 shows these results, together with the analytical solutions of the bearing capacity coefficients of Prandtl and Reissner.

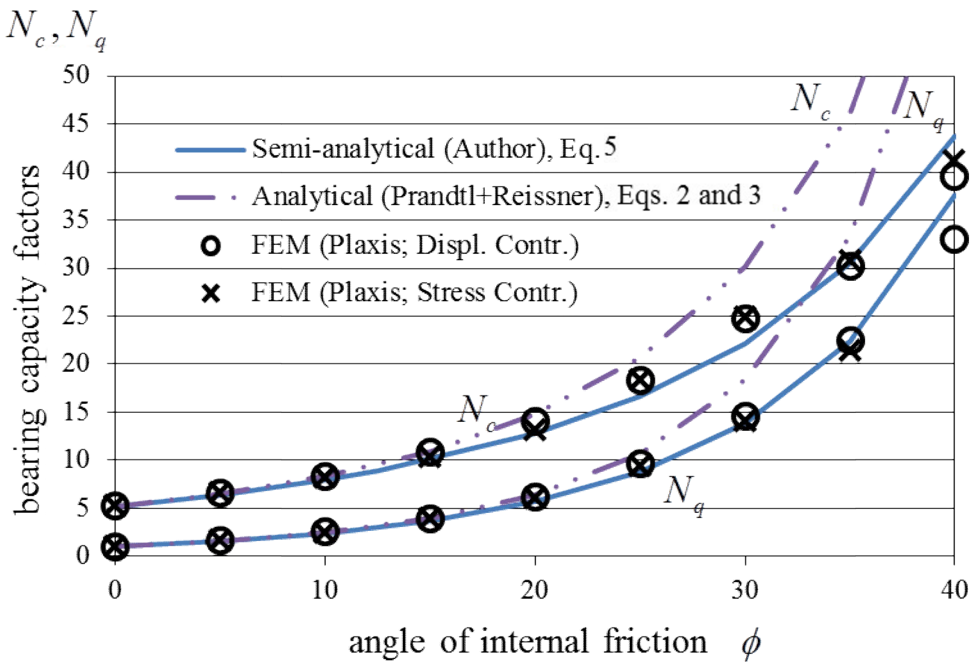

Fig. 3. Bearing capacity factors: Analytical versus FEM.

2.3. ***Discussion***

Figure 3 shows a clear difference in bearing capacity between the numerical and analytical solutions, especially for higher friction angles. The reason for this is that the solutions of Equations 2, 3 and 4 are all based on extreme dilatant (associated) soil, for which the dilatancy angle $\psi = \phi$. Loukidis et al. (2008) already noticed, from their numerical calculations, that non-dilatant (i.e. non-associated) soil is 15% - 30% weaker (lower bearing capacity factors) than extreme dilatant soil, and it has also a rougher (less steady) failure pattern.

The difference between the (extreme dilatant or associated) analytical solutions and the (non-dilatant or non-associated) numerical results has been explained by Knudsen and Mortensen (2016): The higher the friction angle, the wider the logarithmic spiral (larger pitch) of the Prandtl-wedge and the more the stresses reduce during failure. Therefore, the analytical formulas are only kinematically admissible for associated flow. The problem of associated soil is that such a high dilatancy angle is by far unrealistic for natural soils. This means as well that calculating the bearing capacity factor based on the analytical solutions is for higher friction angles also unrealistic, so it is better to use numerically derived bearing capacity factors for non-dilatant soil, such as for example presented by Van Baars (2015, 2016):

$$
\begin{aligned}
&N_q = \cos^2\phi \cdot K_p \cdot \mathrm{e}^{\pi\tan\phi} \\
&N_c = \left(N_q - 1\right)\cot\phi \\
&N_\gamma = 2\left(K_p \cdot e^{\pi\tan\phi} + 1\right)\tan\phi \quad \text{(Rough plate: Vesic)} \\
&N_\gamma = 4\tan\phi \cdot \left(e^{\pi\tan\phi} - 1\right) \quad \text{(Smooth plate)}
\end{aligned}
\tag{5}
$$

3. The influence of superposition on the bearing capacity factors

3.1. *Theory*

Terzaghi (1943) assumed that the three bearing capacity components have the same failure mechanism, so that superposition of the three bearing capacity components is allowed $\left(p = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$.

3.2. *Resuls*

In order to check the influence of this superposition, displacement controlled calculations have been made with the finite element model Plaxis 2D. A large mesh has been used with 15-node elements. The soil has been modelled with a Mohr-Coulomb soil model and. The plate below the load is infinite stiff and rough. First three calculations were made for the three bearing components independently (cohesion, surcharge and soil-weight) (Figure 4).

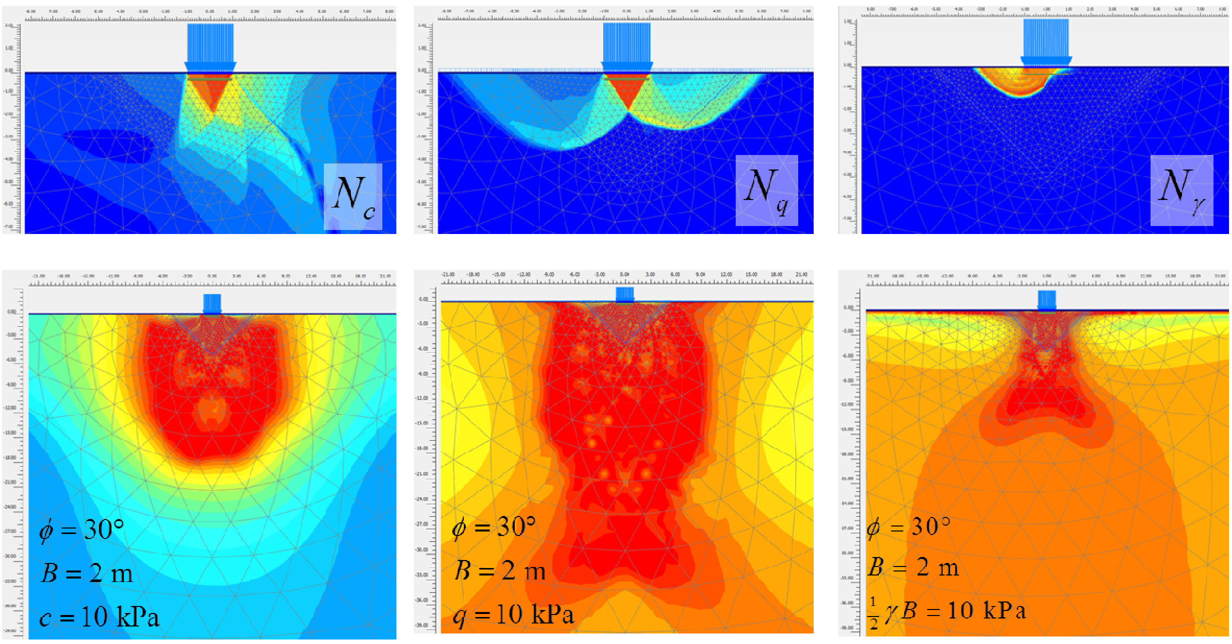

Fig. 4. Failure of footing for the independent components.

Above: Incremental displacements at failure Below: Relative shear stress at failure.

The friction angle (ϕ) is taken 30° for all cases, and the width of the strip load (B) is 2 m. The bearing capacities of the three independent components were found to be: $N_c = 24.5; N_q = 14.6; N_\gamma = 12.6$ (Figure 3).

The upper part of Figure 4 shows the incremental displacements at failure, indicating the different failure mechanisms. The lower pictures show the relative shear stress at failure (size of the Mohr-circle divided by a maximum Mohr-circle touching the Coulomb failure envelope), indicating the different plastic zones.

In order to indicate the error due to the used superposition, another calculation has been made for a footing in which the three bearing components are combined with an equal weight, by using a cohesion, surcharge and soil weight of $c = q = \frac{1}{2}\gamma B' = 10$ kPa (Figure 5).

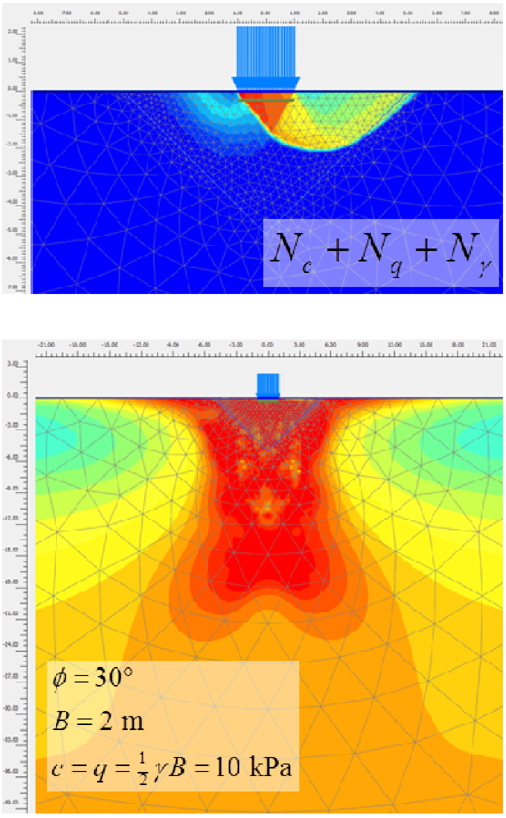

Fig. 5. Failure of footing with combined components.

Above: Incremental displacements at failure Below: Relative shear stress at failure.

3.3. *Discussion*

The different failure patterns in Figure 4 proof that the use of superposition is not correct, because the failure mechanisms belonging to the soil-weight (N_γ) the cohesion (N_c) and the surcharge (N_q), are all three different.

The last calculation, in which the three bearing components are combined, resulted in a combined bearing capacity of $N_c + N_q + N_\gamma = 61.0$, showing a (61.0 - 24.5 - 14.6 - 12.6) / 61.0 = 15.2% underestimation with the superposition rule, which is luckily not too large and leads also to a prediction on the safe side. This error is caused by the fact that the failure mechanism for the combined loading is different from the optimized failure mechanisms for the three individual bearing capacity components.

4. The influence of eccentric loading on the bearing capacity factors

4.1. *Theory*

Until now, nobody proposed correction factors for eccentric loading. The common practice is to use the equation of Terzaghi ($p = cN_c + qN_q + \frac{1}{2}\gamma B' N_\gamma$) and to reduce the contact area of the foundation such that its centroid coincides with that of the load, which means that, the area of the foundation outside the effective area, *B'*, is completely neglected (Figure 6).

The question is, if this reduction of the foundation area is an accurate method to describe the reduction of the bearing capacity due to vertical eccentric loading, or not. Several people have performed finite element calculations of both vertical and inclined eccentric loaded strip footings, but the results are not complete. For example Hjiaj et al. (2004) based their results only on the limit analysis and Knudsen and Mortensen (2016) and independently Khitas et al. (2016) only found results for frictionless soils $(\phi = 0)$. They found that the error of the simplified method, with the reduction of the foundation area, is limited to 5%, in case the eccentricity is limited to: $e/B < 0.30$.

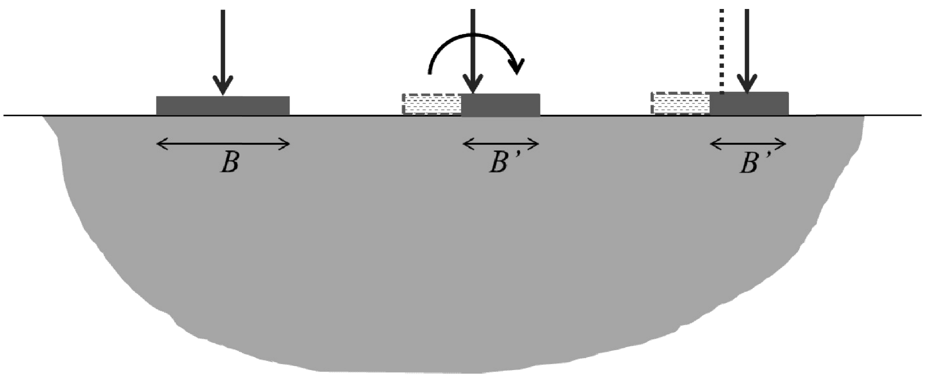

Fig. 6. Effective or reduced foundation, due to eccentric loading.

4.2. *Results*

In order to check this conclusion, the centric example of Paragraph 3 (Case I in Figure 7) has been extended. The plate below the loading has been extended with 0.5 m (Case II) and 1.0 m (Case III), in order to create eccentric loading, but without changing the effective area, *B'*. Also in this case, the calculations have been made for the three bearing components independently, but also for a loaded footing in which the three bearing components are combined such that the three bearing components; cohesion, surcharge and soil weight, are: $c = q = \frac{1}{2}\gamma B' = 10$ kPa .

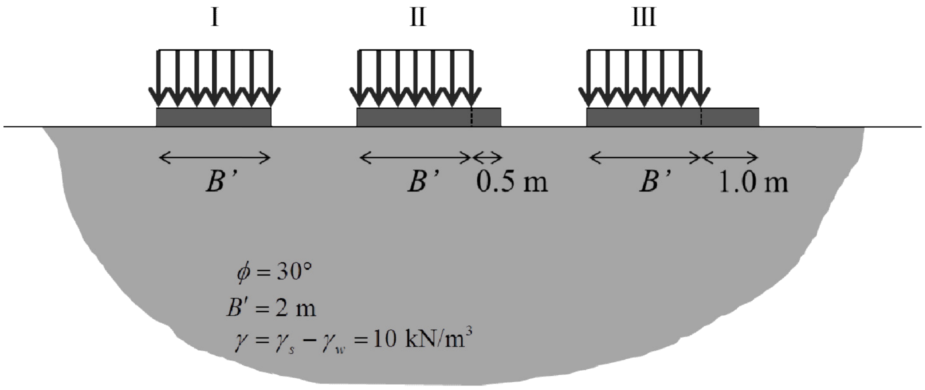

Fig. 7. Eccentric loading: extension of the footing.

Figure 8 shows the numerically calculated bearing capacities, for the individual bearing components, for the three load cases, both centric and eccentric loading.

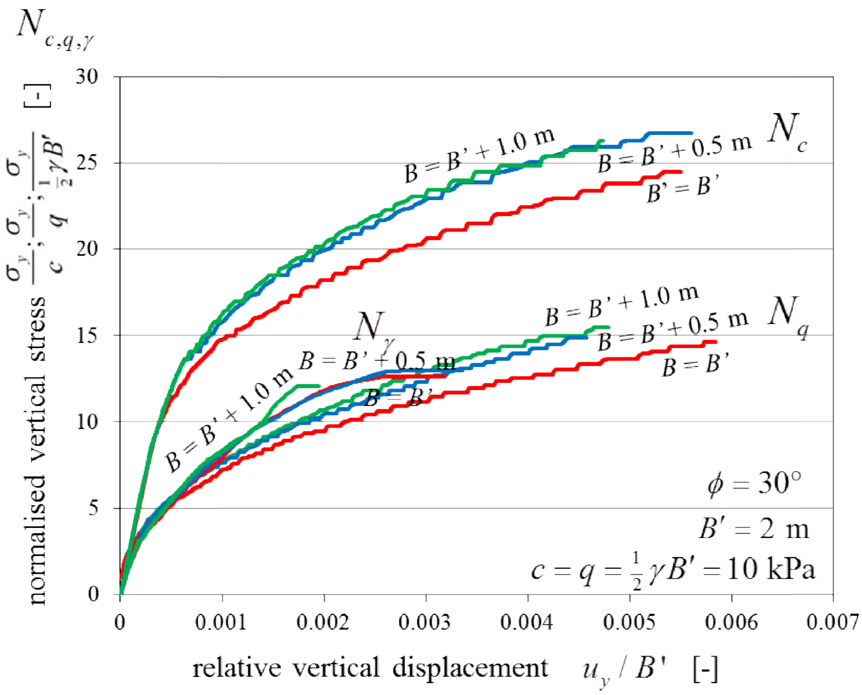

Fig. 8. Individual components: Normalized stress versus relative displacement.

Figure 9 shows the numerically calculated bearing capacity, for the combined bearing components, for the three load cases, both centric and eccentric loading. Figure 10 shows the failure mechanism and the plastic zone for these three load cases (note the different scales and sizes of the plate).

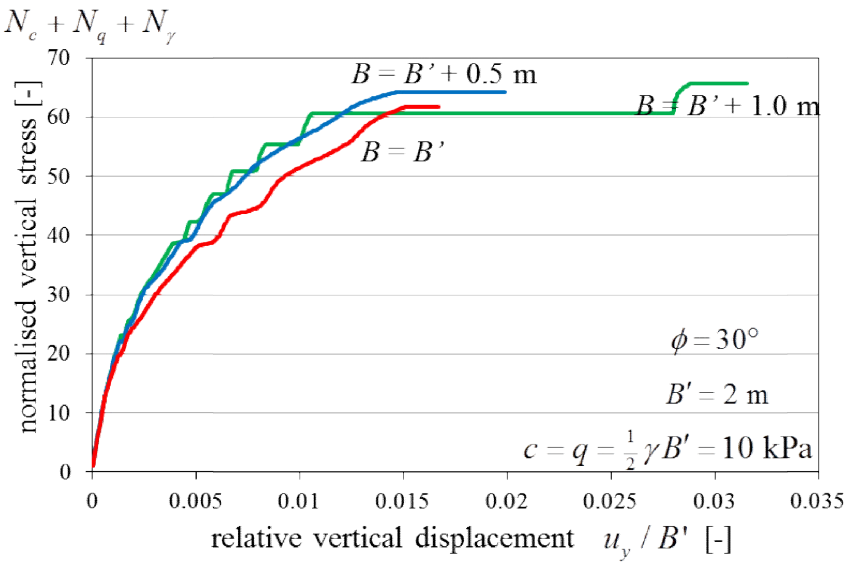

Fig. 9. Combined components: Normalized stress versus relative displacement.

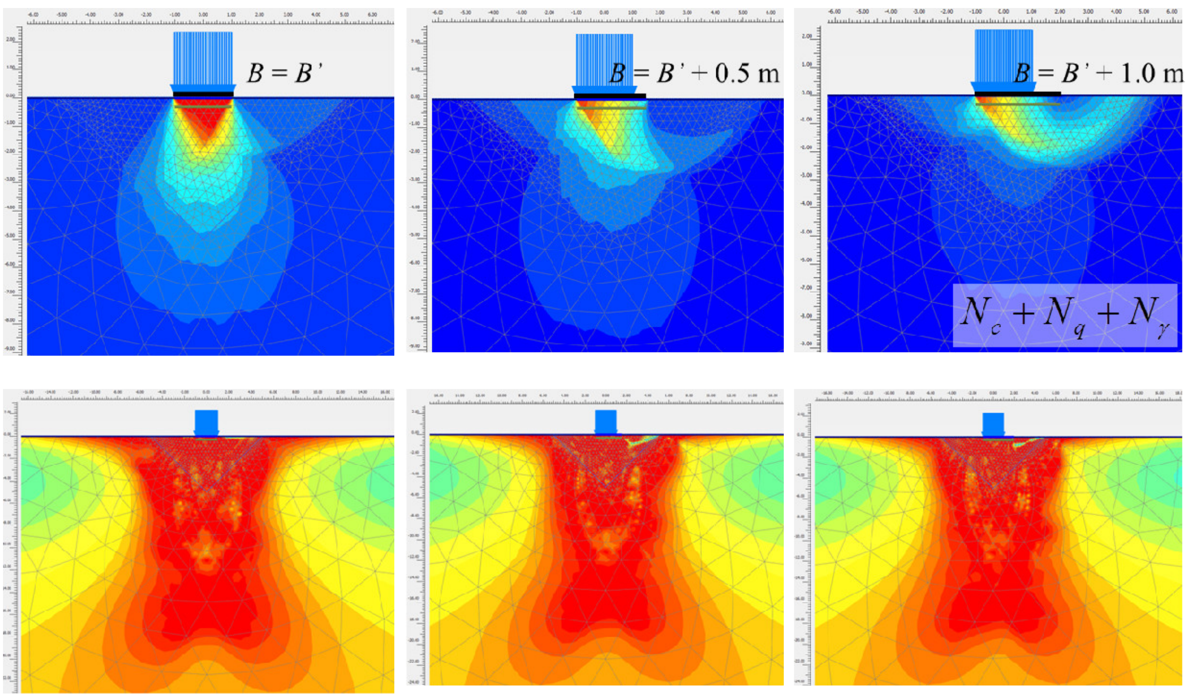

Fig. 10. Influence of eccentric loaded footing with combined components.

Above: Total displacements at failure. Below: Relative shear stress at failure.

4.3. *Discussion*

The numerically calculated bearing capacities, for the individual bearing components for eccentric loading, are found to be rather similar as for the centric loading (Figure 8). Also for the case with the combined components, the bearing capacities for eccentric loading are almost the same as for centric loading (Figure 9). This means that the current method, which reduces the contact area of the foundation for dealing with eccentric loads, is an accurate method. The reason for this is probably that the plastic zone is much larger than the failure mechanism (Figure 10), which means that a change in the failure mechanism (the additional rotation due to the eccentric loading as result of the extension of the plate), does not have to lead to another plastic zone. An extension of the plate has therefore only a small consequence on the bearing capacity (maximum normalized vertical stress).

5. Conclusions

The currently used equations for the bearing capacity factors are, especially for higher friction angles, too high. Therefore more accurate equations have been given for non-dilatant soil.

Terzaghi wrote the equation for the bearing capacity of a shallow foundation as a superposition of three separate bearing capacity components, for the cohesion, surcharge and soil-weight. This is not correct because the three independent components have each their own failure mechanism. The Finite Element calculations with Plaxis 2D presented in this paper show that the used superposition results in an error, which is luckily not too large and leads also to predictions on the safe side.

Other Finite Element calculations show that the reduction of the foundation area is an accurate method to describe the reduction of the bearing capacity due to eccentric loading. The reason for this is probably that the plastic zone is much larger than the failure mechanism, which means that an extension on one side of the foundation plate, without any loading on top, is causing a rotation of the plate, but not an increase in bearing capacity.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

Brinch Hansen, J. A. (1970). Revised and extended formula for bearing capacity, Bulletin No 28, Danish Geotechnical Institute Copenhagen, 5-11.

Caquot, A., & Kerisel, J. (1953). Sur le terme de surface dans le calcul des fondations en milieu pulverulent. Proceedings of the third International Conference on Soil Mechanics and Foundation Engineering, Zurich, Switzerland.

Chen, W. F. (1975). Limit analysis and soil plasticity, Elsevier, Amsterdam, Netherlands.

Hjiaj, M., Lyamin, A. V., & Sloan, S. W. (2004). Bearing capacity of a cohesive-frictional soil under non-eccentric inclined loading. Computers and Geotechnics, 31, 491–516.

Jumikis, A. R. (1956). Rupture surfaces in sand under oblique loads, Journal of Soil Mechanics and Foundation Design, 82(3), 1155-1159

Keverling Buisman, A. S. (1940). Grondmechanica, Waltman, Delft, the Netherlands, 243

Khitas, N. E. H., Mellas, M., Benmeddour, D. & Mabrouki, A. (2016). Bearing capacity of strip footings resting on purely cohesive soil subjected to eccentric and inclined loading, 4th International Conference on New Development in Soil Mech. and Geotechn. Eng., Nicosia, Cyprus.

Knudsen, B. S., & Mortensen, N. (2016). Bearing capacity comparison of results from FEM and DS/EN 1997-1 DK NA 2013. Northern Geotechnical Meeting. Reykjavik(Island).

Loukidis, D., Chakraborty, T., & Salgado, R. (2008). Bearing capacity of strip footings on purely frictional soil under eccentric and inclined loads, Can. Geotech. J. 45, 768–787

Lundgren, H., & Mortensen, K. (1953). Determination by the theory of Plasticity of the bearing capacity of Continuous Footings on Sand, Proceedings of the third International Conference on Conf. Soil Mech, Zürich, Switzerland.

Meyerhof, G. G. (1951). The ultimate bearing capacity of foundations, Géotechnique, 2, 301-332

Meyerhof, G. G. (1953). The bearing capacity of foundations under eccentric and inclined loads. Proceedings of the third International Conference on Soil Mechanics Found. Eng, Zürich, Switzerland.

Meyerhof, G. G. (1963). Some recent research on the bearing capacity of foundations, Canadian Geotechnics Journal. 1(1), 16-26.

Meyerhof, G. G. (1965). Shallow foundations, Journal of the Soil Mechanics and Foundations Division ASCE, 91(2), 21-32.

Muhs, H., & Weiß, K. (1972). Versuche über die Standsicheheit flach gegründeter Einzelfundamente in nichtbindigem Boden, Mitteilungen der Deutschen Forschungsgesellschaft für Bodenmechanik (Degebo) an der Technischen Universität Berlin, Heft 28, 122

Prandtl, L. (1920). Über die Härte plastischer Körper. Nachr. Ges. Wiss. Goettingen, Math.-Phys. Kl., 74–85.

Reissner, H. (1924). Zum Erddruckproblem. Proceedings of the 1st Intest international Congress for Applied Mechanics, Delft, Netherlands.

Selig, E. T. and McKee, K. E. (1961). Static and Dynamic behavior of small footings, Journal of Soil Mechanics and Foundation Design, ASCE, 87(6), 29-50.

Terzaghi, K. (1943). Theoretical soil mechanics, John Wiley and Sons, New York, USA.

Van Baars, S. (2015). The Bearing Capacity of Footings on Cohesionless Soils, The Electronic Journal of Geotechnical Engineering, 20, 12945-12955.

Van Baars, S. (2016). Failure mechanisms and corresponding shape factors of shallow foundations, Proceedings of the 4th International Conference on New Development in Soil Mech. and Geotechn. Eng, Nicosia, Cypres.

Vesic, A. S. (1973). Analysis of ultimate loads of shallow foundations. Journal of Soil Mech. Found. Div, 99(1), 45–76.

Vesic, A. S. (1975). Bearing capacity of shallow foundations, H.F. Winterkorn, H.Y. Fang (Eds.), Foundation Engineering Handbook, Van Nostrand Reinhold, New York, USA, 121–147