UNCERTAINTY QUANTIFICATION FOR SOFT TISSUE BIOMECHANICS

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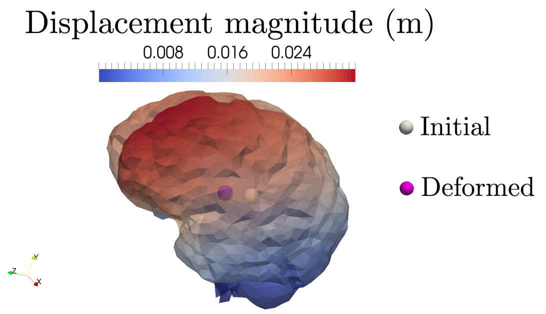
GENERAL AIM OF THE WORK

- Assessing the effects of uncertainty in material parameters in soft tissue models.
- The sensitivity derivative Monte Carlo method provides one to two orders of magnitude better convergence than the standard Monte Carlo method (Fig. 3 and 5).
- Complex models with only few lines of Python code (DOLFIN/FEniCS).

SUMMARY

- Stochastic FE analysis.
- Uncertainty quantification (material properties, loading, geometry, etc.).
- Random variables/fields.
- Global and local sensitivity analysis.
- Biomechanical modeling, simulation and analysis with random parameters.

FIGURE 1

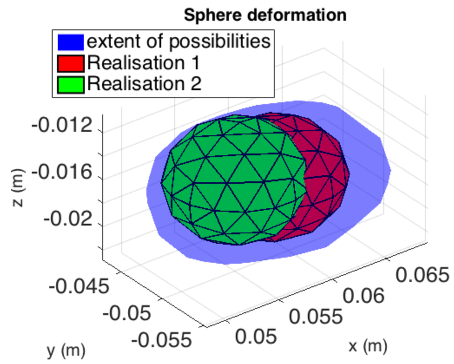


One Monte Carlo realisation (brain deformation).

METHODS

- Monte Carlo and quasi Monte Carlo methods (Caflisch, 1998).
- Accelerating Monte Carlo estimation with sensitivity derivatives (Hauseux, Hale, and Bordas, 2016).
- Non-intrusive multi-level polynomial chaos expansion method.
- Multi Level Monte Carlo methods (Giles, 2015).

FIGURE 2



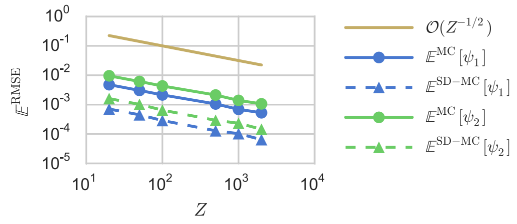
Confidence interval 95%.

NUMERICAL IMPLEMENTATION

- DOLFIN/FEniCS:
  - UFL (Unified Form Language) (Logg, Mardal, and Wells, 2012).
  - Automatically deriving tangent linear models with FEniCS !
- Parallel computing (Ipyparallel and mpi4py).
- Python package for uncertainty quantification (Chaospy, SALib).

GENERALISED BURGERS EQUATION WITH STOCHASTIC VISCOSITY

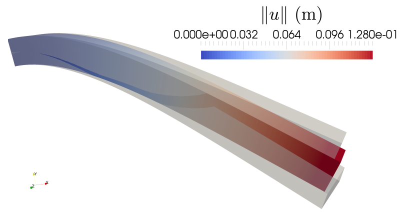
FIGURE 3



Log-log plot (Fig. 3) of relative root-mean-square error (RMSE) for standard Monte Carlo E^{MC} and sensitivity-derivative enhanced Monte Carlo methods $E^{\mathrm{SD-MC}}$.

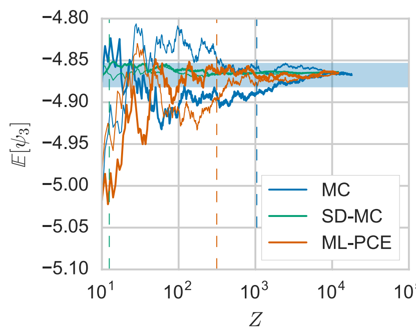
HYPERELASTICITY EQUATION WITH STOCHASTIC MATERIAL PARAMETERS

FIGURE 4



The mean deformed domain is shown coloured with the pointwise magnitude of the mean displacement field (+ and - the standard deviation).

FIGURE 5



Two evolutions of the MC, SD-MC and ML-PCE estimations of ψ_3: the relative displacement of the beam in the y direction as a function of the number of realisations Z.

REFERENCES

Caflisch, C. A. (1998). "Monte carlo and quasi -monte carlo methods". In: *Acta numerica* 7 (53), pp. 1–49. DOI: 10.1017/S0962492900002804.

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