High Frequency Acoustic Scattering in Isogeometric Analysis

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Abstract

There is an emerging need to perform high frequency scattering analysis on high-fidelity models. Conventional Finite Element analysis suffers from irretrievable loss of the boundary accuracy as well as pollution error. Man-made geometries can be represented exactly in Isogeometric Analysis (IGA) with no geometrical loss even with very coarse mesh. The aim of this paper is to analyze the accuracy of IGA for exterior acoustic scattering problems. The numerical results show extremely low pollution error even for very high frequencies.

Keywords: Acoustic scattering, high-frequency, Isogeometric analysis, pollution error

1 Introduction

The pollution error is a limiting factor when analyzing high-frequency scattering problems with a conventional Finite Element Method. In order to achieve a prescribed upper bound for the error, it is necessary to increase the mesh density $n_\lambda = \lambda/h$ faster than the wavenumber k where λ is the wavelength ($\lambda = 2\pi/k$) and h is the element size. Another limitation in using conventional FEM is the loss of geometrical representation of the domain boundaries of the discretized model. The domain discretization in IGA however has no affect on the geometrical accuracy. In fact, the discretization of the IGA model is a result of its parametric definition. As a result the boundaries of the domain are presented exactly in IGA with no artificial facets. In addition, refinement in IGA can be performed without changing the geometry of the domain. To solve an unbounded exterior problem in the context of finite elements it is necessary to truncate the domain artificially. Constructing Absorbing Boundary Conditions (ABCs) and Perfectly Matched Layer (PML) are among the common methods of domain truncation. The truncation of the domain mimics the infinite space in a finite domain at the cost of introducing the truncation error to the numerical solution. In order to study the performance of any numerical scheme for exterior scattering problems it is necessary to separate the pollution error from the truncation error.

2 Problem formulation

Let us consider Ω^- as a two-dimensional circular cylinder $R_0 = 1$ centered at the origin with boundary $\Gamma := \partial\Omega^-$. The associated exterior (i.e. unbounded) domain of propagation is $\Omega^+ := \mathbb{R}^2/\overline{\Omega^-}$. Solving the scattering problem leads to computing the wave field u as the solution to the following Boundary-Value Problem (BVP): given an incident plane wave field u^{inc}, find u such that

$$\begin{aligned} &\Delta u + k^2 u = 0, \quad \text{in} \quad \Omega^+, \\ &\partial_{\mathbf{n}_\Gamma} u = g := -\partial_{\mathbf{n}_\Gamma} u^{\text{inc}}, \quad \text{on} \quad \Gamma, \\ &\lim_{|\mathbf{x}|\to+\infty} |\mathbf{x}|^{(d-1)} \Big(\nabla u \cdot \frac{\mathbf{x}}{|\mathbf{x}|} - iku \Big) = 0, \end{aligned} \tag{1}$$

where Δ is the Laplacian operator, ∇ the gradient operator and $\mathbf{n}_\Gamma$ is the outward-directed unit normal vector to Ω^-. The spatial variable is $\mathbf{x} = (x, y)$. We consider the sound-hard case and apply the Neumann boundary conditions on Γ at $R = 1$ as stated in the second equation of system (1). The Sommerfeld's radiation condition at infinity is applied in the last equation of system (1) which presents the outgoing wave to the domain. We consider an incident plane wave $u^{\text{inc}}(\mathbf{x}) = \mathrm{e}^{ik\mathbf{d}\cdot\mathbf{x}}$, with incidence direction $\mathbf{d} = (1, 0)^T$. To truncate the computational domain, the second-order Bayliss-Turkel ABC is applied on the circle with radius $R_1 = 2$ and is given by [1]:

$$\partial_{\mathbf{n}_\Sigma} u + (-\mathrm{i}k + \frac{\kappa}{2} - \frac{\kappa^2}{8(\kappa - \mathrm{i}k)})u - \frac{1}{2(\kappa - ik)}\partial_s^2 u = 0,$$

where $\mathbf{n}_\Sigma$ is the outward directed unit normal to Σ, $\partial_{\mathbf{n}_\Sigma} := \partial_r$ is the normal derivative, $\kappa = 1/R_1$ is the curvature and $\partial_s^2 := R_1^{-2}\partial_\phi^2$ is the second-order curvilinear derivative on the fictitious boundary at $R = 2$. The domain truncation error was included in the exact solution making it suitable to analyze the pollution and approximation errors and avoid domain truncation error. [2].

3 Numerical results

We generated the crown mesh between $R = 1$ and $R = 2$ by four identical patches in IGA. A Matlab® code was prepared to obtain the numerical results. The real part of the numerical solution is shown in Fig.1 where $k = 100$, degree $p = 4$ in IGA and $n_\lambda = 5$ points per wavelength. The error $|u_h - u^{\mathrm{ex}}|$ is shown in Fig. 2. To examine the performance of IGA for higher k, the evolution of the L_2-norm error vs. k is given in Fig. 3. No pollution error is visible even for k up to 200 for $p = 4$ and higher in IGA.

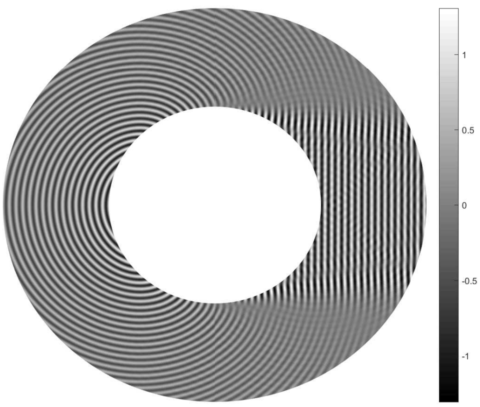

Figure 1: Real part of the numerical IGA solution u_h, $k = 100$, $p = 4$, $n_\lambda = 5$.

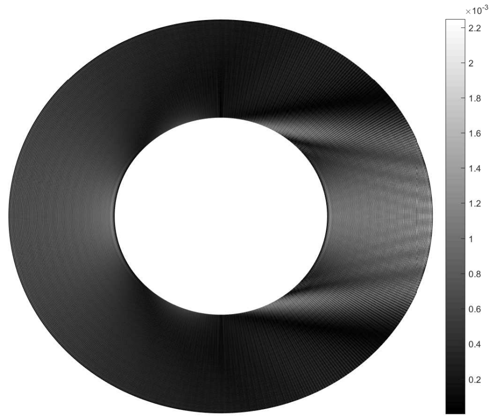

Figure 2: Absolute error $|u_h - u^{\mathrm{ex}}|$.

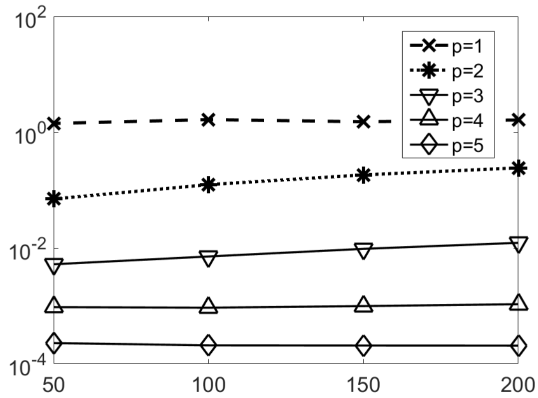

Figure 3: L_2 error vs. k ($p = 1 \cdots 5$).

4 Conclusion

We studied the performance of IGA in solving high frequency scattering problems. By considering the truncation error in the exact solution, we separated the pollution error from the truncation error. Numerical results show no noticeable pollution error even for high k for basis functions of order $p = 3$ and higher. The possibility of exactly presenting domain boundaries in IGA even with very coarse meshes and its convenient refinement makes it an attractive platform for scattering problems.

References

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