Strongly barycentrically associative and preassociative functions

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(joint work with Jean-Luc Marichal)

Let X be a nonempty set and X^* be the free monoid generated by X (that is, X^* is the set of finite words over X equipped with the concatenation operation). Recall that a function $F\colon X^* \to X \cup \{\varepsilon\}$ is barycentrically associative if the function value of a string does not change when replacing every letter of a substring of consecutive letters with the value of this substring. This property is involved in the famous Kolmogoroff - Nagumo's characterization of quasi-arithmetic means.

In this talk, we investigate the stronger property of *strong barycentric associativity* which stipulates that the function value of a string does not change when replacing every letter of *any* substring with the value of this substring. Equivalently, a function $F\colon X^* \to X \cup \{\varepsilon\}$ is strongly barycentrically associative if and only if it satisfies the equation

$$F(\mathbf{xyz}) \;=\; F(F(\mathbf{xz})^{|\mathbf{x}|}\mathbf{y}F(\mathbf{xz})^{|\mathbf{z}|}), \qquad \mathbf{xyz} \in X^*.$$

We also investigate a weaker version of strong barycentric associativity called *strong barycentric preassociativity* which does not involve composition of functions in its definition. We establish links between strong barycentric associativity and strong barycentric preassociativity. We recall a variant of Kolmogoroff-Nagumo's characterization of the class of quasi-arithmetic means based on the strong barycentric associativity property, and we provide a variant of this characterization that involves strongly barycentrically preassociative functions.