Simultaneous Analysis of Strongly-Coupled Composite Energy Harvester-Circuit Systems Driven by Fluid-Structure Interaction

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Talk | outline

- Harvesting electrical energy from vibrations
- Flow-induced vibrations
- Example: flutter of a cantilevered plate

- Mathematical modelling of the coupled multi-field problem
- Governing equations
- Weighted residuals formulation in the space-time domain
- Discretisation using space-time finite elements
- Eigen value problem and forced vibration - frequency domain

- Numerical example: vortex-induced vibration of a plate
- Numerical example: power output of a cantilevered plate

Energy harvesting | aim & methods

scavenging ambient energy from the environment, that otherwise would be lost

drive (low-power) electrical devices/store electrical energy in remote locations

ambient energy sources

- mechanical vibrations,
- solar radiation,
- electromagnetic sources,
- heat and mass flows,
- noise
- …

means of transformation

- piezo-electricity,
- pyro-electricity,
- thermo-electricity,
- electrostatic,
- electro-magnetic induction,
- electro-active polymers,…

Energy harvesting | challenges

- investigation of alternative ambient sources of energy,
- optimisation of power output for varying operational conditions,
- economic utilisation of physical phenomena in functional materials,
- practicability at different length scales,
- optimal control of single and also stacks of energy harvesters

Energy harvesting | our focus

scavenging ambient energy from the environment, that otherwise would be lost

drive (low-power) electrical devices/store electrical energy in remote locations

ambient energy sources

- mechanical vibrations ← fluid flow,
- solar radiation,
- electromagnetic induction,
- heat and mass flows,
- noise
- …

means of transformation

- piezo-electricity,
- pyro-electricity,
- thermo-electricity,
- electrostatic,
- electro-magnetic induction,
- electro-active polymers,…

Flow-induced vibrations | excitations

$C = C(t, Tu)$

external excitation

fluctuating external loadings

e.g. turbulent wind, rough sea (random, periodic)

$C = C(t, St, \omega)$

flow instability

e.g. antennas, cooling towers (periodic vortex shedding)

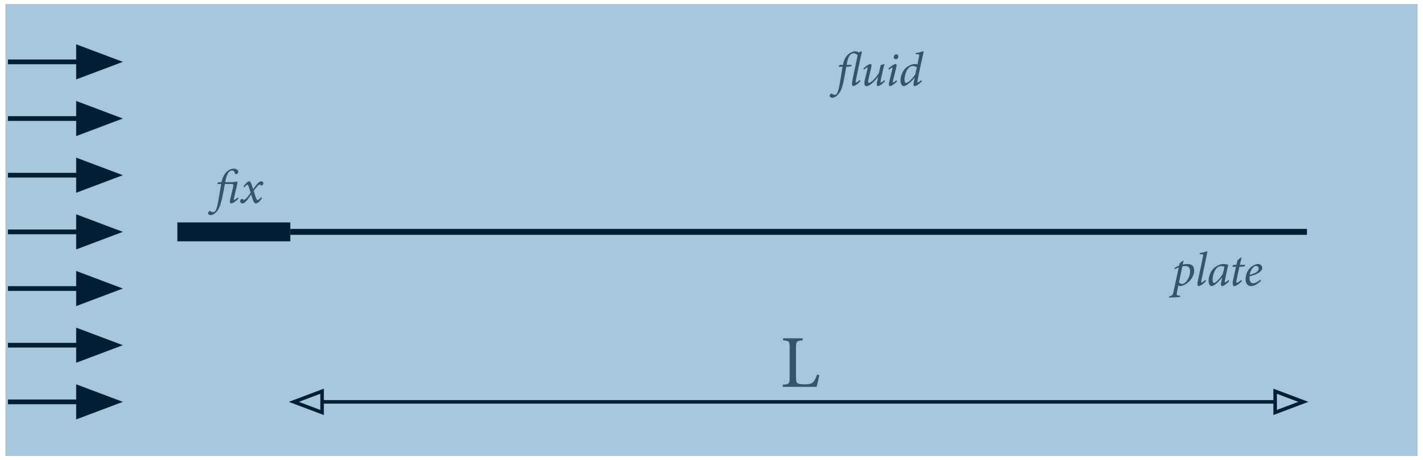
$C = C(x, \dot{x}, \ddot{x})$

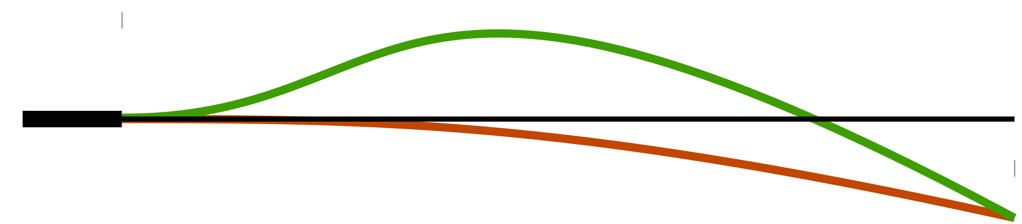
self excitation

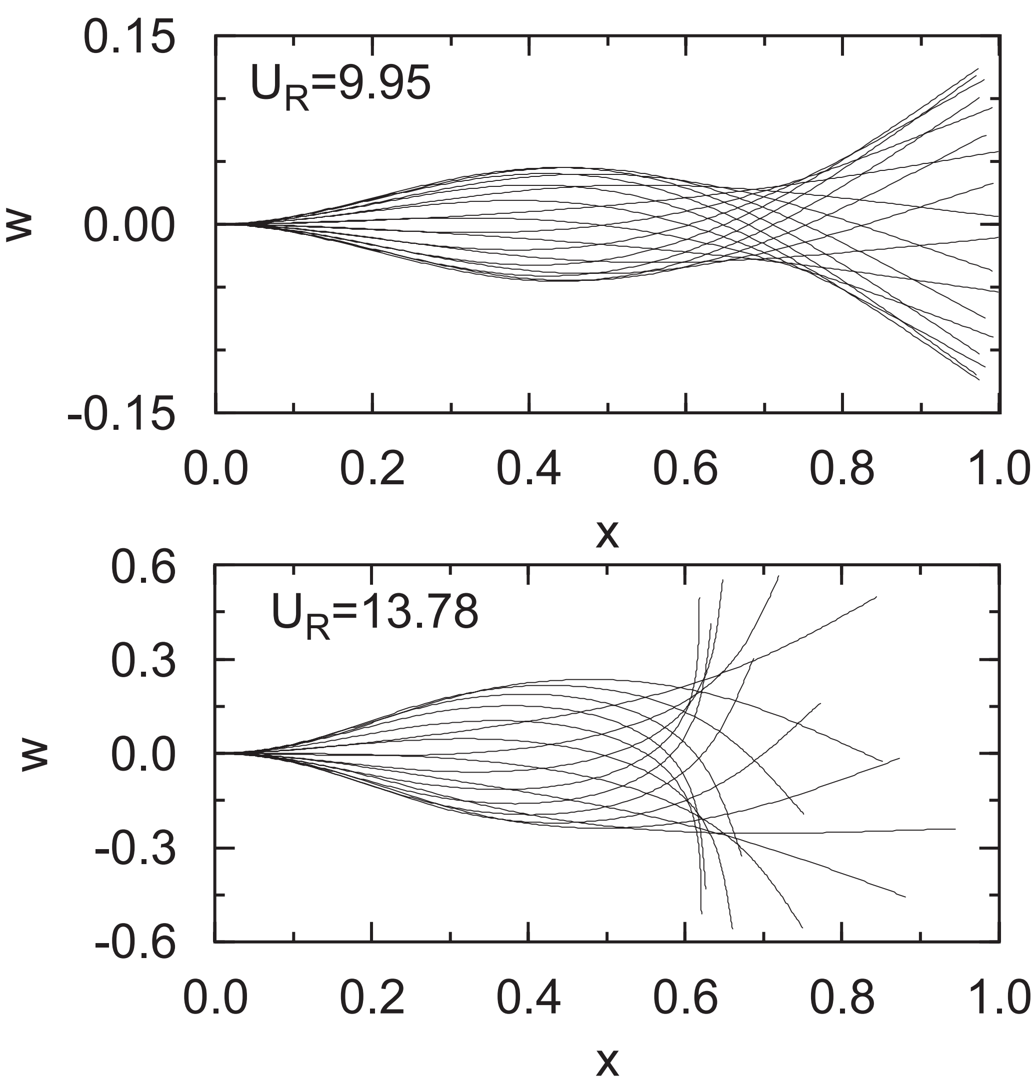
structural motion/deformation

e.g. hanging bridges, membranes (flutter), cables (galloping)

Example | flutter of a cantilevered plate







- semi-analytical models are available
- structural dynamics of linear/nonlinear plate/beam
- fluid dynamics via potential flow/panel methods
- determination of flutter boundary (critical reduced flow velocity U_R)
- flutter instability occurs in the 2nd eigenmode
- VIV occur in 1st eigenmode

[data] Tang, Paidoussis: On the instability and the post-critical behaviour of two-dimensional cantilevered flexible plates in axial flow. J Sound & Vibr. 2007.

Harvest flow energy | flow-induced vibration

- use flow as ambient energy source
- ongoing energy transfer from fluid flow to structural vibrations
- extract energy from controlled flow-induced structural vibrations
- three-way coupling

fluid-structure interaction

vortex-induced vibrations,
movement-induced vibrations

piezo-electric effect

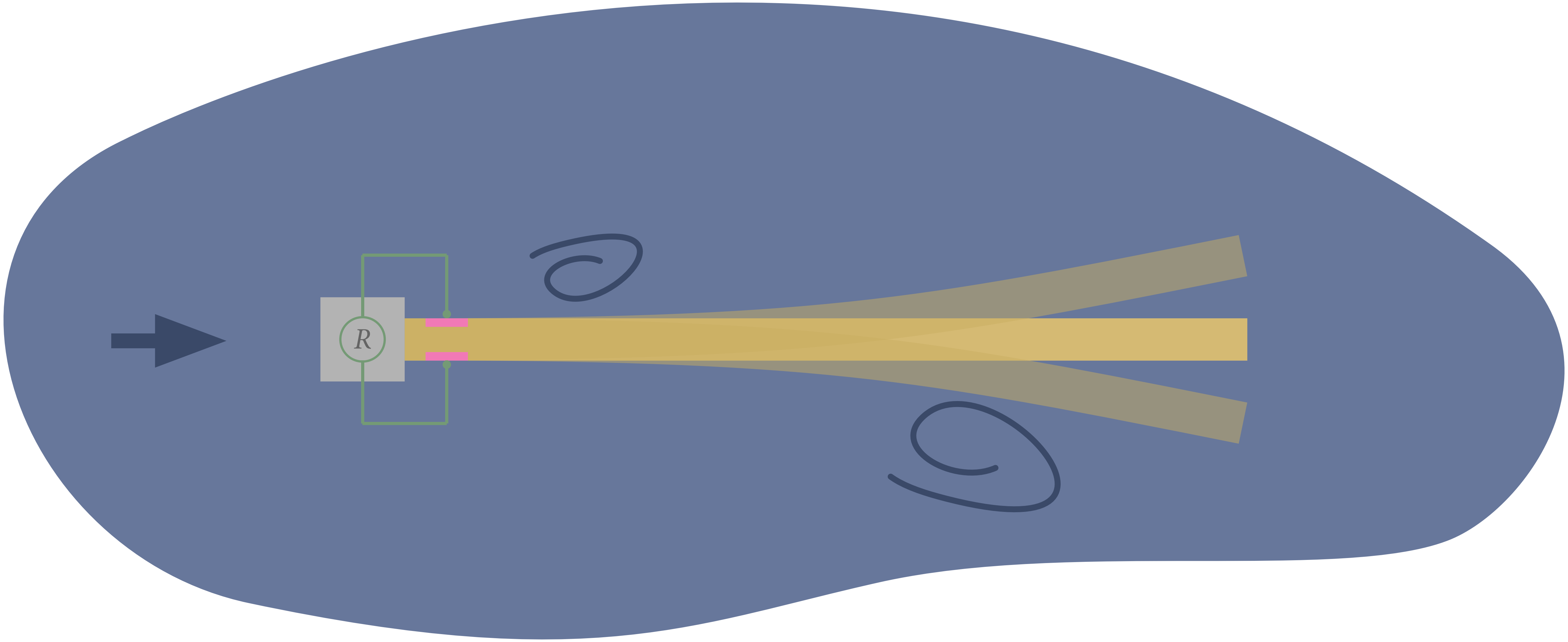
electro-mechanical
coupling

electrical circuit

transformation,
storage

flow energy - strain energy - electric energy

Multi-physics | model



fluid

structure

piezoelectric

circuit

boundary-coupled

volume-coupled

surface/point-coupled

Governing equations | fluid

incompressible Newtonian fluid flow

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) - \nabla\cdot\mathbf{T} - \mathbf{f} = \mathbf{0} \qquad \text{on} \quad Q$$

$$\nabla\cdot\mathbf{v} = 0 \qquad \text{on} \quad Q$$

$$\mathbf{D} - 1/2\,[\nabla\mathbf{v} + (\nabla\mathbf{v})^{\mathsf{T}}] = \mathbf{0} \qquad \text{on} \quad Q$$

$$\mathbf{T} + p\mathbf{I} - 2\mu\mathbf{D} = \mathbf{0} \qquad \text{on} \quad Q$$

$$\mathbf{T}\cdot\mathbf{n} - \mathbf{t} = \mathbf{0} \qquad \text{on} \quad P$$

$$\mathbf{v} - \bar{\mathbf{v}} = \mathbf{0} \qquad \text{on} \quad P^{v}$$

$$\mathbf{t} - \bar{\mathbf{t}} = \mathbf{0} \qquad \text{on} \quad P^{t}$$

- on deformed space-time domain Q

Governing equations | structure & piezo

elastodynamics

$$\rho_0 \dot{\mathbf{v}} - \nabla_0 \cdot (\mathbf{FS}) - \mathbf{f}_0 = \mathbf{0} \quad \text{in} \quad Q_0$$
$$\dot{\mathbf{E}} - 1/2 \left[\nabla_0 \mathbf{v} + (\nabla_0 \mathbf{v})^{\mathsf{T}} + \dots \right] = \mathbf{0} \quad \text{in} \quad Q_0$$
$$\dot{\mathbf{E}} - [s^D] \dot{\mathbf{S}} = \mathbf{0} \quad \text{in} \quad Q_0$$
$$\mathbf{S} \cdot \mathbf{n} - \mathbf{t} = \mathbf{0} \quad \text{on} \quad P_0$$
$$\mathbf{v} - \bar{\mathbf{v}} = \mathbf{0} \quad \text{on} \quad P_0^v$$
$$\mathbf{t} - \bar{\mathbf{t}} = \mathbf{0} \quad \text{on} \quad P_0^t$$

electric field equations

$$\nabla_0 \cdot \tilde{\mathbf{D}}_0 = 0 \quad \text{in} \quad Q_0$$
$$\dot{\tilde{\mathbf{E}}}_0 + \nabla_0 \psi = \mathbf{0} \quad \text{in} \quad Q_0$$
$$\dot{\mathbf{E}} - [s^D] \dot{\mathbf{S}} - [g]^{\mathsf{T}} \dot{\tilde{\mathbf{D}}}_0 = \mathbf{0} \quad \text{in} \quad Q_0$$
$$\dot{\tilde{\mathbf{E}}}_0 + [g] \dot{\mathbf{S}} - [\epsilon^{-1}]^{\mathsf{T}} \dot{\tilde{\mathbf{D}}}_0 = \mathbf{0} \quad \text{in} \quad Q_0$$
$$\tilde{\mathbf{D}}_0 \cdot \mathbf{n} + q = 0 \quad \text{on} \quad P_0$$
$$\psi - \bar{\psi} = 0 \quad \text{on} \quad P_0^{\psi}$$
$$q - \bar{q} = 0 \quad \text{on} \quad P_0^{q}$$

- on reference space-time domain Q_0

Governing equations | electrical circuit

electrode

$$\dot{Q} - \int_{\Gamma^e} \dot{q} \, d\Gamma = 0 \qquad \text{in} \quad I$$

$$\dot{\Phi} - \psi = 0 \qquad \text{in} \quad I$$

- coupling interface between piezoelectric structure and electrical circuit
- equipotential condition

electrical circuit

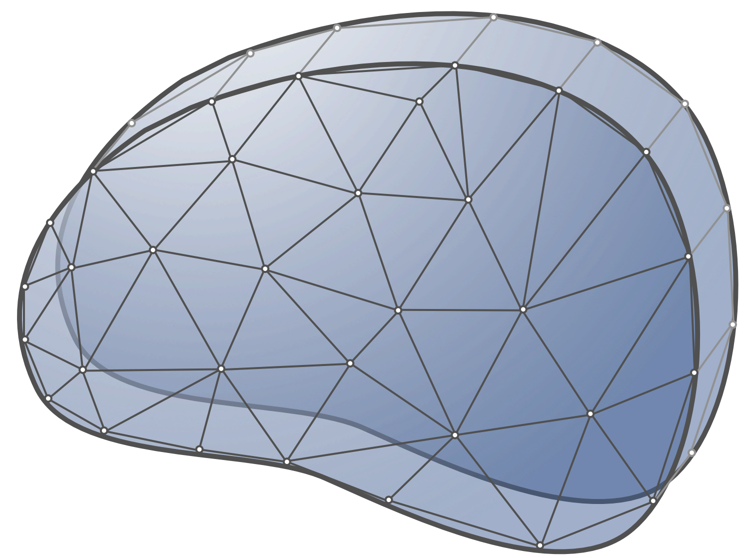
$$I - \dot{Q} = 0 \qquad \text{in} \quad I$$

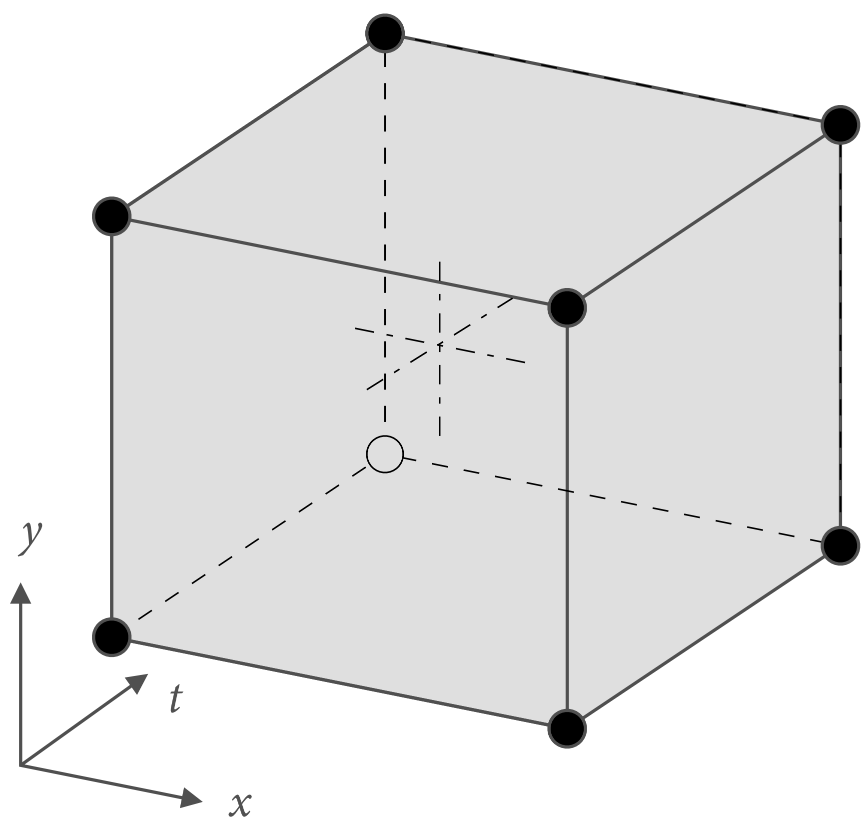
$$\Delta\Phi - R \cdot I = 0 \qquad \text{in} \quad I$$

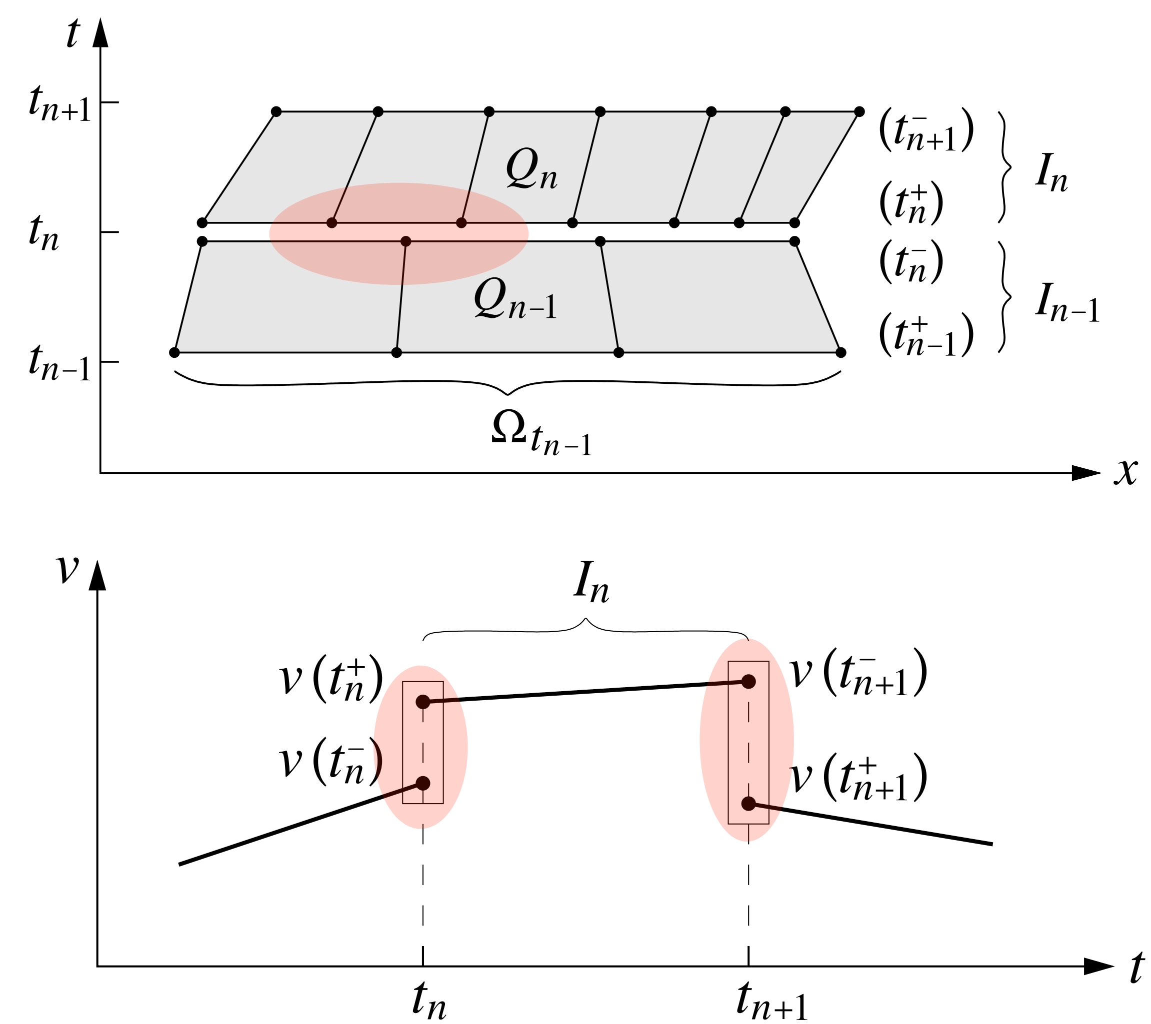
- Ohm’s law
- extendable electrical circuit equation
- here: resistor element

Strongly coupled problem | discretisation

- method of weighted residuals
- velocity-based formulation of structure/fluid
- spatial deformation of the flow domain
- finite elements in space and time
- discontinuous Galerkin method in time







Space-time weak form | fluid

incompressible Newtonian fluid flow

$$
\begin{aligned}
&\int_{Q_n} \delta\mathbf{v} \cdot \rho(\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla\mathbf{v})\, \mathrm{d}Q + 2\mu \int_{Q_n} \mathbf{D}(\delta\mathbf{v}) : \mathbf{D}(\mathbf{v})\, \mathrm{d}Q \\
&- \int_{Q_n} (\nabla \cdot \delta\mathbf{v})\, p\, \mathrm{d}Q - \int_{Q_n} \delta\mathbf{v} \cdot \mathbf{f}\, \mathrm{d}Q \\
&+ \int_{Q_n} \delta p (\nabla \cdot \mathbf{v})\, \mathrm{d}Q \\
&+ \int_{\Omega_n} \delta\mathbf{v}(t_n^+) \cdot \rho\big(\mathbf{v}(t_n^+) - \mathbf{v}(t_n^-)\big)\, \mathrm{d}\Omega \\
&+ [\text{stabilisation terms}] \\
&- \int_{P_n^h} \delta\mathbf{v} \cdot \bar{\mathbf{t}}\, \mathrm{d}P = 0 \qquad \forall\, \delta\mathbf{v},\ \delta p
\end{aligned}
$$

- unknowns: velocity, pressure
- Galerkin LS stabilisation

Space-time weak form | structure

nonlinear elastodynamics

$$
\begin{aligned}
&\int_{Q_{0,n}} \delta\mathbf{v} \cdot \rho_0 \dot{\mathbf{v}} \, \mathrm{d}Q_0 + \int_{Q_{0,n}} \dot{\mathbf{E}}(\delta\mathbf{v}, \mathbf{u}) : \mathbf{S} \, \mathrm{d}Q_0 - \int_{Q_{0,n}} \delta\mathbf{v} \cdot \mathbf{f}_0 \, \mathrm{d}Q_0 \\
&+ \int_{Q^e_{0,n}} \delta\mathbf{S} : \Big([s^D]\dot{\mathbf{S}} - \dot{\mathbf{E}}(\mathbf{v}, \mathbf{u}) \Big) \, \mathrm{d}Q_0 \\
&+ \int_{\Omega_{0,n}} \delta\mathbf{v}(t_n^+) \cdot \Big(\rho_0 \big(\mathbf{v}(t_n^+) - \mathbf{v}(t_n^-) \big) \Big) \, \mathrm{d}\Omega_0 \\
&+ \int_{\Omega^e_{0,n}} \delta\mathbf{S}(t_n^+) : \Big([s^D] \big(\mathbf{S}(t_n^+) - \mathbf{S}(t_n^-) \big) \Big) \, \mathrm{d}\Omega_0 \\
&- \int_{P^t_{0,n}} \delta\mathbf{v} \cdot \bar{\mathbf{t}} \, \mathrm{d}P_0 = 0 \qquad \forall\, \delta\mathbf{v} \text{ and } \delta\mathbf{S}
\end{aligned}
$$

- global unknowns: velocity
- mixed-hybrid formulation

Space-time weak form | piezoelectric patch

piezoelectric structure

$$
\begin{aligned}
&\int_{Q_{0,n}} \delta\mathbf{v}\cdot\rho_0\dot{\mathbf{v}}\,\mathrm{d}Q_0 + \int_{Q_{0,n}} \dot{\mathbf{E}}(\delta\mathbf{v},\mathbf{u}) : \mathbf{S}\,\mathrm{d}Q_0 - \int_{Q_{0,n}} \delta\mathbf{v}\cdot\mathbf{f}_0\,\mathrm{d}Q_0 \\
&- \int_{Q^e_{0,n}} \dot{\tilde{\mathbf{E}}}_0(\delta\psi)\cdot\tilde{\mathbf{D}}_0\,\mathrm{d}Q_0 \\
&+ \int_{Q^e_{0,n}} \delta\mathbf{S} : \Big([s^D]\dot{\mathbf{S}} + [g]^\top\dot{\tilde{\mathbf{D}}}_0 - \dot{\mathbf{E}}(\mathbf{v},\mathbf{u})\Big)\,\mathrm{d}Q_0 + \int_{Q_{0,n}} \delta\tilde{\mathbf{D}}_0\cdot\Big(-[g]\dot{\mathbf{S}} + [\epsilon]\dot{\tilde{\mathbf{D}}}_0 - \dot{\tilde{\mathbf{E}}}_0(\psi)\Big)\,\mathrm{d}Q_0 \\
&+ \int_{\Omega_{0,n}} \delta\mathbf{v}(t_n^+)\cdot\Big(\rho_0\big(\mathbf{v}(t_n^+) - \mathbf{v}(t_n^-)\big)\Big)\,\mathrm{d}\Omega_0 \\
&+ \int_{\Omega^e_{0,n}} \delta\mathbf{S} : \Big([s^D]\big(\mathbf{S}(t_n^+) - \mathbf{S}(t_n^-)\big) + [g]^\top\big(\tilde{\mathbf{D}}_0(t_n^+) - \tilde{\mathbf{D}}_0(t_n^-)\big)\Big)\,\mathrm{d}\Omega_0 \\
&+ \int_{\Omega^e_{0,n}} \delta\tilde{\mathbf{D}}_0\cdot\Big(-[g]\big(\mathbf{S}(t_n^+) - \mathbf{S}(t_n^-)\big) + [\epsilon]\big(\tilde{\mathbf{D}}_0(t_n^+) - \tilde{\mathbf{D}}_0(t_n^-)\big)\Big)\,\mathrm{d}\Omega_0 \\
&- \int_{P^t_{0,n}} \delta\mathbf{v}\cdot\bar{\mathbf{t}}_0\,\mathrm{d}P_0 + \int_{P^\psi_{0,n}} \delta q\bar{\psi}\,\mathrm{d}P_0 = 0 \qquad \forall\,\delta\mathbf{v},\ \delta\psi \text{ and } \delta\mathbf{S},\ \delta\tilde{\mathbf{D}}
\end{aligned}
$$

Space-time weak form | electric circuit

electrode

$$-\int_{P_0^E} \delta q\,\psi\,\mathrm{d}P_0 - \int_{P_0^E} \delta\psi\,q\,\mathrm{d}P_0$$

$$\forall\,\delta\psi,\,\delta q$$

- provides electrode surface charge state via potential rate of the piezoelectric patch
- comparable to Lagrange multiplier

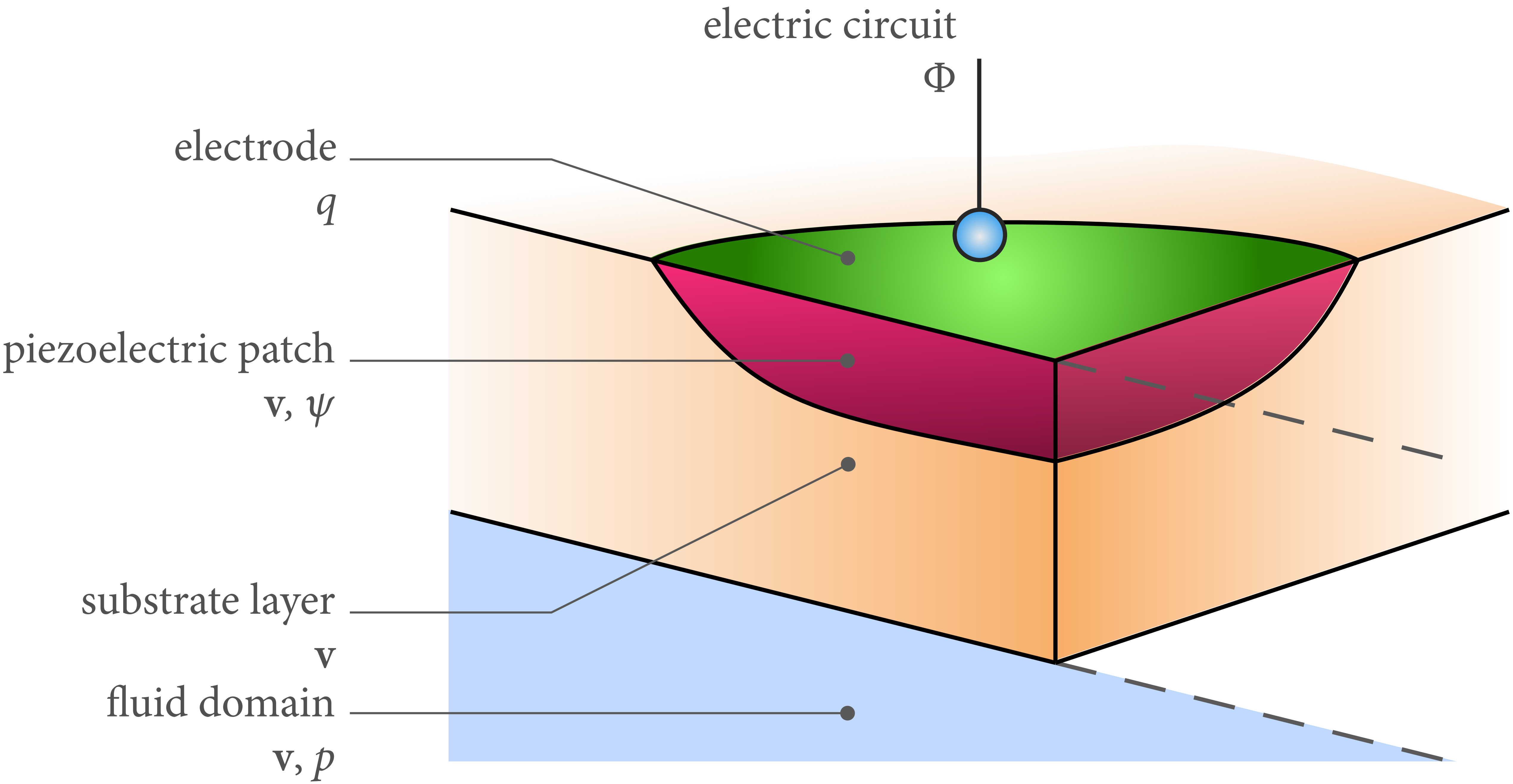
electrical circuit: simple resistor

$$\int_I \delta\Phi\left(\frac{\Phi}{R} - \int_{\Gamma_0^E} \dot{q}\,\mathrm{d}\Gamma\right)\mathrm{d}t = 0$$

$$\forall\,\delta\Phi$$

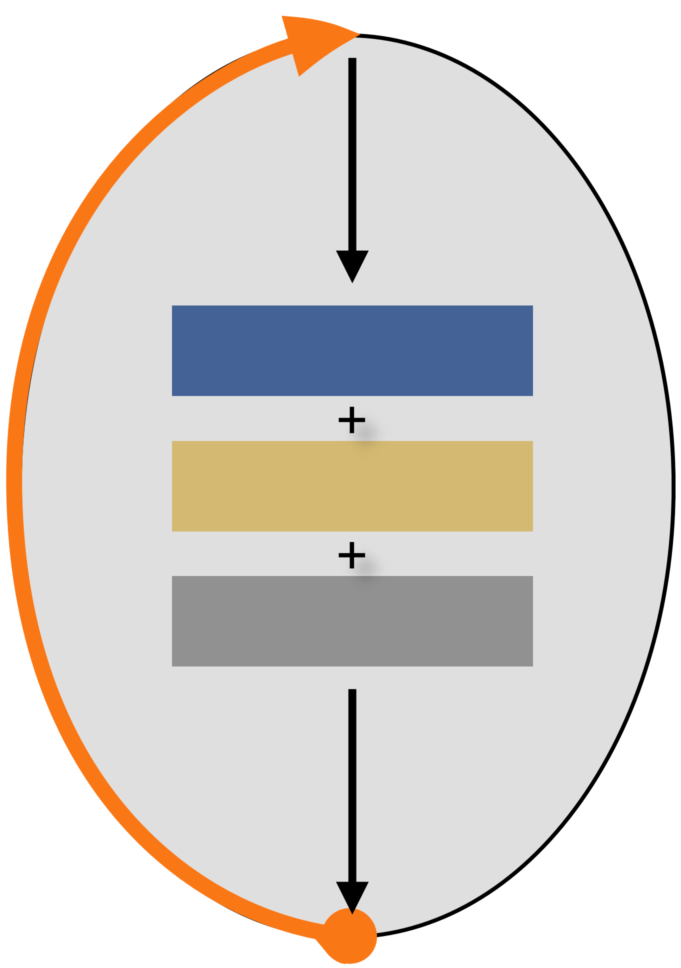
- circuit equation in charge rate (electric current) form
- based on potential state (voltage)

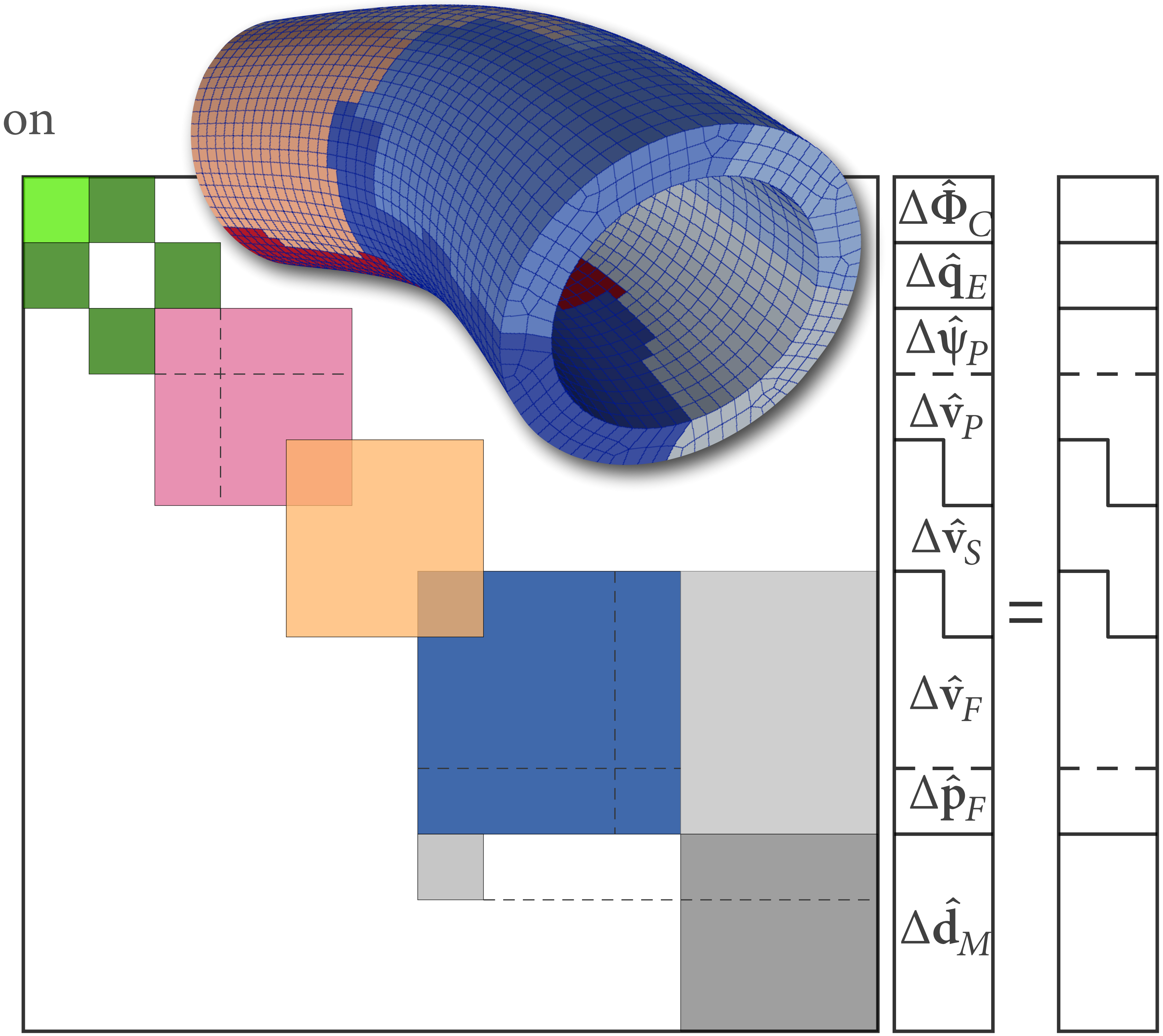
Multi-field problem | uni-morph config



Strongly coupled problem | solution method

- monolithic Newton-Raphson method in the time slab
- parallel solution with preconditioned iterative method

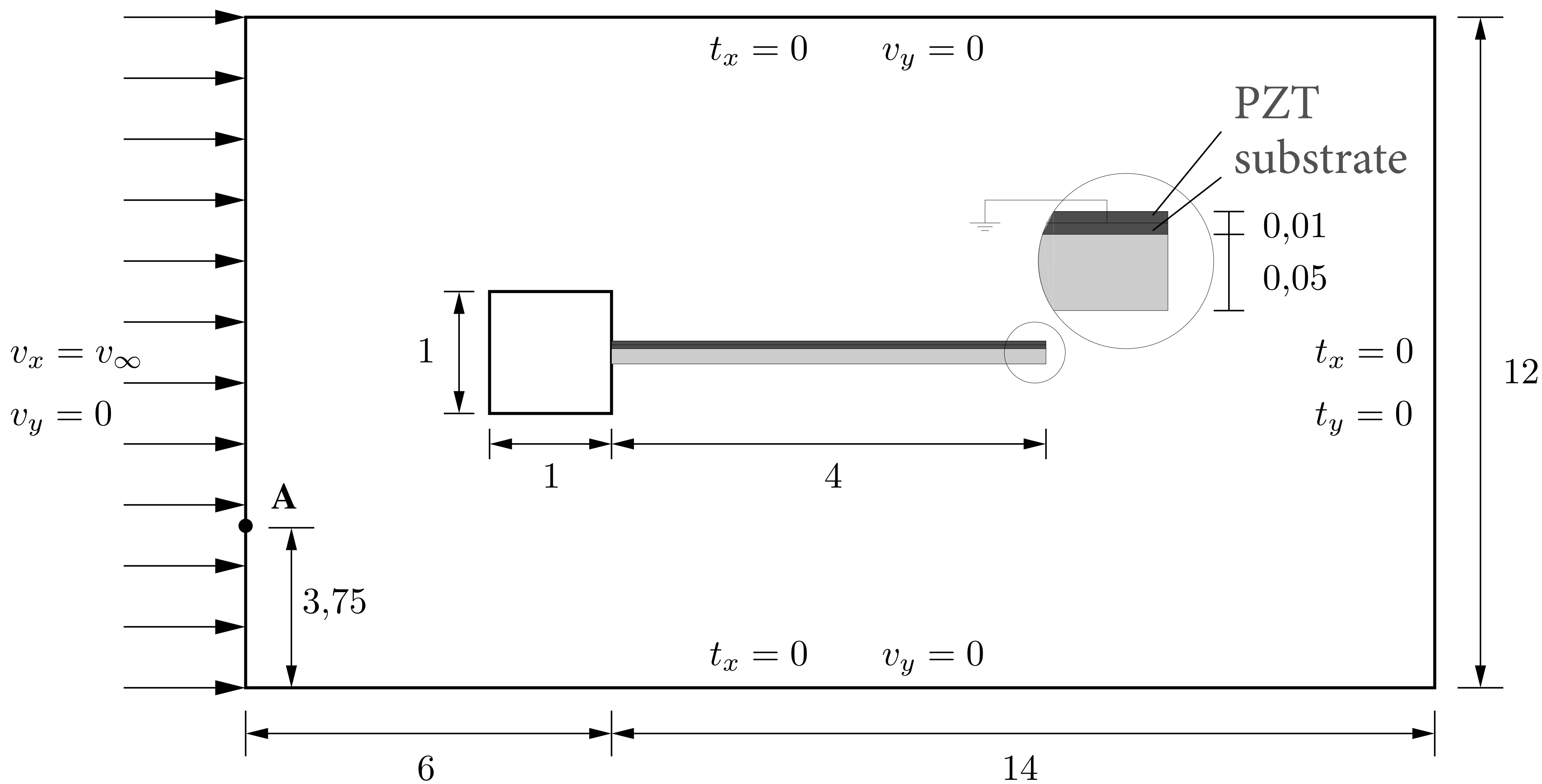




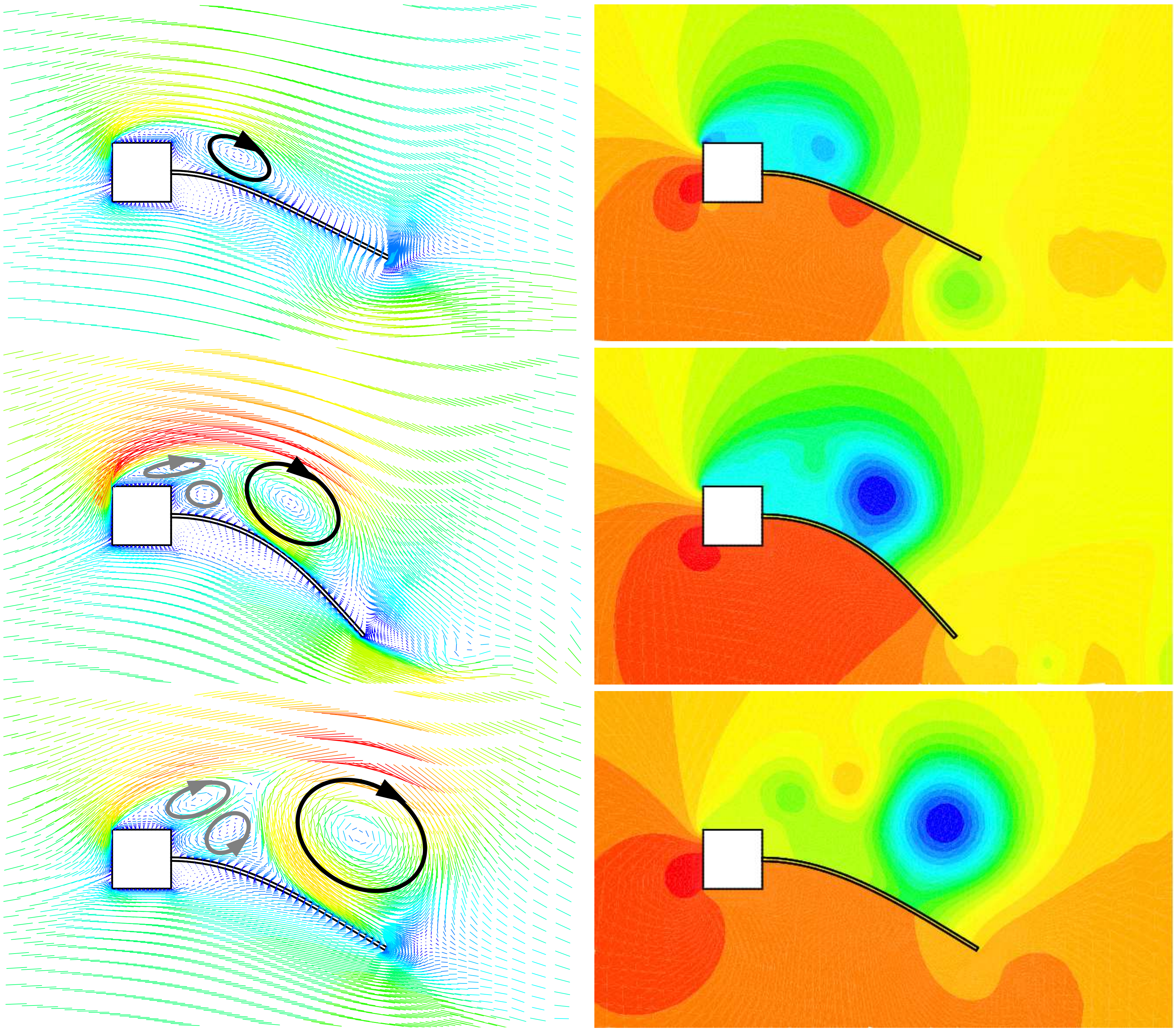
$$\mathbf{A}\left(\hat{\boldsymbol{\Phi}}_C, \hat{\mathbf{q}}_E, \hat{\boldsymbol{\psi}}_P, \hat{\mathbf{v}}_P, \hat{\mathbf{v}}_S, \hat{\mathbf{v}}_F, \hat{\mathbf{p}}_F, \hat{\mathbf{d}}_M\right) \Delta\hat{\mathbf{x}} = \mathbf{r}$$

Example | (fsi) cantilever plate

- plate connected to a leading bluff body
- vortex-induced vibrations of a very flexible plate
- control of large structural deformations



Example | (fsi) cantilever plate II

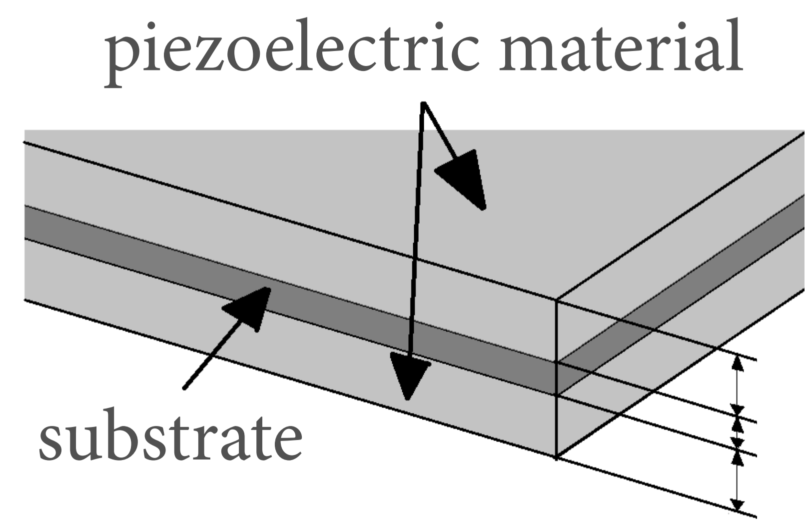


Re = 200
f_1 = 0.75 Hz
f_2 = 4.76 Hz

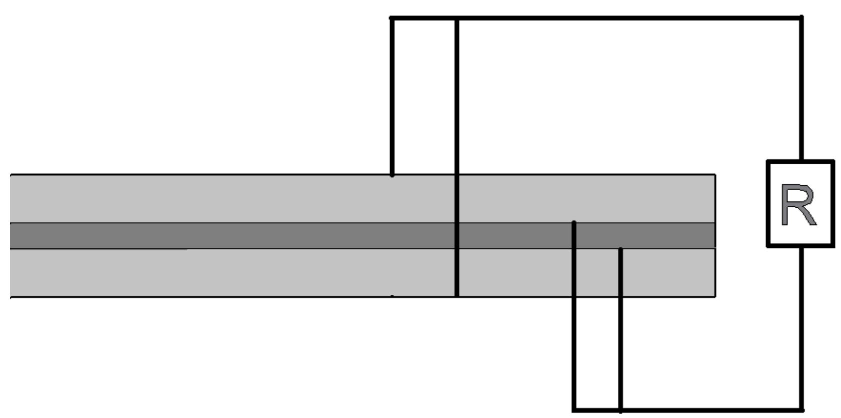
[figure] P Sun. PhD thesis, 2010.

Example | (dry) cantilever plate harvester

- study of harvester power output
- 3D plate subject to harmonic base excitation
- bi-morph configuration
- equipotential electrodes

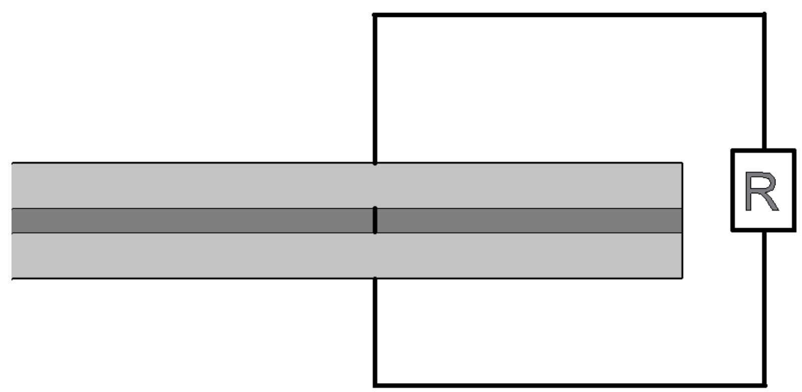


parallel connection



- interface electrodes grounded
- floating potential on top and bottom electrodes

series connection

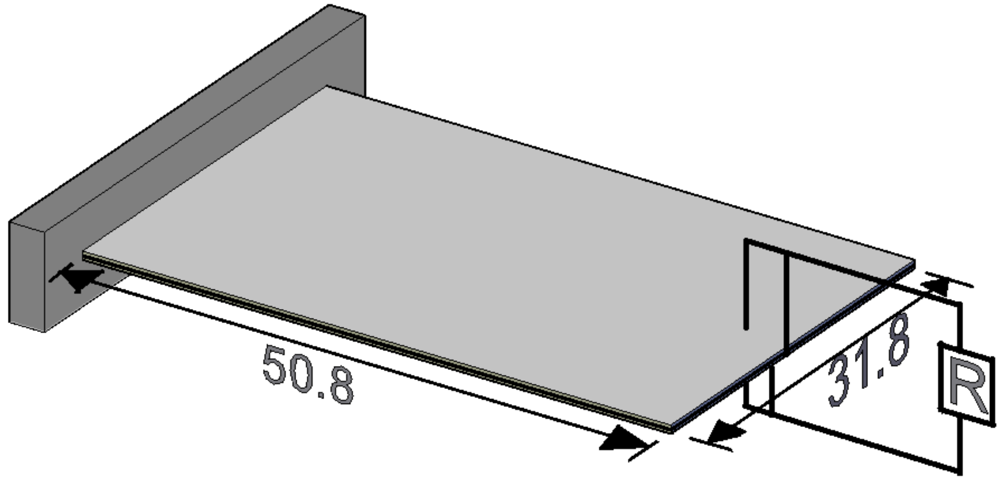


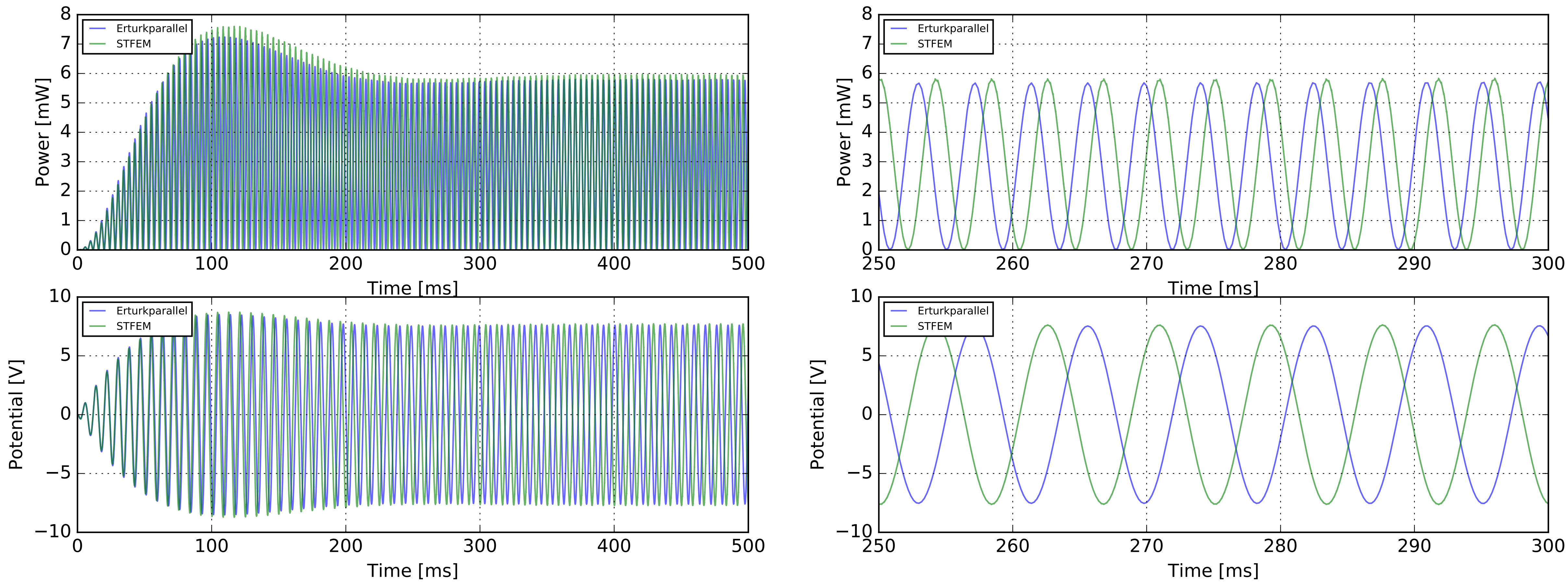
- bottom electrode grounded
- floating potential on top electrode
- no electrodes on interface patches

Example | (dry) cantilever plate harvester I

parallel connection

$L = 50.8\ mm$	Piezo: PZT-5A	$f_{1,3D,R=0} = 119.95\ Hz$
$B = 31.8\ mm$	Substrate: brass	$f_{1,EB,R=0} = 118.6\ Hz$
$h_S = 0.14mm$		
$h_P = 0.26\ mm$	$R = 0.01\ M\Omega$	$\Omega = 119.95\ Hz$





A Erturk and D J Inman. A Distributed Electromechanical Model for cantilevered Piezoelectric Energy Harvesters, Journal of Vibration and Acoustics, 130(4):041002, 2008.

Hybrid velocity-based solid | TDA vs. FDA

- formulation allows time-domain analysis of general nonlinear transient problem
- potentially required: prediction of eigenstates of the associated linear problem

challenge: FDA of solid in mixed velocity-stress formulation with stress hybridisation

Weak form and solution of the homogeneous problem:

$$\int_{\Omega_0} \delta\mathbf{v} \cdot (\rho_0 \dot{\mathbf{v}} + c\mathbf{v})\, \mathrm{d}\Omega_0 + \int_{\Omega_0} \dot{\mathbf{E}}(\delta\mathbf{v}) : \mathbf{S}\, \mathrm{d}\Omega_0 + \int_{\Omega_0^e} \delta\mathbf{S} : \left([s^D]\dot{\mathbf{S}} - \dot{\mathbf{E}}(\mathbf{v}) \right) \mathrm{d}\Omega_0 = 0$$

$\forall\, \delta\mathbf{v},\ \delta\mathbf{S}$

$$\begin{bmatrix} \delta\hat{\mathbf{v}} \\ \delta\hat{\mathbf{s}}_e \end{bmatrix}^{\mathsf{T}} \left(\begin{bmatrix} \mathbf{M}_{vv} & \\ & \mathbf{F}_{ss} \end{bmatrix} \begin{bmatrix} \dot{\hat{\mathbf{v}}} \\ \dot{\hat{\mathbf{s}}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{vv} & \mathbf{B}_{vs} \\ \mathbf{B}_{sv} & \end{bmatrix} \begin{bmatrix} \hat{\mathbf{v}} \\ \hat{\mathbf{s}}_e \end{bmatrix} \right) = 0$$

$$\mathbf{A}\dot{\mathbf{x}} + \mathbf{B}\mathbf{x} = \mathbf{0} \quad \text{and} \quad \dot{\mathbf{x}} = \lambda\mathbf{x} \qquad \rightarrow \qquad (\lambda\mathbf{A} + \mathbf{B})\,\mathbf{x} = \mathbf{0}$$

task: dynamic condensation of element-local stress d.o.f. (discrete problem size!)

FDA | solid - dynamic stress condensation

$$\begin{bmatrix} \mathbf{M}_{vv} & \\ & \mathbf{F}_{ss} \end{bmatrix} \begin{bmatrix} \dot{\hat{\mathbf{v}}} \\ \dot{\hat{\mathbf{s}}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{vv} & \mathbf{B}_{vs} \\ \mathbf{B}_{sv} & \end{bmatrix} \begin{bmatrix} \hat{\mathbf{v}} \\ \hat{\mathbf{s}}_e \end{bmatrix} = \mathbf{0}$$

generalised eigenvalue problem:

$$\begin{bmatrix} \lambda\mathbf{M}_{vv} + \mathbf{C}_{vv} & \mathbf{B}_{vs} \\ \mathbf{B}_{sv} & \lambda\mathbf{F}_{ss} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{v}} \\ \hat{\mathbf{s}}_e \end{bmatrix} = \mathbf{0}$$

dynamic condensation:

$$\left[\lambda\mathbf{M}_{vv} + \mathbf{C}_{vv} - \mathbf{B}_{vs}(\lambda\mathbf{F}_{ss})^{-1}\mathbf{B}_{sv}\right] \hat{\mathbf{v}} = \mathbf{0}$$

$$\left[\lambda\mathbf{M}_{vv} + \mathbf{C}_{vv} - \frac{1}{\lambda}\mathbf{B}_{vs}\mathbf{F}_{ss}^{-1}\mathbf{B}_{sv}\right] \hat{\mathbf{v}} = \mathbf{0}$$

with $\hat{\mathbf{v}} = \lambda\hat{\mathbf{u}}$

$$\left[\lambda^2\mathbf{M}_{vv} + \lambda\mathbf{C}_{vv} - \mathbf{B}_{vs}\mathbf{F}_{ss}^{-1}\mathbf{B}_{sv}\right] \hat{\mathbf{u}} = \mathbf{0}$$

quadratic eigenvalue problem

$$\left[\lambda^2\mathbf{M}_{vv} + \lambda\mathbf{C}_{vv} + \mathbf{K}_{vv}\right] \hat{\mathbf{u}} = \mathbf{0}$$

FDA | piezo/circuit mixed-hybrid

Weak form of the coupled problem:

$$\int_{\Omega_0} \delta\mathbf{v} \cdot (\rho_0 \dot{\mathbf{v}} + c\mathbf{v})\, \mathrm{d}\Omega_0 + \int_{\Omega_0} \dot{\mathbf{E}}(\delta\mathbf{v}) : \mathbf{S}\, \mathrm{d}\Omega_0 - \int_{\Omega_0} \dot{\tilde{\mathbf{E}}}_0(\delta\psi) \cdot \tilde{\mathbf{D}}_0\, \mathrm{d}\Omega_0$$

$$+ \int_{\Omega_0^e} \delta\mathbf{S} : \Big([s^D]\dot{\mathbf{S}} + [g]^\top \dot{\tilde{\mathbf{D}}}_0 - \dot{\mathbf{E}}(\mathbf{v}) \Big)\, \mathrm{d}\Omega_0$$

$$+ \int_{\Omega_0^e} \delta\tilde{\mathbf{D}}_0 : \Big(-[g]\dot{\mathbf{S}} + [\epsilon]\dot{\tilde{\mathbf{D}}}_0 - \dot{\tilde{\mathbf{E}}}_0(\psi) \Big)\, \mathrm{d}\Omega_0$$

$$- \int_{\Gamma_0} \delta q \psi\, \mathrm{d}\Gamma_0 - \int_{\Gamma_0} \delta\psi q\, \mathrm{d}\Gamma_0 + \int_{\Gamma_0} \delta\Phi \dot{q}\, \mathrm{d}\Gamma_0$$

$$+ \int_{\Gamma_0} \delta q \dot{\Phi}\, \mathrm{d}\Gamma_0 + \delta\Phi \frac{\Phi}{R} = 0 \qquad \forall\, \delta\mathbf{v},\, \delta\psi,\, \delta\Phi,\, \delta q,\, \delta\mathbf{S},\, \delta\mathbf{D}$$

FDA | piezo/circuit mixed-hybrid II

$$
\begin{bmatrix} \delta\hat{\mathbf{v}} \\ \delta\hat{\psi} \\ \delta\hat{\Phi} \\ \delta\hat{q} \\ \delta\hat{\mathbf{S}}_e \\ \delta\hat{\mathbf{D}}_e \end{bmatrix}^{\mathsf{T}}
\left(
\begin{bmatrix}
\mathbf{M}_{vv} & & & & & \\
 & & & & & \\
 & & & \mathbf{F}_{\phi q} & & \\
 & & \mathbf{F}_{q\phi} & & & \\
 & & & & \mathbf{F}_{ss} & \mathbf{F}_{sd} \\
 & & & & \mathbf{F}_{ds} & \mathbf{F}_{dd}
\end{bmatrix}
\begin{bmatrix} \dot{\hat{\mathbf{v}}} \\ \dot{\hat{\psi}} \\ \dot{\hat{\Phi}} \\ \dot{\hat{q}} \\ \dot{\hat{\mathbf{S}}}_e \\ \dot{\hat{\mathbf{D}}}_e \end{bmatrix}
+
\begin{bmatrix}
\mathbf{C}_{vv} & & & & \mathbf{B}_{vs} & \\
 & & & \mathbf{B}_{\psi q} & & \mathbf{B}_{\psi d} \\
 & & \mathbf{B}_{\phi\phi} & & & \\
 & \mathbf{B}_{q\psi} & & & & \\
\mathbf{B}_{sv} & & & & & \\
 & \mathbf{B}_{d\psi} & & & &
\end{bmatrix}
\begin{bmatrix} \hat{\mathbf{v}} \\ \hat{\psi} \\ \hat{\Phi} \\ \hat{q} \\ \hat{\mathbf{S}}_e \\ \hat{\mathbf{D}}_e \end{bmatrix}
\right) = \mathbf{0}
$$

$$
\mathbf{A}\dot{\mathbf{x}} + \mathbf{B}\mathbf{x} = \mathbf{0} \quad \text{and} \quad \dot{\mathbf{x}} = \lambda\mathbf{x} \qquad \rightarrow \qquad (\lambda\mathbf{A} + \mathbf{B})\,\mathbf{x} = \mathbf{0}
$$

task: dynamic condensation of element-level dofs (discrete problem size!)

FDA | piezo/circuit dynamic condensation

rational eigenvalue problem:

$$\left(\lambda \begin{bmatrix} \mathbf{M}_{vv} & & & \\ & & & \\ & & & \mathbf{F}_{\psi q} \\ & & \mathbf{F}_{q\psi} & \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{vv} & & & \\ & & & \mathbf{B}_{\psi q} \\ & & \mathbf{B}_{\phi\phi} & \\ & \mathbf{B}_{q\psi} & & \end{bmatrix} + \frac{1}{\lambda} \begin{bmatrix} \; & \; & \; & \; \\ & & & \\ & & & \\ & & & \end{bmatrix} \right) \begin{bmatrix} \hat{\mathbf{v}} \\ \hat{\psi} \\ \hat{\Phi} \\ \hat{q} \end{bmatrix} = \mathbf{0}$$

mechanical + electrical damping matrix

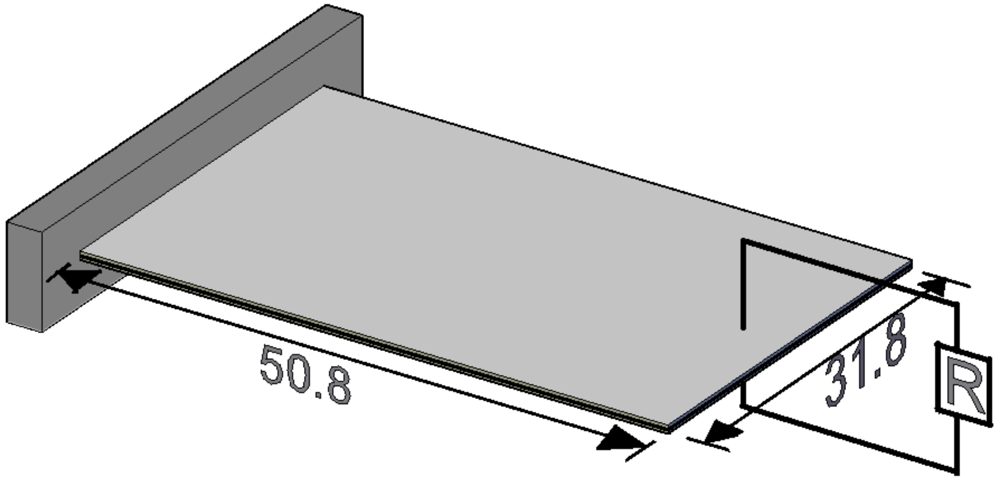
hybridised stiffness matrix

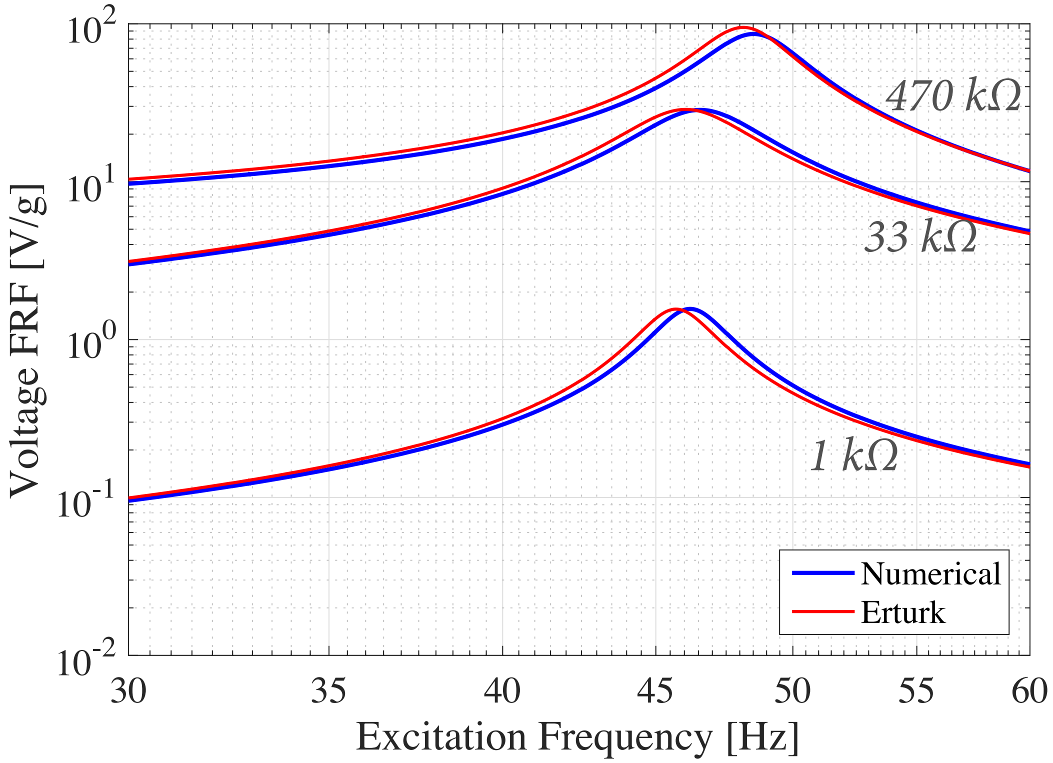
solution: transformation to first order system (state-space form)

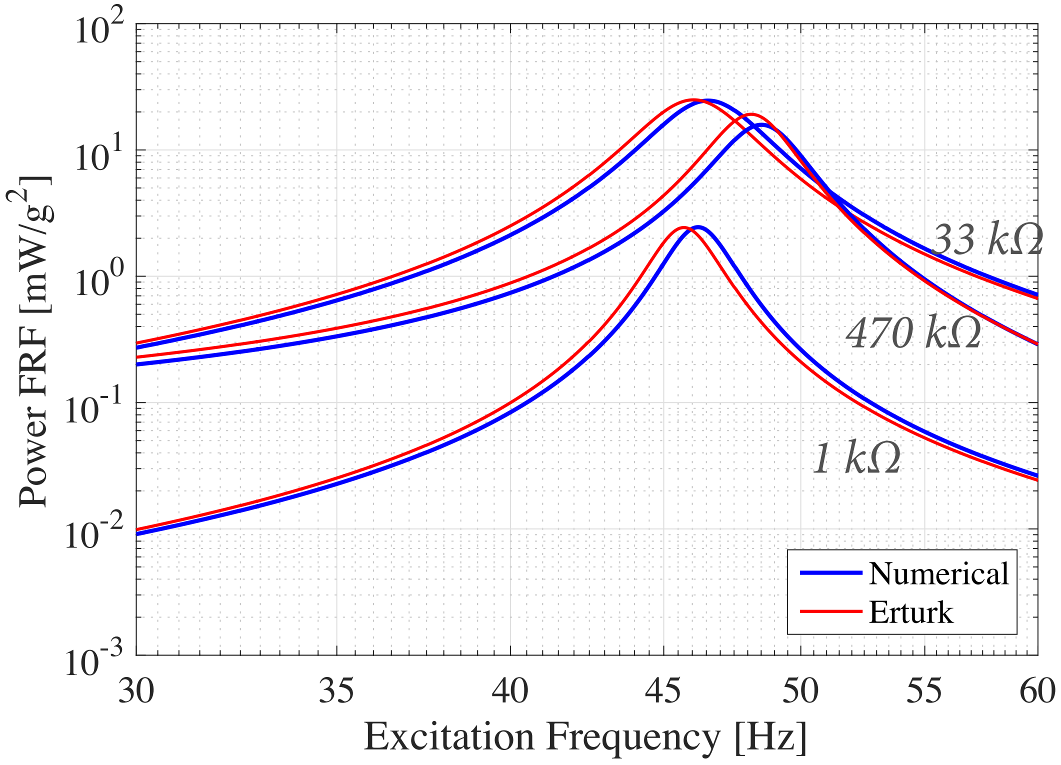
Example | (dry) cantilever plate harvester II

series connection

		$f_{1,3D,R=0}$ = 46.2 Hz
L = 50.8 mm	*Piezo: PZT-5A*	*$f_{1,EB,R=0}$ = 45.7 Hz*
B = 31.8 mm	*Substrate: brass*	
h_S = 0.14mm	*End mass: 12 g*	*$f_{1,3D,R=inf}$ = 48.5 Hz*
h_P = 0.26 mm	*R = 1 kΩ - 470 kΩ*	*$f_{1,EB,R=inf}$ = 48.2 Hz*

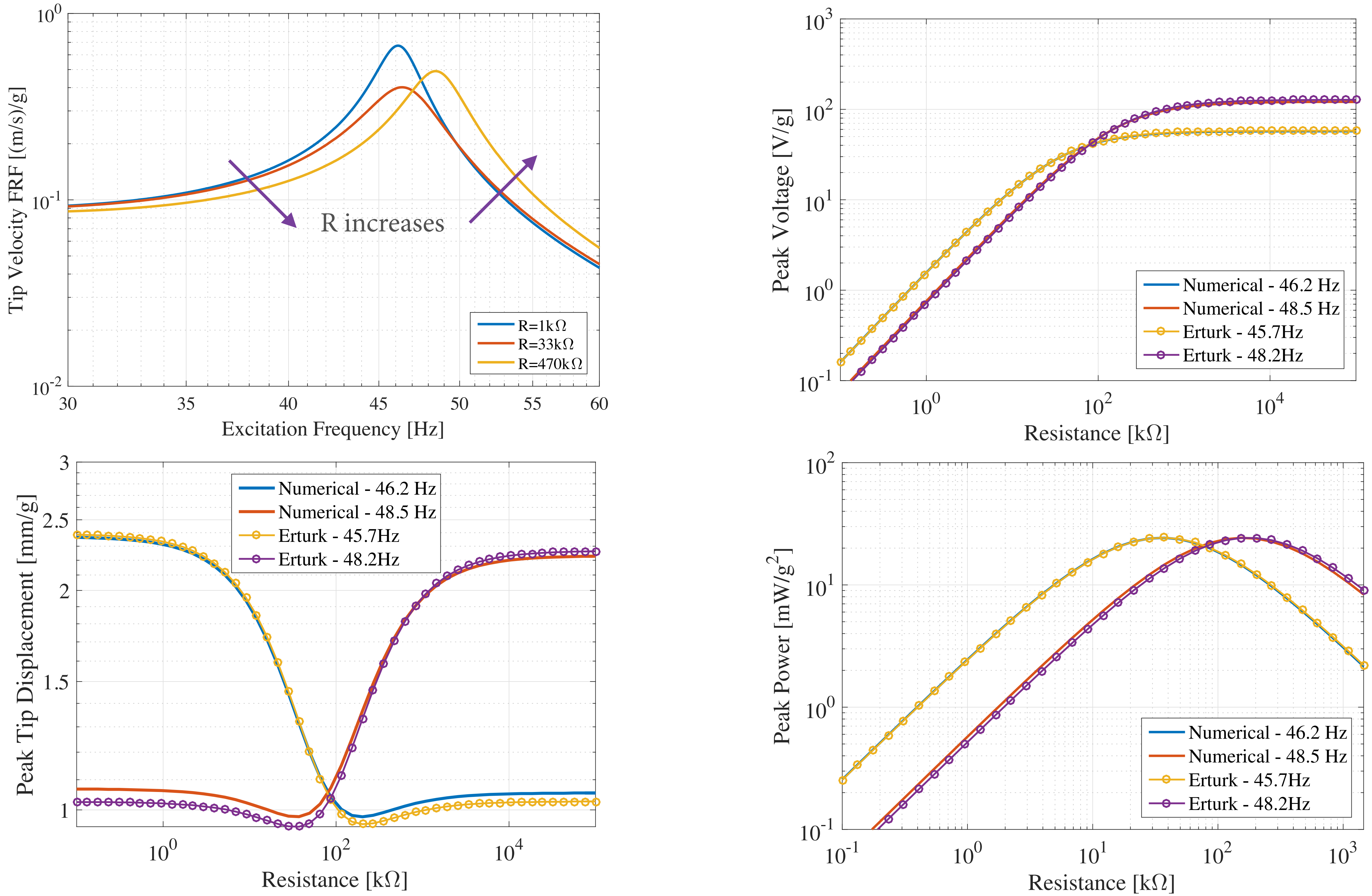






A Erturk and D J Inman. A Distributed Electromechanical Model for cantilevered Piezoelectric Energy Harvesters, Journal of Vibration and Acoustics, 130(4):041002, 2008.

Example | (dry) cantilever plate harvester III



A Erturk and D J Inman. A Distributed Electromechanical Model for cantilevered Piezoelectric Energy Harvesters, Journal of Vibration and Acoustics, 130(4):041002, 2008.

Summary | outlook

- framework for numerical investigation of flow-driven energy harvesters
- based on space-time finite elements
- monolithic approach (solve all multi field equations at once)

- quantification of energy harvested from types of flow-induced vibrations
- study of energy harvested in flutter state (2nd eigenmode)
- influence of patch shape and location / optimisation of energy harvested

Fonds National de la Recherche Luxembourg

MARIE CURIE ACTIONS

SEVENTH FRAMEWORK PROGRAMME