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Strongly-coupled modelling and analysis of energy harvesting devices

Andreas Zilian[1,*] and **Srivathsan Ravi**[1]

[1] University of Luxembourg, 6, rue R. Coudenhove-Kalergi, L-1359 Luxembourg

A monolithic approach is proposed that provides simultaneous modelling and analysis of the harvester, which involves surface-coupled fluid-structure interaction, volume-coupled electro- mechanics and a controlling energy harvesting circuit for applications in energy harvesting. A space-time finite element approximation is used for numerical solution of the weighted residual form of the governing equations of the flow-driven piezoelectric energy harvesting device. This method enables time-domain investigation of different types of structures (plate, shells) subject to exterior/interior flow with varying cross sections, material compositions, and attached electrical circuits with respect to the electrical power output generated.

1 Introduction

A specific class of energy harvester devices for renewable energy resources is investigated, that allow conversion of ambient fluid flow energy to electrical energy via flow-induced vibrations of a piezo-ceramic composite structure positioned in the flow field. In this way, potentially harmful flow fluctuations are harnessed to provide independent power supply to small electrical devices. In order to harvest energy from fluid flows by means of piezoelectric materials the kinetic energy of the fluid first has to be transformed to cyclic straining energy of the piezoelectric material which is then transformed to electrical energy under the presence of an attached electrical circuit representing the powered electrical device or charged battery (see Figure 1). As depicted in Figure 2, this energy converter technology simultaneously involves the interaction of a composite structure and a surrounding fluid, the electric charge accumulated in the piezo-ceramic material and a controlling electrical circuit. In order to predict the efficiency and operational properties of such future devices and to increase their robustness and performance, a mathematical and numerical model of the complex physical system is required to allow systematic computational investigation of the involved phenomena and coupling characteristics.

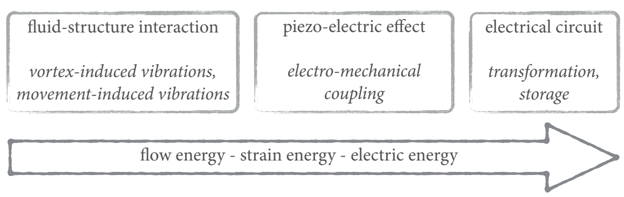

Fig. 1: Multi-physics harvester and energy transformation [2].

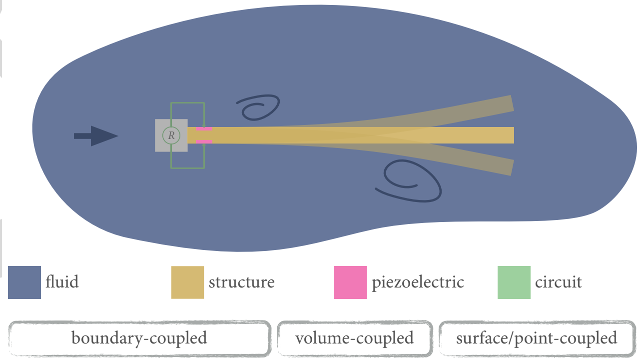

Fig. 2: Components and subdomains of the coupled problem [2].

2 Mathematical Model of a Flow-Driven Energy Harvesting Device

The complete set of mathematical model equations of the coupled problem consists of the strong-form governing equations of the fluid, substrate structure, piezoelectric material (patches) connected to the electrodes, and the electrical circuit. Equations (1)-(7) represent the incompressible Navier-Stokes equations of the fluid in velocity-pressure form defined on the deforming space-time domain $Q = \Omega \times I$ and space-time boundary $P = \Gamma \times I$ with I being the time interval under consideration.

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) - \nabla \cdot \mathbf{T} - \mathbf{f} = \mathbf{0} \quad \text{on} \quad Q \quad (1)$$

$$\nabla \cdot \mathbf{v} = 0 \quad \text{on} \quad Q \quad (2)$$

$$\mathbf{D} - \frac{1}{2} \left[\nabla \mathbf{v} + (\nabla \mathbf{v})^{\mathsf{T}} \right] = \mathbf{0} \quad \text{on} \quad Q \quad (3)$$

$$\mathbf{T} + p\mathbf{I} - 2\mu \mathbf{D} = \mathbf{0} \quad \text{on} \quad Q \quad (4)$$

$$\mathbf{T} \cdot \mathbf{n} - \mathbf{t} = \mathbf{0} \quad \text{on} \quad P \quad (5)$$

$$\mathbf{v} - \bar{\mathbf{v}} = \mathbf{0} \quad \text{on} \quad P^v \quad (6)$$

$$\mathbf{t} - \bar{\mathbf{t}} = \mathbf{0} \quad \text{on} \quad P^t \quad (7)$$

The linear elastic substrate layer is described on the reference domain Q_0 in terms of structural velocities, strain rate and stress, see Equations (8)-(13), using the rate form of the constitutive relation (10). This choice a priori ensures continuity of the

* Corresponding author: e-mail andreas.zilian@uni.lu, phone +352 466644 5220, fax +352 466644 5200

fluid and structural velocity field, avoiding the need for additional coupling constraints along the fluid-structure interface [1]. In the domain occupied by piezo-electric material (patches), the present electrical field is described by Equations (14)-(19) using dielectric displacement, rate of electric field and rate of electric potential. For piezo-electric material, constitutive relations (10) and (16) represent the electro-mechanical coupling through $[g]$.

$$\rho_0 \dot{\mathbf{v}} - \nabla_0 \cdot (\mathbf{F}\mathbf{S}) - \mathbf{f}_0 = \mathbf{0} \quad \text{in} \quad Q_0 \qquad (8)$$

$$\dot{\mathbf{E}} - \frac{1}{2}\left[\nabla_0 \mathbf{v} + (\nabla_0 \mathbf{v})^{\mathsf{T}}\right] = \mathbf{0} \quad \text{in} \quad Q_0 \qquad (9)$$

$$\dot{\mathbf{E}} - [s^D]\dot{\mathbf{S}} - [g]^{\mathsf{T}}\dot{\tilde{\mathbf{D}}}_0 = \mathbf{0} \quad \text{in} \quad Q_0 \qquad (10)$$

$$\mathbf{S} \cdot \mathbf{n} - \mathbf{t} = \mathbf{0} \quad \text{on} \quad P_0 \qquad (11)$$

$$\mathbf{v} - \bar{\mathbf{v}} = \mathbf{0} \quad \text{on} \quad P_0^v \qquad (12)$$

$$\mathbf{t} - \bar{\mathbf{t}} = \mathbf{0} \quad \text{on} \quad P_0^t \qquad (13)$$

$$\nabla_0 \cdot \tilde{\mathbf{D}}_0 = 0 \quad \text{in} \quad Q_0 \qquad (14)$$

$$\dot{\tilde{\mathbf{E}}}_0 + \nabla_0 \psi = \mathbf{0} \quad \text{in} \quad Q_0 \qquad (15)$$

$$\dot{\tilde{\mathbf{E}}}_0 + [g]\dot{\mathbf{S}} - [\epsilon^{-1}]^{\mathsf{T}}\dot{\tilde{\mathbf{D}}}_0 = \mathbf{0} \quad \text{in} \quad Q_0 \qquad (16)$$

$$\tilde{\mathbf{D}}_0 \cdot \mathbf{n} + q = 0 \quad \text{on} \quad P_0 \qquad (17)$$

$$\psi - \bar{\psi} = 0 \quad \text{on} \quad P_0^\psi \qquad (18)$$

$$q - \bar{q} = 0 \quad \text{on} \quad P_0^q \qquad (19)$$

The equi-potentiality condition on the electrodes is introduced with Eq. (20) making use of the surface boundary charge q as defined in Eq. (17) and total circuit charge Q. The connection between electrode and electrical circuit is established via Eqn. (21), while the circuit equations (22)-(23) considers the attached transformation circuitry in terms of a reduced model using a resistor of resistance R.

$$\dot{Q} - \int_{\Gamma^e} \dot{q}\, d\Gamma = 0 \quad \text{in} \quad I \qquad (20)$$

$$\dot{\Phi} - \psi = 0 \quad \text{in} \quad I \qquad (21)$$

$$I - \dot{Q} = 0 \quad \text{in} \quad I \qquad (22)$$

$$\Delta\Phi - R \cdot I = 0 \quad \text{in} \quad I \qquad (23)$$

3 Space-Time Finite Element Analysis

The weighted residual method is applied to the above set of governing equations, leading to the weak form involving the unknown physical fields of fluid velocity $\mathbf{v}_F$ and pressure p_F, substrate layer velocity $\mathbf{v}_S$, velocity $\mathbf{v}_P$ and potential rate ψ_F within the piezoelectric patch, surface boundary charge q_E on the electrode and total potential Φ_C of the electrical circuit as well as the deformation $\mathbf{d}_M$ of the fluid mesh (not discussed here) as shown in Figure 3. Following a discretisation with space-time finite elements [2] one obtains a monolithic description of the (non-linear) multi-physics problem in the time-domain that is solved by means of the Newton-Raphson scheme (see Figure 4) and using preconditioned iterative solvers for the large sparse linear systems of discrete algebraic equations (24).

$$\mathbf{A}\left(\hat{\mathbf{\Phi}}_C, \hat{\mathbf{q}}_E, \hat{\psi}_P, \hat{\mathbf{v}}_P, \hat{\mathbf{v}}_S, \hat{\mathbf{v}}_F, \hat{\mathbf{p}}_F, \hat{\mathbf{d}}_M\right) \Delta\hat{\mathbf{x}} = \mathbf{r} \qquad (24)$$

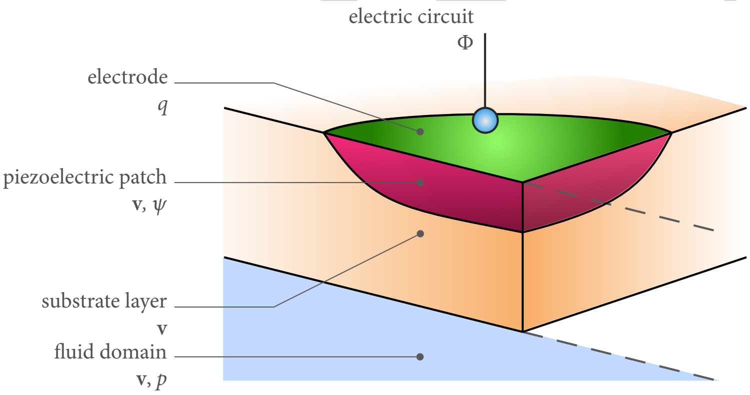

Fig. 3: Involved physical fields of the harvester [2].

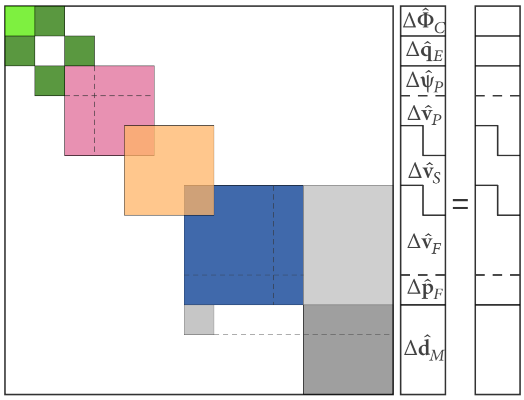

Fig. 4: Tangent matrix of Newton-Raphson method [2].

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