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Preface

Over the last decades the theory and applications of aggregation functions have become more and more important in a large number of disciplines such as applied mathematics, computer sciences, statistics, economics, and engineering sciences. The considerable amount of knowledge that has been collected up to now in this area has led the researchers to extend their investigation to wider fields, including aggregation of more abstract structures such as ordered sets, structured data, etc. The 'International Symposium on Aggregation on Bounded Lattices' (ABLAT 2014, Trabzon, Turkey) was a first conference organized in this direction and provided a medium for the exchange of ideas between theoreticians and practitioners in aggregation on lattices and related areas.

The 'International Symposium on Aggregation and Structures' (ISAS 2016, Luxembourg, Luxembourg) aims at pursuing this tradition of investigating the aggregation problem in a very wide sense. More specifically this symposium constitutes a forum for presenting the latest trends in both aggregation *on* structures and aggregation *of* structures. ISAS 2016 is organized by the Mathematics Research Unit of the University of Luxembourg with the support of the ILIAS laboratory. We hope that this initiative will be followed in the future and that more and more researchers will join this community.

We are very grateful to all those who contributed to the success of ISAS 2016. In particular our special thanks go to the authors and reviewers of the submitted abstracts. We have selected 28 submissions that fit best to the objectives of this symposium. Also, four invited contributions provide new trends in extended aggregation theory. In order to help the attendees and participants interact with the various presentations we have not organized any parallel session.

We wish to all participants a very pleasant symposium full of fruitful projects of collaborations.

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Contents

Penalty-Based Aggregation of Complex Data and Their Applications in Data Analysis

Marek Gagolewski[1,2]

[1] Systems Research Institute, Polish Academy of Sciences,
ul. Newelska 6, 01-447 Warsaw, Poland
[2] Faculty of Mathematics and Information Science, Warsaw University of Technology,
ul. Koszykowa 75, 00-662 Warsaw, Poland
gagolews@ibspan.waw.pl, http://www.gagolewski.com

Abstract

Since the 1980s, studies of aggregation functions most often focus on the construction and formal analysis of diverse ways to summarize numerical lists with elements in some real interval. Quite recently, we also observe an increasing interest in aggregation *of* and aggregation *on* generic partially ordered sets.

However, in many practical applications, we have no natural ordering of given data items. Thus, in this talk we review various aggregation methods in spaces equipped merely with a semimetric (distance, see [3]). These include the concept of such penalty minimizers as the centroid, 1-median, 1-center, medoid, and their generalizations – all leading to idempotent fusion functions (see, e.g., [1]). Special emphasis is placed on procedures to summarize vectors in $\mathbb{R}^d$ for $d \geq 2$ (e.g., rows in numeric data frames) as well as character strings (e.g., DNA sequences), but of course the list of other interesting domains could go on forever (rankings, graphs, images, time series, and so on).

We discuss some of their formal properties, exact or approximate (if the underlying optimization task is hard) algorithms to compute them (see, e.g., [4]) and their applications in clustering and classification tasks [2,5,6].

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Integer means and nonassociative calculus

Michel Grabisch
(with Miguel Couceiro)

Paris School of Economics, Université Paris I Panthéon-Sorbonne,
Centre d'économie de la Sorbonne, 106 – 112, bd de l'Hôpital, 75013 Paris,France
michel.grabisch@univ-paris1.fr

Abstract

Integer-valued means aggregate values in Z, and satisfy symmetry (anonymity), internality, monotonicity and decomposability in the sense of Kolmogoroff. It has been recently proved by Bennett et al. that all integer means are extremal, i.e., the result is a function of the minimum and maximum entries. This result shows that the restriction to integers prevents from using all entries, and can be seen as a negative result. We try in this work to overcome this limitation, by weakening the decomposability axiom in order to use not only extremal values but also second extremal values, etc. Besides, the extension of the maximum on Z leads to the symmetric maximum, which is a nonassociative operator. By defining rules of computation (i.e., systematic ways of putting parentheses), one can overcome nonassociativity. It turns out that these extended symmetric maximum operators precisely use extremal elements up to some rank and can be considered as a new family of integer means. We identify which type of weak decomposability is satisfied by the symmetric maximum operators, and give axiomatizations of these operators.

Beyond pixels: when visual information escapes the matrix

Carlos Lopez-Molina

GIARA, Universidad Pública de Navarra
Pamplona 31006, Spain carlos.lopez@unavarra.es

Abstract

Digital imagery has always be represented as a matrix of pixels, each of them holding a tone. Although innovation has led to a wide variety of tasks, goals and applications, very few authors have analyzed the representation and processing of images in a shape other than matrices. Although mathematically convenient, we find this representation inadequate to mimic human behaviour. In this talk we analyze the superpixel paradigm, from its biological roots to its requirements in terms of mathematical modelling. We pay special attention to the relationship with soft computing and non-standard data fusion.

Compatible group decisions

Gabriella Pigozzi
(with E. Awad, M. Caminada, M. Podlaszewski, and I. Rahwan)

LAMSADE - Université Paris Dauphine
Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France
gabriella.pigozzi@dauphine.fr

An inconsistent knowledge base can be abstracted as a set of arguments and a defeat relation among them. There can be more than one consistent way to evaluate such an argumentation graph. Collective argument evaluation is the problem of aggregating the opinions of multiple agents on how a given set of arguments should be evaluated. It is crucial not only to ensure that the outcome is logically consistent, but also satisfies measures of social optimality and immunity to strategic manipulation. This is because agents have their individual preferences about what the outcome ought to be. In this talk I will present three argument-based aggregation operators and they analysis with respect to Pareto optimality and strategy proofness under different general classes of agent preferences.

An Order Obtained from Nullnorms and Its Properties

Emel Aşıcı

Department of Software Engineering, Faculty of Technology, Karadeniz Technical University, 61830 Trabzon, Turkey
E-mail: emelkalin@hotmail.com

Abstract. In this paper, an order induced by nullnorms on bounded lattices is given and discussed. We define the set of incomparable elements with respect to the order induced by a nullnorm. Also, by defining such an order, an equivalence relation on the class of nullnorms is defined and this equivalence is deeply investigated.

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Directional monotonicity of fuzzy implications

Michał Baczyński and Katarzyna Miś

Institute of Mathematics, University of Silesia,
40-007 Katowice, ul. Bankowa 14, Poland
michal.baczynski@us.edu.pl
kmis@us.edu.pl

Standard monotonicity is one of the key properties of aggregation operations. For example t-norms, t-conorms and copulas are increasingly monotone in each variable (cf. [3]). But fuzzy implication functions, which are very useful in fuzzy logic and fuzzy control, are hybrid monotonic – they are decreasing in the first variable and increasing in the second one. Our motivation for this work is the article [2], where the authors discussed directional monotonicity for fusion functions.

Definition 0.1 ([2]). *Let* $F\colon [0,1]^n \to [0,1]$ *and* $\mathbb{R}^n \ni r = (r_1, \ldots, r_n) \neq (0, \ldots, 0)$.

(i) *F is r-increasing, if for all $x \in [0,1]^n$ and all $c > 0$ such that $x + cr \in [0,1]$, it holds that*

$$F(x + cr) \geq F(x).$$

(ii) *F is r-decreasing, if for all $x \in [0,1]^n$ and all $c > 0$ such that $x + cr \in [0,1]$, it holds that*

$$F(x + cr) \leq F(x).$$

Fuzzy implications (see [1]) are in fact $(1,0)$-decreasing and $(0,1)$-increasing functions. Please note that the $(\varepsilon, \varepsilon)$-increasing fuzzy implications, for $\varepsilon > 0$, have been investigated in [4]. This type of increasingness is also called weak increasingness (see [5]).

Definition 0.2. *A fuzzy implication I is said to be special, if for any $\varepsilon > 0$ and for all $x, y \in [0,1]$ such that $x + \varepsilon, y + \varepsilon \in [0,1]$ the following condition is satisfied:*

$$I(x, y) \leq I(x + \varepsilon, y + \varepsilon).$$

In [4] the authors have obtained a characterization of general binary operations whose residuals become special (in particular R-implications). In our contribution we present main results connected with special implications, discuss directional monotonicity for fuzzy implications and we analyze $(\varepsilon_1, \varepsilon_2)$-increasing fuzzy implications and also $(\varepsilon, \varepsilon)$-decreasing fuzzy implication functions.

Key words: aggregation operations; functional equations; fuzzy implications; directional monotonicity

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Some constructions of uninorms and nullnorms on bounded lattices

Slavka Bodjanova and Martin Kalina

[1] Texas A&M University-Kingsville, Department of Mathematics
MSC 172, Kingsville TX 78363, U.S.A.
kfsb000@tamuk.edu
[2] Slovak University of Technology in Bratislava
Faculty of Civil Engineering, Department of Mathematics
Radlinského 11, Sk-810 05 Bratislava, Slovakia
kalina@math.sk

This contribution is a continuation of papers [1, 4, 5]. In [1] we have constructed uninorms on some special types of bounded lattices. In [4] a pre-order generated by uninorms was introduced. This pre-order was inspired by a partial order induced by t-norms which was suggested by Karaçal and Kesicioğlu in [6]. Finally, as shown in [5], it is possible to construct operations on a lattice L which are both, proper uninorms and proper nullnorms, if there exist incomparable elements $\mathbf{b}_1 \in L$ and $\mathbf{b}_2 \in L$ such that the following conditions are fulfilled

$$\mathbf{b}_1 \wedge \mathbf{b}_2 = \mathbf{0}, \quad \mathbf{b}_1 \vee \mathbf{b}_2 = \mathbf{1},$$
$$(\forall x \in L)(x = (x \wedge \mathbf{b}_1) \vee (x \wedge \mathbf{b}_2)).$$

We will discus other possible conditions under which it is possible to construct such operations. We will be interested also in (pre-)order induced by these operations.

In [2] the authors showed that on the unit interval there is no proper uninorm solving the well-known Frank functional equation (see [3]). We point out that this is not the case when considering uninorms on $[0, 1]^n$ for $n > 1$.

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A Dimensionality Reduction Approach for Qualitative Preference Aggregation

Quentin Brabant[1], Miguel Couceiro[1], Fabien Labernia[2], Amedeo Napoli[1]

[1] LORIA (CNRS, Inira Nancy Grand Est - Université de Lorraine
{quentin.brabant, miguel.couceiro, amedeo.napoli}@inria.fr
[2] LAMSADE (CNRS - Université Paris-Dauphine)
fabien.labernia@dauphine.fr

1 Qualitative preference aggregation models

In this paper we briefly present a method for reducing the dimensionality of data in a qualitative preference aggregation framework. For a more complete description of this approach, see [4]. For an alternative approach based on rough sets theory, see [1].

We consider the following setting. X is a set of alternatives that are evaluated according to a set of criteria represented by there indices: $[n] = \{1, \dots, n\}$. For an alternative $x \in X$ we denote by $(x_1, \dots, x_n) \in L^n$ the tuple of the evaluations of x in each crieterion. L is called the *evaluation space*, and is a distributive lattice for which we denote respectively by 0 and 1 the minimal and maximal element. We consider a binary preference relation $\preccurlyeq$ between the alternatives that can be expressed in terms of a *utility function*:

$$\forall x, y \in X : x \preccurlyeq y \Rightarrow U(x) \leq U(y),$$

where $U : X \to L$ associates a global evaluation on L to each alternative, and is obtained through the aggregation of the evaluations in criteria by a Sugeno integral $\mathcal{S}_\mu : L^n \to L$. In other words we have $U(x) = \mathcal{S}_\mu(x_1, \dots, x_n)$. The Sugeno integral defined over distributive lattices [3], is expressed

$$\mathcal{S}_\mu(x_1, \dots, x_n) = \bigvee_{I \subseteq [n]} \mu(I) \bigwedge_{i \in I} x_i,$$

where $\mu : 2^{[n]} \to L$ a capacity, that is to say a non-decreasing set function on $[n]$, with $\mu(\emptyset) = 0$ and $\mu([n]) = 1$. Capacities (and Sugeno integrals) are defined by a value on L for each subset of $[n]$, and therefore carry an intrinsic complexity, that grows exponentially with n. We now consider a set $\mathscr{D} = \{(\mathbf{x}^1, y^1), \dots, (\mathbf{x}^m, y^m)\} \subseteq L^n \times L$, where each $\mathbf{x}^i = (x^i_1, \dots, x^i_n) \in L^n$ is a tuple of evaluations in n critieria, and y^i is a utility value associated to $\mathbf{x}^j$. We want to learn a Sugeno integral $\mathcal{S}_\mu$ that generalizes these data. Ideally this function would be such that $\mathcal{S}_\mu(\mathbf{x}^j) = y^j$ for any $j \in \{1, \dots, m\}$. However, it is very common that no such function exists: in that case $\mathscr{D}$ is said to be *inconsistent*, and we aim at learning a Sugeno integral that realizes the prediction of y^j for each element, with an error as low as possible. Because of the nature of capacities, this optimization problem is on 2^n variables, and is therefore hard to solve when a high number of criteria is considered.

2 Dimensionnality reduction based on quality measure

By a *quality measure* over $\mathscr{D}$ we mean a degree with which $\mathscr{D}$ satisfies a certain hypothesis. In this presentation we consider two of such measures.

The first quality measure is the *monotonicity degree*, that is, the ratio of pairs $\{i, j\} \subseteq \{1, \ldots, m\}$ that satisfy the following condition

$$y^i > y^j \Rightarrow \exists k \in [n] : y^i_k > y^j_k.$$

This condition can be seen as a generalization of the Pareto condition to partially ordered evaluation spaces. The second quality measure is the *compatibility degree*, that is, the ratio of pairs satisfying the the condition

$$\exists \mathcal{S}_\mu : [\mathcal{S}_\mu(\mathbf{x}^i) = y^i \text{ and } \mathcal{S}_\mu(\mathbf{x}^j) = y^j]. \tag{1}$$

This condition is justified by results from [2] that apply only when L is totally ordered. Indeed it can be shown that $\mathscr{D}$ is consistent if and only if (2) is true for any pair from $\mathscr{D}$. Moreover, for a given pair this condition can be checked in a linear time w.r.t. n. Hence, provided that L is totally ordered, the compatibility degree is both theoretically meaningful and practically interesting. If L is not totally ordered, the monotonicity degree is the quality measure that makes sense.

The principle of the algorithm for dimensionality reduction that we propose is to iteratively remove a criterion, in order to minimize the decrease of the quality of the dataset at each step. Criteria are deleted until it is impossible to remove a criterion without decreasing the quality of the data below a certain ratio α. This algorithm was tested on empirical data [1] and allowed a reduction of the number of criteria from 7 to 3. Aggregation models trained on original data and on data reduced to 3 criteria showed to have similar accuracy. On the other hand, models trained on data with only 2 criteria left had significantly worse accuracy, suggesting that a reduction to 3 criteria constitutes the best compromise between simplicity and accuracy for these data.

Future research work should include further empirical studies and should aim to determining a procedure for deciding the optimal value of α, currently being set by hand.

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Directional monotonicity and ordered directional monotonicity

H. Bustince[a,b⋆], J. Fernandez[a,b], R.Mesiar[c], A. Kolesárová[d], B. Bedregal[e], G. P. Dimuro[f], and E. Barrenechea[a,b]

[a] Dept. de Automática y Computación, Universidad Pública de Navarra, Campus Arrosadia, s/n, 31.006 Pamplona, Spain
[b] Institute of Smart Cities, Universidad Pública de Navarra, Campus Arrosadia, s/n, 31.006 Pamplona, Spain
[c] Slovak University of Technology, Radlinskeho 11, Bratislava, Slovakia
[d]Institute of Information Engineering, Automation and Mathematics, Slovak University of Technology, Radlinskeho 9, Bratislava, Slovakia
[e]Departamento de Informática e Matemática Aplicada, Universidade Federal do Rio Grande do Norte, Campus Universitario, s/n, Lagoa Nova, CEP 59078-970, Natal, Brazil
[f] Centro de Ciências Computacionais, Universidade Federal do Rio Grande, Av. Itália, km 08, Campus Carreiros, 96201-900 Rio Grande, Brazil

bustince@unavarra.es
fcojavier.fernandez@unavarra.es
anna.kolesarova@stuba.sk
bedregal@dimap.ufrn.br
gracaliz@furg.br

Monotonicity is one of the key features of aggregation functions ([3]) both in the real setting and when dealing with extensions of fuzzy sets. Its requirement is natural in many applied fields, as, for instance, image processing or decision making. However, in some cases such requirement can be too strong. Consider, for instance, the case of the mode in image processing. Although it provides a very good tool to remove some kinds of noise, it is not monotone, so it can not be considered to be an aggregation function ([1]). In this talk, and based on the idea of directional monotonicity [2], we introduce the notion of a pre-aggregation function([4]), as a function which satisfies the same boundary conditions as an aggregation function but for which only monotonicity along a fixed direction rather than monotonicity along any direction is required. In this sense, pre-aggregation functions encompass many relevant examples of functions which do not fall into the scope of usual aggregation functions, as, for instance, the (properly defined) mode. We also discuss three different construction methods of such pre-aggregation functions and we present several examples of pre-aggregation functions which are not aggregation functions. In particular, one of this method is based on the usual definition of the Choquet integral on a discrete setting, but

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replacing the product by other functions. The pre-aggregation functions built in this way can be used to design a fuzzy rule-based classification system which outperforms some of the examples of such systems which can be found in the literature.

Furthermore, and taking into account that for some applications such as image processing directions along which variation data may change from one point ot another and may depend on the relative size of the inputs, we also discuss the notion of ordered directionally monotone function.

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A Generalized Form of a Functional Equation Related to Scientific Laws

Juan C. Candeal

Universidad de Zaragoza. Departamento de Análisis Económico
Gran Vía 2, 50005-Zaragoza (SPAIN).
candeal@unizar.es

Abstract. A very general functional equation which is reminiscent of that established for certain 'laws of sciences' is considered. Specifically, this equation involves partial independence with respect to ordinal scales for the input variables and a generalized form of an interval scale for the output variable. The equation is established in the setting of aggregation operators which means that both the input and the output variables are real-valued functions defined on an abstract space. The solution is based upon a characterization of aggregation operators which are comparison meaningful with respect to independent ordinal scales, nondecreasing, and idempotent. Some other related functional equations are analyzed and an application to the theory of social choice is also shown.

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The HOGS procedure: a sampling approach to aggregate subgroups of a poset

G. Caperna, G. Boccuzzo

University of Padova, Department of Statistical Science

Abstract

The simplification of complex and unobservable concepts is the reason of the interest for synthetic measures in applied sciences. This interest led to different approaches of formalization and methodologies [1].
Despite the use of synthetic indicators is widespread, many methodological issues arise. Especially in social science, many concepts are subjective, and are often measured on an ordinal or dichotomous scale in large surveys that involve thousands of individuals.
The moving issues, arise from specific needs due to the application to social and official statistics: the use of ordinal data, the implementation of profiles' frequency in order to take the distribution of observed variables into account, the complexity of the poset structure in the case of big datasets, and the dimensional limitation of software respect to the mean of the rank of profiles among linear extensions (*average rank*), and the approximation of the average rank.
The HOGS procedure (Height Of Groups by Sampling) is conceived to handle a big set of observations using the results developed in the theory of partially ordered sets, overcoming some of the usual limitations of the approaches based on the approximation of average rank.
This new procedure computes the mean of average rank of groups of units, identified respect to common external explanatory variables, and allows the investigation of the relations between the ranks and the explanatory variables.
The utilization of poset theory allows the management of ordinal data while our method adapts it to the case of big datasets characterized by complex distributions.
The procedure consists in the observation of sub-samples of the population. In every sub-sample the average rank of every vertex (also called *profile* in its application to statistics where they are defined by the vectors of observed data) is approximated [2,3], with methods based on the dimension of the down/up sets and the set of incomparables as defined in [4]. Then the elements of every sub-sample are stratified by an external variable (gender, region, . . .), and then aggregated computing the mean of average ranks in every stratum. We apply this method to study the relation of socio-economic conditions to life satisfaction.

This work proposes specific advantages: first of all a new approach for the computation of an aggregated measure of an unobservable concept observed on multiple ordinal (or mixed) variables, secondly a procedure to study the effect of

explanatory (external) variables on the different levels of such a concept. Finally, the HOGS procedure allows to handle big data in small portions, offering a solution to take into account the frequency distribution of data and reducing the issues determined by the dimension of social surveys, often made by tens of thousands of observations.

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Preferences and aggregation operators over property spaces

Marta Cardin

Department of Economics
Università Ca' Foscari Venezia

Decision problems are characterized by a plurality of points of view. We have to consider the different dimensions from which the alternatives can be viewed in a multi-attribute decision model or the preferences of voters in a social choice problem.

In order to solve a decision problem we have to compare and rank a set of alternatives. In this note we consider the model of abstract Arrowian aggregation introduced in [10] that represents a decision problem in term of a set of Boolean properties specifying for every alternative a list of properties that are satisfied.

A *property space* is a pair $(X;\mathcal{H})$ where X is a non-empty set and $\mathcal{H}$ is a collection of non-empty subsets of X and if $x, y \in X$ and $x \neq y$ there exists $H \in \mathcal{H}$ such that $x \in H$ and $y \notin H$. The elements of $\mathcal{H}$ are referred to as properties and if $x \in H$ we say that x has property represented by the subset H. Our definition is slightly more general than that of [10] and we do not assume that the set X is finite as in [10].

The "property space" model has received attention in the literature on judgement aggregation for studying the problem of aggregating sets of logically interconnected propositions. Moreover it provides a general framework for representing preferences and then aggregation of preferences.

We prove that every property space defines a lattice structure on the set X and also that every distributive lattice is a property space characterized by the set $\mathcal{H}$ of prime filters.

Then we can provide an axiomatic characterization of property space structure. We then focus on aggregation operators $f\colon X^n \to X$ where $(X;\mathcal{H})$ is a property space and we consider operators that are componentwise compatible with the structure of property space of X. We study compatible aggregation operators that satisfy properties of monotonicity and independence and we obtain a characterization of Sugeno integral in the framework of property spaces.

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On a New Class of Uninorms on Bounded Lattices

Gül Deniz Çaylı[1], Funda Karaçal[2], and Radko Mesiar[3] ⋆

[1] Department of Mathematics, Faculty of Sciences, Karadeniz Technical University, 61080 Trabzon, Turkey
E-mail: guldeniz.cayli@ktu.edu.tr

[2] Department of Mathematics, Faculty of Sciences, Karadeniz Technical University, 61080 Trabzon, Turkey
E-mail: fkaracal@yahoo.com

[3] Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 81005 Bratislava, Slovakia
E-mail: mesiar@math.sk

Abstract. We study and propose some new construction methods to obtain uninorms on bounded lattices. Considering an arbitrary bounded lattice L, we show the existence of idempotent uninorms on L for any element $e \in L\backslash\{0,1\}$ playing the role of a neutral element. By our construction method, we obtain the smallest idempotent uninorm and the greatest idempotent uninorm with the neutral element $e \in L\backslash\{0,1\}$. We see that the obtained uninorms are conjunctive and disjunctive uninorms, respectively. On the other hand, if L is not a chain, we also provide an example of an idempotent uninorm which is neither conjunctive nor disjunctive.

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Quasideviation means in insurance

Jacek Chudziak

Faculty of Mathematics and Natural Sciences, University of Rzeszów,
ul. Prof. St. Pigonia 1, 35-310 Rzeszów, Poland

Abstract. Let $I \subseteq \mathbb{R}$ be an open interval. A function $D : I \times I \to \mathbb{R}$ is said to be *a quasideviation* provided it satisfies the following three conditions:
(D1) for every $x, y \in I$, $D(x, y)$ is of the same sign as $x - y$;
(D2) for every $x \in I$, the function $I \ni t \to D(x, t) \in \mathbb{R}$, is continuous;
(D3) for every $x, y \in I$ such that $x < y$, the function

$$(x, y) \ni t \to \frac{D(y, t)}{D(x, t)} \in \mathbb{R},$$

is strictly increasing.
The notion of a quasideviation has been introduced by Zs. Páles [2]. Quasideviations are generalizations of deviations, considered earlier by Z. Daróczy [1]. In [3] it has been proved that if $D : I \times I \to \mathbb{R}$ is a quasideviation, then for every $n \in \mathbb{N}$, $x_1, ..., x_n \in I$ and $\lambda_1, ..., \lambda_n \in [0, \infty)$ with $\sum_{i=1}^{n} \lambda_i > 0$, equation

$$\sum_{i=1}^{n} \lambda_i D(x_i, t) = 0 \qquad (1)$$

has a unique solution $t_0 \in [\min\{x_i : i \in \{1, .., n\}\}, \max\{x_i : i \in \{1, .., n\}\}]$. In this way, equation (1) defines a mean, called *a quasideviation mean of* $x_1, ..., x_n$ *weighted by* $\lambda_1, ..., \lambda_n$. It turns out that some quasideviation means are closely related to an important notion of insurance mathematics, namely *the zero utility principle*. Applying the results in [3], we prove several properties of that principle.

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Using aggregation functions on structured data: a use case in the FIGHT-HF project

Kevin Dalleau[1], Miguel Couceiro[1], Marie-Dominique Devignes[1], Chedy Raissi[1], and Malika Smail-Tabbone[1]

LORIA (CNRS, Inria NGE, University of Lorraine), Nancy, France
kevin.dalleau@loria.fr

Abstract. Heart Failure is a serious condition, affecting more and more people every year. The project FIGHT-HF aims at getting a better understanding of the disease and finding innovative ways to prevent and treat it. In order to enable an evidence-based decision making and design novel therapeutic strategies from the collected data, efficient data integration and mining tools must be used. We propose a method to aggregate complex graphs into simple weighted graphs, and to apply it on these data, containing complex relations between entities of various nature.

Keywords: Graph aggregation, Complex graphs, Clustering, Heart Failure

Heart Failure (HF) is a pandemic heart disease and a major public health issue, particularly on elderly people. The ambition of "FIGHT-HF", a French national research program coordinated by Prof. P. Rossignol (Regional University Hospital, Nancy, France), is to engage the battle against heart failure, by getting a better understanding of the origins of the disease, identifying and validating new biotargets, improving the current classification (nosography), and finding novel ways to prevent and treat the disease. A lot of data are available and will be generated, leading to the need for cutting-edge data integration and mining methods and tools.

We have at hand complex and heterogeneous data. The data include, among other, data regarding sociodemographical aspects of patients, biological and clinical features, drugs taken, and genetic profile. Once integrated, the data are schemaless, as the patient follow-up and measured variables are different from one data source to another. Consequently, the patient data will be represented as graphs, due to their capacity to represent schema-less data along with their relationships with background domain knowledge (protein functions and interactions, biological pathways, etc.). Indeed, taking advantage of the domain knowledge, new relations can be added. An example of how using this knowledge might be useful in connecting data (before aggregation) is presented fig. 1. The global graph can be viewed as a complex graph where a subset of vertices contains the patients nodes and other vertices correspond to attributes nodes. The first step of this work consisted in designing an aggregation method, to aggregate such complex graphs into a single graph containing only patient nodes, connected with weighted edges according to their similarity. This is a case of data fusion, where data of heterogeneous types are aggregated into one single graph.

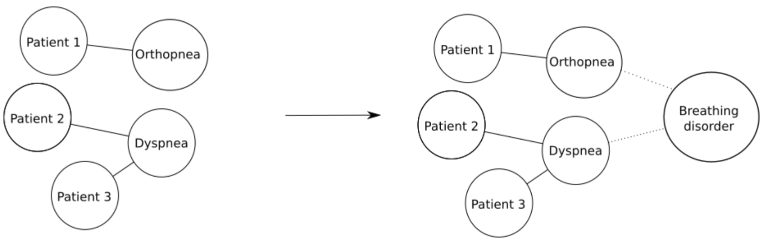

Fig. 1. A novel connection between patients is added, taking advantage of the domain knowledge

The application of this aggregation method on patient enables the use of specific clustering algorithms on the data, and find groups of patients. Each cluster should correspond to a subgroup of patients sharing the same form of heart failure, thus requiring a specific care strategy.

Transforming multiple patient data graphs (corresponding to multiple points of view) into more synthetic ones may lead to the proposal of new classifications of the considered disease as well as a consensual classification, using the principles of consensus theories [1].

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Ranking rules characterized by means of monometrics and consensus states

Bernard De Baets and Raúl Pérez-Fernández

KERMIT, Department of Mathematical Modelling, Statistics and Bioinformatics, Ghent University, Coupure links 653, B-9000 Gent, Belgium
{bernard.debaets,raul.perezfernandez}@ugent.be

Abstract. Social choice theory is the science studying what conclusions can be drawn from the preferences expressed by several voters over a set of candidates. Here, we consider the problem of ranking candidates, meaning we are dealing with the social choice subdiscipline of ranking rules. Given a set of preferences provided in the form of a profile of rankings (list of rankings), ranking rules obtain the ranking that best fits the profile. Many ranking rules have been proposed since the eighteenth century, when the works of Rousseau [15], Borda [2] and Condorcet [4] laid the foundations of social choice theory and, for some profiles, depending on the choice of ranking rule, one can obtain a different ranking as output. It will be no surprise then that there is no single absolute and universal 'best' ranking rule, as Arrow proved in [1]. Arrow's Theorem states that there is no ranking rule simultaneously satisfying a set of properties (non-dictatorship, unanimity and independence of irrelevant alternatives) that can be considered natural and desirable. Rather, the choice of a 'correct' ranking rule depends on the nature of the problem and which natural or desirable properties are prioritized. Some of the most well-known ranking rules in social choice theory are plurality [16], the Borda count [2] and the Kemeny rule [5].

However, not only the choice of the 'correct' ranking rule is important but also the notion of consensus state (or simply consensus). In general, a profile is said to be in a consensus state when determining a winning ranking is obvious. A consensus state can be seen as the domain of a partially defined ranking rule on the set of candidates. A trivial consensus state is unanimity [5], where each voter has the exact same preferences on the set of candidates. Another slightly more involved one is the existence of a Condorcet ranking [4], which is a ranking where every candidate is preferred by more than half of the voters to all the candidates ranked after him/her. Several authors such as Nitzan [10], Lerer and Nitzan [7], Campbell and Nitzan [3] and Meskanen and Nurmi [8,9] have advocated that ranking rules can be characterized by a consensus state and a distance function. In a recent paper [13], we stated that distance functions are actually too restrictive, and we introduced monometrics, a new type of functions that better fits with the nature of the problem. Like a distance function, a monometric satisfies the axioms of non-negativity and coincidence, but a monometric requires compatibility with a betweenness relation [11] and does not impose symmetry nor the triangle inequality.

The introduction of a betweenness relation determines a notion of closeness that needs to be preserved. The betweenness relation defined by Kemeny [5] seems to be the most natural one in social choice theory. The best-known ranking rule that can be characterized as minimizing the distance to a consensus state for some appropriate monometric is the Kemeny rule [5], where the search for the profile of rankings that is the closest to becoming unanimous is addressed considering a function that is both a distance function and a monometric w.r.t. Kemeny's betweeness relation [5]: the Kendall (tau) distance [6]. As another example of a recent such ranking rule, Rademaker and De Baets proposed in [14] a ranking rule that amounts to finding the ranking for which it holds that the votes are closest to satisfying a natural property: monotonicity. For a ranking $a \succ b \succ c$, monotonicity means that the number of voters preferring a to c should not be less than both the number of voters preferring a to b and the number of voters preferring b to c. In [12], this ranking rule was addressed considering monotonicity of the votrix [4,17] as the consensus state and proposing the use of monometrics to measure the distance to such consensus state. As discussed in [13], monometrics and consensus states play a key role in social choice theory characterizing most ranking rules.

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Different Types of De Morgan Identities in Nilpotent Systems and a Universal Description

J. Dombi[1] and O. Csiszár[2]

[1] University of Szeged
dombi@inf.u-szeged.hu
[2] Óbuda University, Institute of Informatics & Applied Mathematics
csiszar.orsolya@nik.uni-obuda.hu

In human thinking, averaging operators, where a high input can for a lower one compensate, play a significant role. The aggregative operator was first introduced in 1982 by Dombi, by selecting a set of minimal concepts that must be fulfilled by an evaluation-like operator. The concept of uninorms was introduced by Yager and Rybalov, as a generalization of both t-norms and t-conorms. By adjusting its neutral element, a uninorm is a t-norm if $\nu_* = 1$ and a t-conorm if $\nu_* = 0$. Uninorms turned out to be useful in many fields like expert systems, aggregation and fuzzy integral. The main difference in the definition of the uninorms and aggregative operators is that the self-duality requirement does not appear in uninorms, and the neutral element property is not in the definition for the aggregative operators. Now we distinguish between logical operators (with classical logical values on the boundaries, e.g. conjunction, disjunction, implication) and multicriteria decision tools (e.g. means, preferences) and here we consider multicriteria decision tools. Our main purpose is to consider generated nilpotent operators in an integrative frame and to examine the nilpotent self-dual generated operators. A general parametric framework for the nilpotent conjunctive, disjunctive, aggregative and negation operators is given and it is showed, how the nilpotent generated operator can be used for preference modelling. First we show that by shifting the generator function of a disjunction, we can get a conjunction and also operators that fulfil the self De Morgan property. We provide a general parametric formula for these operators, in which the conjunction, disjunction and the so-called aggregative operator differ only in one single parameter. This parameter has the semantical meaning of the level of expectancy.

Definition 1. *Let $f : [0,1] \to [0,1]$ be a strictly increasing bijection, $\nu \in [0,1]$, and $\boldsymbol{x} = (x_1, \ldots, x_n)$, where $x_i \in [0,1]$ and let us define the general operator by*

$$o_\nu(\boldsymbol{x}) = f^{-1}\left[\sum_{i=1}^{n}\left(f(x_i) - f(\nu)\right) + f(\nu)\right] = f^{-1}\left[\sum_{i=1}^{n} f(x_i) - (n-1)f(\nu)\right]. \tag{1}$$

A more general, weighted form of this operator is also examined.

Definition 2. *Let* $\boldsymbol{w} = (w_1, \ldots, w_n)$, $w_i > 0$ *real parameters,* $f : [0,1] \to [0,1]$ *a strictly increasing bijection,* $\nu \in [0,1]$. *The weighted generated operator is defined by*

$$a_{\nu,\boldsymbol{w}}(\boldsymbol{x}) := f^{-1}\left[\sum_{i=1}^{n} w_i(f(x_i) - f(\nu)) + f(\nu)\right]. \tag{2}$$

We examine the followiing question: for which parameter values satisfy the above-defined general operator for the De Morgan property with respect to the negation generated by $f(x)$. A commutative weighted generated operator fulfils the self De Morgan property if and only if $w = \frac{1}{n}$ or $\nu = \nu^*$, where $f(\nu^*) = \frac{1}{2}$; i.e. it has one of the following forms:

$$f^{-1}\left[\frac{1}{n}\sum_{i=1}^{n} f(x_i)\right] \tag{3}$$

or

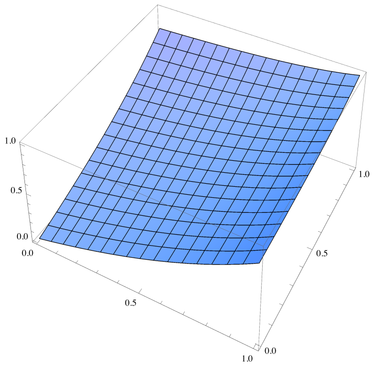
$$f^{-1}\left[w\left(\sum_{i=1}^{n} f(x_i) - \frac{n}{2}\right) + \frac{1}{2}\right]. \tag{4}$$

The weighted generated operator of the form $f^{-1}\left[w\left(\sum_{i=1}^{n} f(x_i) - \frac{n}{2}\right) + \frac{1}{2}\right]$, is commutative and satisfies the self De Morgan property. For its nice properties it is sensible to give it a distinctive name. The operator

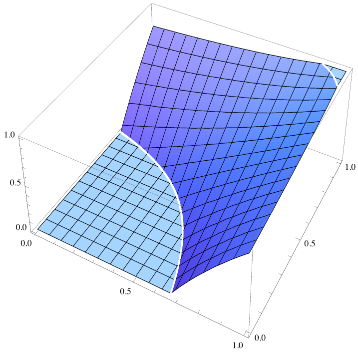
$$a_w(\mathbf{x}) = f^{-1}\left[w\left(\sum_{i=1}^{n} f(x_i) - \frac{n}{2}\right) + \frac{1}{2}\right], \tag{5}$$

where $w > 0$ is called weighted aggregative operator.

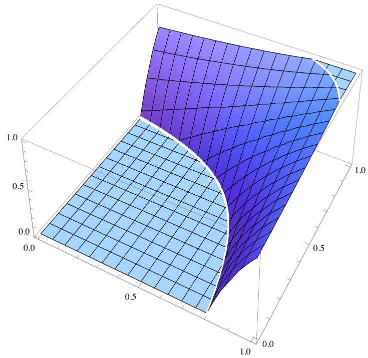
An important property of aggregation functions concerns the grouping character; i.e. whether it is possible to build a partial aggregation for subgroups of input values, and then to get the overall value by combining these partial results. We show that the weighted aggregative operator with weights $w \leq \frac{1}{n}$ is bisymmetric.



(a) $w = 0.5$



(b) $w = 0.75$



(c) $w = 1$

Fig. 1. The weighted aggregative operator a_w for $f(x) = \dfrac{1}{1 + \frac{\nu_d}{1-\nu_d}\frac{1-x}{x}}$, $\nu_d = 0.8$

We thoroughly examine the weighted aggregative operator of two variables. We show that $a_1(x, y)$ has a uninorm-like property and it satisfies the self De Morgan property as well. However, it is not associative (since $a_1(0, 1) = a_1(1, 0) = f^{-1}\left(\frac{1}{2}\right) = \nu^*$), and therefore cannot be a uninorm.

By substituting $n(x)$ and y in the commutative self De Morgan weighted aggregative operator, the operator $a(n(x), y)$ has certain properties which are similar to those expected of a preference operator.

Properties of uninorms and their generalization

Paweł Drygaś

Faculty of Mathematics and Natural Sciences, University of Rzeszów
35-959 Rzeszów, ul. Pigonia 1, Poland
paweldrs@ur.edu.pl

Uninorms were introduced by Yager and Rybalov in 1996 [8]. They are important generalizations of triangular norms and conorms because they allow a neutral element to lie anywhere in the unit interval rather than at zero or one as in the case of t-norm and t-conorm.

Fodor, Yager and Rybalov examined the general structure of uninorms, for example, the frame structure of uninorms and characterization of representable uninorms are presented in [3]. In the next papers we can find other properties (see [1, 2, 7]).

In this paper we present some properties of increasing, associative binary operations in the unit interval with a neutral element $e \in (0, 1)$. More specifically, we ask which of the property of uninorms will be preserved, if we omit some of the conditions in the definition of uninorms. We will deal with, among others, properties related to associativity, commutativity or a neutral element. This means that they will be discussed and compare certain properties of uninorms, week uninorms, semi-uninorms, pseudo-uninorms (see [8, 4, 6]) and summarized the relationship between different groups of assumptions from the definition of uninorm and its properties.

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Aggregation in logical consequence: revisiting from graded context

Soma Dutta[§], Mihir K. Chakraborty[⋆], and Andrzej Skowron[*]

[§]Vistula University, Warsaw, Poland
[*,§]Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland
[⋆]Jadavpur University, Kolkata, India
somadutta9@gmail.com mihirc4@gmail.com skowron@mimuw.edu.pl

Use of aggregation [1] has different purposes, and accordingly different treatments. Statistical nature of a data requires to be handled with typical statistical tools of aggregation, e.g., average, weighted average etcetera. Information expressed in terms of finitely many sentences of a language, or attributes of a concept, is aggregated using connectives like conjunction, disjunction. In case of consequence in logic, a formula α follows from a set of formulae X if for all possible valuation function T_i's if T_i is a model of X (i.e., every member of X is true under T_i or T_i satisfies every member of X), then α is also true under T_i (or T_i satisfies α). That is, in order to come to a decision that whether α follows from X, one needs to first check whether for a T_i, X is satisfied by T_i implies α is satisfied by T_i, and then aggregate the case for all possible T_i. Intuitively, if we consider each T_i as a source or agent or expert, who has different opinion regarding whether *if every member of X is true then α is true*, then the quantifier 'all' works as an aggregation operation accumulating the opinion of each T_i.

In this presentation we would concentrate on the nature of aggregation, which is required to have a faithful representation of the notion of logical consequence. In this regard, we shall pass on from the classical context of consequence to a graded context, as to learn the general nature of such an aggregation operation, instead of two values, dealing with many values would be more insighful.

Classically, the notion of semantic consequence, denoted as $\models$, is defined as $X \models \alpha$ iff for all possible valuation functions $T_i : \mathcal{F} \mapsto \{0,1\}$, from the set of all formulas to the value set $\{0,1\}$, if every member of X receives the value 1 (true) then α also receives the same. Identifying T_i with a set, consisting of those formulas from $\mathcal{F}$ which receives 1 under the function T_i, $X \models \alpha$ turns out to be $\forall_{T_i}\{\forall(x \in X \rightarrow x \in T_i) \rightarrow \alpha \in T_i\} \ldots (\Sigma)$.

That a formula is a logical consequence of a set of formulas, involves two meta-level operations; one is universal quantification ($\forall$), and the other is implication ($\rightarrow$). The above definition, given by (Σ), went through a generalization when Shoesmith and Smiley [4] proposed to consider a collection of $\{T_i\}_{i\in I}$ instead of all possible valuation functions, and the notion $X \models \alpha$ is relativized by $X \models_{\{T_i\}_{i\in I}} \alpha$. Based on that generalization, next generalization appears when instead of a collection of two-valued functions, $\{T_i\}_{i\in I}$ is considered to be a collection of fuzzy sets or functions from $\mathcal{F}$ to a general (complete) lattice structure L bounded by the top (1) and the least (0). That is, given any formula α, $\alpha \in T_i$ is now having a value, possibly other than the top or the least, in L. So, both the right hand side expression ($\alpha \in T_i$) and the left hand side expression ($\forall_x(x \in X \rightarrow x \in T_i)$) of $\rightarrow$ present in (Σ) get some values in L. That is, α follows from X, which may be denoted as $X \mid\!\approx_{\{T_i\}_{i\in I}} \alpha$, is now a many-valued notion. So $X \mid\!\approx_{\{T_i\}_{i\in I}} \alpha$

has a value, denoted as $gr(X \mid\approx_{\{T_i\}_{i\in I}} \alpha)$, in L. The value $gr(X \mid\approx_{\{T_i\}_{i\in I}} \alpha)$ is supposed to be the value of the sentence (Σ) with respect to the value set L endowed with some algebraic operation for the connective $\to$ present in (Σ).

As we go beyond the two-valued set up, operations for computing $\forall$ and $\to$ are required. Like the algebraic semantics of classical logic is captured in a Boolean algebra, and that of intuitionistic logic is obtained in a Heyting algebra, a complete residuated lattice structure $(L, *_m, \to_m, 0, 1)$ is required to have a sound interpretation for a notion of graded consequence, which is a fuzzy relation $\mid\sim$ from the set of all subsets of formulae ($P(\mathcal{F})$) to $\mathcal{F}$, satisying following conditions.
(GC1) If $\alpha \in X$, then $gr(X \mid\sim \alpha) = 1$.
(GC2) If $X \subseteq Y$, then $gr(X \mid\sim \alpha) \leq gr(Y \mid\sim \alpha)$.
(GC3) $\inf_{\beta\in Y} gr(X \mid\sim \beta) *_m gr(X \cup Y \mid\sim \alpha) \leq gr(X \mid\sim \alpha)$.

These are respectively the generalization of the properties, namely overlap, dilution, and cut of a classical consequence relation in the context of graded consequence relation. With respect to the value set L endowed with the above mentioned algebraic structure, the value of the expression (Σ), in graded context, is computed (algebraically) using the operators 'inf' and '$\to_m$' for $\forall$ and $\to$ respectively, and it becomes $gr(X \mid\approx \alpha)$ = $\inf_{i\in I}\{\inf_{x\in X} T_i(x) \to_m T_i(\alpha)\}$ ...(Σ').

The theory of graded consequence, based on the above set up, is developed to a considerable length [2, 3]. Here, we take an attempt to look back some of the drawbacks of the way of aggregation, proposed in (Σ'), from a practical perspective. The 'infimum' operation of a complete lattice is considered to be the algebraic translation of the linguistic quantifier $\forall$. Sometimes, $\forall$ is considered to be a generalization of conjunctive aggregation operations, which are algebraically represented by t-norms. Infimum (inf) is a special kind of t-norm having the property of idempotence, and is the greatest among all t-norms. From the practical perspective, if T_i's are considered to be the experts, who evaluate whether α follows from X (in terms of assigning values to $(\forall_x(x \in X \to x \in T_i) \to \alpha \in T_i)$ from L), then while aggregating all the experts' opinion using infimum, it boils down to the lowest; even if that lowest value does not appear as someone's opinion, or appears as an isolated value among a majority of greater-valued opinions, the lowest prevails in the final aggregation. So, aggregation using 'inf' often does not fit well in practical context. We hence, in this presentation, look for a different notion of aggregation which can be faithful to the theoretical interpretation of $\models$, and takes care of the practicality of aggregation too.

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A Group Decision Making Procedure in the Context of Non-Uniform Qualitative Scales

José Luis García-Lapresta[1] and David Pérez-Román[2]

[1] PRESAD Research Group, BORDA Research Unit, IMUVA, Dept. de Economía Aplicada, Universidad de Valladolid, Spain
[2] PRESAD Research Group, BORDA Research Unit, Dep. de Organización de Empresas y C.I.M., Universidad de Valladolid, Spain

In this paper, we introduce a new voting system in the context of ordered qualitative scales. The process is conducted in a purely ordinal way by considering an ordinal proximity measure that assigns an ordinal degree of proximity to each pair of linguistic terms of the qualitative scale. Once the agents assess the alternatives through the qualitative scale, the alternatives are ranked according to the medians of the ordinal degrees of proximity between the obtained individual assessments and the highest linguistic term of the scale. Since some alternatives may share the same median, a tie-breaking procedure is introduced; it is based on an appropriate linear order on the set of feasible medians. Some properties of the proposed voting system have been provided.

We first recall the notion of ordinal proximity between linguistic terms with values on a finite chain (linear order), introduced by García-Lapresta and Pérez-Román [GL-PR]. Consider an ordered qualitative scale $\mathcal{L} = \{l_1, \ldots, l_g\}$ such that $l_1 < \cdots < l_g$ and $g \geq 3$, whose elements are linguistic terms, and a chain $\Delta = \{\delta_1, \ldots, \delta_h\}$, with $\delta_1 \succ \cdots \succ \delta_h$. The elements of Δ have no meaning and they only represent different degrees of proximity, being δ_1 and δ_h the maximum and minimum degrees, respectively.

Definition 1. ([GL-PR]) An *ordinal proximity measure on $\mathcal{L}$ with values in Δ* is a mapping $\pi : \mathcal{L}^2 \longrightarrow \Delta$, where $\pi(l_r, l_s) = \pi_{rs}$ means the degree of proximity between l_r and l_s, satisfying the following conditions:

1. *Exhaustiveness*: For every $\delta \in \Delta$, there exist $l_r, l_s \in \mathcal{L}$ such that $\delta = \pi_{rs}$.
2. *Symmetry*: $\pi_{sr} = \pi_{rs}$, for all $r, s \in \{1, \ldots, g\}$.
3. *Maximum proximity*: $\pi_{rs} = \delta_1 \Leftrightarrow r = s$, for all $r, s \in \{1, \ldots, g\}$.
4. *Monotonicity*: $\pi_{rs} \succ \pi_{rt}$ and $\pi_{st} \succ \pi_{rt}$, for all $r, s, t \in \{1, \ldots, g\}$ such that $r < s < t$.

Consider a set of agents $A = \{1, \ldots, m\}$, with $m \geq 2$, that have to evaluate a set of alternatives $X = \{x_1, \ldots, x_n\}$, with $n \geq 2$, through an ordered qualitative scale $\mathcal{L} = \{l_1, \ldots, l_g\}$, $l_1 < \cdots < l_g$, with $g \geq 3$, and an ordinal proximity measure $\pi : \mathcal{L}^2 \longrightarrow \Delta$. The agents' judgments on the alternatives are collected in a *profile*, that is a matrix $V = (v_i^a)$ consisting of m rows and n columns of linguistic terms, where the element $v_i^a \in \mathcal{L}$ represents the linguistic assessment given by the agent $a \in A$ to the alternative $x_i \in X$.

For ranking the alternatives, the procedure is divided in the following steps.

1. For each alternative $x_i \in X$, consider the assessments obtained by x_i for all the agents: $v_i^1, \ldots, v_i^m \in \mathcal{L}$ (column i of the profile).
2. For each alternative $x_i \in X$, calculate the ordinal proximities between the assessments obtained by x_i and the highest linguistic term l_g:

$$\pi\left(v_i^1, l_g\right), \ldots, \pi\left(v_i^m, l_g\right) \in \Delta.$$

3. For each alternative $x_i \in X$, arrange the previous ordinal degrees in a decreasing fashion and select the median(s), M_i:
   (a) If the number of assessments is odd, then we duplicate the median. Thus, $M_i = (\delta_r, \delta_r)$ for some $r \in \{1, \ldots, h\}$.
   (b) If the number of assessments is even, then we take into account the two medians. Thus, $M_i = (\delta_r, \delta_s)$ for some $r, s \in \{1, \ldots, h\}$ such that $r \leq s$.
   Consequently, $M_i \in \Delta_2$, where Δ_2 is the *set of feasible medians*:

$$\Delta_2 = \{(\delta_r, \delta_s) \in \Delta^2 \mid r \leq s\}.$$

4. For ordering the medians of ordinal proximities obtained by different alternatives in the previous step, consider the linear order $\succeq$ on Δ_2 defined as

$$(\delta_r, \delta_s) \succeq (\delta_t, \delta_u) \;\Leftrightarrow\; \begin{cases} r+s < t+u \\ \text{or} \\ r+s = t+u \;\text{ and }\; s-r \leq u-t, \end{cases} \qquad (1)$$

   for all $(\delta_r, \delta_s), (\delta_t, \delta_u) \in \Delta_2$.
5. Finally, the alternatives are ranked according to the weak order $\succcurlyeq$ on X defined as $x_i \succcurlyeq x_j \;\Leftrightarrow\; M_i \succeq M_j$.

Since some alternatives can share the same median(s), it is necessary to devise a tie-breaking process for ordering the alternatives. We propose to use a sequential procedure based on Balinski and Laraki [BL]. It consists of dropping the median(s) of the respective alternatives that are in a tie, and then select the new median(s) of the remaining ordinal degrees for the corresponding alternatives and applying the procedure given in (1). The process continues until the ties are broken. It is important noticing that alternatives with different assessments never are in a final tie.

We note that the devised group decision making procedure satisfies anonymity, neutrality, independence of irrelevant alternatives, unanimity, monotonicity and replication invariance, among other properties.

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Interval-valued Extended Bonferroni Mean Operator and Its Applications in Multi-criteria Decision Making

Debashree Guha[1] and Radko Mesiar[2]

[1]Indian Institute of Technology Patna, Department of Mathematics,
Patna 801103, India
[2]Slovak University of Technology, Fac. of Civil Engineering,
Radlinského 11, 810 05 Bratislava, Slovakia
{debashree@iitp.ac.in;radko.mesiar@stuba.sk}

In recent years, there has been a growing research interest in analyzing extended Bonferroni mean (EBM) operator [5, 3] and applying it for solving decision making problems. The aim of EBM operator is to capture the heterogeneous relationship among the input data. Its use for the extension of fuzzy sets, such as, interval valued fuzzy sets or Atanassov's Intuitionistic fuzzy sets [4] are becoming very popular in the literature. For many of these applications we need to present a general method that also helps to build EBM for all environments. For this purpose, we first study the point-wise operations for intervals [1]. We discuss the concept of admissible orders between intervals [2] in terms of two aggregation functions. Next, we present the definition of interval-valued EBM operators based on a fixed admissible order and point-wise operations for intervals. In addition, we investigate several desirable properties of the proposed operators and we prove that some known specific aggregation operators are special cases of the proposed interval-valued EBM operators. The influence of the interaction among data on the proposed EBM is analyzed by observing the variations of the aggregated value with respect to the changes of interrelationship structure. We also investigate the behaviour of the proposed operators by using suitable examples. Then we apply the proposed operators in multi-criteria decision making problem.

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Lattices with a smallest set of aggregation functions

Radomír Halaš[1,⋆], Jan Kühr[1,⋆⋆], and Jozef Pócs[1,2,⋆⋆⋆]

[1] Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc, Czech Republic

[2] Mathematical Institute, Slovak Academy of Sciences, Košice, Slovakia

In a recent paper [1] we have studied aggregation functions $\mathcal{C}_L$ on a lattice L by a clone theory approach. Recall that a clone is a set of functions closed under projections and their composition. For any finite n-element lattice L we presented a set of at most $2n+2$ aggregation functions on L from which the clone $\mathcal{C}_L$ is generated.

The aim of our talk is to present a characterization of all finite lattices L for which the clone $\mathcal{C}_L$ is as small as possible, i.e. when it coincides with the clone of $0,1$-polynomial functions on L. Clearly, this problem is closely related to a well-known description of so-called order polynomially complete lattices [2]. These are shown to be completely determined by their tolerances, also several sufficient purely lattice conditions will be presented. In particular, all simple relatively complemented lattices or simple lattices for which the join (meet) of atoms (coatoms) is 1 (0) are of this kind.

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Generalized comonotonicity and a characterization of discrete Sugeno integrals on bounded distributive lattices

Radomír Halaš[1,⋆], Radko Mesiar[2,⋆⋆], and Jozef Pócs[1,3,⋆⋆⋆]

[1] Department of Algebra and Geometry, Faculty of Science, Palacký University Olomouc, Czech Republic. radomir.halas@upol.cz

[2] Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Slovakia radko.mesiar@stuba.sk

[3] Mathematical Institute, Slovak Academy of Sciences, Košice, Slovakia. pocs@saske.sk

Sugeno integral was introduced by Sugeno in [3] for fusion of information obtained in a fuzzy set characterized by its membership function. Recall that the Sugeno integral is formally introduced as the Lebesgue and Choquet integrals, replacing the standard arithmetic operations + and · on the real unit interval $[0,1]$ by the lattice operations supremum and infimum respectively. Considering its disjunctive normal representation enables to extend the original definition of the Sugeno integral to the more general case, i.e., where the interval $[0,1]$ is replaced by any bounded distributive lattice L, cf. [1] and [2].

If L is a bounded chain, it is well-known that the Sugeno integral on L can be characterized as a comonotone maxitive and min-homogeneous aggregation function. Our aim is to study a similar characterization for the Sugeno integral on any bounded distributive lattice L. We say that a pair $\mathbf{x}, \mathbf{y} \in L^n$ of vectors is called generalized comonotone if for every pair $i, j \in \{1, \ldots, n\}$ we have

$$(x_i \vee y_i) \wedge (x_j \vee y_j) = (x_i \wedge x_j) \vee (y_i \wedge y_j).$$

We show that generalized comonotonicity of L-valued vectors, generalizes the notions of comonotonicity as well as comparability of vectors in L^n. Based on this notion an axiomatization of L-valued Sugeno integrals is introduced.

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Some Notes on Nullnorms on Bounded Lattices

Mehmet Akif İnce [1] , Funda Karaçal [2] and Radko Mesiar [3]

[1] *Recep Tayyip Erdoğan University*
Department of Mathematics, 53100 Rize, Turkey
E-mail: ma-ince@hotmail.com

[2] *Karadeniz Technical University*
Department of Mathematics, 61080, Trabzon, Turkey
E-mail: fkaracal@yahoo.com

[3] *Slovak University of Technology*
Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Radlinského 11, 81 005 Bratislava, Slovakia
E-mail: mesiar@math.sk

Nullnorms are aggregation functions which are generalizations of triangular norms and triangular conorms with a zero element in the interior of the unit interval. In this study, we work on nullnorms which are defined on an arbitrary bounded lattice and we obtain interesting results. We introduce a general median-based method for constructing nullnorms by means of triangular norms and triangular conorms. Furthermore, we highlight a significant difference between the (existence and representation of) (idempotent) nullnorms on chains, distributive bounded lattices and general bounded lattices.

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Elicitation of fuzzy partial orders from incomplete preferences

Simon James

School of Information Technology, Deakin University, Australia
sjames@deakin.edu.au

Abstract. Recently we have proposed the framework of fuzzy partial order based preference relations for expression and aggregation of preferences. For a set of n alternatives/options, a FPO-based preference relation is represented as an $n \times n$ matrix A where each entry a_{ij} represents the degree to which option i is preferred to option j. While this kind of representation has been researched extensively, e.g. with multiplicative and additive relations, the key difference here is that a value of $a_{ij} = 1$ is interpreted as indicating option i is preferred to j, a value of $a_{ij} = 0$ means that option i is not preferred to j and values in-between represent partial preference. We therefore have the restriction that $a_{ij} > 0$ implies $a_{ji} = 0$, and the maximum expression of strength of preference is only crisp preference.

The perceived advantage of such a representation is that the aggregation of such matrices is less susceptible to extreme opinions, corresponding with a fuzzy version of the Kemeny distance. It also should align more with a natural expression of preference that is not as dependent on individual interpretations of a ratings scale.

While we have developed methods for obtaining final rankings of alternatives through aggregation and for repairing inconsistent matrices, a remaining problem is how to deal with large datasets involving many alternatives. In these situations, the elicitation of preferences becomes quite onerous on the decision maker and, on the computation side, the number of corresponding partial orders becomes expensively large. We propose to use a subset of triplets of comparison data, i.e. rankings provided between 3 alternatives, in order to obtain a final ranking of the alternatives. Our goal is to reduce the amount of information and effort required from the decision maker but still be able to obtain an acceptable ranking. Once the theory behind this process is developed, it can be evaluated on human subjects in terms of ease of preference elicitation and their agreement with the final ranking.

A New Ordering based on Fuzzy Implications

Balasubramaniam Jayaram

Department of Mathematics,
Indian Institute of Technology Hyderabad, INDIA
jbala@iith.ac.in

Through out this work, let $(\mathbb{P}, \leq, 0, 1)$ denote a bounded lattice. Let $\otimes, I : \mathbb{P} \times \mathbb{P} \to \mathbb{P}$ be a t-norm and an implication on $\mathbb{P}$, respectively. For definitions, please see [1], [2].

It is well-known that if I has the ordering property, viz.,

$$x \leq y \Longleftrightarrow I(x, y) = 1 \ . \tag{OP}$$

then one reclaims the underlying order on $\mathbb{P}$.

However, in [3], [4] Kesicioglu and Mesiar introduced an ordering on $\mathbb{P}$ based on an implication (See [2], Definition 1.1.1) as follows:

Definition 1 (Definition 8, [3]). *Let $I : \mathbb{P} \times \mathbb{P} \to \mathbb{P}$ be an implication on $\mathbb{P}$. For $x, y \in \mathbb{P}$ we say that*

$$x \preceq_I y \Longleftrightarrow \exists \ell \in \mathbb{P} \ni I(\ell, y) = x \ . \tag{1}$$

Theorem 1 (Propositions 1 & 2, [3]). *Let $(\mathbb{P}, \leq, 0, 1)$ be a bounded lattice and let I be an implication on $\mathbb{P}$. For all $a, b, c \in \mathbb{P}$, let I satisfy the following properties:*

$$I(b, I(a, c)) = I(a, I(b, c)) \ , \tag{EP}$$

$$I(a, b) = I(N_I(b), N_I(a)) \ , \tag{CP}$$

where N_I is the natural negation of I, viz., $N_I(a) = I(a, 0)$, and is involutive.

Then $\preceq_I$ as in (1) defines an order on $\mathbb{P}$. Further, $x \preceq_I y \Longrightarrow y \leq x$.

Firstly, it is easy to see that the greatest element in $(\mathbb{P}, \leq)$ becomes the least element in $(\mathbb{P}, \preceq_I)$, see **Remark 1(i)** in [3]. Further, the ordering of comparable elements w.r.to $\preceq_I$ is the reverse of the original ordering that they have in the underlying lattice.

In this work, given a bounded lattice $\mathbb{P}$ and an implication on it, taking into account the mixed monotonicity of fuzzy implications, we present an alternate way of obtaining order on the underlying $\mathbb{P}$, as follows:

Definition 2. *Let $I : \mathbb{P} \times \mathbb{P} \to \mathbb{P}$ be an implication on $\mathbb{P}$. For $x, y \in \mathbb{P}$ we say that*

$$x \sqsubseteq_I y \Longleftrightarrow \exists \ell \in \mathbb{P} \ni I(\ell, x) = y \ . \tag{2}$$

Theorem 2. *Let* $(\mathbb{P}, \leq, 0, 1)$ *be a bounded lattice and* $\otimes$ *be a t-norm and* I *an implication on* $\mathbb{P}$*, respectively. For all* $a, b, c \in \mathbb{P}$*, let* I *satisfy the following:*

$$I(a \otimes b, c) = I(a, I(b, c)) \ , \tag{LI}$$

$$I(1, b) = b \ . \tag{NP}$$

Then

(i) $\sqsubseteq_I$ *as in* (2) *defines an order on* $\mathbb{P}$*.*
(ii) Further, $x \sqsubseteq_I y \Longrightarrow x \leq y$*.*

A few interesting aspects of the above ordering are worthy of note:

- ⊕ Comparable elements preserve the ordering as in the underlying poset.
- ⊕ Note that the conditions in Theorem 2 are only sufficient and not necessary. Further, the conditions required for $\sqsubseteq_I$ to define an ordering are different and possibly more lenient than the ones required for $\preceq_I$. See Proposition 1 below and note that while (LI) implies (EP), in the absence of an involutive N_I it need not be true that I satisfies (CP)(N_I).
- ⊕ Thus any implication I on $\mathbb{P}$ that imposes the order $\preceq_I$ on $\mathbb{P}$ can also generate the ordering $\sqsubseteq_I$ on $\mathbb{P}$. If $\mathbb{I}_{\preceq}, \mathbb{I}_{\lessdot}$ denote the sets of all implications I on $\mathbb{P}$ that impose the orders $\preceq_I, \sqsubseteq_I$ on $\mathbb{P}$, respectively, then $\mathbb{I}_{\preceq} \subsetneq \mathbb{I}_{\lessdot}$.
- ⊕ Using Theorem 3, it can be shown that it is possible to obtain the original order on $\mathbb{P}$ even if I does not satisfy (OP).

Proposition 1. *Let* $(\mathbb{P}, \leq, 0, 1)$ *be a bounded lattice and* I *an implication on* $\mathbb{P}$ *such that* N_I *is strong. The following are equivelent:*

(i) I *satisfies* (CP) *w.r.to* N_I *and* (EP)*,*
(ii) There exists a t-norm $\otimes$ *on* $\mathbb{P}$ *such that* I *satisfies* (LI) *and* (NP)*.*

Theorem 3. *Let* $I : \mathbb{P} \times \mathbb{P} \to \mathbb{P}$ *be an implication on* $\mathbb{P}$ *and let* $\sqsubseteq_I$ *as in* (2) *define an order on* $\mathbb{P}$*. For any fixed* $\alpha \in \mathbb{P}$*, let us define the following:*

(i) $I_\alpha : \mathbb{P} \to \mathbb{P}$ *is the partial function* $I_\alpha(\gamma) = I(\gamma, \alpha)$*, and*
(ii) $F_\alpha = \{\beta \in \mathbb{P} | \alpha \leq \beta\}$*.*

The following are equivalent:

(i) $x \leq y \Longrightarrow x \sqsubseteq_I y$*.*
(ii) For each $\alpha \in \mathbb{P}$*,* $F_\alpha \subseteq \mathcal{R}an(I_\alpha) = \{\delta \in \mathbb{P} | \exists \gamma \in \mathbb{P} \ni I_\alpha(\gamma) = \delta\}$*.*

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Ordinal Sum of t-norms on Bounded Lattices

Funda Karaçal[1], Ümit Ertuğrul[2], and Gül Deniz Çaylı[3]

[1] Department of Mathematics, Faculty of Sciences, Karadeniz Technical University, 61080 Trabzon, Turkey
E-mail: fkaracal@yahoo.com
[2] Department of Mathematics, Faculty of Sciences, Karadeniz Technical University, 61080 Trabzon, Turkey
E-mail: uertugrul@ktu.edu.tr
[3] Department of Mathematics, Faculty of Sciences, Karadeniz Technical University, 61080 Trabzon, Turkey
E-mail: guldeniz.cayli@ktu.edu.tr

Abstract. In this study, a new method to construct a t-norm from t-norms defined on discrete bounded lattices is proposed. Underlying idea of this method is to combine bounded lattices over bounded lattice index set putting bounded lattices on indexes of bounded index set. This construction method is considered for semi-groups. Moreover this construction method is modified to construct t-norm from t-norms defined on sub-intervals of $[0, 1]$. Whether it is preserving continuity is investigated. Some illustrative examples are added for clarity.

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A perturbative approach to convergence theorems for nonlinear integrals

Jun Kawabe

Shinshu University, 4-17-1 Wakasato, Nagano 3808553, Japan
jkawabe@shinshu-u.ac.jp

Abstract. In this talk we present a unified approach to convergence theorems for nonlinear integrals that may be considered as aggregation functions of infinite inputs. A key tool is a perturbation of functional.

Keywords: nonadditive measure, nonlinear integral, convergence theorem, perturbation

1 Introduction

Aggregation functions are used for aggregating a finite or infinite number of inputs into a single output value. In multicriteria decision making the Choquet and the Sugeno discrete integrals are typical examples of aggregation functions of finite inputs and they are obviously continuous with respect to each of their inputs. This property is a guarantee for certain robustness and consistency and a non chaotic behavior.

By contrast the Choquet integral $\mathrm{Ch}(\mu, f)$ and the Sugeno integral $\mathrm{Su}(\mu, f)$ for a measurable function $f\colon X \to [0,\infty]$ and a nonadditive (also called monotone) measure $\mu\colon \mathcal{A} \to [0,\infty]$ on a measurable space $(X, \mathcal{A})$ may be considered as aggregation functions of infinite inputs. For those aggregation integrals their continuity corresponds to the convergence theorem of integrals, which means that the limit of the integrals of a sequence of functions is the integral of the limit function. Thus many attempts have been made to formulate the monotone, bounded, and dominated convergence theorems for nonlinear integrals such as the Choquet, the Šipoš, the Sugeno, and the Shilkret. However, to the best of knowledge, there is no unified approach to such convergence theorems in literature that are simultaneously applicable to both the Lebesgue integral as a linear integral and the Choquet, the Šipoš, the Sugeno, and the Shilkret integrals as nonlinear integrals. Thus the purpose of this talk is to present a unified approach to convergence theorems for such linear and nonlinear integrals.

A nonlinear integral may be viewed as a nonlinear functional $I\colon \mathcal{M}(X) \times \mathcal{F}^+(X) \to [0,\infty]$, where $\mathcal{M}(X)$ is the set of all nonadditive measures $\mu\colon \mathcal{A} \to [0,\infty]$ and $\mathcal{F}^+(X)$ is the set of all $\mathcal{A}$-measurable functions $f\colon X \to [0,\infty]$. So we formulate our general type of convergence theorem for such a functional. In particular we announce that the monotone, bounded, and dominated convergence theorems for nonlinear integrals follow from our convergence theorems for functionals regardless of the type of nonlinear integrals. A key tool is a perturbation of functional that manages not only the monotonicity of a functional I but also the small change of the value $I(\mu, f)$ arising as a result of adding small amounts to a measure μ and a function f in the domain of I.

2 One of main results

To state mathematically one of our main results we first collect some necessary definitions. A *nonadditive measure* is a set function $\mu\colon \mathcal{A} \to [0,\infty]$ such that $\mu(\emptyset) = 0$ and $\mu(A) \le \mu(B)$ whenever $A, B \in \mathcal{A}$ and $A \subset B$. It is called *finite* if $\mu(X) < \infty$.

Definition 1. *A functional $I\colon \mathcal{M}(X) \times \mathcal{F}^+(X) \to [0,\infty]$ is called an integral if it satisfies the following conditions:*

(i) $I(\mu, 0) = I(0, f) = 0$ *for every* $\mu \in \mathcal{M}(X)$ *and* $f \in \mathcal{F}^+(X)$.
(ii) I *is jointly monotone, that is,* $I(\mu, f) \le I(\nu, g)$ *for every* $\mu, \nu \in \mathcal{M}(X)$ *with* $\mu \le \nu$ *and* $f, g \in \mathcal{F}^+(X)$ *with* $f \le g$.

Definition 2. *Let $\mu, \nu\colon \mathcal{A} \to [0,\infty]$ be set functions and $f, g \in \mathcal{F}^+(X)$. The pair (μ, f) is called dominated by (ν, g) and written $(\mu, f) \prec (\nu, g)$ if $\mu(\{f \ge t\}) \le \nu(\{g \ge t\})$ for every $t \in \mathbb{R}$.*

Let Φ denote the set of all functions $\varphi\colon [0,\infty) \to [0,\infty)$ satisfying $\varphi(0) = \lim_{t\to+0} \varphi(t) = 0$. A function belonging to Φ is called a *control function.*

Definition 3. *An integral functional $I\colon \mathcal{M}(X) \times \mathcal{F}^+(X) \to [0,\infty]$ is called perturbative if, for each $p, q > 0$, there are control functions $\varphi_{p,q}, \psi_{p,q} \in \Phi$ satisfying the following perturbation: for any $\mu \in \mathcal{M}(X)$, $f, g \in \mathcal{F}^+(X)$, $\varepsilon \ge 0$, and $\delta \ge 0$, it holds that*

$$I(\mu, f) \le I(\mu, g) + \varphi_{p,q}(\delta) + \psi_{p,q}(\varepsilon)$$

whenever $\|f\|_\mu < p$, $\|g\|_\mu < p$, $\mu(X) < q$, and $(\mu, f) \prec (\mu+\delta, g+\varepsilon)$, where $\|f\|_\mu$ is the μ-essential supremum of f.

The following is one of our main results and gives a unified formulation of the bounded convergence theorem for linear and nonlinear integrals.

Theorem 1. *Let $I\colon \mathcal{M}(X) \times \mathcal{F}^+(X) \to [0,\infty]$ be an integral functional. Let $\mu \in \mathcal{M}(X)$ be finite. Assume that μ is autocontinuous, that is, $\mu(A \cup B_n) \to \mu(A)$ and $\mu(A \setminus B_n) \to \mu(A)$ whenever $A, B_n \in \mathcal{A}$ $(n = 1, 2, \dots)$ and $\mu(B_n) \to 0$. If I is perturbative, then the bounded convergence theorem holds for I with respect to μ, that is, for any uniformly μ-essentially bounded sequence $\{f_n\}_{n\in\mathbb{N}} \subset \mathcal{F}^+(X)$, if f_n converges in μ-measure to a function $f \in \mathcal{F}^+(X)$, then f is μ-essentially bounded and $I(\mu, f_n) \to I(\mu, f)$.*

The Lebesgue, the Choquet, the Šipoš, the Sugeno, and the Shilkret integrals are all perturbative. Other types of convergence theorems will be also announced during the talk.

Aggregation on some posets of aggregation functions

Anna Kolesárová[1] and Radko Mesiar[2]

[1]Slovak University of Technology, Fac. of Chemical and Food Technology, Radlinského 9, 812 37 Bratislava, Slovakia
[2]Slovak University of Technology, Fac. of Civil Engineering, Radlinského 11, 810 05 Bratislava, Slovakia
{anna.kolesarova;radko.mesiar}@stuba.sk

For a fixed natural number $n \geq 2$, the set $\mathcal{A}_n$ of all n–ary aggregation functions $A\colon [0,1]^n \to [0,1]$, equipped with the standard partial ordering $\leq$ of n–ary real functions, is a complete lattice. More details on aggregation functions can be found in [1, 2, 6, 7].

There are several subposets of $\mathcal{A}_n$ which form a bounded poset or even a bounded (complete) lattice. The aim of this contribution is to discuss aggregation on some of these posets. We recall several aggregation functions on the lattice $\mathcal{S}_n$ of all semicopulas [4, 5], on the lattice $\mathcal{Q}_2$ of all binary quasi-copulas [10], and also on the poset $\mathcal{C}_2$ of all binary copulas [10]. As particular examples recall aggregation of quasi-copulas based on aggregation of the corresponding adjoint operators [5] and the Darsow product of copulas [3].

Another distinguished lattice in $\mathcal{A}_n$ is formed by OWA operators [11] (for $n = 2$ we even get a chain). Aggregation of n–ary OWA operators is discussed from several points of view. In some cases the orness/andness parameters of OWA operators are also considered. Some details can be found in [9]. A similar discussion of aggregation of the Choquet integrals is also provided. We also introduce examples with rather poor results. This is, e.g., the case of the poset of triangular norms (or triangular conorms) [8], or the anti-chain of all weighted arithmetic means. Moreover, some open problems are outlined.

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Some properties of (C, I)-equivalences

Anna Król

Interdisciplinary Centre for Computational Modelling,
Faculty of Mathematics and Natural Sciences,
University of Rzeszów, Pigonia 1, 35-310 Rzeszów, Poland
annakrol@ur.edu.pl

This contribution deals with the notion of a fuzzy equivalence, as one of fuzzy connectives, whose definition depends on a fuzzy conjunction and implication. More precisely, let C, I be a fuzzy conjunction and implication, respectively. The function $E : [0,1]^2 \to [0,1]$ given by the formula

$$E_{C,I}(x, y) = C(I(x, y), I(y, x)),\ \ x, y \in [0,1]$$

will be called (C, I)-equivalence. Some properties of (C, I)-equivalences according to axioms of other notions of a fuzzy equivalence are presented. Some additional properties of (C, I)-equivalences, taking into consideration relevant properties of generators C and I are examined. Moreover, preservation of properties of (C, I)-equivalences in an aggregation process is indicated. Examination of preservation of axioms and properties of fuzzy connectives finds its applications e.g. in decision making, approximate reasoning and fuzzy control.

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Extremal weighted aggregation

Radko Mesiar and Andrea Stupňanová

Slovak University of Technology, Faculty of Civil Engineering
Radlinského 11,
810 05 Bratislava, Slovak Republic
{radko.mesiar,andrea.stupnanova}@stuba.sk

Weighted aggregation functions allow to introduce weights or importances of single inputs into the global aggregation. For more details see Chapter 6.4. of monograph [1]. In the standard introduction of weights we consider two n- tuples: the weighting vector $\mathbf{w} = (w_1, \ldots, w_n)$ with some constraints ($\mathbf{w} \in [0,1]^n$ and $\sum\limits_{i=1}^{n} w_i = 1$, or $\bigvee\limits_{i=1}^{n} w_i = 1$, etc.), and the input vector $\mathbf{x} = (x_1, \ldots, x_n)$ (mostly $\mathbf{x} \in [0,1]^n$ or $\mathbf{x} \in \mathbb{R}^n$). The order of weights and inputs is usually fixed, see, for example weighted (quasi-)arithmetic means. In some cases, only the order of weights is fixed, see OWA operators, for example.
The aim of this contribution is to discuss the case when neither the ordering of weights nor of the inputs is fixed, i.e., when any weight w_i can be assigned to any input x_j. Formally, we have $(n!)^2$ different situations to be discussed, characterized by

$$(\mathbf{w}_\sigma, \mathbf{x}_\tau) = ((w_{\sigma(1)}, x_{\tau(1)}), \ldots, (w_{\sigma(n)}, x_{\tau(n)})),$$

where $\sigma, \tau : \{1, \ldots, n\} \to \{1, \ldots, n\}$ are permutations.
We denote by $\mathcal{P}_n$ the set of all such permutations. For a considered weighted aggregation function $A : [0,1]^n \times [0,1]^n \to [0,1]$, we introduce two new aggregation functions $A^*, A_* : [0,1]^n \times [0,1]^n \to [0,1]$ given by

$$A^*(\mathbf{w}, \mathbf{x}) = \max\{A(\mathbf{w}_\sigma, \mathbf{x}_\tau) \mid \sigma, \tau \in \mathcal{P}_n\}$$

and (1)

$$A_*(\mathbf{w}, \mathbf{x}) = \min\{A(\mathbf{w}_\sigma, \mathbf{x}_\tau) \mid \sigma, \tau \in \mathcal{P}_n\}$$

Clearly, these extremal weighted aggregation functions satisfy

$$A_*(\mathbf{w}, \mathbf{x}) \le A(\mathbf{w}_\sigma, \mathbf{x}_\tau) \le A^*(\mathbf{w}, \mathbf{x})$$

for all $\mathbf{w}, \mathbf{x} \in [0,1]^n$ and $\sigma, \tau \in \mathcal{P}_n$.

We discuss and study some particular cases. So, for example, consider the weighted arithmetic mean $W : [0,1]^n \times [0,1]^n \to [0,1]$ given by

$$W(\mathbf{w}, \mathbf{x}) = \sum_{i=1}^{n} w_i x_i \qquad \text{(where} \sum_{i=1}^{n} w_i = 1).$$

Then

$$W^*(\mathbf{w}, \mathbf{x}) = W(\mathbf{w}_\alpha, \mathbf{x}_\beta)$$

for any permutations $\alpha, \beta \in \mathcal{P}_n$ such that the n-tuples $\mathbf{w}_\alpha$ and $\mathbf{x}_\beta$ are comonotone. In particular,

$$W^*(\mathbf{w}, \mathbf{x}) = W(\mathbf{w}^*_{\mathbf{x}}, \mathbf{x}),$$

where $\mathbf{w}^*_{\mathbf{x}} = \mathbf{w}_\gamma$ is comonotone with $\mathbf{x}, \gamma \in \mathcal{P}_n$. Similarly,

$$W_*(\mathbf{w}, \mathbf{x}) = W(\mathbf{w}_{\mathbf{x}*}, \mathbf{x}),$$

where $\mathbf{w}_{\mathbf{x}*} = \mathbf{w}_{\gamma^{-1}}$ is countermonotone with $\mathbf{x}$.
For OWA operators we have

$$OWA^*(\mathbf{w}, \mathbf{x}) = OWA(\mathbf{w}^*, \mathbf{x}),$$

where $\mathbf{w}^* = \mathbf{w}_\delta$, $w_{\delta(1)} \geq \cdots \geq w_{\delta(n)}$, and

$$OWA_*(\mathbf{w}, \mathbf{x}) = OWA(\mathbf{w}_*, \mathbf{x}),$$

where $\mathbf{w}_* = \bar{\mathbf{w}}_\delta = (w_{\delta(n)}, \ldots, w_{\delta(1)})$. Obviously, for any weighted aggregation function A, both A^* and A_* are symmetric aggregation functions. More, the idempotency of A, A^* and A_* either holds for all three aggregation functions, or for none of them.

We include also a discussion on extremal capacity - based integrals, where instead of an n-dimensional weighting vector $\mathbf{w} \in [0,1]^n$, a capacity $m : 2^{\{1,\ldots,n\}} \to [0,1]$ is considered [2],[3],[4].

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Monotonicity: A cornerstone of social choice theory

Raúl Pérez-Fernández and Bernard De Baets

KERMIT, Department of Mathematical Modelling, Statistics and Bioinformatics, Ghent University, Coupure links 653, B-9000 Gent, Belgium
{raul.perezfernandez,bernard.debaets}@ugent.be

Abstract. Electing a winner based on the preferences expressed by several voters over a set of candidates has been a relevant matter of study for centuries. Although some examples of voting procedures have been proved to be used already in the Ancient Greece [19], it is in the eighteenth century, with the works of Rousseau [17], Borda [2] and Condorcet [4], that social choice theory became a theoretical framework calling the attention of the scientific community.

In particular, we discuss here the sub-problem of ranking candidates according to the preferences of the voters, where each voter expresses his/her preferences in the form of a profile of rankings (list of rankings) on the set of candidates. Unfortunately, for some profiles, depending on the choice of ranking rule (function selecting a winning ranking given any profile of rankings), one can obtain a different ranking as output. Actually, Arrow's Theorem [1] states that there is no ranking rule simultaneously satisfying a set of properties (non-dictatorship, unanimity and independence of irrelevant alternatives) that can be considered natural and desirable. This fact leads to a vast literature where many different ranking rules are proposed: plurality [19], majority [8,17], the Borda count [2], veto [18], Condorcet's least reversals [4,5,10] or Kemeny's method [6], just mentioning some of the most relevant ones.

Several authors such as Nitzan [11], Lerer and Nitzan [7], Campbell and Nitzan [3], Nurmi [12], Meskanen and Nurmi [9,10] and Pérez-Fernández et al. [15] have called attention to the fact that not only the choice of a ranking rule is necessary but also agreeing on what we understand by consensus state (or simply consensus). Most ranking rules have been proved [3,10] to be characterized by a consensus state and a distance function[1]. In case the given profile of rankings is in the chosen consensus state it is selected as the winning ranking; otherwise the search for the closest profile of rankings in the chosen consensus state is addressed.

A trivial consensus state is unanimity [6], where each voter has the exact same preferences on the set of candidates. Another slightly more involved one is the existence of a weak Condorcet ranking [4], which is a ranking where every candidate is preferred by at least half of the voters to all the candidates ranked after him/her. Another example of consensus state is

[1] In [15], we stated that distance functions are actually too strict and proposed to consider monometrics instead.

based on the notion of monotonicity of the votrix[2] [13,16]. For a ranking $a \succ b \succ c$, monotonicity of the votrix means that the number of voters preferring a to c should not be less than both the number of voters preferring a to b and the number of voters preferring b to c. Relative positions between candidates are not gathered by the votrix and, in order to take this hitherto unexploited information into account, we proposed in [13] a new representation of votes, the votex, that is a natural extension of the votrix. After all, when a voter is providing a ranking $a \succ b \succ c$, he is actually declaring that he supports a over c stronger than both a over b and b over c, something that is not gathered by the votrix. Monotonicity of the votrix and monotonicity of the votex lead to two consensus states that are located in between unanimity (which is a too restrictive consensus state) and the presence of a weak Condorcet ranking (which is a too weak consensus state usually leading to the well-known 'voting paradox').

Anyhow, monotonicity can also be considered with respect to other representations of votes, such us the scorix [14] or the (contracted) profile. All these monotonicity-based consensus states lead to different types of compromises and the relation between all of them will be the main focus of the presentation.

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[2] A votrix is a reciprocal matrix that is the voting matrix [4,20] (also known as matrix of pairwise comparisons) of at least one profile of rankings.

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