# Large-deformation lattice model for dry-woven fabrics including contact

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1. The QC method in a nutshell

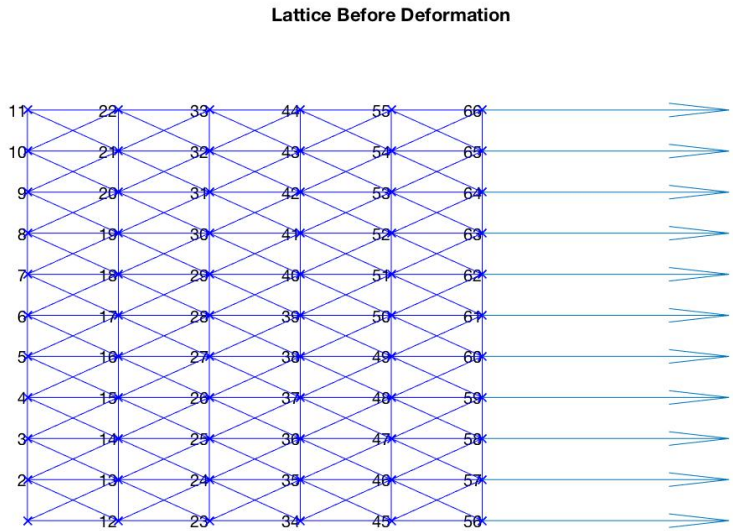
2. Future Work

# Scope of application of QC method

Used to determine the influence of localised microscale phenomena on large-scale responses:

- Atomistic computations, in particular for:
  1. presence of a defect in a solid (e.g. inclusion) [5]
  2. Indentation problem
- Crack initiation and propagation [4]
- Simulations of wovens (mesoscale) [2]

# Case of a network linear elastic trusses (conservative)




For this simple case, the energy of a lattice node is given by [1]:

$$E_i = \frac{1}{2}\sum_{j \in Bi} E_{ij} = \frac{1}{4}\sum_{j \in Bi} k_{ij}(\Delta L_{ij}{}^2) \tag{1}$$

with

$$\Delta L_{ij} = \sqrt{(\overrightarrow{x_j} - \overrightarrow{x_i})^2} - \sqrt{(\overrightarrow{X_j} - \overrightarrow{X_i})^2} \tag{2}$$

knowing that [1] :

$$E_{tot} = \sum_{node} E_i - \mathbf{f}^{\mathbf{t}}_{\mathbf{ext}}\mathbf{u} = E_{int} - \mathbf{f}^{\mathbf{t}}_{\mathbf{ext}}\mathbf{u} \tag{3}$$

we can determine the min of $E_{tot}$ using the variational principle:

$$\delta E_{tot} = 0 \; \forall \delta u \tag{4}$$

This problem can be solved using a standard Newton-Raphson algorithm [1]:

$$\delta\mathbf{u}(\mathbf{f}(\mathbf{u}^*) + \mathbf{K}(\mathbf{u}^*)\mathbf{d}(\mathbf{u})) = \mathbf{0} \tag{5}$$

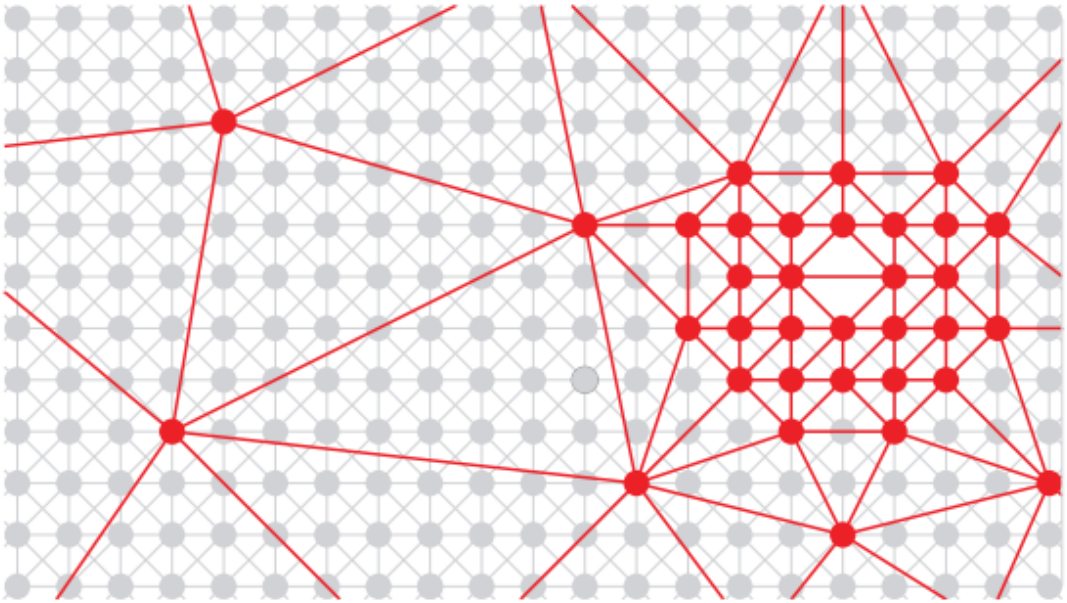
With:

- $\mathbf{f}(\mathbf{u}) = \frac{\partial \mathbf{E_{tot}}}{\partial \mathbf{u}}$
- $\mathbf{K}$ the stiffness matrix
- $\mathbf{u}^*$ the displacement vector at the previous iteration

Inefficiency comes from:

1. The displacement of all the nodes of the lattice has to be found
2. Every node has to be taken discretely into account construct the governing equations.

# Interpolation of the displacements within the triangles



Use of interpolation functions: $\mathbf{u} = \mathbf{u_r}\boldsymbol{\Psi}$

- The displacement of all the nodes depends only on the displacement of the representative node
- Since $r \ll n$ the solution space is reduced

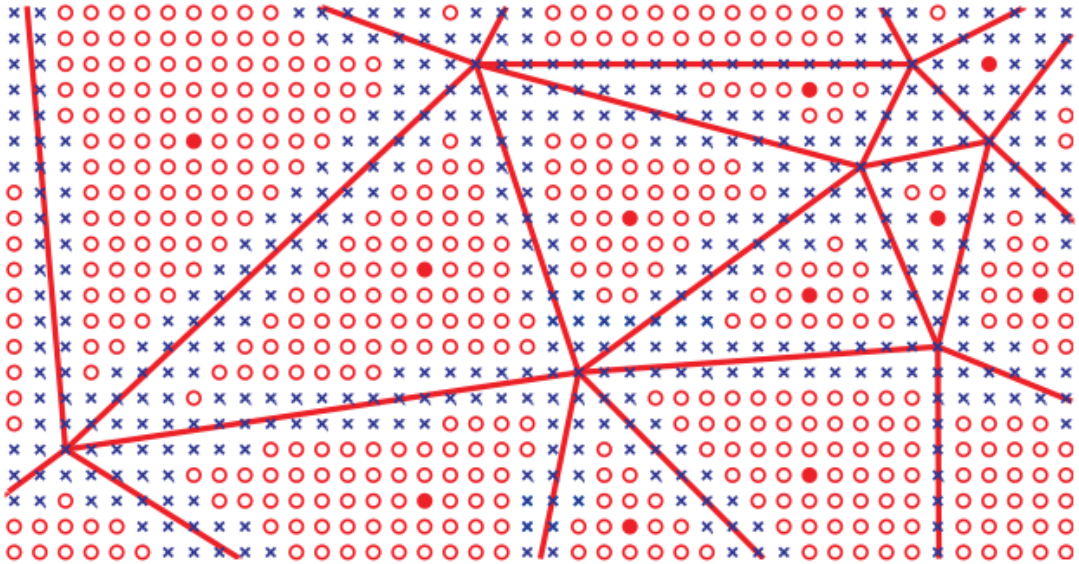
The equation to solve become [1]:

$$\delta \mathbf{u_r^T}(\Psi^T \mathbf{f}(\mathbf{u}^*) + \mathbf{\Psi^T K}(\mathbf{u}^*)\mathbf{\Psi d}(\mathbf{u_r})) = \mathbf{0}\, \forall \delta \mathbf{u_r} \tag{6}$$

But still all the lattice node must be visited to construct $\mathbf{f}$ and $\mathbf{K}$!

# Summation rule

Various summation rule exist.



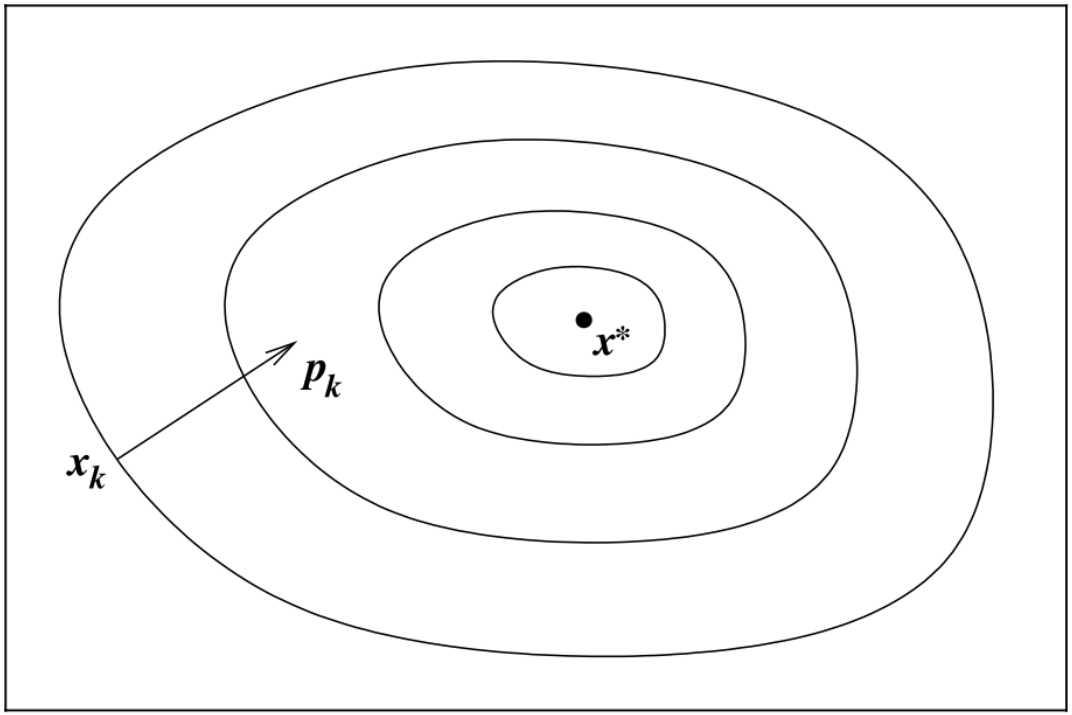
For the *central summation rule* nodes represent other node within the same interp. triangle through weights $\omega$ [1]:

$$f^{c,s}(\mathbf{u_r}) = \sum_{\mathbf{i}\in\mathbf{S}} \omega_\mathbf{i} \boldsymbol{\Psi}^\mathbf{T} \mathbf{f_i} \tag{7}$$

$$\mathbf{K}^{\mathbf{c},\mathbf{s}}(\mathbf{u_r}) = \sum_{\mathbf{i}\in\mathbf{S}} \omega_\mathbf{i} \boldsymbol{\Psi}^\mathbf{T} \mathbf{K_i} \boldsymbol{\Psi} \tag{8}$$

# Ill-posed problems

NR not suited for Ill-posed problem (e.g. rigid body motions)
Alternative like the Deepest Descent with Back track Line Search and Armijo Rule for unconstrained optimisation [3]



# Future Work

- Implement Trust Region Algorithm
- Inclusion of contacts (inequality constraints)
- Modelling of real materials subjected to large deformations

# References I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[3] Jorge Nocedal and Stephen J Wright.
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# References II

[4] VB Shenoy, R Miller, EB Tadmor, R Phillips, and M Ortiz.
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[5] Ellad B Tadmor, Michael Ortiz, and Rob Phillips.
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*Thanks for your attention*