Erratum to “On nonstrict means” [Aequationes Math. 54 (3)(1997) 308–327]

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Theorems 3, 4, 7, and 8 are incorrectly stated. The correct versions are as follows.

Theorem 3. *$M \in \mathcal{B}_{a,b,a}$ if and only if*

- *either*
$$M(x,y) = \min(x,y), \quad \forall\, x,y \in [a,b],$$
- *or*
$$M(x,y) = g^{-1}\sqrt{g(x)g(y)}, \quad \forall\, x,y \in [a,b],$$
where g is any continuous strictly increasing function on $[a,b]$, with $g(a)=0$,
- *or there exists a countable index set K and a family of disjoint subintervals $\{(a_k,b_k) : k \in K\}$ of $[a,b]$ such that*
$$M(x,y) = \begin{cases} g_k^{-1}\sqrt{g_k[\min(x,b_k)]\,g_k[\min(y,b_k)]} & \text{if there exists } k \in K \text{ such that} \\ & \min(x,y) \in (a_k,b_k), \\ \min(x,y) & \text{otherwise,} \end{cases}$$
where g_k is any continuous strictly increasing function on $[a_k,b_k]$, with $g_k(a_k)=0$.

Theorem 4. *$M \in \mathcal{B}_{a,b,b}$ if and only if*

- *either*
$$M(x,y) = \max(x,y), \quad \forall\, x,y \in [a,b],$$
- *or*
$$M(x,y) = g^{-1}\sqrt{g(x)g(y)}, \quad \forall\, x,y \in [a,b],$$
where g is any continuous strictly decreasing function on $[a,b]$, with $g(b)=0$,
- *or there exists a countable index set K and a family of disjoint subintervals $\{(a_k,b_k) : k \in K\}$ of $[a,b]$ such that*
$$M(x,y) = \begin{cases} g_k^{-1}\sqrt{g_k[\max(a_k,x)]\,g_k[\max(a_k,y)]} & \text{if there exists } k \in K \text{ such that} \\ & \max(x,y) \in (a_k,b_k), \\ \max(x,y) & \text{otherwise,} \end{cases}$$
where g_k is any continuous strictly decreasing function on $[a_k,b_k]$, with $g_k(b_k)=0$.

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Theorem 7. *$M \in \mathcal{D}_{a,b,a}$ if and only if*

- *either, for all $m \in \mathbb{N}_0$,*

$$M(x_1,\ldots,x_m) = \min_i x_i, \quad \forall\,(x_1,\ldots,x_m) \in [a,b]^m,$$

- *or, for all $m \in \mathbb{N}_0$,*

$$M(x_1,\ldots,x_m) = g^{-1}\sqrt[m]{\prod_i g(x_i)}, \quad \forall\,(x_1,\ldots,x_m) \in [a,b]^m,$$

  where g is any continuous strictly increasing function on $[a,b]$, with $g(a)=0$,

- *or there exists a countable index set K and a family of disjoint subintervals $\{(a_k,b_k) : k \in K\}$ of $[a,b]$ such that, for all $m \in \mathbb{N}_0$,*

$$M(x_1,\ldots,x_m) = \begin{cases} g_k^{-1}\sqrt[m]{\prod_i g_k[\min(x_i,b_k)]} & \text{if there exists } k \in K \text{ such that} \\ & \min_i x_i \in (a_k,b_k), \\ \min_i x_i & \text{otherwise,} \end{cases}$$

  where g_k is any continuous strictly increasing function on $[a_k,b_k]$, with $g_k(a_k)=0$.

Theorem 8. *$M \in \mathcal{D}_{a,b,b}$ if and only if*

- *either, for all $m \in \mathbb{N}_0$,*

$$M(x_1,\ldots,x_m) = \max_i x_i, \quad \forall\,(x_1,\ldots,x_m) \in [a,b]^m,$$

- *or, for all $m \in \mathbb{N}_0$,*

$$M(x_1,\ldots,x_m) = g^{-1}\sqrt[m]{\prod_i g(x_i)}, \quad \forall\,(x_1,\ldots,x_m) \in [a,b]^m,$$

  where g is any continuous strictly decreasing function on $[a,b]$, with $g(b)=0$,

- *or there exists a countable index set K and a family of disjoint subintervals $\{(a_k,b_k) : k \in K\}$ of $[a,b]$ such that, for all $m \in \mathbb{N}_0$,*

$$M(x_1,\ldots,x_m) = \begin{cases} g_k^{-1}\sqrt[m]{\prod_i g_k[\max(a_k,x_i)]} & \text{if there exists } k \in K \text{ such that} \\ & \max_i x_i \in (a_k,b_k), \\ \max_i x_i & \text{otherwise,} \end{cases}$$

  where g_k is any continuous strictly decreasing function on $[a_k,b_k]$, with $g_k(b_k)=0$.

It is noteworthy that, in each of the above theorems, the third case includes the first two. Indeed, we get the first case when no subinterval (a_k,b_k) is considered and we get the second case when only one subinterval is considered and if it coincides with (a,b). Consequently, only the third case could have been stated, the first two cases being merely degenerations of the third one.

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