

# Linear smoothing over arbitrary polytopes

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The conventional constant strain smoothing technique yields less accurate solutions than other techniques such as the conventional polygonal finite element method [1, 2]. In this work, we propose a linear strain smoothing scheme that improves the accuracy of linear and quadratic approximations over convex polytopes. The method relies on sub-division of the polytope into simplicial subcells; however instead of using a constant smoothing function, we employ a linear smoothing function over each subcell. This gives a new definition for the strain to compute the stiffness matrix. The convergence properties and accuracy of the proposed scheme are discussed by solving few benchmark problems. Numerical results show that the proposed linear strain smoothing scheme makes the approximation based on polytopes able to deliver the optimal convergence rate as in traditional quadrilateral and hexahedral finite elements. The accuracy is also improved, and all the methods tested pass the patch test to machine precision.

**Keywords:** Smoothed finite element method, polytope, patch test, polygonal finite element method.

## References

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