Extended Finite Element Method with Global Enrichment

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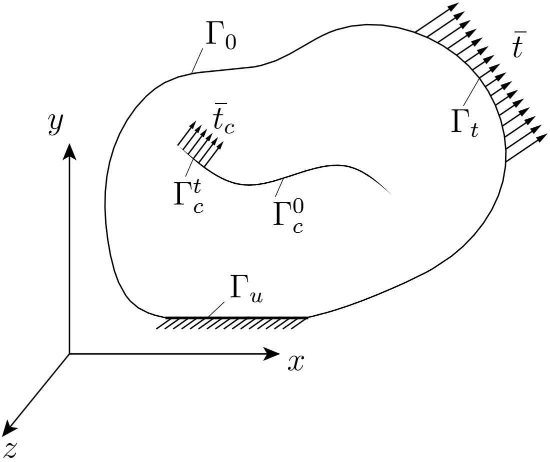
Outline

3D body geomery



$$\Gamma = \Gamma_0 \cup \Gamma_u \cup \Gamma_t \cup \Gamma_c$$

$$\Gamma_c = \Gamma_c^t \cup \Gamma_c^0$$

Governing equations

Equilibrium equations and boundary conditions:

$$
\begin{aligned}
\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} &= \mathbf{0} && \text{in} \quad \Omega \\
\boldsymbol{u} &= \bar{\boldsymbol{u}} && \text{on} \quad \Gamma_u \\
\boldsymbol{\sigma} \cdot \mathbf{n} &= \bar{\boldsymbol{t}} && \text{on} \quad \Gamma_t \\
\boldsymbol{\sigma} \cdot \mathbf{n} &= \mathbf{0} && \text{on} \quad \Gamma_c^0 \\
\boldsymbol{\sigma} \cdot \mathbf{n} &= \bar{\boldsymbol{t}}_c && \text{on} \quad \Gamma_c^t
\end{aligned}
$$

Kinematic equations:

$$\boldsymbol{\epsilon} = \nabla_s \boldsymbol{u}$$

Constitutive equations:

$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\epsilon}$$

Weak form of equilibrium equations

Find $\boldsymbol{u} \in \mathcal{U}$ such that $\forall \boldsymbol{v} \in \mathcal{V}^0$

$$\int_\Omega \boldsymbol{\sigma}(\boldsymbol{u}) : \epsilon(\boldsymbol{v})\ d\Omega = \int_\Omega \boldsymbol{b} \cdot \boldsymbol{v}\ d\Omega + \int_{\Gamma_t} \bar{\boldsymbol{t}} \cdot \boldsymbol{v}\ d\Gamma + \int_{\Gamma_c^t} \bar{\boldsymbol{t}}_c \cdot \boldsymbol{v}\ d\Gamma_c^t$$

where :

$$\mathcal{U} = \left\{ \boldsymbol{u} | \boldsymbol{u} \in \left(H^1(\Omega) \right)^3, \boldsymbol{u} = \bar{\boldsymbol{u}} \text{ on } \Gamma_u \right\}$$

and

$$\mathcal{V} = \left\{ \boldsymbol{v} | \boldsymbol{v} \in \left(H^1(\Omega) \right)^3, \boldsymbol{v} = 0 \text{ on } \Gamma_u \right\}$$

Motivation

- XFEM for industrially relevant (3D) crack problems
  - Requires robust methods for stress intensity evaluation.
  - Requires low solution times and ease of use.
- but standard XFEM leads to
  - Ill-conditioning of the stiffness matrix for “large” enrichment domains.
  - Lack of smoothness and accuracy of the stress intensity factor field along the crack front.
  - Blending issues close at the boundary of the enriched region.
  - Problem size for propagating cracks (“old” front-dofs must be kept for stability of time integration schemes).

Global enrichment XFEM

There exists different approaches to alleviate the above difficulties:

- Preconditioning (e.g. Moës; Menk and Bordas)
- Ghost penalty (Burman)
- Stable XFEM/GFEM (Banerjee, Duarte, Babuška, Paladim, Bordas) - behaviour for realistic 3D crack not clear.
- Corrected XFEM/GFEM (Fries, Loehnert)
- SIF-oriented (goal-oriented) error estimation methods for SIFs (Ródenas, Estrada, Ladevèze, Chamoin, Bordas)
- Restrict the variability of the enrichment within the enriched domain: doc-gathering, cut-off XFEM (Laborde, Renard, Chahine, Salün and the French team ;-)

Global enrichment XFEM

An XFEM variant is introduced which:

- Extends dof gathering to 3D through global enrichment.
- Employs point-wise matching of displacements.
- Employs integral matching of displacements.
- Enables the application of geometrical enrichment to 3D.

Related works

Similar concepts to the ones introduced herein can be found:

- In the work of Laborde et al.
  - $\rightarrow$ dof gathering
  - $\rightarrow$ point-wise matching

  (Laborde, Pommier, Renard, & Salaün, 2005)

- In the work of Chahine et al.
  - $\rightarrow$ integral matching

  (Chahine, Laborde, & Renard, 2011)

Related works

- In the work of Langlois et al.
  $\rightarrow$ discretization along the crack front
  (Langlois, Gravouil, Baieto, & Réthoré, 2014)

- In the s-finite element method
  $\rightarrow$ superimposed mesh
  (Fish, 1992)

Crack representation

Level set functions:

- $\phi(\mathbf{x})$ is the signed distance from the crack surface.
- $\psi(\mathbf{x})$ is a signed distance function such that:

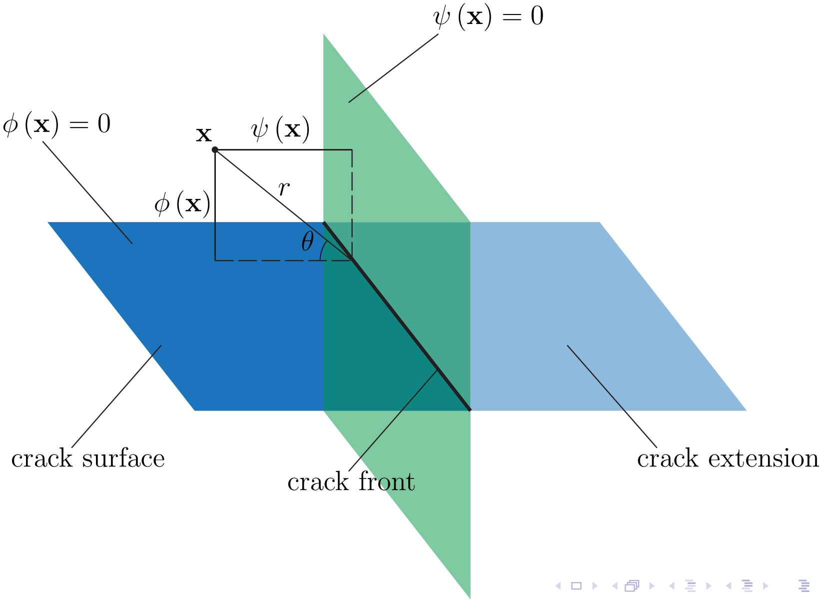
  $\rightarrow$ $\nabla\phi \cdot \nabla\psi = 0$

  $\rightarrow$ $\phi(\mathbf{x}) = 0$ and $\psi(\mathbf{x}) = 0$ defines the crack front

Polar coordinates:

$$r = \sqrt{\phi^2 + \psi^2}, \qquad \theta = \arctan\left(\frac{\phi}{\psi}\right)$$

Crack representation



Tip enrichment

Enriched part of the approximation for tip elements:

$$\mathbf{u_{te}}(\mathbf{x}) = \sum_K N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x}) \mathbf{c}_{Kj}$$

N_K^g are the global shape functions to be defined.

Tip enrichment functions:

$$F_j(\mathbf{x}) \equiv F_j(r, \theta) = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

Geometrical enrichment

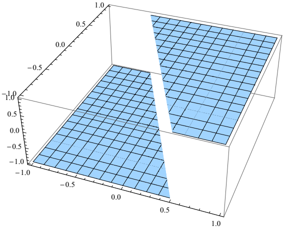
- Enrichment radius r_e is defined.
- Nodal values r_i of variable r are computed.
- The condition $r_i < r_e$ is tested.
- If true for all nodes of an element, the element is tip enriched.

Jump enrichment

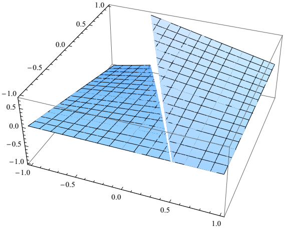
Jump enrichment function definition:

$$H(\phi) = \begin{cases} 1 & \text{for } \phi > 0 \\ -1 & \text{for } \phi < 0 \end{cases}$$

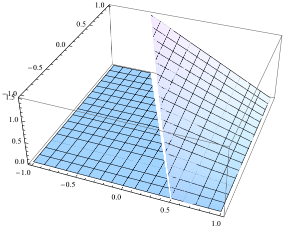
Shifted jump enrichment functions are used throughout this work.



$H(\phi)$



$N_1 H(\phi)$



$N_1(H(\phi) - H_1)$

Enrichment strategy

Motivation for an alternative enrichment strategy:

- Tip enrichment functions are derived from the first term of the Williams expansion.
- Displacements consist of higher order terms as well.
- Those terms are represented by:

  $\rightarrow$ the FE part

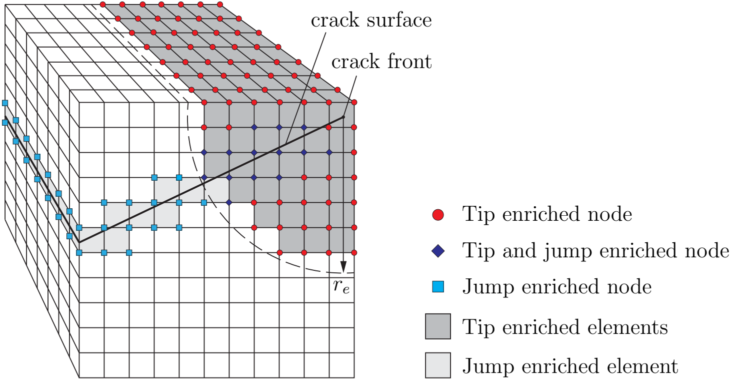
  $\rightarrow$ spatial variation of the tip enrichment functions

Enrichment strategy

- In the proposed method:
  - $\rightarrow$ no spatial variation is allowed
  - $\rightarrow$ higher order terms can only be approximated by the FE part
- Higher order displacement jumps can not be represented in tip elements.

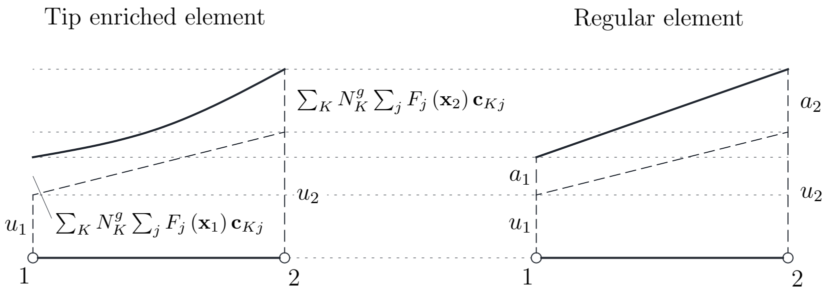
Enrichment strategy

Proposed enrichment strategy:



Both tip and jump enrichment is used for tip elements that contain the crack.

Tip and Regular Elements

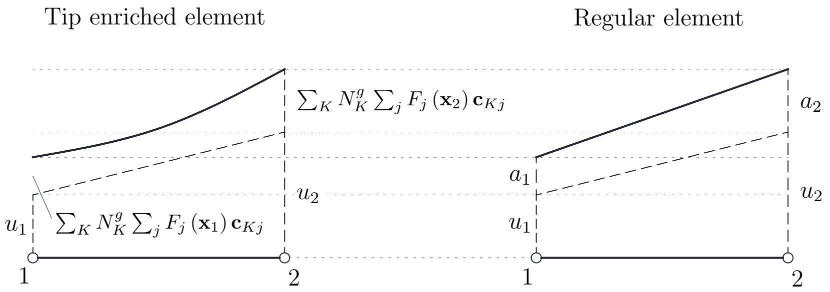


Displacement approximations of regular and tip elements:

$$\mathbf{u_r}(\mathbf{x}) = \sum_I N_I(\mathbf{x})\,\mathbf{u}_I + \sum_J N_J(\mathbf{x})\,\mathbf{a}_J$$

$$\mathbf{u_t}(\mathbf{x}) = \sum_I N_I(\mathbf{x})\,\mathbf{u}_I + \sum_K N_K^g(\mathbf{x})\sum_j F_j(\mathbf{x})\,\mathbf{c}_{Kj}$$

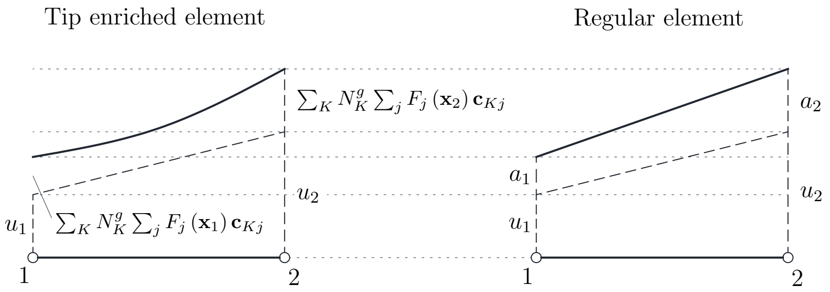
Tip and Regular Elements



Displacements are matched by imposing the condition:

$$\mathbf{u_r}\left(\mathbf{x}_I\right) = \mathbf{u_t}\left(\mathbf{x}_I\right)$$

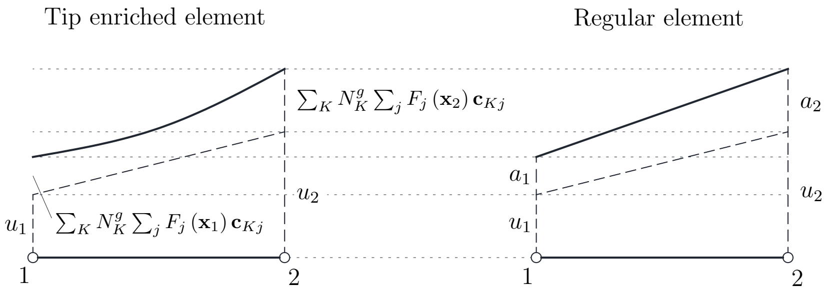
Tip and Regular Elements



Parameters $\mathbf{a}_I$ are obtained:

$$\mathbf{a}_I = \sum_K N_K^g(\mathbf{X}_I) \sum_j F_j(\mathbf{X}_I)\mathbf{c}_{Kj}$$

Tip and Regular Elements



Parameters $\mathbf{a}_I$ can be expressed as:

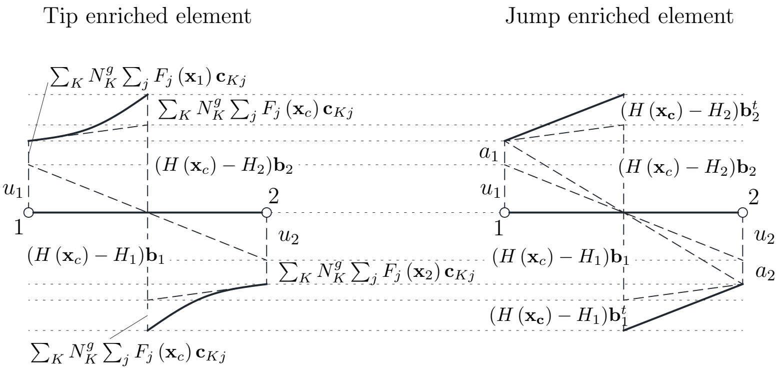
$$\mathbf{a}_I = \sum_K \sum_j T_{IKj}^{t-r}\mathbf{c}_{Kj}$$

Tip and Jump Elements

Displacement approximations of tip and jump elements:

$$
\begin{aligned}
\mathbf{u_j}(\mathbf{x}) &= \sum_I N_I(\mathbf{x})\,\mathbf{u}_I + \sum_J N_J(\mathbf{x})\,\mathbf{a}_J + \sum_L N_L(\mathbf{x})\,(H(\mathbf{x}) - H_L)\mathbf{b}_L + \\
&+ \sum_M N_M(\mathbf{x})\,(H(\mathbf{x}) - H_M)\mathbf{b}_M^t \;, \\
\mathbf{u_t}(\mathbf{x}) &= \sum_I N_I(\mathbf{x})\,\mathbf{u}_I + \sum_J N_J(\mathbf{x})\,(H(\mathbf{x}) - H_J)\mathbf{b}_J + \\
&+ \sum_K N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x})\,\mathbf{c}_{Kj}
\end{aligned}
$$

Tip and Jump Elements



Point-wise matching condition:

$$\mathbf{u_j}(\mathbf{x}_n) = \mathbf{u_t}(\mathbf{x}_n)$$

Tip and Jump Elements

The condition is imposed:

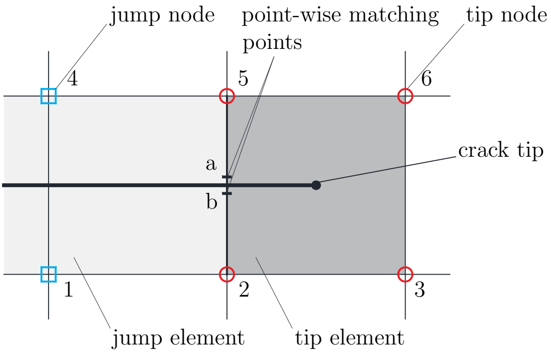
- at nodes $\rightarrow$ parameters $\mathbf{a}_I$ are obtained
- at additional points $\rightarrow$ parameters $\mathbf{b}_I^t$ are obtained:

$$(H(\mathbf{X}_I) - H_I)\,\mathbf{b}_I^t = \sum_K N_K^g(\mathbf{X}_I) \sum_j F_j(\mathbf{X}_I)\mathbf{c}_{Kj} - \sum_I N_I(\mathbf{X}_I)\mathbf{a}_I$$

Parameters $\mathbf{b}_I^t$ can be reformulated as:

$$\mathbf{b}_I^t = \sum_K \sum_j T_{IKj}^{t-j}\mathbf{c}_{Kj}$$

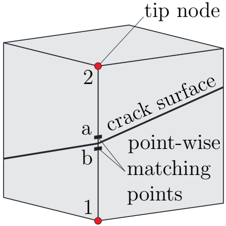
Selection of additional points



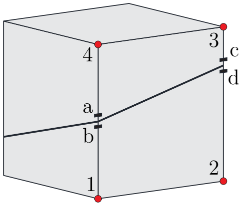
The condition is imposed at the points where the crack intersects element edges or faces.

Selection of additional points

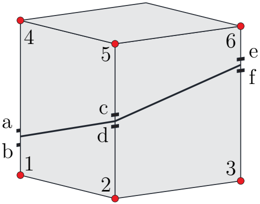
3D case:



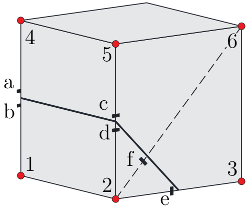
a) Point-wise matching at an edge



b) Point-wise matching at a face



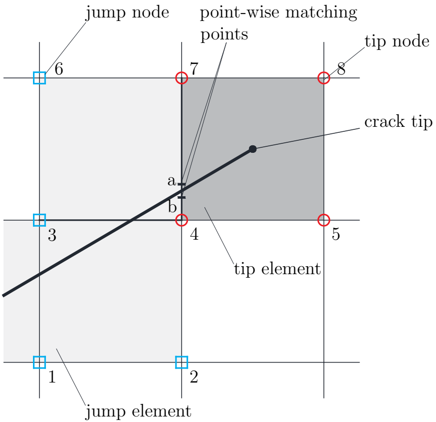
c) Point-wise matching at several faces



d) Point-wise matching at several faces

Selection of additional points

Special case:



- Edge 3-4 does not belong to a tip element.
- Evaluating the tip enrichment functions at 3-4 leads to errors.
- The values obtained from edge 4-7 will be used for 3-4.

Selection of additional points

In order to implement the above procedure:

- Point-wise matching elements are looped upon prior to the assembly.
- Parameters $\mathbf{b}_i^t$ are computed and stored.

Parameters $\mathbf{b}_i^t$ can be computed for all nodes.

The whole procedure is computationally inexpensive.

Integral matching

Motivation:

- For P1 elements and topological enrichment a loss of accuracy occurs.
- The effect is more pronounced for mode I loading.
- This is attributed to the displacement jump between regular and tip elements.
- A possible solution is the addition of one layer of tip elements.

Hierarchical functions

The addition of hierarchical blending functions is proposed.

Those functions:

- Eliminate the displacement jump in a weak sense.
- For linear quadrilateral elements assume the form:

$$N^h\left(\xi_1, \xi_2\right) = \frac{\left(1 - |\xi_1|\right)\left(1 + \xi_2\right)}{2}$$

Integral matching

Displacements along the edges of regular and jump elements:

$$\begin{aligned}\mathbf{u_r}(\xi_1,\xi_2) &= \sum_I N_I(\xi_1,\xi_2)\,\mathbf{u}_I + \sum_J N_J(\xi_1,\xi_2)\,\mathbf{a}_J + N^h(\xi_1,\xi_2)\,\mathbf{a}^h \\ \mathbf{u_t}(\xi_1,\xi_2) &= \sum_I N_I(\xi_1,\xi_2)\,\mathbf{u}_I + \sum_K N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x})\,\mathbf{c}_{Kj}\end{aligned}$$

Integral matching condition:

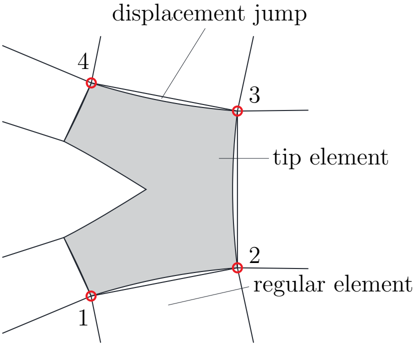
$$\int_S (\mathbf{u_r} - \mathbf{u_t})\,dS = 0$$

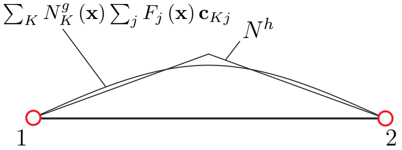
Coefficients $\mathbf{a}^h$ are obtained as:

$$\mathbf{a}_i^h = \sum_K \sum_j T_{iKj}^h \mathbf{c}_{Kj}$$

Integral matching-Mode I

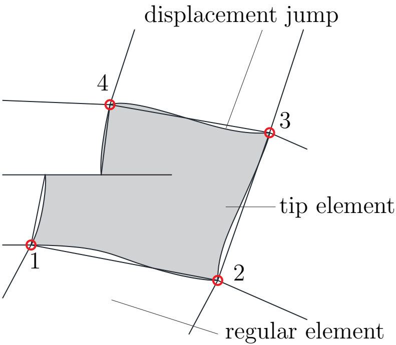
Mode I, hierarchical functions are used to eliminate displacement jumps in a weak sense:

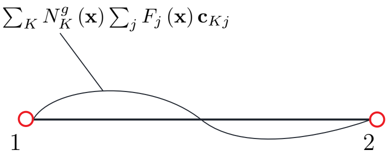




Integral matching-Mode II

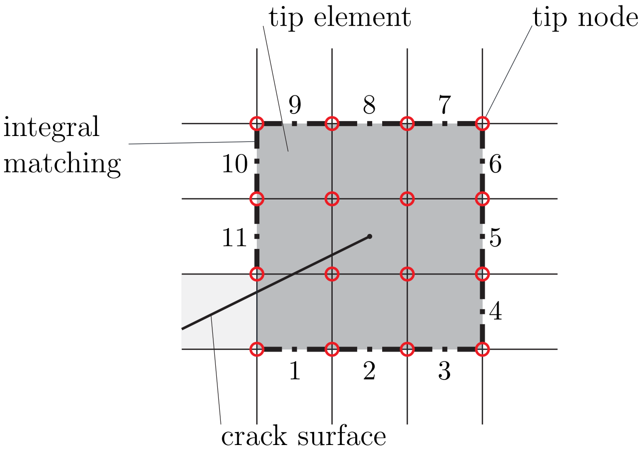
Mode II, displacement jumps almost vanish in a weak sense:





Integral matching

Imposition of integral matching condition:



Displacement approximation

Displacement approximation for the whole domain:

$$
\begin{aligned}
\mathbf{u}(\mathbf{x}) &= \sum_{I\in\mathcal{N}} N_I(\mathbf{x})\,\mathbf{u}_I + \sum_{J\in\mathcal{N}^j} N_J(\mathbf{x})\,(H(\mathbf{x}) - H_J)\mathbf{b}_J + \\
&+ \sum_{K\in\mathcal{N}^s} N_K^g(\mathbf{x}) \sum_j F_j(\mathbf{x})\,\mathbf{c}_{Kj} + \mathbf{u}^{pm}(\mathbf{x}) + \mathbf{u}^{im}(\mathbf{x}) \\
\mathbf{u}^{pm}(\mathbf{x}) &= \sum_{I\in\mathcal{N}^{t1}} N_I(\mathbf{x}) \sum_K \sum_j T_{IKj}^{t-r}\mathbf{c}_{Kj} + \\
&+ \sum_{J\in\mathcal{N}^{t2}} N_J(\mathbf{x})\,(H(\mathbf{x}) - H_J) \sum_K \sum_j T_{IKj}^{t-j}\mathbf{c}_{Kj} \\
\mathbf{u}^{im}(\mathbf{x}) &= \sum_{I\in\mathcal{N}^h} N_I^h(\mathbf{x}) \sum_K \sum_j T_{IKj}^{h}\mathbf{c}_{Kj}
\end{aligned}
$$

Nodal sets:

$\mathcal{N}$ set of all nodes in the FE mesh.

$\mathcal{N}^j$ set of jump enriched nodes.

$\mathcal{N}^s$ set of superimposed nodes which will be described next.

$\mathcal{N}^{t1}$ set of transition nodes between tip and regular elements.

$\mathcal{N}^{t2}$ set of transition nodes between tip and jump elements.

$\mathcal{N}^h$ set of edges where the blending functions are added.

Front elements

A superimposed mesh is used to provide a basis for weighting tip enrichment functions.

Desired properties:

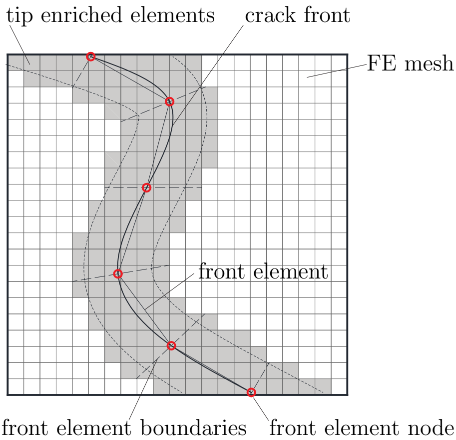
- Satisfaction of the partition of unity property.
- Spatial variation only along the direction of the crack front.

Front elements

Special elements are employed which are both:

- 1D $\rightarrow$ shape functions vary only along one dimension
- 3D $\rightarrow$ they are defined in a three-dimensional domain

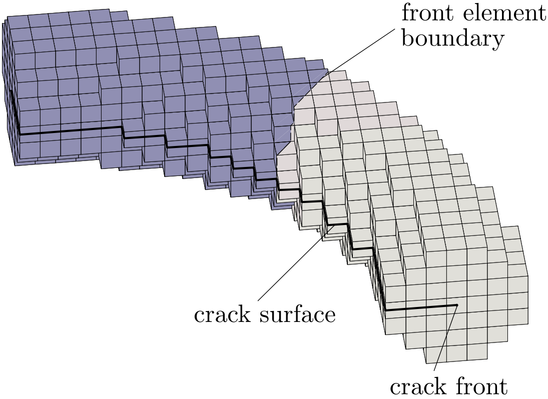
Front elements



- A set of nodes along the crack front is defined.
- Such points are also required for SIF evaluation.
- A good starting point for front element thickness is h.

Front elements

Volume corresponding to two consecutive front elements.



Different element colors correspond to different front elements.

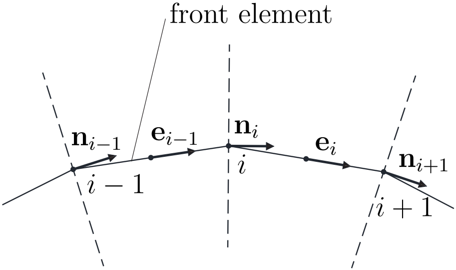
Open crack fronts

Front element definition:

- Unit vectors $\mathbf{e_i}$ are defined parallel to the element directions: $\mathbf{e_i} = \frac{\mathbf{x_{i+1}} - \mathbf{x_i}}{|\mathbf{x_{i+1}} - \mathbf{x_i}|}$.
- For every nodal point i a unit vector $\mathbf{n_i}$ is defined: $\mathbf{n_i} = \frac{\mathbf{e_i} + \mathbf{e_{i-1}}}{|\mathbf{e_i} + \mathbf{e_{i-1}}|}$.
- A plane is defined that passes through the node: $\mathbf{n_i} \cdot (\mathbf{x_0} - \mathbf{x_i}) = 0$.
- The element volume is defined by the planes corresponding to its nodes.

Open crack fronts

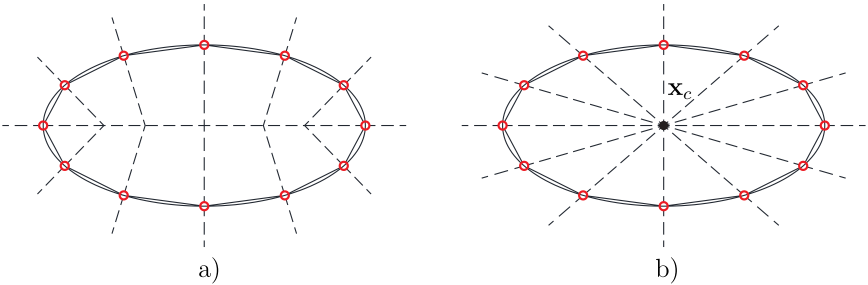
Vectors associated with front elements.



Closed crack fronts

a) Application of the method used for open crack fronts to closed crack fronts $\rightarrow$ front elements overlap.

b) Method used for closed crack fronts $\rightarrow$ overlaps are avoided.



Closed crack fronts

Element definition using an additional point ($\mathbf{x_c}$):

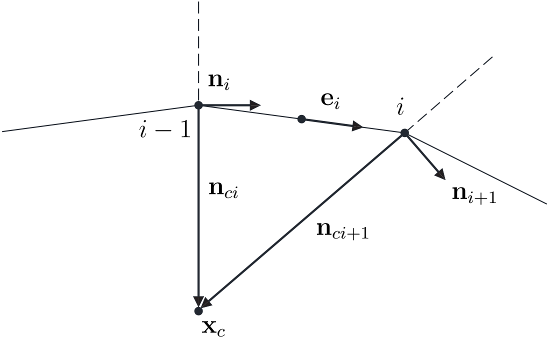
- Vectors $\mathbf{e_i}$ are defined for every element.
- Point $\mathbf{x_c}$ is defined as: $\mathbf{x_c} = \frac{\sum_{i=1}^{n} \mathbf{x_c}}{n}$.
- Vectors $\mathbf{n_{ci}}$ joining points i to the internal point $\mathbf{x_c}$ are defined: $\mathbf{n_{ci}} = \mathbf{x_c} - \mathbf{x_i}$.

Closed crack fronts

- Vectors $\mathbf{n_{ni}}$ normal to vectors $\mathbf{e_i}$ and $\mathbf{n_{ci}}$ are defined: $\mathbf{n_{ni}} = \mathbf{e_i} \times \mathbf{n_{ci}}$.
- Vectors $\mathbf{n_i}$ are defined: $\mathbf{n_i} = \frac{\mathbf{n_{ti}} \times \mathbf{n_{ci}}}{|\mathbf{n_{ti}} \times \mathbf{n_{ci}}|}$.
- Planes normal to the vectors $\mathbf{n_i}$ are defined: $\mathbf{n_i} \cdot (\mathbf{x_0} - \mathbf{x_i}) = 0$.
- Element volumes are defined as in the open crack front case.

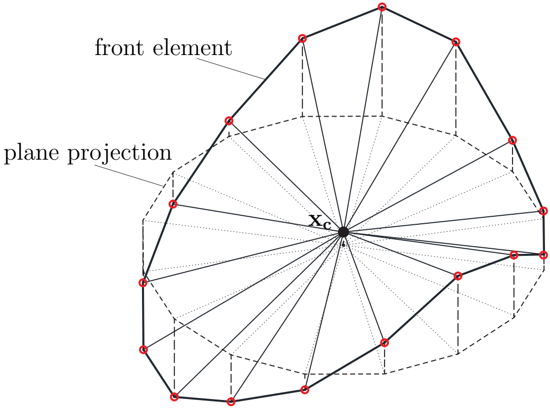
Closed crack fronts

Vectors used in the definition of front elements.



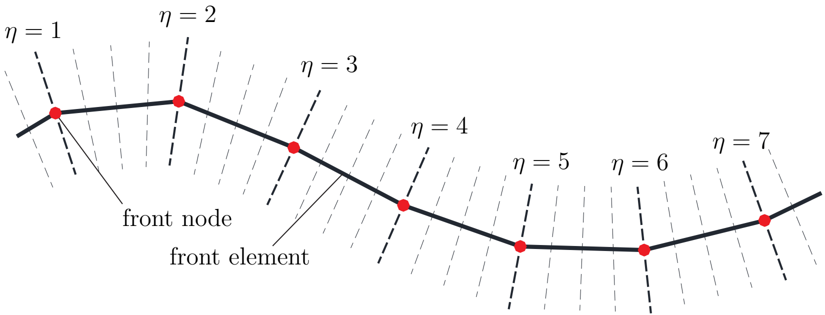
Closed crack fronts

Discretization of a non-planar closed crack front using an additional point $\mathbf{x_c}$.



Front element parameter

A function similar to the level sets is defined which varies along the crack front.



Front element parameter

Evaluation of the parameter for a point $\mathbf{x_0}$:

Plane equations corresponding to the nodes of each element are evaluated:

$$\begin{aligned} f_i(\mathbf{x_0}) &= \mathbf{n_i} \cdot (\mathbf{x_0} - \mathbf{x_i}) \\ f_{i+1}(\mathbf{x_0}) &= \mathbf{n_{i+1}} \cdot (\mathbf{x_0} - \mathbf{x_{i+1}}) \end{aligned}$$

Front element parameter

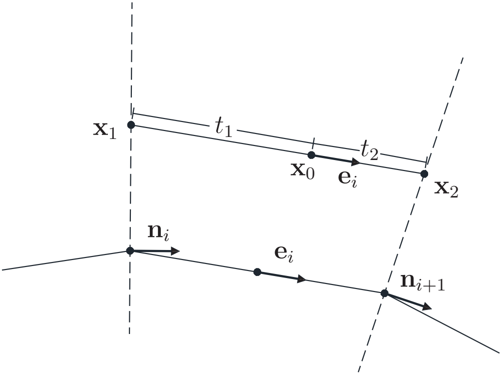
Once f_i and f_{i+1} are obtained:

- If $f_i < 0$ or $f_{i+1} > 0$ the point lies outside the element
- If $f_i = 0$ or $f_{i+1} = 0$ the point lies on the plane corresponding to node i or $i + 1$: $\eta = i$ or $\eta = i + 1$
- If $f_i > 0$ and $f_{i+1} < 0$ the point lies inside the element

Front element parameter

For points lying inside front elements:

- Integer Part: $\eta_i = i$
- Fractional part:



$$\mathbf{x} = \mathbf{x_0} + t\mathbf{e_i} \qquad t \in \mathbb{R}$$

$$t_1 = \frac{\mathbf{n_i} \cdot (\mathbf{x_0} - \mathbf{x_i})}{\mathbf{n_i} \cdot \mathbf{e_i}}$$

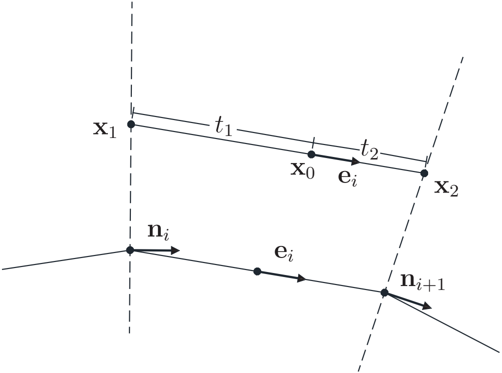
$$t_2 = \frac{\mathbf{n_{i+1}} \cdot (\mathbf{x_0} - \mathbf{x_{i+1}})}{\mathbf{n_{i+1}} \cdot \mathbf{e_i}}$$

$$\mathbf{x_1} = \mathbf{x_0} + t_1\mathbf{e_i}$$

$$\mathbf{x_2} = \mathbf{x_0} + t_2\mathbf{e_i}$$

Front element parameter

- Fractional part:



$$\mathbf{x_1} = \mathbf{x_0} + t_1\mathbf{e_i}$$
$$\mathbf{x_2} = \mathbf{x_0} + t_2\mathbf{e_i}$$

$$\mathbf{x_{10}} = \mathbf{x_0} - \mathbf{x_1}$$
$$\mathbf{x_{12}} = \mathbf{x_2} - \mathbf{x_1}$$

$$\eta_f = \frac{|\mathbf{x_{10}}|}{|\mathbf{x_{12}}|}$$

Finally:

$$\eta = \eta_i + \eta_f$$

Front element shape functions

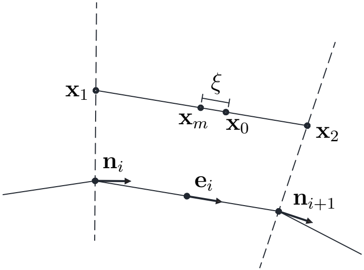
Linear 1D shape functions are used:

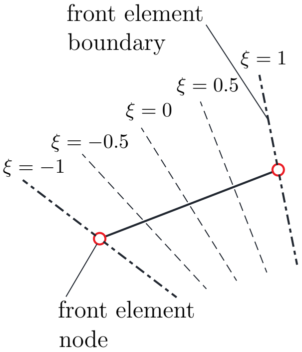
$$\mathbf{N}^{g}(\xi) = \begin{bmatrix} \frac{1-\xi}{2} & \frac{1+\xi}{2} \end{bmatrix}$$

- ξ is the local coordinate of the superimposed element.
- Those functions are used to weight tip enrichment functions.

Front element shape functions

Definition of the front element parameter used for shape function evaluation.





Front element shape functions

The evaluation of ξ is almost identical to the evaluation of η_f:

$$\xi = \frac{2\,\mathbf{x_{12}} \cdot \mathbf{x_{m0}}}{\left|\mathbf{x_{12}}\right|^2}$$

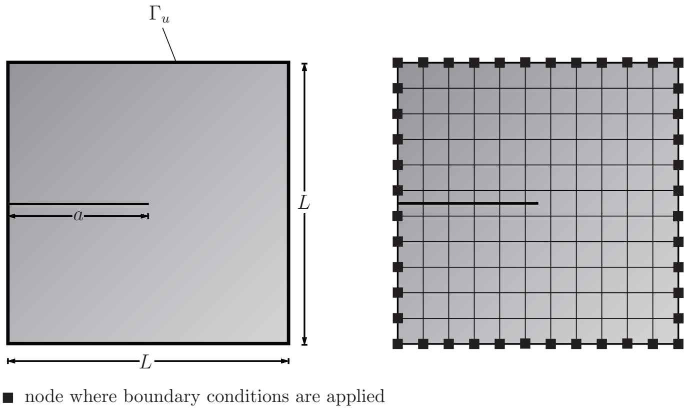
where

$$\begin{aligned} \mathbf{x_{12}} &= \mathbf{x_2} - \mathbf{x_1} \\ \mathbf{x_{m0}} &= \mathbf{x_0} - \mathbf{x_m} \\ \mathbf{x_m} &= \frac{\mathbf{x_1} + \mathbf{x_2}}{2} \end{aligned}$$

2D convergence study

- An $L \times L$ square domain with an edge crack of length a is considered.
- Boundary conditions are provided by the Griffith problem.
- Both topological and geometrical enrichment are used.
- The alternative jump enrichment strategy is not used.

2D convergence study



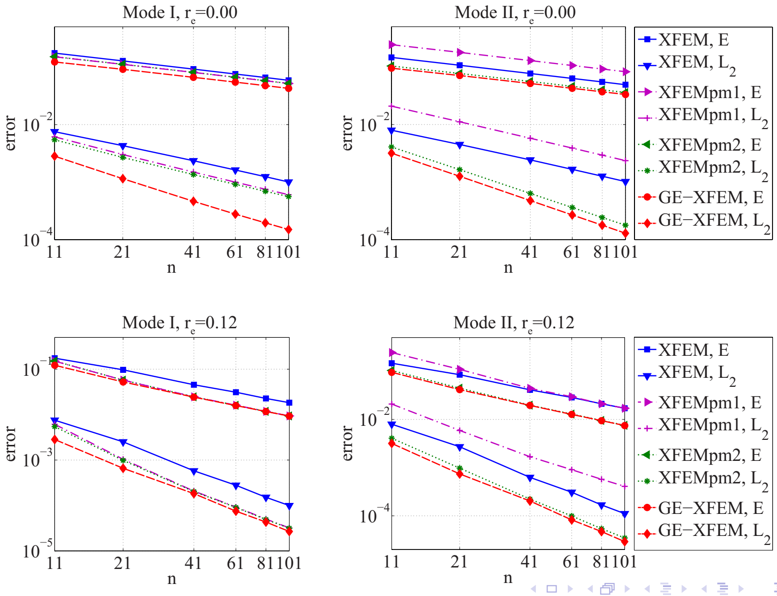
- Dimensions of the problem: $L = 1$ unit, $a = 0.5$ units.
- Material parameters: $E = 100$ units and $\nu = 0.0$.
- Mesh consists of $n \times n$ linear quadrilateral elements, $n = 11, 21, 41, 61, 81, 101$.

2D convergence study

Acronyms used for the 2D convergence study

Acronym	Description
FEM	The FE part of the approximation
XFEM	Standard XFEM (with shifted enrichment functions)
XFEMpm1	XFEM using dof gathering and point-wise matching
XFEMpm2	XFEMpm1 with the additional p.m. condition of subsection
GE-XFEM	XFEMpm2 with integral matching (Global Enrichment XFEM)

L_2 and energy norms

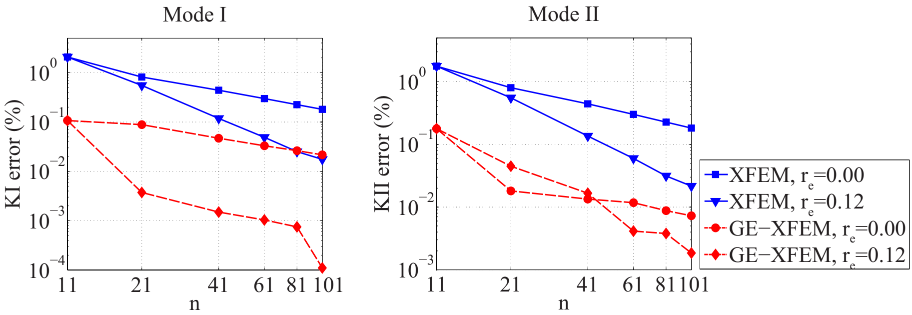


L_2 and energy norms

Convergence rates

	$r_e = 0.00$		$r_e = 0.12$	
	Mode I	Mode II	Mode I	Mode II
XFEM E	0.491	0.493	1.030	0.982
XFEM L_2	0.908	0.928	1.980	1.955
XFEMpm1 E	0.483	0.489	1.243	1.211
XFEMpm1 L_2	1.044	0.984	2.355	1.773
XFEMpm2 E	0.483	0.479	1.245	1.179
XFEMpm2 L_2	1.022	1.414	2.311	2.151
GE-XFEM E	**0.477**	**0.476**	**1.156**	**1.140**
GE-XFEM L_2	**1.326**	**1.446**	**2.086**	**2.100**

Stress intensity factors

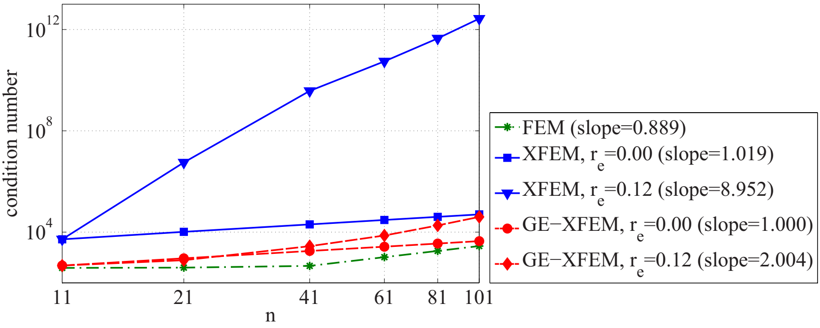


Convergence rates for the SIFs

	$r = 0.00$		$r = 0.12$	
	Mode I	Mode II	Mode I	Mode II
XFEM	1.071	1.005	2.195	2.021
GE-XFEM	0.759	1.246	2.545	2.029

Conditioning

Condition numbers of the system matrices produced by XFEM and GE-XFEM.



Condition numbers of the FE part are also plotted.

3D convergence study

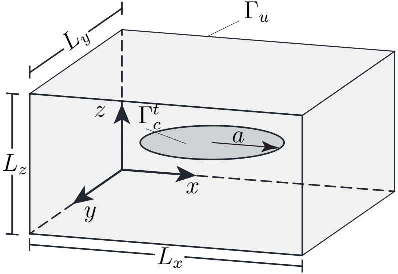
A benchmark problem is proposed which:

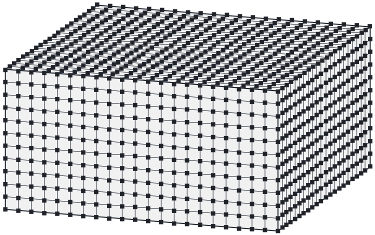
- Includes the full solution for the whole crack.
- Involves variation of the SIFs along the crack front.
- Involves a curved crack front.

3D convergence study

- A penny crack in an infinite solid is considered.
- Evaluation of L_2 and energy norms is possible.
- An $L_x \times L_y \times L_z$ parallelepiped domain with a penny crack of radius a is used.
- Analytical displacements are imposed as boundary conditions.
- A uniform normal and shear load is applied at the crack faces.

3D convergence study





■ node where boundary conditions are applied

- Uniform normal and shear loads of magnitude 1 are applied at Γ_c^t.
- Problem dimensions: $L_x = L_y = 2L_z = 0.4$ units and $a = 0.1$ unit.
- Material parameters: $E = 100$ units and $\nu = 0.3$.
- Mesh consists of $n_x \times n_y \times n_z$ hexahedral elements, $n_x = n_y = 2n_z = n$ and $n \in \{21, 41, 61, 81, 101\}$.

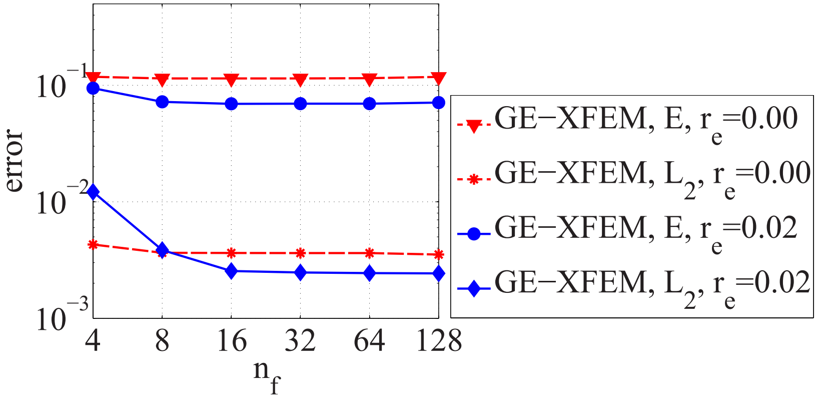
3D convergence study

Acronyms used for the 3D convergence study

Acronym	Description
XFEM	Standard XFEM (with shifted enrichment functions)
GE-XFEM	The proposed method (Global Enrichment XFEM)
GE-XFEM1	The proposed method without the new enrichment strategy

L_2 and energy norms

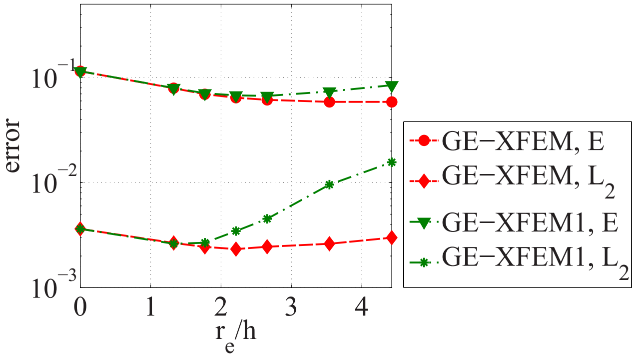
Influence of the crack front mesh density in the energy (E) and L_2 norms.



n_f is the number of elements along the front.

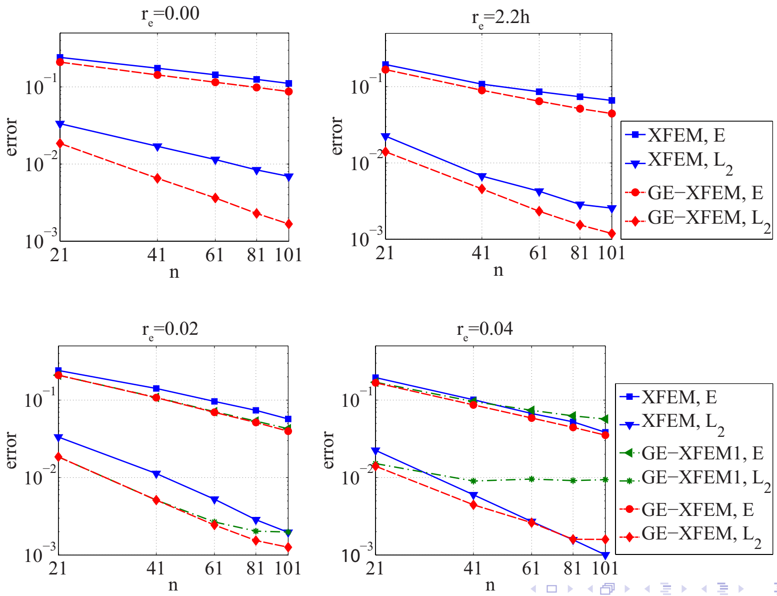
L_2 and energy norms

Influence of the enrichment radius (r_e) in the energy (E) and L_2 norms for the $31 \times 61 \times 61$ mesh.



The proposed enrichment strategy improves the behavior of the solution.

L_2 and energy norms



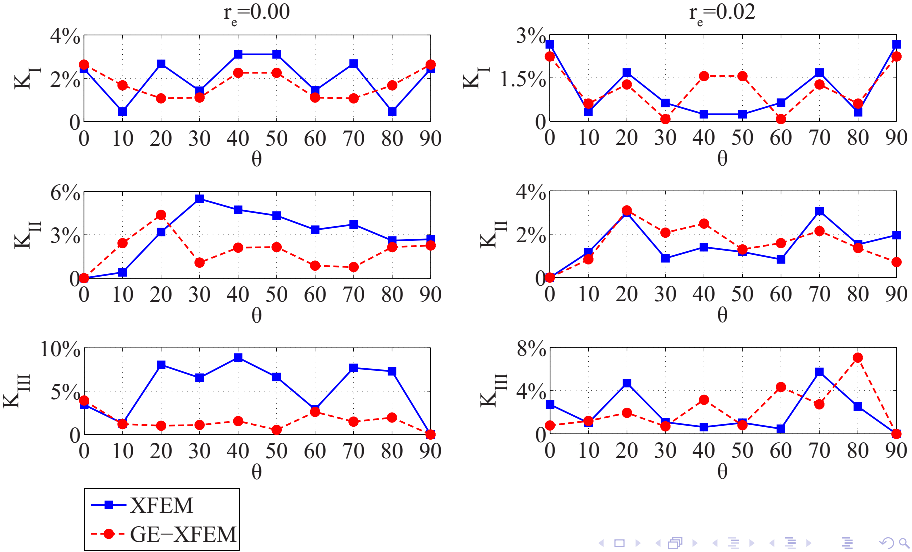
L_2 and energy norms

Convergence rates

	$r_e = 0.00$	$r_e = 2.2h$	$r_e = 0.02$	$r_e = 0.04$
XFEM E	0.492	0.686	0.911	1.015
XFEM L_2	1.009	1.405	1.824	1.976
GE-XFEM1 E	-	-	1.016	0.706
GE-XFEM1 L_2	-	-	1.481	0.289
GE-XFEM E	**0.558**	**0.850**	**1.057**	**0.988**
GE-XFEM L_2	**1.535**	**1.594**	**1.753**	**1.448**

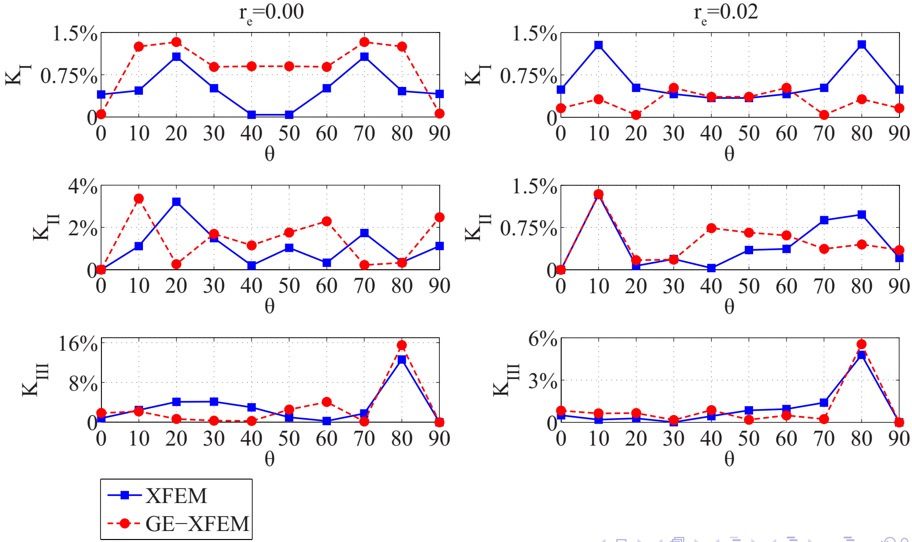
Stress intensity factors

Mode I, II and III stress intensity factors for the $21 \times 41 \times 41$ mesh.



Stress intensity factors

Mode I, II and III stress intensity factors for the $41 \times 81 \times 81$ mesh.

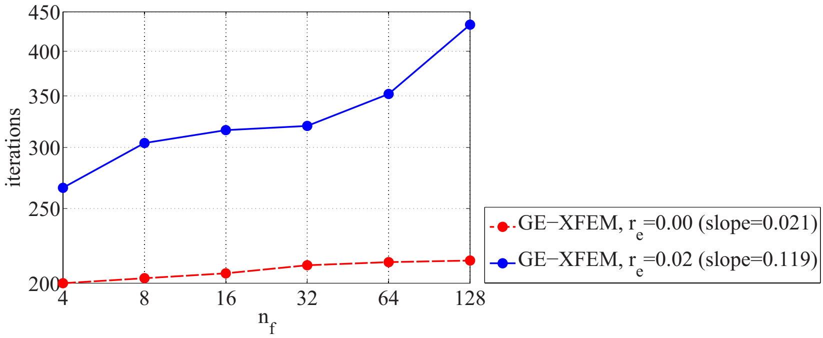


Conditioning

- Conditioning of the proposed method is compared to XFEM.
- The number of iterations required by the solver is used as an estimate.
- A comparison of the time needed to solve the resulting systems of equations is also provided.
- A CG solver with a diagonal preconditioner is used.

Conditioning

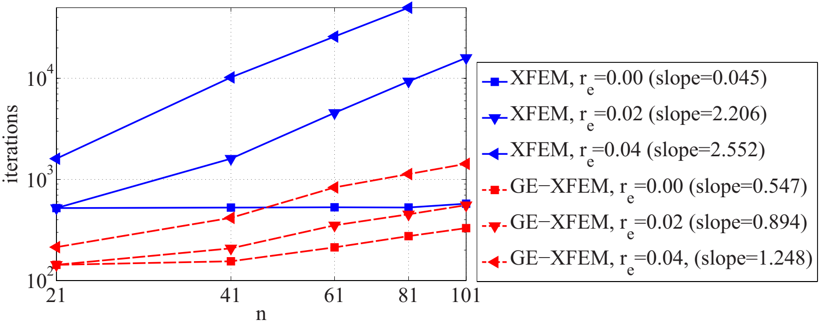
Influence of the crack front mesh density in the number of iterations for the $31 \times 61 \times 61$ mesh.



n_f is the number of elements along the front.

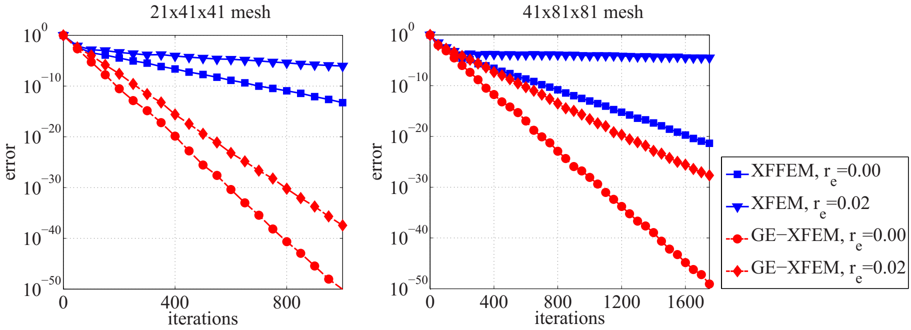
Conditioning

Number of iterations required for three different enrichment radii.



Conditioning

Performance of the PCG solver.



Number of additional dofs

Total number of enriched dofs

Mesh	FE dofs	XFEM dofs ($r_e = 0.00$)	XFEM dofs ($r_e = 0.02$)	XFEM dofs ($r_e = 0.04$)	GE-XFEM dofs
$11 \times 21 \times 21$	17,424	2,232	2,232	5,856	696
$21 \times 41 \times 41$	116,424	5,376	11,904	42,288	1,920
$31 \times 61 \times 61$	369,024	9,456	37,752	137,280	4,464
$41 \times 81 \times 81$	847,224	14,424	84,696	320,664	7,512
$51 \times 101 \times 101$	1,623,024	20,376	162,528	620,184	11,544

Conclusions

A method was introduced which:

- Employs point-wise and integral matching.
- Uses a novel enrichment strategy.
- Generalizes and extends the dof gathering approach to 3D.
- Is applicable to general 3D problems.

Conclusions

A benchmark problem was proposed which:

- Involves a curved crack front.
- Enables the computation of L_2 and energy norms for the 3D case.

Conclusions

Advantages of the method:

- It improves accuracy almost in every case.
- Enables the application of geometrical enrichment in 3d applications.
- Reduces the number of additional dofs.
- Reduces computational cost in every case.

Conclusions

Possible disadvantages:

- When the enrichment radius exceeds a certain value, the $L2$ norm increases.
- The method is not straightforward to implement in existing XFEM codes.
- The additional point wise-matching constraints are complex to implement for higher order elements.

Bibliography

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