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IMAM
Institute of Mechanics and Advanced Materials

An Adaptive Multiscale Method For Fracture

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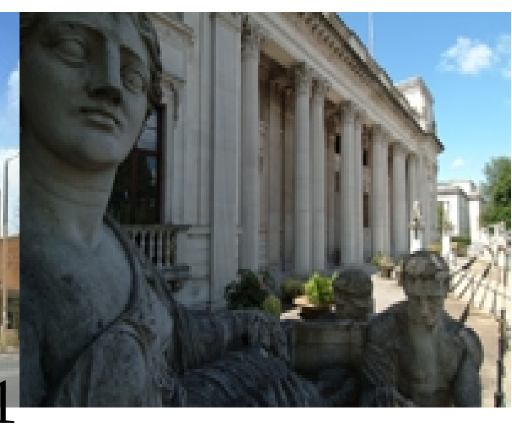
Supervisors:

Prof. Stéphane Bordas, Dr. Pierre Kerfriden

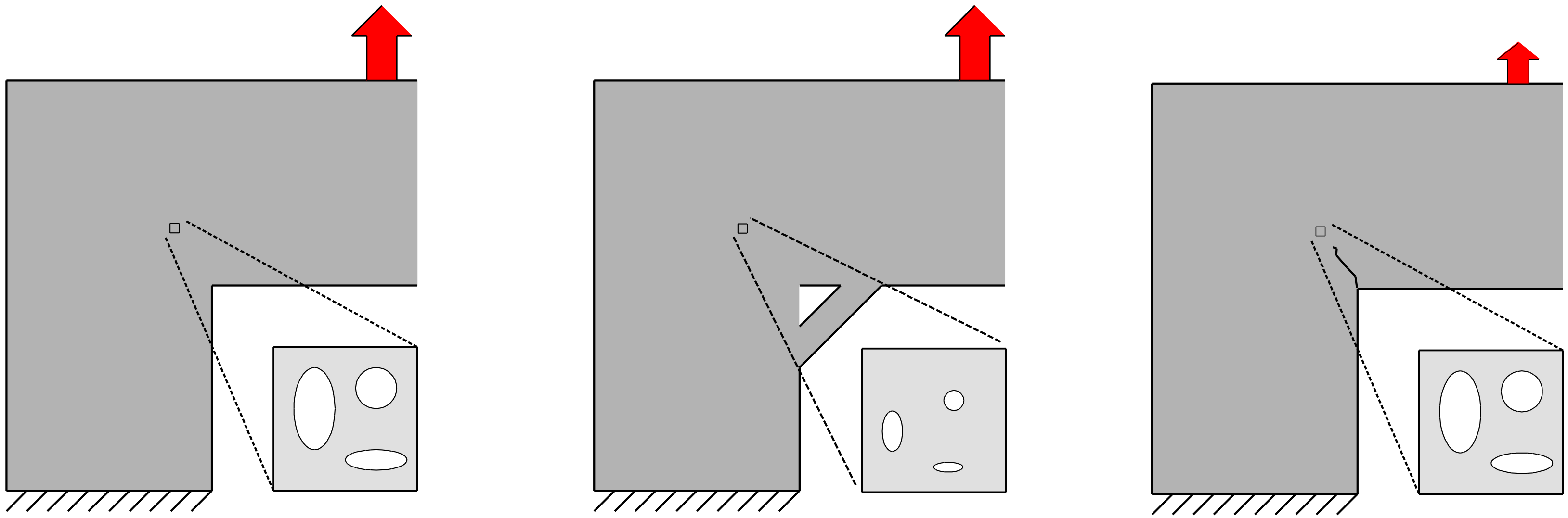


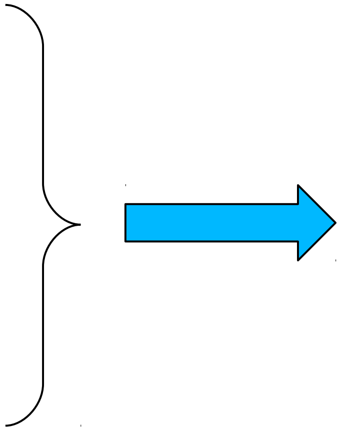
Outline:

1. Introduction
2. Scale separation and multiscale
3. FE2 Multiscale method
4. FE2 and its difficulty in fracture mechanics
5. Hybrid multiscale method
6. Adaptive multiscale method
7. Conclusion
8. Open research areas

Mechanics of materials

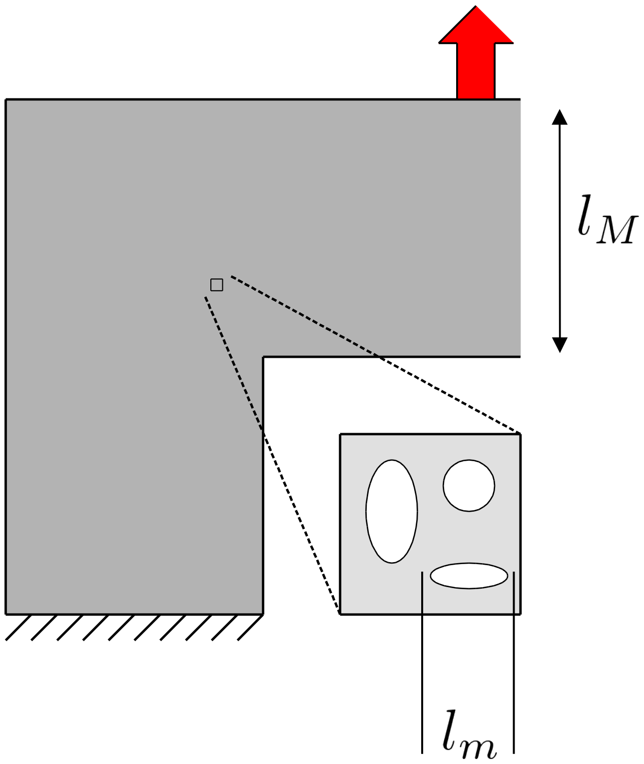


- Geometry
- Load
- Microscopic response



- Macroscopic response

Modelling of microscopic response



- $l_M > l_m$ (Separable scales)

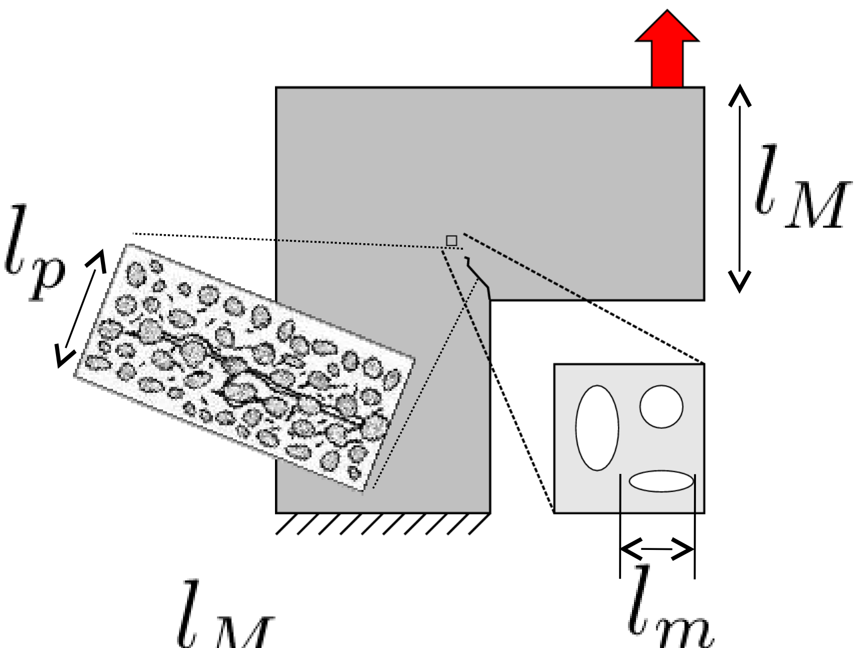
Effective macroscopic model

- Analytical approaches
- Averaging technique approaches

- $l_M \approx l_m$ (Inseparable scales)

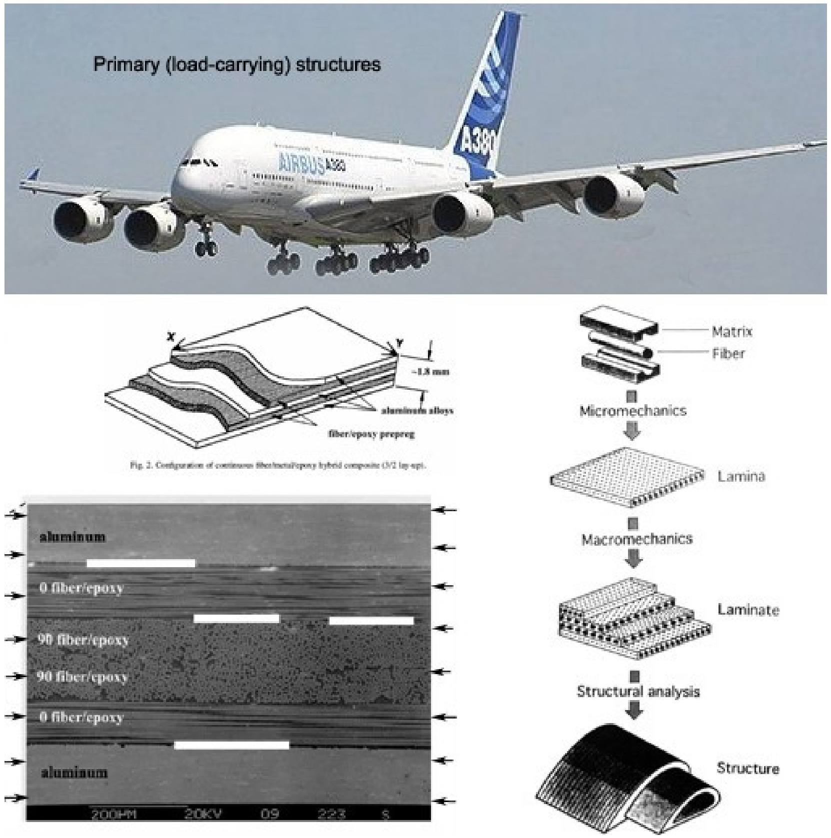
Direct microscale model

Fracture



$$\frac{l_M}{l_m}$$

$$\frac{l_M}{l_p} \quad ?$$



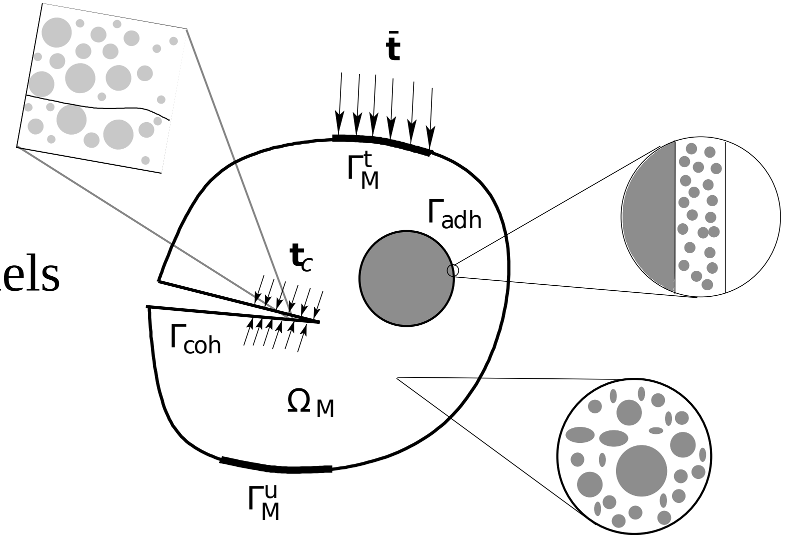
Cross-section of carbon-fibre aluminium laminate, a part of primary structure of Airbus A380 [from CEMINACS centre, University of Aberdeen]

- Non-concurrent approaches

$$\frac{l_M}{l_p} > 1$$

(Separable scales)

Macroscopic crack modelling:
-Local or non-local damage models
-Homogenisation of damage
-Crack direction obtained by macroscopic assumptions



From [Nguyen et al. 2010]

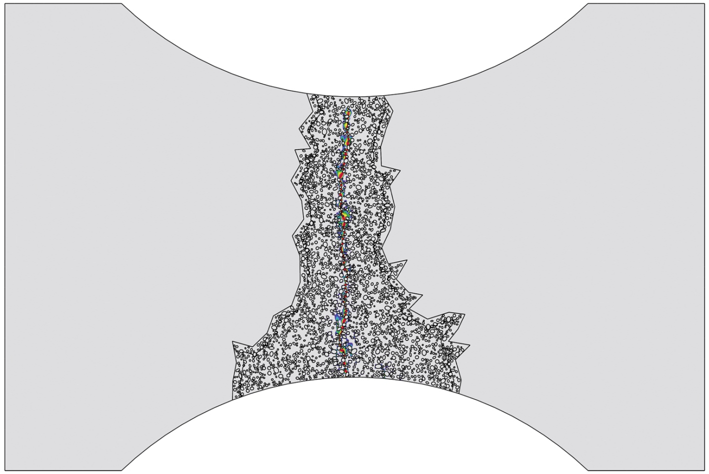
- Concurrent approaches

$$\frac{l_M}{l_p} \approx 1$$

(Inseparable scales)

Fracture is modelled at the microscale

Expensive

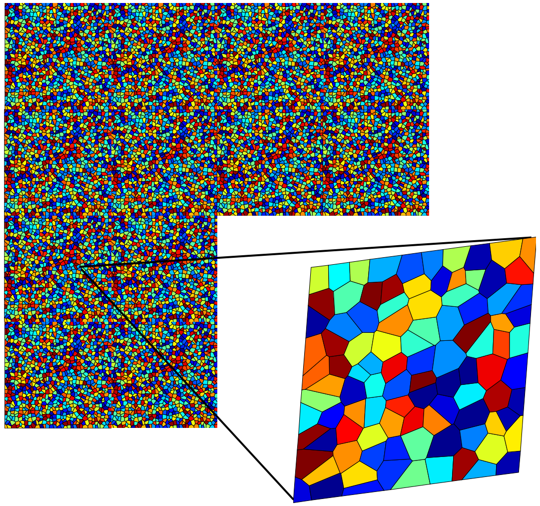


From [Unger and Eckart 2011]

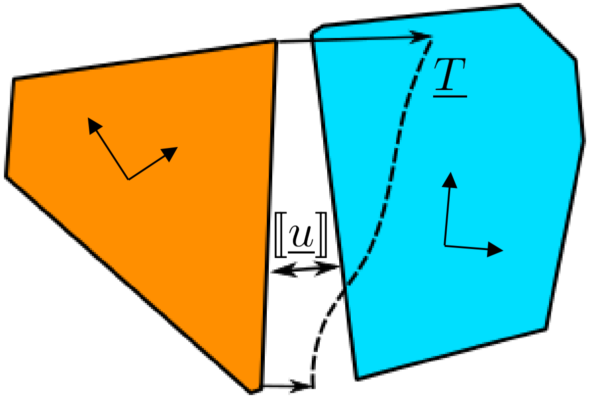
Adaptive Multiscale method

microscopic model

$$\int_{\Omega/\Gamma_c} \boldsymbol{\sigma}(\mathbf{u}) : \delta\varepsilon \, d\Omega + \int_{\Gamma_c} \mathbf{T} \cdot [\![\delta\mathbf{u}]\!] d\Omega$$
$$= \int_{\partial\Omega} \mathbf{f} \cdot \delta\mathbf{u} d\Gamma$$



- Orthotropic grains
- Cohesive interface

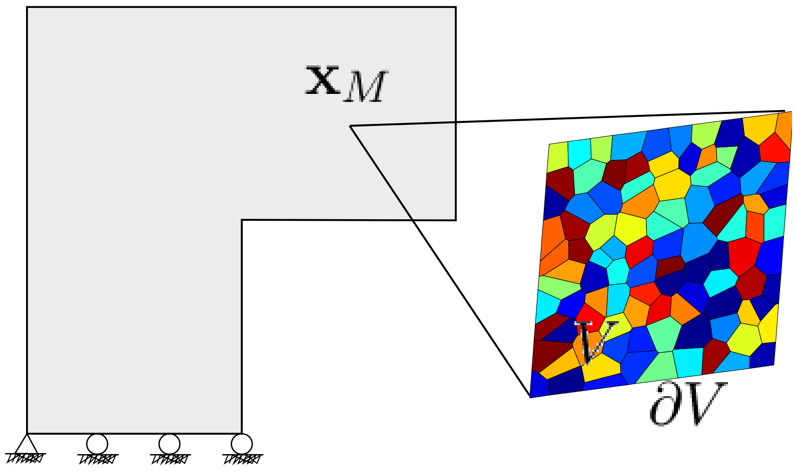


$$\mathbf{T} = \mathbb{k}[\![\mathbf{u}]\!]$$

$$\mathbb{k} = \begin{bmatrix} k_n^+ (1 - d_n) H([\![\tilde{u}_1]\!]) + k_n^- H(-[\![\tilde{u}_1]\!]) & 0 \\ 0 & k_t(1 - d_t) \end{bmatrix}$$

Macroscale problem:

$$\int_{\Omega} \boldsymbol{\sigma}^c(\mathbf{u}) : \delta\boldsymbol{\varepsilon}^c \, \mathrm{d}\Omega = \int_{\Gamma_t} \mathbf{f} \cdot \delta\mathbf{u}^c \mathrm{d}\Gamma$$



Constitutive relation is obtained by FE2 scheme

Coupling of macroscopic and microscopic levels

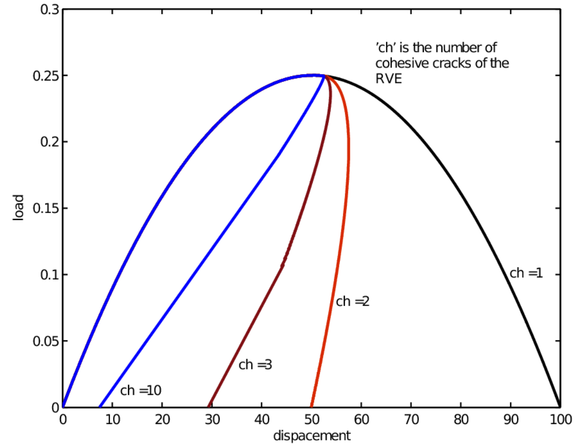
The volume averaging theorem is postulated for:

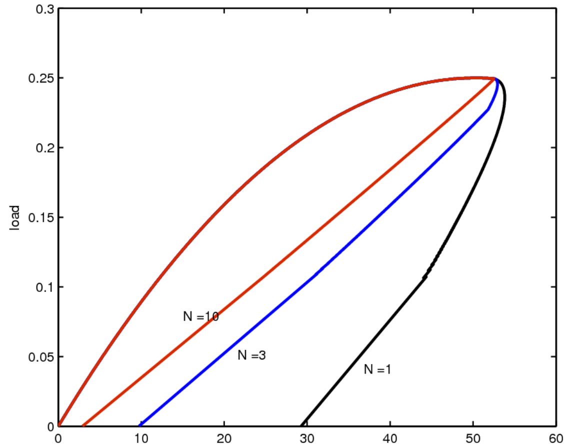
- Strain tensor $$\langle \varepsilon_{ij} \rangle = \frac{1}{V} \int_{\partial V} \frac{1}{2}(u_i n_j + u_j n_i) \, \mathrm{d}A$$
- Virtual work (Hill-Mandel condition) $$\langle \sigma_{ij} \rangle \langle \varepsilon_{ij} \rangle = \frac{1}{V} \int_V \sigma_{ij} \, \varepsilon_{ij} \, \mathrm{d}V$$
- Stress tensor $$\langle \sigma_{ij} \rangle = \frac{1}{V} \int_V \sigma_{ij} \, \mathrm{d}V$$

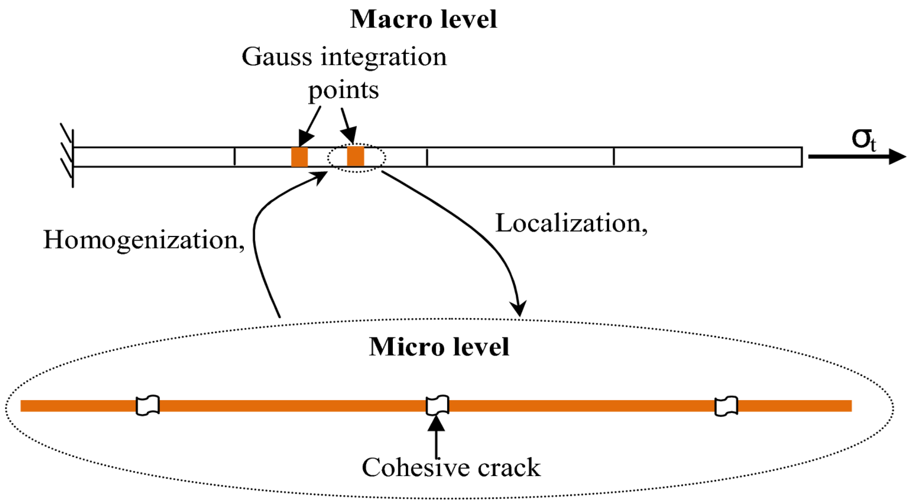
Limitation of Computational homogenization

In softening regime:

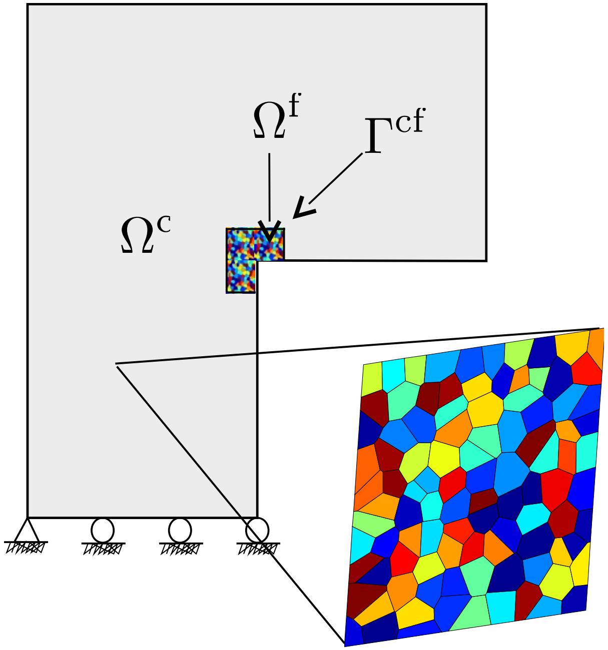
- Lack of scale separation or (the RVE is not valid for Homogenization)







Hybrid multiscale method



- FE2 for safe region (hierarchical multiscale)
- Domain decomposition for damage region (concurrent multiscale)

$$\begin{cases} \int_{\Omega^s} \boldsymbol{\sigma}^c : \boldsymbol{\delta\epsilon}^c \, d\Omega = \int_{\partial\Omega_F^s} \mathbf{F}_d . \boldsymbol{\delta u}^c \, d\Gamma - \int_{\Gamma_{cf}} \boldsymbol{\lambda}^p . \boldsymbol{\delta u}^c \, d\Gamma \\ \sum_{G \in \mathcal{G}^p} \int_G \boldsymbol{\epsilon}^f : \mathbb{H}^{(G)} : \boldsymbol{\delta\epsilon}^f \, d\Omega + \sum_{G \in \mathcal{G}^p} \frac{1}{2} \int_{\partial G \setminus \partial\Omega^p} \mathbf{t}^{(G)} . [\![\boldsymbol{\delta u}^f]\!]^{(G)} \, d\Gamma = \int_{\Gamma_{cf}} \boldsymbol{\lambda}^p . \boldsymbol{\delta u}^f \, d\Gamma \end{cases}$$

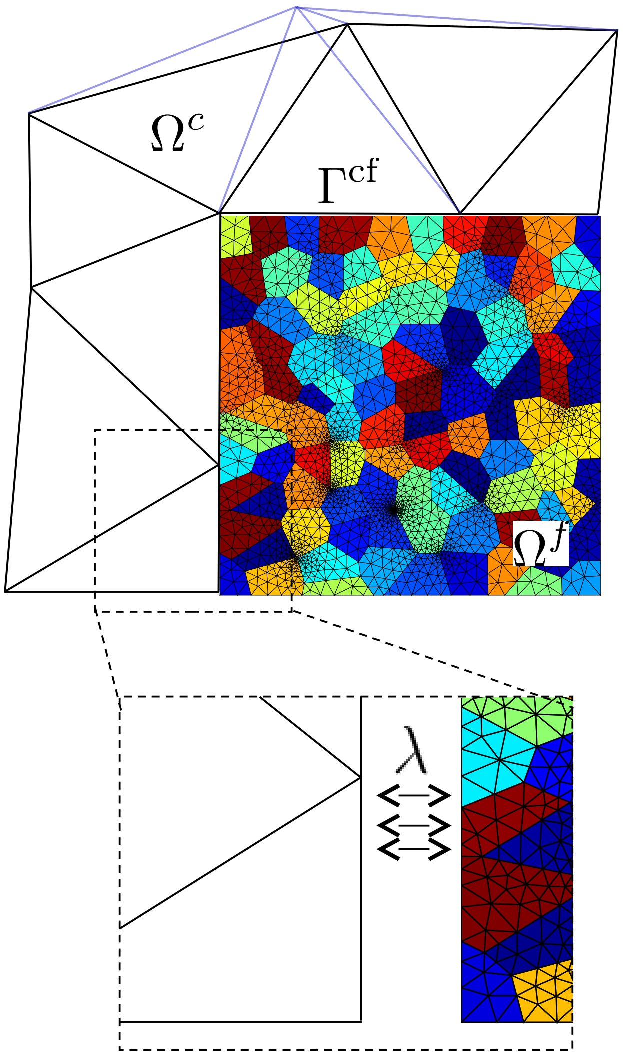
Displacement continuity at the interface $\mathbf{u}^c \big|_{\Gamma_{cf}} = \mathbf{u}^f \big|_{\Gamma_{cf}}$

Hybrid multiscale method

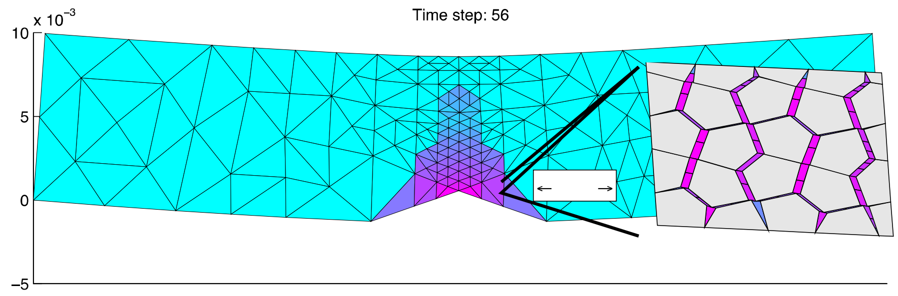
- Implicit incremental time integration

- Space discretisation at time t_n:
  - P^1 FE in Ω^c (3-nodes triangles),
  - P^1 FE + cohesive elements in Ω^f
  - Interface constraint: each node of Ω^c correspond to a node of Ω^f

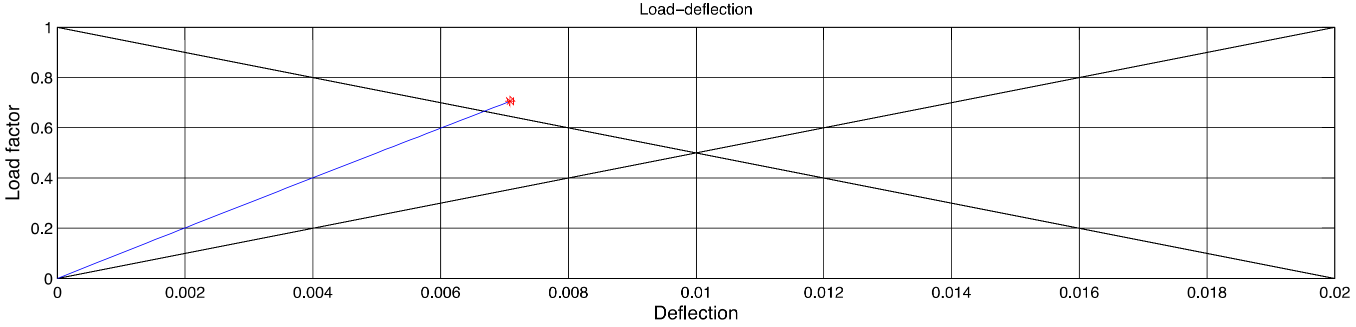
- Local arc-length technique
  - Control maximum opening of Cohesive interface cracks



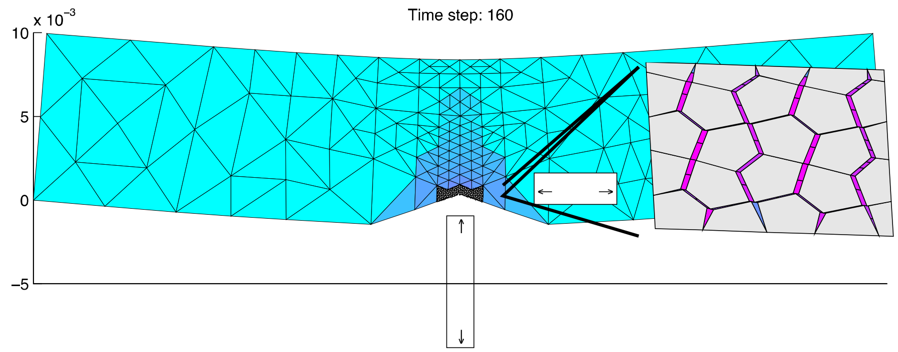
Example: Hybrid multiscale method

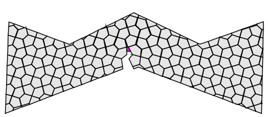


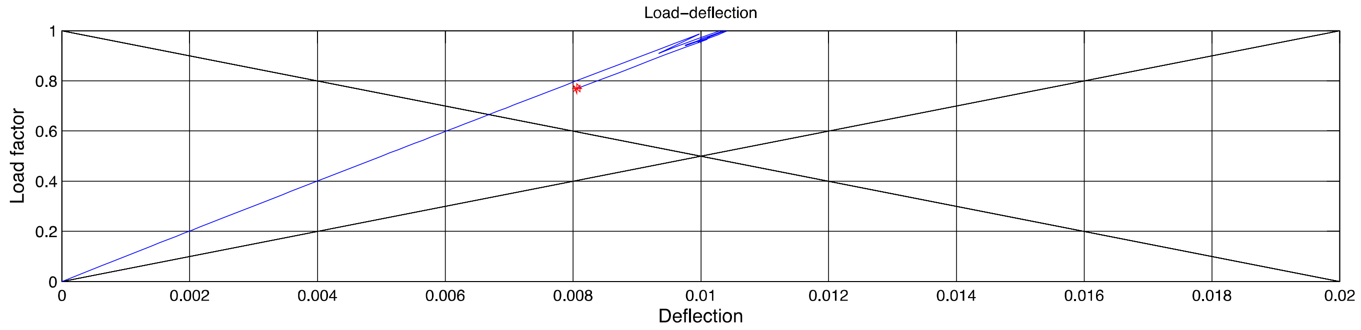
Macro/micro Model refinement criterion: relative distance to the loss of ellipticity



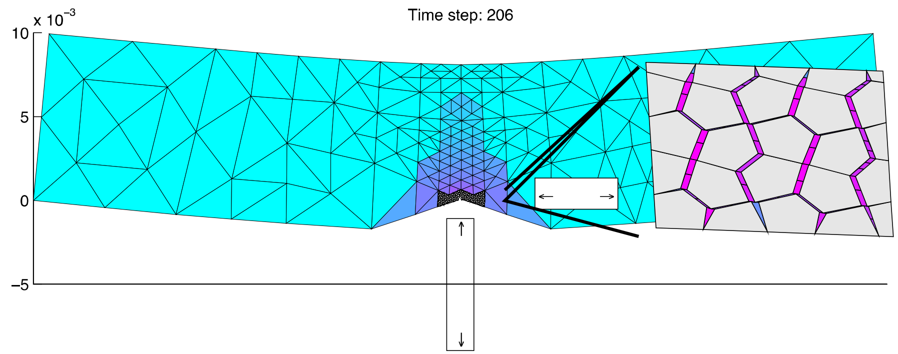
Example: Hybrid multiscale method

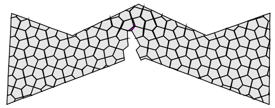


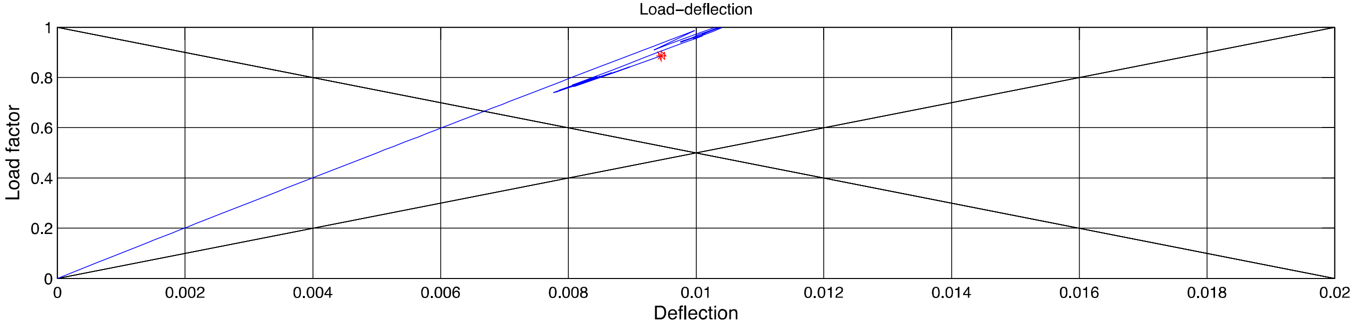




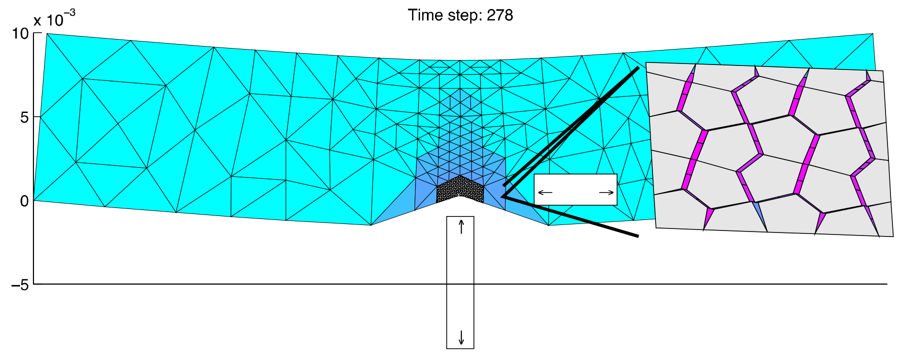
Example: Hybrid multiscale method

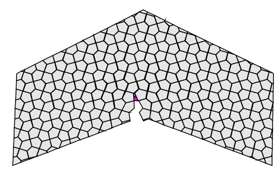


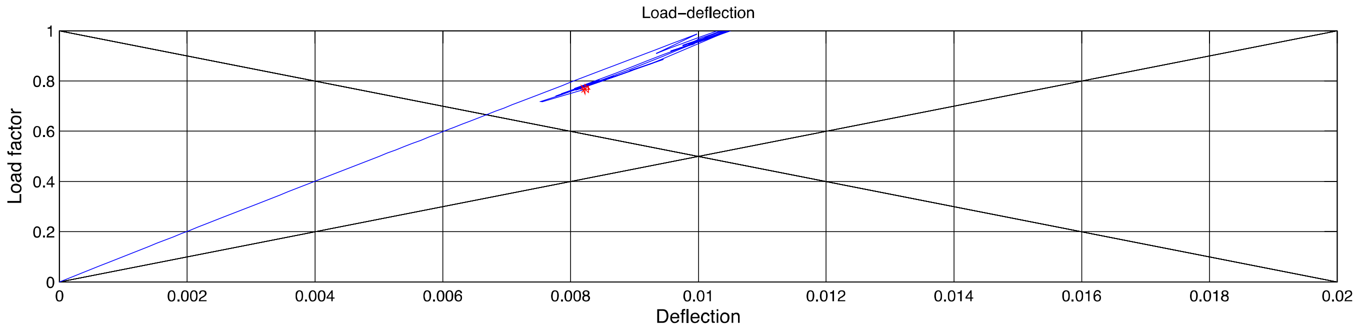




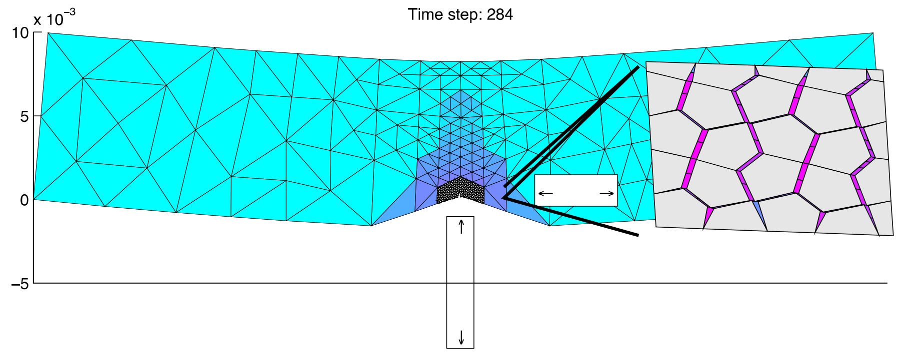
Example: Hybrid multiscale method

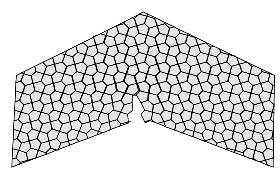


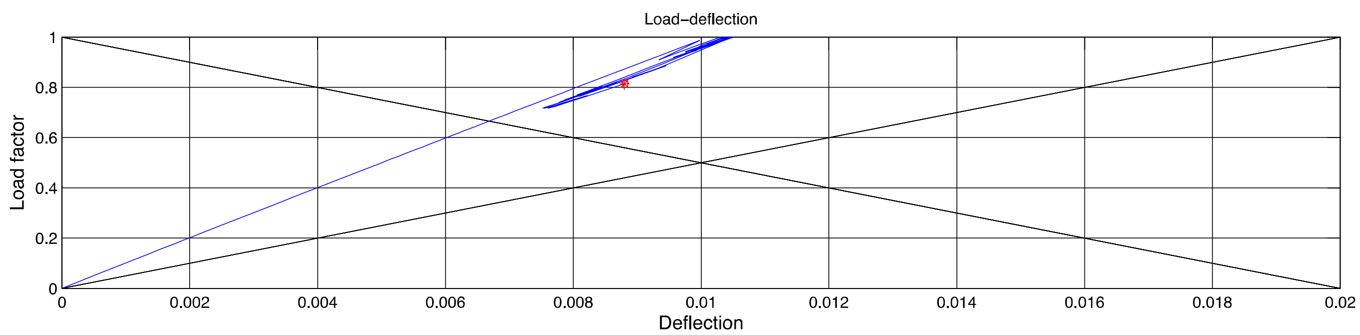




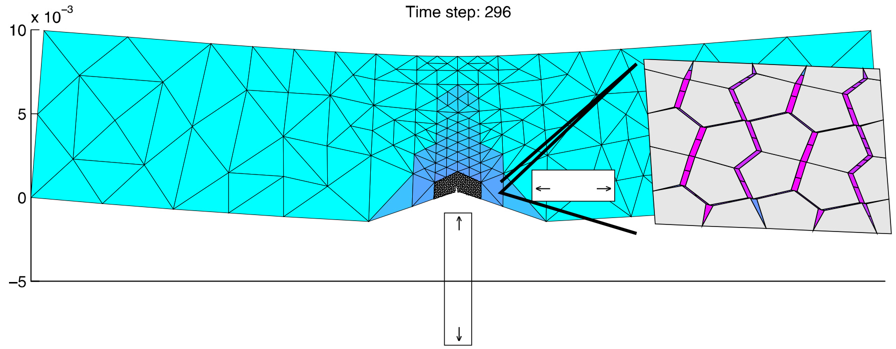
Example: Hybrid multiscale method

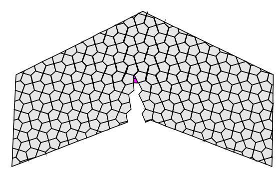


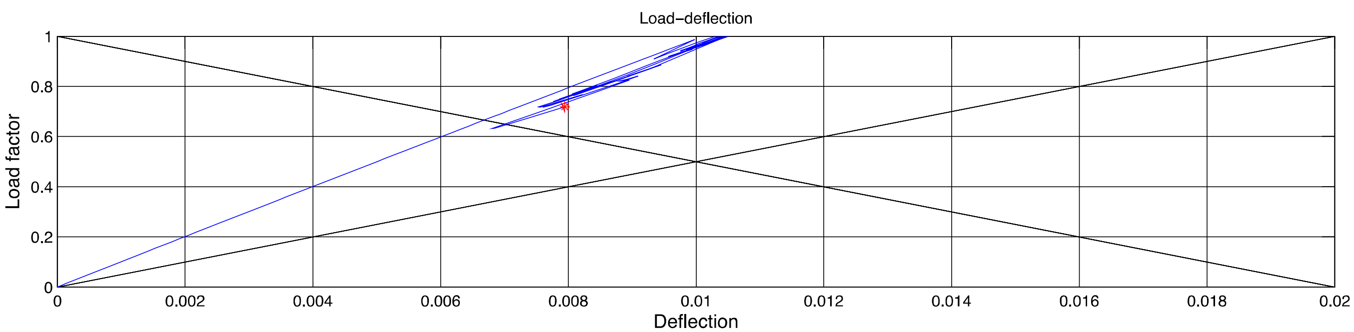




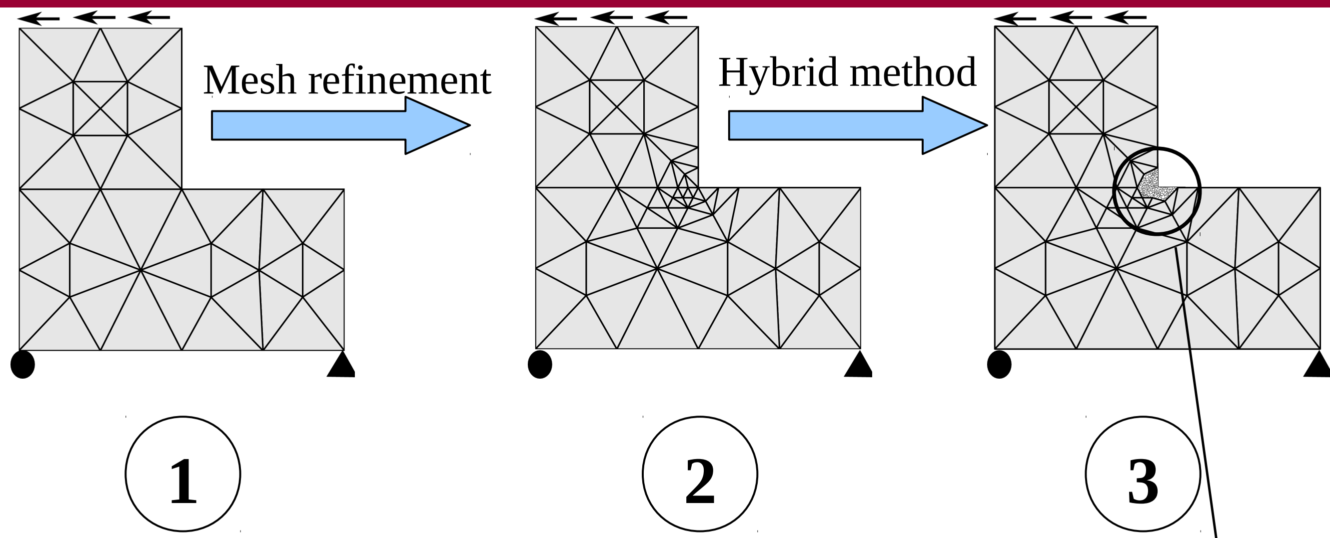
Example: Hybrid multiscale method

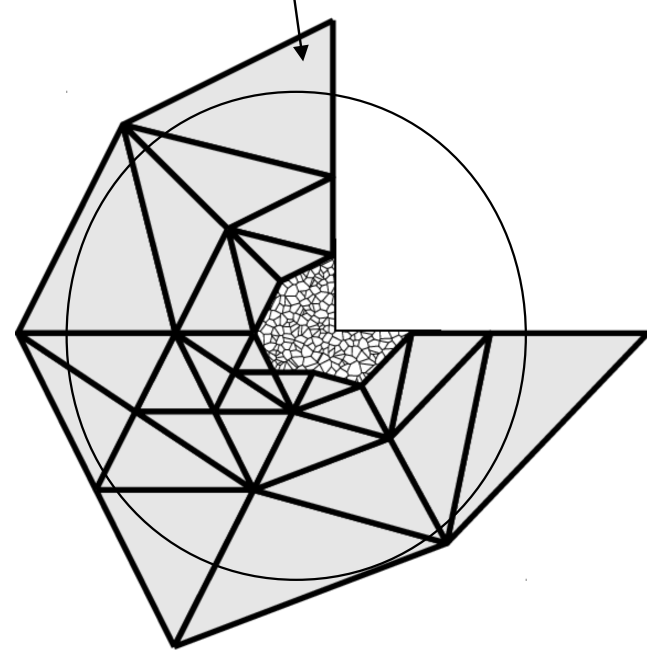


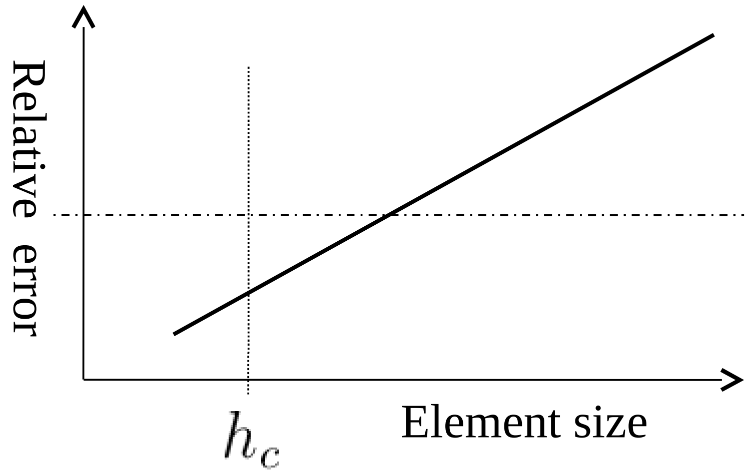




Adaptive mesh refinement



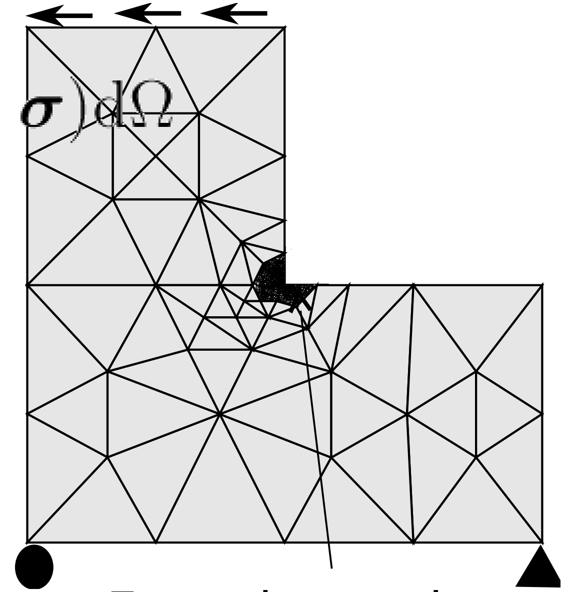




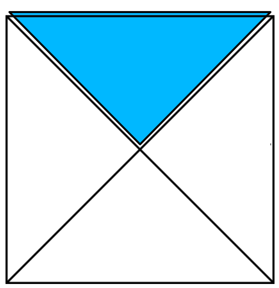
Macroscopic discretisation error

- Error estimation by Zienkiewicz-Zhu-type recovery technique

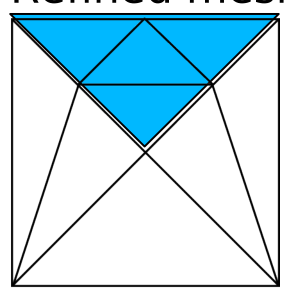
$$\|e\| = \int_{\Omega_c} (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}) : \left(\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}} \Big|_{\mathbf{u}^c} \right)^{-1} : (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}) \, d\Omega$$



Element to refine



Refined mesh

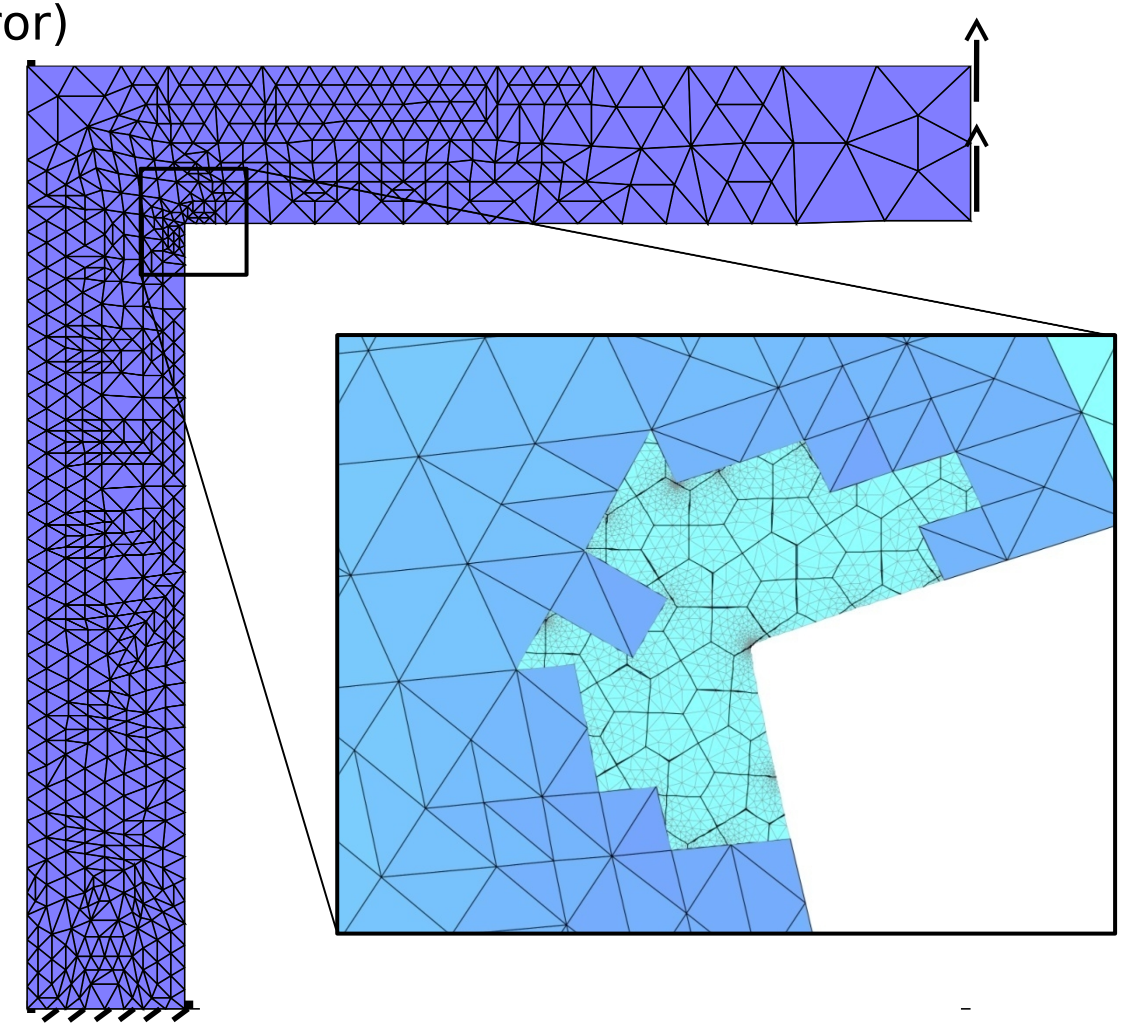


- Convergence criterion:

$$\frac{\|e\|}{\|\sigma\|} < Tol$$

Initial mesh/model refinement

Initial mesh, refined hierarchically using ZZ (15% error)



Example: Adaptive multiscale method

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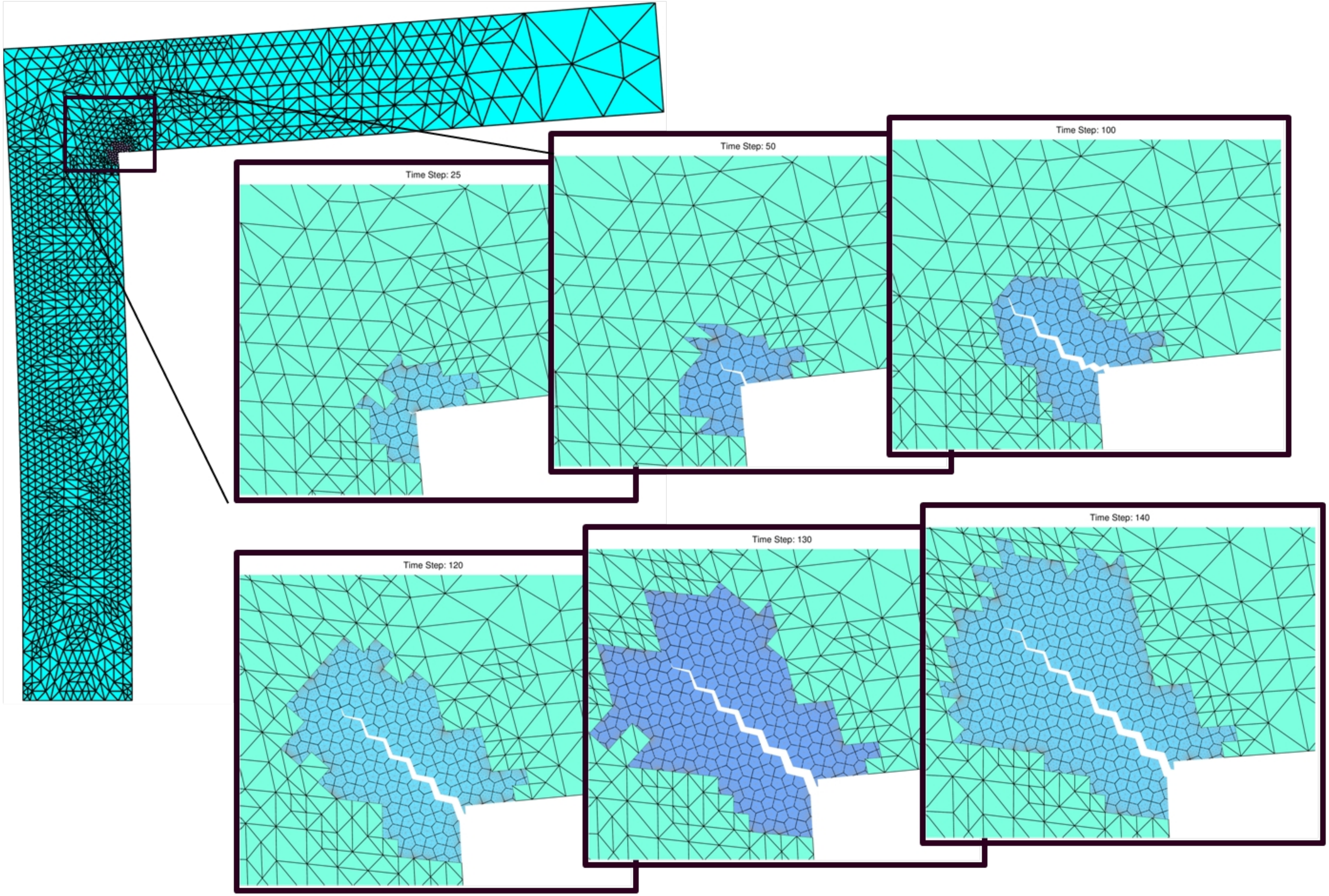
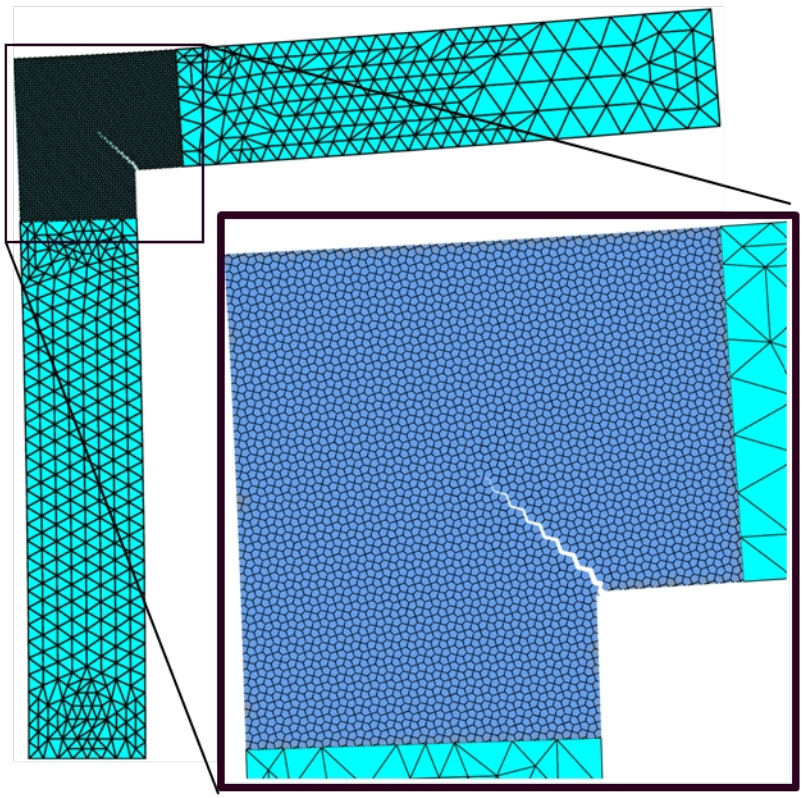
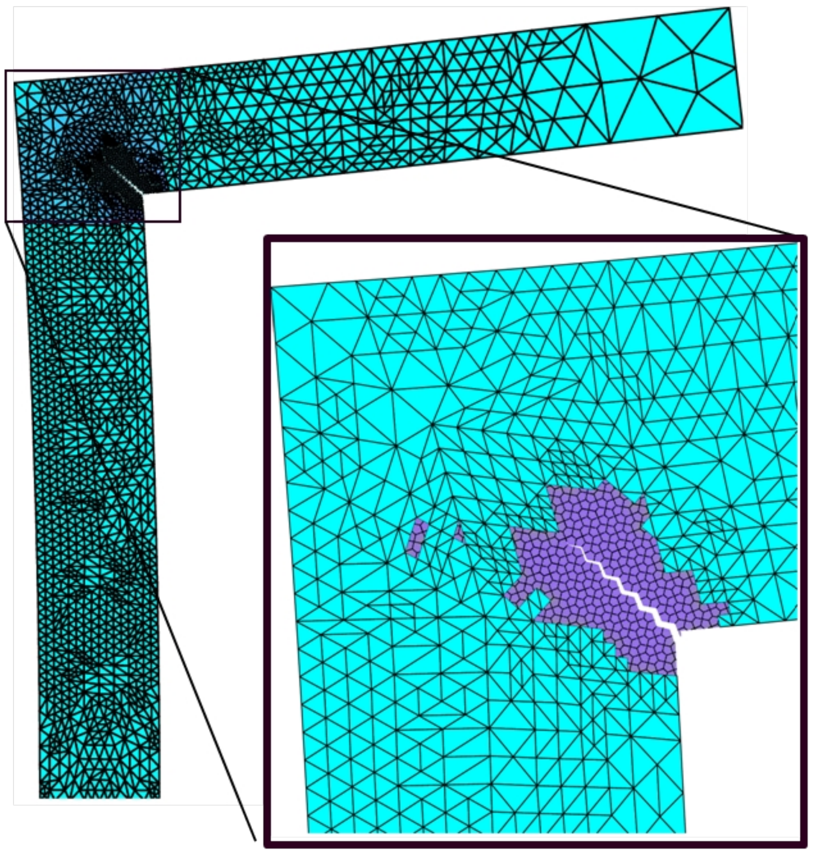


Verification of the Adaptive multiscale method

Direct Numerical Solution



Adaptive Multiscale method



Conclusion

An adaptive multiscale method was developed for modelling of fracture in polycrystalline materials:

- An unstructured mesh is used for the coarse scale of concurrent multiscale .
- A local arc-length was used to control crack speed at process zones.
- A recovery based error indicator was employed to improve the mesh at each time step.
- The robustness of the method was shown by an example.

Open areas for research

- Coarsening the damaged region where the fine scale study is not necessary.

- Improving the error indicator in the vicinity of the interface of the coarse and fine meshes.

- Developing a goal-oriented error estimator which refines the mesh with respect to a quantity of interest, e.g. damage at micro level.

Thanks for your attention!