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JEL Classification: G12

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Risk and Expected Returns of Private Equity Investments: Evidence Based on Market Prices*

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Abstract

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I. Introduction

Private equity (PE) refers to equity securities in private companies that are not publicly traded. Private equity funds specialize in PE investments and provide a mechanism for the distribution of this asset class to institutional investors and other capital market participants. The early success of some large PE funds led to the rapid growth of private equity funds. According to Preqin (2009), capital commitments to new private equity funds have grown from approximately $6 billion in 1991 to more than $340 billion in 2008. As of 2008, this same report indicates that private equity funds manage approximately $2.5 trillion.[1]

Although PE has experienced rapid growth, the risk and return profile of this asset class is not well understood. Many news stories suggest that PE investments provide higher returns than traditional asset classes.[2] A news release by Thomson Financial and the National Venture Capital Association (NVCA) announced that Thomson Reuters' All Venture Private Equity Performance Index[3] for the United States (US) "across all horizons outperformed public market indices, NASDAQ and the S&P 500, through December 31, 2008."

This paper examines the risk and expected return of private equity investments using market prices for two samples of publicly traded firms that invest in private equity. The first sample contains 24 publicly traded funds of funds (FoFs) that predominantly invest as limited partners of unlisted private equity funds. FoFs holding unlisted PE funds trade on exchanges outside the United States (US), including the London Stock Exchange and exchanges in

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[1] See http://www.preqin.com/ to download PESpotlight_February09_NonSubscribers.pdf.

[2] See Phalippou and Gottschalg (2009) for examples of several news articles that report high expectations for returns from PE investments. For instance, the *Financial Times* (September 26, 2005) reports that a survey of large UK investors found that these "investors hope to make an average annual net return of 12.8 percent from their private equity investments."

[3] See http://www.nvca.org/ to download Q408VCPerformanceRelease.pdf. The All Venture Private Equity Performance Index is computed using "the latest quarterly statistics from Thomson Reuters' Private Equity Performance Database analyzing the cash flows and returns for over 1,266 US venture capital partnerships with a capitalization of $224 billion. Sources are financial documents and schedules from Limited Partner investors and General Partners."

Continental Europe. Even though these FoFs are listed outside the US, their investments consist largely of unlisted PE funds located in the US (on a value-weighted basis). The FoFs themselves usually charge a management fee in addition to the fee collected by the manager of the unlisted private equity fund held by the FoF. The usefulness of FoFs as a gauge of the risk and performance characteristics of unlisted PE funds depends on how closely the underlying portfolio of unlisted PE funds held by FoFs mimics the unlisted PE fund universe. We find that the average internal rate of return (IRR) for the unlisted PE funds held by the FoFs is not statistically different from the appropriate benchmark IRR. While we cannot rule out the possibility that the portfolio of PE funds held by FoFs is somehow different from the unlisted PE fund universe, we do not see any statistical evidence supporting this possibility.

The second sample contains 129 publicly traded PE funds that participate in private equity transactions. We refer to these funds as listed private equity funds (LPEs). The main difference between the LPE sample and the FoF sample is that LPEs primarily invest directly through PE transactions rather than holding limited partnerships in unlisted PE funds, and therefore, LPEs do not have a second layer of fees. The managers of LPEs are compensated through management and performance fees similar to unlisted PE funds. To the extent that excess returns are available to skilled managers who specialize in private equity transactions, these LPEs have the same opportunity sets as unlisted PE funds. However, there are differences in the organizational structure of LPEs compared to unlisted PE funds that could affect risk and performance characteristics. In particular, a LPE is a closed-end fund with an indefinite life. In contrast, since unlisted PE funds have a finite life, the management company must approach investors periodically to raise money for the next generation of unlisted PE funds.

We follow a widely accepted methodology in finance. We estimate risk factor loadings for FoFs or LPEs using realized returns calculated from market prices. Of course, if the market is sufficiently inefficient, it is possible that such estimates of the factor loadings may not accurately reflect the risk characteristics of the underlying portfolio of PE. Nevertheless, this estimation procedure is the starting point for many asset pricing tests, and thus, it is important to bring this same methodology to the analysis of private equity performance.

The existing literature attempts to estimate the market risk of PE investments using some combination of cash flows to investors and intermediate valuations of investments. Because it is difficult to determine the intermediate market valuation of all investments made by PE funds using cash distributions or follow-up financing rounds, additional assumptions are necessary to determine the risk of these investments. The estimates of market risk vary widely depending on the assumptions, sample characteristics, and underlying data. Indeed, market loadings range from about 0.3 for buyout funds using the S&P 500 as the market proxy in Driessen, Lin and Philappou (2012) to 4.7 for venture capital (VC) funds using the NASDAQ as the market proxy in Peng (2001).

Several papers use data from individual investments made by PE funds to estimate systematic risk. This approach requires careful treatment of investment outcomes that are not observed because many investments fail. Cochrane (2005) presents a Bayesian model to statistically estimate intermediate valuations of investments based on the characteristics of the investment sample and the subset of investments for which future valuations are observed either at subsequent financing rounds or at successful exits. Cochrane finds a market loading of 1.9 for an IPO/acquisition sample and a market loading of 0.6 for the intermediate financing sample. Korteweg and Sorensen (2009) extend Cochrane's methodology in several ways and find that the market loading is approximately 2.7. Apparently, the estimate of market risk is quite sensitive to the details of the Bayesian methodology. Franzoni, Nowak and Phalippou (2013) use similar data augmented with intermediate cash flows, but follow a portfolio approach ignoring investments with unobserved exits. This paper finds market loadings ranging from 0.95 to 1.4. Franzoni et al. (2013) assume that the cash flows from the investments are reinvested in the S&P 500 until the termination date of all investments in each monthly cohort, and thus, their estimates of systematic risk are based on an unknown and time-varying combination of PE and the market index. All three of these papers estimate risk using investment level data rather than fund level data. Investment level cash flows and valuations are gross of fees charged by the general partners (GPs) of the PE funds and therefore, they do not perfectly reflect the risks borne by limited partners (LPs).

In addition, these analyses use datasets that may suffer from several problems due to potential selection biases. For example, these datasets are compiled from information self-reported by the GPs of the funds and/or from data voluntarily provided by large private equity investors participating as limited partners LPs.[4] It is quite likely that GPs and LPs who do not have good experiences choose not to report their performance at all or choose not to report performance in sufficient detail. In fact, a large majority of the funds in the typical dataset used in the literature do not report cash flows. For instance, only 19% of the funds in the database used by Phallipou and Gottschalg (2009) report cash flows for vintage years from 1994 to 2002.[5]

[4] For example, Venture Economics "is based on voluntary reporting of fund returns by the private equity firms (or general partners (GPs)) as well as their limited partners (LPs)" according to Kaplan and Schoar (2005); Burgiss "is derived entirely from institutional investors (the limited partners or LPs)" that use Burgiss "for record keeping and fund investment monitoring" according to Harris, Jenkison, and Kaplan (2013); and the proprietary dataset used by Robinson and Sensoy (2011) is from "one large institutional limited partner."

[5] In addition, Stucke (2011) finds that 43% of funds with cash flows in VentureXpert have incomplete cash flow information, that is, the cash flow history is often truncated at a point during the active lifetime of the fund.

Since these funds cannot be used to estimate systematic risk or historical performance for PE, the potential for bias due to missing data is considerable.

In contrast to the existing literature, both our samples of listed vehicles avoid the various selection bias problems encountered in this literature for two reasons. First, intermediate valuation is observable using market prices. Second, the samples contain all publicly traded FoFs and LPEs meeting basic criteria. Therefore, we are able to directly estimate the risk characteristics of PE funds using the observed returns of FoF and LPE indices. In addition to market risk, we estimate the exposures of PE funds to other factors, such as the Fama-French risk factors and the momentum factor. We also examine the sensitivity of the value of PE investments to important characteristics of the macroeconomic environment.

Our results indicate that the market risk loadings for both FoFs and LPEs are not significantly different from one. Splitting the sample based on investment type, i.e., buyout vehicles compared to VC vehicles, we find that the point estimates of market risk exposure for buyout vehicles are slightly lower than the analogous point estimates for VC vehicles. However, these point estimates are not statistically different from each other or statistically different from one. Augmenting our analysis with additional factors, we find significantly positive loadings on the Fama-French SMB factor. After controlling for the stock market, the FoF performance index and the LPE index are negatively related to the credit spread and the estimated coefficient for GDP growth changes sign and it is usually not statistically significant. Our results also show that the role of the credit spread is greater for buyout vehicles compared to VC vehicles.

We find that the systematic risk estimate based on the private equity performance index (PEPI) for unlisted PE funds reported by the NVCA is considerably lower than the estimates based on market prices. The NVCA computes the PEPI using cash flows and intermediate valuations reported by PE funds. We find that the stock returns for FoFs predict future returns to the PEPI. Therefore, it is critical to use market prices to estimate the risk and return profile of private equity because the reported net asset values used for the intermediate valuation attenuate any sharp movements in the true value of these assets.

The extensive literature regarding abnormal performance estimates for private equity includes a variety of assumptions to take into account systematic risk. For example, Kaplan and Schoar (2005) assume that beta is equal to one when they examine the performance of unlisted PE funds. However, they acknowledge that "the average return results are potentially biased because we do not control for differences in market risk and because of possible sample selection biases." Phalippou and Gottschalg (2009) conjecture that "the assumption of a beta as one is likely to overstate relative performance" and use a number of other assumptions to adjust for risk. They find that the abnormal returns of PE funds range from approximately zero when beta is assumed to be one to approximately -6% using other benchmarks. The differences in findings due to variation in the assumptions regarding the appropriate benchmark indicate that the typical approach to analyzing the performance of PE funds depends critically on risk adjustment. Thus, our estimates of systematic risk are quite informative.

In addition to estimating systematic risk, we analyze the expected abnormal performance of private equity from a unique perspective. The intuition behind our approach is straightforward. The listed FoFs in our sample are structured as closed-end funds. The relation between the market value of these funds and the amount of money they invest in unlisted PE funds provides a measure of the value added by holding the underlying PE funds net of the extra layer of fees charged by the FoFs. After taking into account the present value of these fees, we infer the abnormal return that the underlying PE funds are expected to earn from the market value of the equity invested in unlisted PE funds. The difference between the amount of cash that raised by a FoF and the combined value of its shares and the present value of fees is a function of the abnormal returns that the market expects underlying unlisted PE funds to earn.

Since we observe the amount of cash that the funds raise and the market value of FoF equity, the only missing information is the present value of FoF fees. We follow several approaches to determine plausible values for FoF fees and extract market's abnormal return expectations that are consistent with the market prices of equity that we observe. We find that a relatively narrow range of abnormal return expectations would be consistent with the observed market prices regardless of the parameters used to compute the present value of FoF fees.

We present an analytically tractable model to determine upper and lower bounds for abnormal returns expectations from underlying PE funds that is consistent with observed market prices and the typical FoF fee structure. We find that the market expects unlisted PE funds to earn abnormal returns between -0.25% and 1.75% before FoF fees. In another set of tests, we use simulations to capture the actual fee structure of each individual FoF as well as the observed fund discount to narrow the range of estimated abnormal return expectations. The results of our simulations indicate that the market's abnormal return expectation is between 0.25% and 0.75% for plausible payout ratios.

We apply the same approach to the LPE sample. Since LPEs do not charge a second layer of fees, our model can be simplified further. The results indicate that the market expects the portfolio of LPE funds to earn abnormal returns between -0.5% and 0.25%. Essentially, the market anticipates that listed private equity funds will earn approximately zero abnormal returns net of fees. Similar to our results for FoFs, market prices of LPEs are inconsistent with either significant negative expected abnormal returns or significant positive expected abnormal returns for private equity in the long run.

The extant literature regarding the abnormal performance of private equity is fairly extensive. Estimates of abnormal performance range from -6% in Phalippou and Gottschalg (2009) to 32% in Cochrane (2005). Higson and Stucke (2012) criticizes the quality of the data used by many previous studies and reports that private equity investments outperformed the S&P 500 by 4.5% per year. This estimate is similar to the findings in Ljungqvist and Richardson (2003). However, Franzoni, Nowak, and Phalippou (2013), Kaplan and Schoar (2005), as well as Peng (2001) find that private equity outperforms the S&P 500 by more modest amounts. Since the estimates of expected returns based on commercial PE datasets vary so widely, it is important to explore an alternative approach based on market prices.

This literature also acknowledges the potential impact of various aspects of selection bias on measured abnormal performance. For example, Cochrane (2005) states that “Overcoming selection bias is the central hurdle in evaluating such investments”. Phalippou and Gottschalg (2009) examine a particular facet of selection bias and indicate that the “commonly used dataset for private equity performance contains funds that perform better than average.” Alternatively, Stucke (2011) finds that “identified errors have a systematic and persistent character” for the same commonly used dataset and these errors “do not just increase noise but result in a significant downward bias of aggregated performance”. Consequently, the direction and magnitude of the aggregate bias regarding measured performance is an open question. Any attempt to address these issues using datasets of PE transactions or of unlisted PE performance assembled through voluntary disclosure or self-reported information will at best be imperfect.

Our approach avoids several potential selection biases encountered in this literature because it contains all publicly traded eligible FoFs and LPEs and uses market prices instead of voluntary reports of net asset values and/or cash flows. The methodology for estimating expected returns is also different along another important dimension. We extract the market's ex-ante expectation of abnormal returns for PE investments from market prices rather than analyze the historical abnormal performance of private equity.

The remainder of this paper is organized as follows: Section II describes the data, Section III examines the risk characteristics of PE investments, Section IV provides estimates of the market's ex-ante expected abnormal returns for unlisted PE funds, Section V discusses whether the FoF sample is an accurate representation of the PE fund universe, and Section VI concludes.

II. Sample

Unlisted PE funds are typically organized as limited partnerships. Outside investors have partnership interests in the funds as LPs and fund managers as GPs. Private equity FoFs are intermediaries that raise capital from investors to finance investments in unlisted PE funds as LPs. In the US, only large institutions and qualified investors, who meet certain minimum wealth and income criteria, are allowed to invest in unlisted PE funds. Many other countries do not have similar restrictions. Therefore, small investors who may not meet US regulatory restrictions or the minimum investment thresholds stipulated by unlisted funds can invest through FoFs in other countries. Some of the FoFs holding unlisted PE funds are actively traded on stock exchanges outside of the US.

To find these publicly traded FoFs, we start by identifying FoFs from the VentureXpert database and the Dow Jones Private Equity Funds of Funds database. Next, we augment this list using the components from the S&P Listed Private Equity Index, PowerShares Listed Private

Equity Fund, Power Shares International Listed Private Equity Fund, Listed Private Equity Index, and International Listed Private Equity Index. We match the names on this augmented list to the universe of traded stocks on Datastream. For the subset of FoFs that can be matched to trading data, we obtain annual reports from company websites and from industry sources for the sample period from 1994 to 2008. We examine each annual report to identify FoFs that invest at least 50% of their capital in unlisted PE funds. Our final sample contains 24 listed funds of funds for private equity.[6]

Table 1 presents descriptive statistics for our sample of FoFs. Nine FoFs in the sample are listed in London, 13 are listed in Continental Europe, and two are listed in Australia. Although these FoFs are traded outside the US, Panel C shows that 10 of them focus primarily on North American PE funds. In terms of the value of investments, 59% of the assets are managed by FoFs that focus on unlisted private equity funds in North America. Therefore, although these FoFs are listed outside the US, unlisted private equity funds in North America dominate the underlying portfolio. The annual reports of these FoFs indicate that the funds invest in a wide variety of unlisted PE funds raised by well-known PE groups.[7]

Table 2 presents the market capitalization, the fee structure, and the number of fund holdings. The average market capitalization one year after the initial public offering (IPO) is $385.6 million and the range for market capitalization is from $11 million to $1.8 billion. Since the median (average) number of distinct underlying funds held by each fund of funds is 19 (55), the combined portfolio of unlisted private equity funds appears to be well diversified. We also obtain the proportion of FoF shares that are held by shareholders with more than 5% ownership

[6] If we require more than 75% of invested capital be allocated to unlisted PE funds, our estimates of systematic risk and the average FoF discount remain virtually unchanged.
[7] Among others: Berkshire Fund VII, Blackstone Capital Partners V, Carlyle Partners V, KKR 2006 Fund, Silver Lake Partners III, TPG Partners V, and Warburg Pincus Private Equity X.

from the annual reports. The average ownership by large shareholders is 34.9% for the 17 FoFs disclosing this information. Thus, large shareholders, who are presumably sophisticated investors, own substantial stakes in these funds.

We also examine the risk and expected returns of LPEs. Our primary source for LPEs is the VentureXpert database. The managers of listed private equity vehicles classified as "funds" in VentureXpert are compensated through management fees similar in structure to those of unlisted PE funds.[8] Table 3 presents descriptive statistics for this sample of 129 LPEs. Of these funds, 99 are listed in London and only 6 are listed on US exchanges. In terms of the value of investments, Panel C indicates that 36% of the assets are managed by LPEs that focus on North America. Compared to FoFs, the LPEs in our sample invest more heavily in private equity outside of North America.

III. Risk Characteristics of Private Equity

This section examines the risk profile of PE investments. The wide range of risk estimates for PE in the extant literature highlights the sensitivity of these estimates to the choice of datasets and to the underlying assumptions. To a large extent, this dependence arises from the fact that the existing estimates for systematic risk utilize realized cash flows rather than market prices and must rely upon explicit assumptions (e.g., intermediate valuation of investments from Bayesian approaches) or implicit assumptions (e.g., PE funds without performance information) about missing data. In turn, the imprecise estimation of the risk for PE precludes the reliable evaluation of the performance of PE. Since we have market prices, we are able to reliably estimate multiple dimensions of exposure to systematic risk using traditional time series regressions and value-

[8] To verify the completeness of VentureXpert, we examine a subsample of listed entities that were not classified in VentureXpert as "funds" but were included as components of the S&P Listed Private Equity Index. These entities are usually holding companies for separate operating companies that do not compensate their managers using the typical PE management fee structure.

weighted indices of FoF and LPE performance. We measure systematic risk using both single factor and multi-factor models. In addition, we analyze the sensitivity of FoF and LPE performance indices to aspects of macroeconomic activity. Understanding the systematic risk characteristics of PE is critically important for the asset allocation decisions of large institutional investors, including pension funds and endowments.

A. Private Equity Indices Using Market Prices

To examine the systematic risk of various categories of PE funds, we generate value-weighted indices for various categories of listed entities in our sample. Specifically, we form the following six indices: all FoFs, buyout FoFs, VC FoFs, all LPEs, buyout LPEs, and VC LPEs. To create the subcategories for FoFs, we classify each FoF as buyout or VC using the asset allocation in the annual report filed at the end of the previous year. If this annual report for a FoF indicates that a greater percentage of assets under management are allocated to unlisted buyout (VC) funds rather than unlisted VC (buyout) funds, it is deemed to be a buyout (VC) FoF. To create subcategories for LPEs, we classify each LPE fund as buyout or VC using the stage focus in VentureXpert.

Since all of the listed entities in our sample are organized as closed-end funds that hold private equity, we must address a minor complication in our methodology. The existing literature about closed-end funds indicates that the average closed-end fund exhibits substantial underperformance in the subsequent months following the IPO. For example, Weiss (1989) and Peavy (1990) find that closed-end funds holding publicly traded securities underperform the market by more than 10% during the six months following issuance. Indeed, our sample of FoFs has a substantial negative average cumulative return during the months immediately after the IPO. In addition, Weiss (1989) indicates that the typical equity closed-end fund is issued at a premium to net asset value (NAV), but the market price falls in the post-IPO period, and thus, the average closed-end equity fund trades at a discount of approximately 10% within six months of the IPO. The evolution of the average discount for the listed entities in our sample follows a similar path to the average discount for closed-end funds in the US holding publicly traded securities.[9] FoFs and LPEs are usually issued at a significant premium, but they typically trade at a substantial discount within one year. To avoid any potential for this pattern of underperformance following an IPO to influence the estimates of systematic risk, each listed entity enters the relevant index 12 months after its IPO.[10]

B. Estimates of Systematic Risk from Factor Models

We examine systematic risk in the context of the CAPM and a 4-factor model including the three risk factors from Fama and French (1993) as well as a momentum factor. Since both the FoF and LPE samples contain international PE investments, we use the MSCI world index as the appropriate proxy for the market factor. We also examine the systematic risk with respect to the S&P 500 index to examine the sensitivity of our indices to the US market.

We consider the following time series regressions to estimate systematic risk:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,\text{SMB}} SMB_t + \beta_{i,HML} HML_t + \beta_{i,\text{MOM}} MOM_t + \varepsilon_{i,t}\,, \qquad (1)$$

where $R_{i,t}$ is the index return of the FoF index or the LPE index. Depending on the specification, $R_{m,t}$ is the return for the MSCI world index or the return for the S&P 500 index. $R_{f,t}$ is the one-month US Treasury bill rate. In the single factor specification, we exclude the size, book-to-market and momentum factors, labeled SMB, HML, and MOM, respectively. In

[9] We discuss the post-IPO evolution of the average closed-end fund discount for FoFs and LPEs in greater detail as part of the analysis of expected abnormal performance in Section IV.
[10] The inclusion of data for each listed entity during the first year following an IPO does not substantially alter the estimates for the systematic factor loadings.

other specifications, we use factor returns from Kenneth French's website. We note that these three factors are constructed only using stocks listed in the US. Nevertheless, since the US equity market is the largest part of the world portfolio, the sensitivity of the fund indices to the US factors will shed important insights into the nature of the risks associated with PE investments.

Table 4 presents the regression estimates and corresponding Newey-West standard errors with six lags. In the 1-factor model, the systematic risk estimates for FoFs and LPEs based on the MSCI World index are 0.82 and 1.14, and based on the S&P 500 index are 0.71 and 1.05, respectively. Since the underlying funds have substantial international exposures, both FoFs and LPEs have higher sensitivity to the MSCI World index and the MSCI World index has greater explanatory power than the S&P 500 index. Even though all of the estimates of market risk are systematically different from zero, none of these estimates is significantly different from one.

The model presented in Section IV indicates that the magnitude of each risk factor loading for the FoF index is proportional to the magnitude of the analogous factor loading for the underlying portfolio of PE funds. Given the parameter values determined by the characteristics of the FoFs in our data, this scaling parameter is approximately equal to 1. Thus, the observed factor loadings for the FoF index are similar in magnitude to the factor loadings of the underlying portfolio of unlisted PE funds.

The results in Table 4 also include estimates from specifications of the four-factor model. With the MSCI index as the market factor, the loadings with respect to the SMB factor are 0.49 and 0.48 for the FoF and LPE indices, respectively. Both these estimates are significantly greater than zero. Therefore, the performance of both FoFs and LPEs behaves more like small-cap stocks than large-cap stocks. This finding is intuitive since most PE funds invest in firms that are smaller than the average listed firm. The factor loading with respect to HML is 0.35 for FoFs and

0.27 for LPEs. Although these point estimates are not significantly different from each other, the HML loading for FoFs is not significantly different from zero while the analogous loading for LPEs is marginally significant. There is suggestive evidence that both FoFs and LPEs are slightly more sensitive to value stocks than growth stocks. One possible explanation for this finding is that these listed entities have significant investments in buyouts and targets of buyouts are more likely to be value than growth firms. The loading for the momentum factor is not statistically different from zero for FoFs or LPEs and excluding this factor from the specifications does not qualitatively affect the findings.

Kaplan and Schoar (2005) acknowledge the difficulty in estimating the stock market beta for PE funds due to "the lack of true market values for fund investments until the investments are exited" and they assume that beta is equal to one. Phalippou and Gottschalg (2009), however, conjecture that "the assumption of a beta as 1 is likely to overstate relative performance" and they use an industry/size-matched cost-of-capital benchmark. Our findings indicate that the point estimates of systematic risk for FoFs and LPEs are indeed close to one and not significantly different from one. Hence, the benchmark selected by Kaplan and Schoar (2005) appears to be quite reasonable.

The estimate of the intercept for each of these regressions is the average ex-post abnormal return relative to a specific factor model for a given PE index during the sample period. The estimates for both FoFs and LPEs are not significantly different from zero using either the CAPM or the four-factor benchmark. Therefore, these listed vehicles performed as expected on average, conditional on the realizations of the risk factors. Since ex-post abnormal performance for these two indices is not statistically different from zero, the market prices for these entities must have been a relatively unbiased reflection of risk-adjusted expected future cash flows generated by the PE investments during the sample period. Thus, there is no persistent mispricing of the average FoF or the average LPE in these indices.

We must emphasize that the intercept estimates should not be interpreted as a measure of whether or not PE investments themselves earn abnormal returns. In an efficient market, investors rationally anticipate any potential ability of PE funds to earn abnormal returns above their cost of capital, and hence any abnormal returns that PE funds may earn would be reflected in post-issue market prices. We develop a methodology to extract estimates of expected abnormal performance from the discounts of FoFs and LPEs in Section IV.

C. Private Equity Performance and the Macroeconomic Environment

This section examines the relation between PE fund performance and macroeconomic activity after controlling for market returns. We use GDP growth and the credit spread to capture macroeconomic activity.

GDP growth could have a positive impact on fund performance after controlling for contemporaneous market returns if higher macroeconomic activity has a greater effect on early stage firms than on more mature firms that form a large part of the market index. Substantial GDP growth could potentially allow PE funds to profitably exit some underlying investments via IPO and lead to better performance for PE funds. However, economic growth could also result in greater competition between PE funds. For instance, Gompers and Lerner (2000) find that the valuation of investment opportunities increases during periods of expansion, and hence, the increased competition that comes with economic growth may have a negative impact on fund performance. Kaplan and Schoar (2005) report that funds raised during periods of increased competition tend to underperform funds raised during other periods, and find support for the hypothesis that performance suffers whenever money chases deals. The net effect of the positive impact of economic growth and the negative impact of increased competition on the covariance between economic growth and performance is determined by the relative impact of these two offsetting forces.

The credit spread is another measure of macroeconomic conditions. As Fama (1990) suggests, a widening spread generally signals deteriorating business conditions, which would make it more difficult for PE funds to exit underlying investments via IPO. Moreover, a high credit spread implies a high cost of raising new risky debt, and hence, it would likely have an adverse impact on the performance of PE funds attempting to finance new transactions involving substantial leverage. Therefore, we expect a negative relation between the performance of PE funds and the credit spread.

We obtain data for GDP growth and the credit spread from the Federal Reserve Economic Data (FRED) website. Table 5 reports the estimates for the following equation:

$$\begin{aligned} R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,\mathrm{SMB}} SMB_t + \beta_{i,\mathrm{HML}} HML_t + \beta_{i,\mathrm{MOM}} MOM_t \\ + \beta_{i,\Delta GDP} \Delta GDP_t + \beta_{i,\mathrm{CS}} CS_t + \varepsilon_{i,t} \end{aligned}, \qquad (2)$$

where $R_{i,t}$ is the index return of the FoF index or the LPE index, $R_{m,t}$ is the return for the MSCI world index, and $R_{f,t}$ is the one-month US Treasury bill rate. Size, book-to-market and momentum factors are labeled SMB, HML, and MOM, respectively. Percentage real GDP growth is labeled as ΔGDP_t and CS_t is the difference between the yield on Baa-rated and Aaa-rated corporate bonds. For ease of interpretation, we standardize both ΔGDP_t and CS_t by demeaning both variables and dividing by their respective standard deviations. Therefore, the slope coefficient for each variable may be interpreted as the change in performance associated with an increase of one standard deviation in the variable of interest.

We find that the credit spread is negatively related to FoF and LPE returns, after controlling for market returns, factor realizations, and GDP growth. For instance, a one standard deviation change in the credit spread is associated with a 1.9% decrease in the excess return of the FoF index according to the estimates in Column 4 of Table 5. Essentially, restrictive credit conditions reduce the performance of private equity. This result is also consistent with the possibility that the performance of private equity depends significantly on credit risk.[11] The coefficient estimate for GDP growth is not significant in many specifications and the estimate changes sign in some specifications after controlling for the credit spread.

D. Comparison of Buyout Funds and Venture Capital Funds

Table 6 reports the factor loadings for buyout funds and VC funds separately for FoFs and Table 7 reports the analogous factor loadings for LPEs. In columns 1 and 2 of these tables, we find qualitatively similar market risk loadings for buyout vehicles and VC vehicles. Market risk estimates for buyout and VC entities are not significantly different from one and they are not significantly different from each other for both FoFs and LPEs. However, the point estimates of the market loading are somewhat smaller for buyout entities than for VC entities. The market loadings for buyout entities are 0.68 (FoFs) and 0.95 (LPEs) while the loading for VC entities are 0.98 (FoFs) and 1.19 (LPEs).

Columns 3 and 4 of these two tables report four-factor risk loadings separately for buyout entities and VC entities. For both FoFs and LPEs, the coefficients on the size factor are significantly positive for buyout vehicles as well as for VC entities. While the point estimates of the SMB factor loading are slightly higher for buyout vehicles compared to VC vehicles, the

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[11] We find similar results using the S&P 500 index instead of the MSCI World as the market portfolio. In addition, IPO volume is not significantly related to the performance of the FoF index or the LPE index after controlling for GDP growth and the credit spread and the inclusion of IPO volume does not qualitatively change the estimates for the other explanatory variables.

magnitudes are not significantly different from each other for FoFs or LPEs. The results indicate that PE investments, regardless of type, share the risk characteristics of small stocks. The point estimates of the HML factor loading are higher for buyout vehicles compared to VC vehicles. While this difference is not significant for FoFs, the results for LPEs indicate that the magnitude of the HML loading for buyout entities is significantly different from zero and significantly different from the analogous estimate for VC entities. Interestingly, the point estimates for the HML loading are still positive, albeit not significant, for VC entities. Therefore, the evidence suggests that the systematic risk of buyout entities is related to the value factor, while VC vehicles do not share the systematic risk characteristics of growth stocks.

Columns 5 through 8 in both tables analyze the relation between performance and the macroeconomic environment separately for buyout entities and VC entities. The point estimates for the GDP growth coefficient are usually higher for buyout entities compared to VC entities, but the differences are not statistically significant. The results for the credit spread are somewhat more informative. The coefficient estimates are more negative for buyout vehicles compared to VC vehicles and this difference is marginally significant at the 10% level for both FoFs and LPEs. Thus, favorable credit market conditions appear to be more important for buyout entities. This result is consistent with the differential importance of leverage for buyout transactions compared to VC transactions. Essentially, there must be a pronounced reduction in the performance of buyout vehicles when credit conditions are restrictive because buyout transactions depend much more heavily on the use of leverage.

E. Systematic Risk of the Private Equity Performance Index

Practitioners frequently use the Private Equity Performance Index (PEPI) for VC funds and buyout funds to measure the performance of the PE industry. Thomson Reuters computes PEPI based voluntary reports from GPs of fund NAV and cash flow data on a quarterly basis from PE funds in the Thomson database. The index does use NAV data to calculate for interim returns. Setting aside any directional biases, self-reported NAVs may not reflect market valuation changes in a timely manner. For instance, these values reported by funds may only partially adjust to changes in their true value. Under this partial adjustment hypothesis, the PEPI does not reflect the actual changes in the value of PE investments in a timely manner since smoothed NAVs are used to compute the PEPI. In contrast to the PEPI, we compute the index for FoFs using market prices. Since market prices should reflect fundamental values quickly, indices based on market prices should be able to predict changes in the PEPI. To examine whether the FoF index can predict future changes in book values embedded in the PEPI, we examine the relation between the PEPI return and contemporaneous and lagged values of the MSCI World index return as well as the FoF index return.

Table 8 reports the regression results for a variety of specifications. Since PEPI is published quarterly, we also use quarterly versions of each explanatory variable. The slope coefficient from the regression of PEPI on the MSCI World index is statistically significant. However, the estimated coefficient of 0.45 for the systematic risk is considerably smaller than the analogous estimates of systematic risk of 0.82 and 1.08 for FoFs and LPEs, respectively. The partial adjustment of the NAVs used in the computation of PEPI will result in a smaller slope coefficient because the slope coefficient estimate for contemporaneous returns will understate the true sensitivity of PE funds to the stock market.[12]

To further test the partial adjustment hypothesis, we incorporate the contemporaneous and lagged MSCI World returns in the regression specification as well as the contemporaneous

[12] The relatively low estimate of systematic risk for the private equity performance index (PEPI) is not related to our choice of the MSCI World index as the measure of public equity performance. We find a virtually identical estimate of systematic risk using the S&P 500 index instead.

and lagged return for the FoF index in a variety of different specifications. Since FoF returns reflect changes in the value that are unique to the PE sector, lagged FoF returns should be able to incrementally predict PEPI returns under the partial adjustment hypothesis. The results in column 6 of Table 8 indicate that the slope coefficients on lagged MSCI World and FoF index returns are 0.07 and 0.10, respectively, and both are statistically significant. Therefore, both the lagged MSCI World index and the lagged FoF index are useful in predicting PEPI returns. Since the results remain largely unchanged even when the lagged performance of the PEPI index is included, these results are not largely due to the autocorrelation of the dependent variable. The findings support the partial adjustment hypothesis and imply that the FoF index provides crucial information about the performance of the unlisted PE industry. In unreported regressions we find similar results using the LPE index in place of the FoF index.

IV. Expected Abnormal Performance of Private Equity

In this section, we develop a methodology to extract market’s abnormal return expectation for PE investments based on the market prices of FoFs and LPEs. In contrast to the existing literature, we estimate ex-ante expected abnormal returns since we observe the amount of money raised by the FoFs and LPEs as well as the market value of these vehicles after the IPO. The difference between the market value and the amount of capital used to invest in PE is related to the net present value (NPV) of the underlying investment in PE. We use this information to estimate expected abnormal returns.

A. Approach and Model

While the theoretical basis for our methodology is straightforward, there are a few practical issues that need to be addressed. FoFs charge fees for managing the capital they raise. These fees are in addition to the fees charged by the unlisted private equity funds, and therefore, the market capitalization of a FoF reflects the present value of cash flows net of the second layer of fees. Consequently, the market value of the underlying portfolio of limited partnerships in unlisted PE funds held by a FoF is the sum of the market capitalization based on the listed shares of the FoF and the present value of fees owed to the manager of the FoF. In contrast, the market value of the shares of an LPE fund is directly observable and is the equivalent of a LP in an unlisted PE fund. There is no second layer of fees to consider for LPEs.

Figure 1 presents the intuition for our approach with respect to FoFs. These entities invest the amount they raise at IPO (after issuance costs) in unlisted PE funds. This is the book value of the initial investment in PE. The unobserved market value of this investment in unlisted PE funds consists of two components. The first component is the market value of FoF equity, i.e., the aggregate value of the investors' shares, and the second component is the present value of FoF fees. The difference between the total market value of the investments in unlisted PE funds and the book value of these funds is the NPV of the underlying portfolio of unlisted PE funds. If the market expects positive abnormal returns from PE investments then the total market value is greater than the book value (Panel A). However, if the market expects PE investments to earn negative abnormal returns, then the total market value is less than the book value (Panel B). Holding all else constant, the magnitude of this NPV reflects the magnitude of the expected abnormal return for underlying PE funds in the long run. Given the assumptions of the model, we translate this NPV into an estimate of the expected abnormal return. For a LPE fund, the NPV only depends on the market capitalization and the book value of the underlying investments.

The computation of the market value of the traded shares is based on the price and number of shares outstanding. The inclusion of the present value of FoF fees, however, is less straightforward since claims to these fees are not traded. We use two approaches to incorporate the value of FoF fees and extract market expectations of abnormal returns for PE investments. In the first approach, we use a theoretical model with analytic formulas for FoF fees as a function of the market value of traded shares and the fee structure. We show that the results of this model and observed FoF discounts provide upper and lower bounds on the magnitude of abnormal returns that the market expects PE funds to earn. We find that the distance between these bounds is fairly narrow and that these bounds exclude many estimates in the literature. In addition, a special case of this approach in which the two components of the fee structure are set to zero provides the basis for extracting an estimate of expected abnormal performance from LPE discounts. In the second approach, we use Monte Carlo simulations to compute the present value of FoF fees based on the actual fee structure of each FoF because some of the features of these fee structures are not analytically tractable. We estimate the market's abnormal return expectation for PE funds using our simulation results and the corresponding FoF discounts.

Our model is specified as follows. Let NAV_t be the net asset value at time t and let l be the base management fee expressed as a fraction of NAV_t. Let $R_{u,t}$ be the return of the underlying PE fund portfolio held by the FoF (gross of FoF fees but net of fees paid to the GPs of the underlying funds). We assume that $\{R_{u,t}\}$ is independently and identically distributed. Let p be the proportional incentive fee, which is computed as a fraction of the return after the base management fee is paid. For the sake of analytical tractability, we assume that the incentive fee is not subject to a hurdle rate or high watermark.[13] Similarly, let q be the proportional

[13] Several funds apply a hurdle rate. The incentive fee is a percentage of the profits that exceed a specific level determined by the hurdle rate. Often, the incentive fees are payable only if the NAV exceeds a high watermark, which is the highest NAV that the FoF had previously reached.

reinvestment rate (retention ratio). It is also defined as a fraction of the return after the base management fee is paid. Thus, each FoF makes the following distributions at time *t+1*:

$$
\begin{aligned}
&\text{Base Managment Fee} && = \lambda NAV_t \\
&\text{Incentive Fee} && = \pi NAV_t \left(R_{u,t+1} - \lambda \right) \\
&\text{Distribution to FoF Shareholders} && = \left(1 - \theta - \pi\right) NAV_t \left(R_{u,t+1} - \lambda \right) \\
&\text{Retained Earnings} && = \theta NAV_t \left(R_{u,t+1} - \lambda \right).
\end{aligned}
\tag{3}
$$

To ensure that investors receive positive distributions on average, the sum of the retention ratio (θ) and the proportional incentive fee (π) must be less than unity. Consequently, the retention ratio has a range from zero to $1-\pi$ and the payout ratio ($1-\theta$) has a range from π to unity. The payout ratio is the proportion of $NAV_t\left(R_{u,t+1}-\lambda\right)$ that is distributed to investors or paid to the manager as an incentive fee. Thus, the evolution of the net asset value is:

$$NAV_{t+1} = \left(1+\theta\left(R_{u,t+1}-\lambda\right)\right)NAV_t. \tag{4}$$

In general, the base management fee inherits some of the underlying fund risk as long as the fund retains a portion of its earnings for reinvestment. However, if all earnings are distributed to managers and shareholders, i.e., if $\theta = 0$, then the base management fee is riskless.[14]

Let α be the abnormal return that the market expects the underlying portfolio of PE funds to earn. The expected return for the underlying PE portfolio is determined by the cost of capital according to a *K*-factor model plus this expected abnormal return:[15]

$$E_t\left[R_u\right] - R_f = \alpha + \sum_{k=1}^{K} \beta_{k,u}\left(E_t\left[R_k\right] - R_f\right), \tag{5}$$

---

[14] Implicitly, this specification assumes that the FoF manager pays the fund if the fund return is lower than λ. However, FoF managers do not make such payments in practice. Instead, the high watermark reduces subsequent incentive compensation by the amount corresponding to any return shortfall. While our model is analytically appealing, it changes the timing of cash flows to managers slightly. We relax this assumption in our simulation analysis to address any potential issues stemming from this assumption.

[15] Since PE funds invest at privately negotiated prices and may also implement operational changes, the expected return for these investments can deviate from the CAPM depending on the level of managerial skill.

where $\beta_{k,u}$ is the underlying portfolio loading with respect to factor *k*, $E_t\left[R_k\right]$ is the expected return for factor *k*, and R_f is the risk-free rate. We assume that all of these moments are constant for convenience. The return earned by the underlying portfolio is defined in this context using the quantity of money invested in the PE funds and not the market value of the underlying portfolio. The market value of the underlying portfolio would incorporate the effect of any anticipated outperformance, and hence, in an efficient market the expected abnormal return based on the market value of the underlying PE portfolio would always be zero.

It is possible that investors must pay a premium in equilibrium to hold the liquid version of a typically illiquid LP investment in the underlying PE funds. If such a premium is outside the specified factor model for traded risky assets, then this premium would be reflected in the expected abnormal return for the underlying PE funds relative to the factor model. Thus, a in our model incorporates any illiquidity premium earned by the underlying unlisted PE funds that is not captured by risk premium associated with the factor loadings.

We assume that the valuation of each claim to the cash flows of the FoF is consistent with a *K*-factor model of systematic risk. Essentially, the present value of distributions to investors and the present value of the management fees from the perspective of investors are determined by participants in an efficient market governed by a *K*-factor model. The market value of FoF equity is equal to the present value of the distributions to investors (net of the fees paid to the management company of the FoF) and is given by the following equation:

$$V_t = E_t\left[\sum_{s=1}^{\infty}\frac{\left(1-\theta-\pi\right)\left(R_{t+s}-\lambda\right)NAV_{t+s-1}}{\left(1+E_t\left[R_e\right]\right)^s}\right], \tag{6}$$

where $E_t\left[R_e\right]$ is the discount rate for equity. Since distributions to shareholders are made after payment of base management fees, $E_t\left[R_e\right]$ will usually be different from the discount rate for the underlying PE funds.

Proposition 1 below provides an analytic solution for the present value of distributions to investors stated in terms of the discount for the FoF as well as an expression for the endogenously determined value of $E_t\left[R_e\right]$.

Proposition 1. *The FoF discount and the associated discount rate for equity are specified by the following two equations:*

$$D_t = 1 - \frac{V_t}{NAV_t} = 1 - \frac{(1-\theta-\pi)\left(\alpha-\lambda+R_f\right)}{R_f - \theta\left(\alpha-\lambda+R_f\right)} \tag{7}$$

and

$$E_t\left[R_e\right] = R_f + \left(\frac{R_f}{\alpha-\lambda+R_f}\right)\sum_{k=1}^{K}\beta_{k,u}\left(E_t\left[R_k\right]-R_f\right). \tag{8}$$

Proof: See appendix.

Equation (7) indicates that the FoF discount is a decreasing function of the expected abnormal return and an increasing function of both the base management fee and the incentive fee.

Equation (8) provides the discount rate for FoF equity. This equation implies that the systematic factor loadings for the return net of fees is equal to a scaled version of the systematic factor loadings for the underlying portfolio. The scaling parameter is equal to $R_f/\left(\alpha-\lambda+R_f\right)$. Since the base management fee is a fixed proportion of NAV_t, this fee magnifies the risk borne by equity holders. However, the impact of the base management fee on the scaling parameter is offset by the expected abnormal return of the underlying portfolio because a higher expected abnormal return raises the market value of the claim relative to the quantity of systematic risk.

Indeed, if the base management fee charged by the FoF and the expected abnormal return to the underlying portfolio are similar in magnitude, then the observed factor loadings for the FoF will be similar to the factor loadings of the underlying portfolio.

Interestingly, Equation (8) also indicates that the proportional reinvestment policy does not change the systematic risk of the investors' claims. Instead, the proportional reinvestment policy only changes the systematic risk of the base management fee. Since the incentive fee is treated as shares of equity in the FoF, the incentive fee also does not alter the factor loadings of the equity claim. For the many FoFs in our sample that do not have an incentive fee and the few FoFs that use founder shares for the incentive fee, our model applies exactly as written. For the other half of the sample, the model is only approximate because the incentive fee is characterized as an equity claim rather than calculated using the appropriate hurdle rate with a high watermark.

Although the model utilizes a simplified fee structure, the analytic results we derive provide upper and lower bounds on market's expectation of abnormal returns under the actual fee structures we observe in practice. First, we determine a lower bound by answering the following question: given the level of FoF discount we observe, what is the lowest possible level of market's expectation of abnormal returns?

As we discussed in the context of Figure 1, the difference between the NAV and the total market value of the underlying portfolio held by the FoF is positively related to the expected abnormal return. We observe the NAV and the market value of equity. The only unobservable component of total market value is the value of FoF fees. For any given NAV and market value of equity, a smaller value of FoF fees would imply a smaller expected abnormal return. Therefore, if we find a lower bound for the value of FoF fees, then we also have a lower bound for the expected abnormal return. If we assume that the incentive fee is zero, i.e., $p = 0$, then we have determined a lower bound for FoF fees and the hurdle rate and high watermark become irrelevant. Rearranging Equation (7) and setting $\rho = 0$, yields the following lower bound α_l:

$$\alpha_l = \lambda - R_f + R_f \left(\frac{1 - D_t}{1 - \theta D_t} \right). \tag{9}$$

In addition to finding a lower bound for expected abnormal performance in the context of FoFs, if we also assume that the base management fee is zero in Equation (9), i.e., $\lambda = 0,$ then the equation provides an exact relation between the expected abnormal return and the discount for LPEs because there is no second layer of fees for these entities.

$$\alpha = R_f \left(\frac{1 - D_t}{1 - \theta D_t} \right) - R_f . \tag{10}$$

Thus, for the LPE sample, Equation (10) indicates that if the fund discount is zero then expected abnormal return for the underlying portfolio PE investments is also zero.

If we find an upper bound for the value of FoF fees, then there exists a corresponding upper bound for α. The value of FoF fees without a hurdle rate will be greater than the value with a hurdle rate. Since the hurdle rate is always non-negative, the analytic present value of the incentive fee is an upper bound on the present value of the incentive fee in practice.[16] Therefore, the α in Equation (7) is itself an upper bound for the expected abnormal return because this equation was derived while ignoring the hurdle rate. Specifically, we get the following upper bound α_h by rearranging Equation (7):

$$\alpha_h = \lambda - R_f + R_f \left(\frac{1 - D_t}{1 - \rho - \theta D_t} \right). \tag{11}$$

[16] This discussion overlooks the mismatch between the timing of incentive fee payments in the model relative to standard practice whenever the underlying portfolio return is sufficiently low. The value of the incentive component of the management fee is understated due to the time value of money because negative incentive payments in the model are delayed in the real world to offset subsequent profits. However, for plausible hurdle rates, disregarding the hurdle rate entirely more than counterbalances this understatement.

The ratio of the price of a fund and its net asset value, a transformation of the fund discount, is broadly due to two features in the model. The first feature is the fees charged by the FoFs and the second feature is the expected abnormal return for the underlying PE investments. Previous work by Lee, Shleifer, and Thaler (1991) indicates that the fund discounts for closed-end funds holding publicly traded securities vary over time and such variations may be related to behavioral factors. Although the evidence may suggest that fund discounts are larger when the investors are pessimistic and smaller when investors are optimistic, the extensive literature about closed-end fund discounts does not claim that the unconditional average discount is incorrect or that discounts are not largely driven by basic economic factors such as fund fees and the expected abnormal performance of the underlying portfolio.

The idea that economic factors explain the average closed-end fund discount is also supported by the empirical evidence in Ross (2005). Ross analytically derives the theoretical discount based on the present value of fees and the assumption that the underlying portfolio has no abnormal performance. He finds that the average theoretical discount for closed-end funds based on the present value of fees is equal to the average actual closed-end fund discount of 7.7%. Therefore, he concludes that factors other than the present value of fees are not important determinants of the unconditional closed-end fund discount. While Ross's (2005) continuous-time model assumes that fees are proportional to assets under management and that expected abnormal return for the underlying portfolio is zero, our discrete-time model incorporates the possibility that funds holding PE may be able to earn expected abnormal returns and includes the possibility of an incentive fee. The generalization regarding expected abnormal performance is particularly important because we examine whether or not the market expects the underlying portfolio to exhibit substantial abnormal performance.

Extracting estimates of the expected abnormal return in our context does rely on the assumption that the unconditional discount one year after the IPO is an accurate reflection of economic factors. The evidence in Table 4 (discussed above) indicates that the ex-post monthly performance, in which we exclude the first twelve months of performance for each listing, is not significantly different from zero for FoFs or LPEs using either the CAPM or the four-factor benchmark. Therefore, the market prices for these entities from that point in time onwards must have been a relatively unbiased reflection of risk-adjusted expected future cash flows generated by the PE investments unconditionally. This evidence directly supports the assumption underlying the extraction approach.

B. Extracting Expected Abnormal Performance from the Average Discount

Since the PE vehicles in our sample are organized as closed-end funds, the market value of equity and book value of equity for these entities are key inputs for determining the market's expectation of abnormal returns for investments in PE. Because the amount available for investment is initially raised in cash, the most accurate measure of the amount actually invested in PE is the NAV known at the time of the IPO. Therefore, in much of the following analysis we will focus on valuation of these PE vehicles in the months immediately following the IPO.

We examine the event-time evolution of fund discount, D_t, defined as:

$$D_t = 1 - \frac{V_t}{NAV_t}, \tag{12}$$

where V_t is the price and NAV_t is the net asset value per share t months after the IPO date. We obtain prices from Datastream and NAV information from Bloomberg. We obtain the first NAV from the IPO prospectuses for FoFs.

Table 9 presents the patterns for the average discount for FoFs and LPEs during the first 12 months in event time after the IPO. The results for FoFs are contained in column 1. These entities are issued at a premium of 4% and trade at a small discount within a month of the IPO. The average discount increases to more than 6% three months after the IPO and continues to increase gradually to approximately 12% by month ten and fluctuates around this level for the remainder of the year. The evolution of the average FoF discount following IPOs in our sample follows a similar path to the average discount for closed-end mutual funds in the US that hold publicly traded stocks and bonds documented by Weiss (1989) and Peavy (1990). In our context and for closed-end funds in general, the pattern for discounts is driven by the underperformance of the shares rather than changes in NAV.

Column 3 of Table 9 contains the average discount during the first 12 months in event time after the IPO for LPEs. These vehicles trade at a 2% premium to NAV at the end of the first month. However, the average premium declines to about 1.6% six months after the IPO. By the end of the first year after the IPO, LPEs trade at an average discount of 3.5%. However, the average discount is not significantly different from zero at any point in event time, and the transition from premium to discount is more gradual. These results indicate that although the IPO premium eventually disappears, the average LPE price is near its NAV, and consequently, expected abnormal performance is near zero.

We use Equation (9) and Equation (11) to determine the upper and lower bounds on market's abnormal return expectations for the underlying PE funds held by FoFs. To do so, we select the values of various parameters in the equations based on the corresponding values in the data during our sample period. We set the risk-free rate (R_f) equal to 6%, which is near the mid-point of the average 20-Year Treasury Constant Maturity of 5.74%, and the average yield for

Aaa-rated corporate bonds of 6.55% during our sample period. We set the base management fee (l) equal to 1% based on the average base management fee reported in Table 2. To determine the upper bound we also set the incentive fee (p) equal to 10% (the average incentive fee for FoFs with an incentive fee).[17]

With these parameters, we can determine the market's abnormal return expectation based on any observed discount. Since the FoF discount is -4% at the IPO and eventually becomes 12% twelve months after the IPO, extracting the expected abnormal return at the time of the IPO is an interesting starting point. The pair of lines labeled "IPO Discount" in Figure 2 plots the bounds on the expected abnormal return for IPO investors. We find that a -4% discount implies an expected abnormal return between 1% and 2%. Is this expected abnormal return estimate based on the IPO investors unbiased? Investors purchasing closed-end funds at a premium underperform the market by more than 10% during the six months following the IPO according to Weiss (1989) and Peavy (1990). We find similar underperformance for IPO investors in the case of FoFs. Given this initial underperformance, it seems untenable to assume that the expectations of these initial investors are equivalent to the market's expectations. The secondary market price, however, eventually reflects aggregate market expectations, and investors trading at the post-issue price receive an expected abnormal return of zero, regardless of the expected abnormal return on the underlying portfolio. Therefore, we use the average FoF discount 12 months after the IPO to infer the expected abnormal return for unlisted PE funds.

The average FoF discount twelve months after IPO is approximately 12% and the two standard deviation confidence interval around this point estimate ranges from 5% to 19%. We examine the implied values of a for these three levels of fund discounts. The final parameter

[17] This value for the incentive fee overstates the average incentive fee for our sample because the sample includes several FoFs that do not charge any incentive fees. Hence, it provides an upper bound compared to the average incentive fee for the full sample that includes FoFs that do not charge an incentive fee.

that we need to specify is the market's expectation of the payout ratio $(1-\theta)$. The average payout ratio in the sample is approximately 0.3, but this average is not necessarily the market's expectation of the payout ratio in the future. Because many of the funds in our sample are early in their life cycle, they might choose to have low payout ratios during this time period. We examine the implied values of a for a wide range of payout ratios, ranging from 0.25 to 1. Since there is no statistical difference between the average discounts for FoFs that predominantly hold buyout funds and for FoFs that predominantly hold VC funds, the market's abnormal return expectation is similar for both types of PE funds.

Figure 2 plots the upper and lower bounds for expected abnormal performance. This figure indicates that the lower bound for a is always greater than -0.25% and the upper bound is always less than 1.75%.[18] The difference between the upper and lower bounds for the expected abnormal return, given a specific discount level and payout ratio, is less than 1%. The model produces a relatively narrow range for the market's expectation of a compared to the wide range of estimates offered in the literature. Our range, however, indicates that the market considers investing as an LP in unlisted PE funds to be a positive NPV project for a range of plausible payout ratios. Hence, reasonable assumptions rule out negative expected abnormal returns.

We also examine the expected abnormal return for the PE investments of LPEs. Since LPEs and unlisted PE funds both conduct private equity transactions, they have similar goals. The managers of LPEs are also compensated through base management fees and, in some cases, incentive fees. However, there are several differences in the organizational structure that could lead to performance differences between LPEs and unlisted PEs. In particular, a LPE is a closed-end fund with an indefinite life. In contrast, since unlisted PE funds have a finite life, the

[18] If we set R_f equal to 4%, the analogous lower bound for α is always greater than 0.2% and the analogous upper bound is always less than 1.5%. In general, the bounds for α are not sensitive to the choice of the risk-free rate.

management company must approach investors periodically to raise money for the next generation of unlisted PE funds. These ongoing interactions with investors create a reputational concern that could provide an added incentive to perform well. While these issues potentially affect the performance of LPEs relative to unlisted PE funds, the findings suggest that the expected abnormal performance for LPEs is not dramatically different from the expected performance for the portfolio of unlisted funds held by FoFs.

We apply the methodology we used to extract expectations of abnormal performance from the observed discount for FoFs to LPEs. Since it is not necessary to account for a second layer of fees when analyzing LPEs, we use Equation (10). From Table 9, the average LPE discount 12 months after the IPO is about 3% and the two standard deviation confidence interval around this point estimate ranges from -2% to 8%. Figure 3 plots α for various values of payout ratios. The market expectation of α for LPE investments ranges from about -0.5% to about 0.25%. This range is slightly below the range of α for unlisted PE funds that we extracted from FoF discounts. The point estimates suggest that the organizational structure of LPEs might have a mild negative impact on the performance of private equity investments of LPEs compared to the performance of unlisted PE funds. However, this difference is not statistically noticeable.

C. Simulation Results

This subsection presents the results of our simulations designed to estimate the market's expectation of the abnormal return for the portfolio of unlisted PE funds held by FoFs. These simulations incorporate the hurdle rate and the high watermark features of the typical FoF fee structure that we observe in practice in the context of a single factor model describing asset returns. This approach also correctly accounts for the fact that incentive fees are non-negative even if the underlying portfolio earns a negative return.

Our simulation methodology requires assumptions regarding several parameter values. As above, we assume that the risk-free rate is equal to 6% based on the average 20-Year Treasury Constant Maturity rate during our sample period. Further, we assume that the market risk premium is 4% and the standard deviation of stock market returns is 15% per year. In addition, we assume that the systematic risk loading for the single risk factor is equal to 1, that is, $\beta_{1,u} = 1$ for the underlying portfolio of PE investments, and that the fund-specific standard deviation is 15% per year. We choose these parameters to be consistent with the actual values we observe in the data, but the results are not particularly sensitive to these parameter values.

We simulate the annual return for the underlying portfolio of unlisted PE funds for each FoF using the single factor model with an adjustment for the level of the expected abnormal return. Each component of this annual portfolio return is drawn independently and identically over time from a normal distribution. We calculate the base management fee and the incentive fee using the fund-specific base management rate, incentive rate, and hurdle rate for each particular FoF. The base fee is a fixed proportion of the underlying portfolio value at the beginning of the period and is collected by the fund manager at the end of each period. The incentive fee is the maximum of zero and the fraction of the underlying portfolio proceeds minus the base management fee that is above a level specified by the hurdle rate and high watermark. The payout ratio, in conjunction with the fund-specific base management rate, determines the distribution to investors. The lower bound of the distribution to investors is zero. Each FoF reinvests the residual after all fees and distributions, i.e., retained earnings. The sample path for returns is simulated for 300 years and we repeat this process 5,000 times for each fund, holding the expected abnormal return and payout ratio constant.

Using the simulated distributions, we calculate the present value of all distributions to investors using a constant discount rate. The discount rate for the distribution to investors depends on the systematic risk of these distributions. Since we derive the relevant theoretical beta in the last section under simplifying assumptions, the theoretical beta does not reflect the true beta for these distributions because the model does not include the hurdle rate and high watermark provisions. Instead, we find the appropriate beta for these distributions using an iterative methodology. We use the betas from the analytic solution as the starting point. In this iterative process, we calculate the return for each period using the present value of subsequent distributions at the beginning of the current period, the present value of subsequent distributions at the beginning of the subsequent period, and the distribution at the end of the period. Then, we regress this return series on the simulated market return series. The coefficient estimate from this regression is the next iterative estimate for beta. We continue this process until the difference between consecutive iterative estimates for beta is less than 0.001, i.e., until the sequence converges. We find that the simulated beta for the investor's claim is close to the beta of the underlying portfolio as the expected abnormal return from 0% to 2%.

Once we have the appropriate beta for the distributions to investors for a particular FoF, we calculate the relevant discount rate and the present value of these distributions for each sample path. The average of these present value calculations is used to compute the simulated FoF discount. We repeat this exercise for the expected abnormal return ranging from -3% to 3% in increments of 1% and for the payout ratio ranging from 0.25 to 1 in increments of 0.05.[19]

Table 10 presents the average of simulated discounts across FoFs for expected abnormal returns ranging from -3% to +3% and various levels of the payout ratio. As we would anticipate,

[19] Since some FoFs have an incentive fee of 20%, the lower bound for the payout ratio to investors for these funds must be greater than 0.2 (or 20%).

the average FoF discount decreases as a increases because investors are willing to pay more if they expect the underlying PE funds to earn higher returns, holding all else constant. For a relatively low expected abnormal return, the discount decreases with the payout ratio. For example, if a is equal to -2%, the average discount is 59% for a payout ratio of 0.25 compared to a discount of 27% when the payout ratio is 1. Similarly, if a is equal to 0%, the average discount for a payout ratio of 0.25 is 33% compared to a discount of 11% when the payout ratio is 1. Intuitively, if abnormal performance is low relative to the fees of a FoF, the market would prefer that the FoF increase its payout ratio because reinvestment destroys value.

Analogously, the discount increases or the premium decreases with the payout ratio for large expected abnormal returns. For example, if a is equal to 2%, the average premium is 44% (i.e., a discount of -44%) if the payout ratio is 0.25 compared to a premium of only 5% if the payout ratio is 1. This result is also intuitive. The market would prefer that each FoF decreases its payout ratio whenever abnormal performance is high relative to FoF fees because reinvestment in the underlying PE funds is a positive NPV project.

We search for the levels of a that are consistent with the average FoF discount that we observe in Table 9 one year after the IPO. Each curve in Figure 4 plots the simulated FoF discount for a given expected abnormal return as a function of the payout ratio, i.e., each curve represents an iso-a curve. The red horizontal line in Figure 4, labeled “Post-IPO Discount”, reflects the average FoF discount of 12% one year after the IPO. Each intersection of an iso-a curve with this horizontal line is a possible combination of the expected abnormal return and the payout ratio that is consistent with post-IPO market prices.

Figure 4 indicates that there is a relatively narrow range of possible expected abnormal returns that are consistent with the observed FoF discount of 12% one year after IPO. First, we consider the plausibility of substantial negative expected abnormal returns. According to the figure, if the market expects the underlying PE funds to have an average abnormal return of less than -1% per year, the FoF discount would be at least 19%, or approximately two standard deviations greater than our point estimate of 12%. The horizontal line for a discount of 12% is below the iso-a curve for $a = 0$ whenever the payout ratio is less than 0.9, and hence, the market expects positive abnormal returns from the underlying portfolio of PE funds for plausible payout ratios. Hence, estimates of the expected abnormal return between -6% and -3% in Phalippou and Gottschalg (2009) are significantly smaller than the market's ex-ante expectation. Indeed, if the expected abnormal return was -3%, then Figure 4 indicates that the average discount should be at least 35%, i.e., more than 6 standard deviations from the observed average FoF discount.

Figure 4 also implies that if a is greater than or equal to 2%, FoFs would trade at a premium, rather than at a discount, for any expectation of the payout ratio. Intuitively, if a equals 2%, then the abnormal returns PE funds earn are expected to more than compensate for FoF fees. In this case, the smallest possible premium is 4.8% (discount of -4.8%), i.e., more than four standard deviations away from the post-IPO average discount of 12%. Therefore, many of the large estimates of a based on ex-post performance in the literature far exceed the level anticipated by the market during our sample period.

While Figure 4 does not extend the curves to include payout ratios lower than 0.25 due to technical restrictions imposed by the large incentive fees charged by some FoFs in our sample, analyzing such a payout ratio for funds with lower incentive fees would not change our findings. Note that iso-a curves outside the range of 0% to 1% diverge from the horizontal line as the payout ratio decreases to 0.25. This divergence pattern would continue for lower payout ratios because the impact of abnormal performance relative to fees is magnified by reinvestment.

Hence, our results indicate that the post-IPO market expects a to be between 0 and 1% regardless of the market's expectation of the payout ratio. If we consider a plausible payout ratio range, such as 0.3 to 0.7, a more detailed version of Figure 4 would indicate that the expected abnormal return is bounded between 0.25% and 0.75%, or approximately 0.5%.

In contrast to the post-IPO average discount for FoFs, the green horizontal line in Figure 4, labeled "IPO Discount", reflects the average FoF discount of -4%, that is, a 4% premium, during the IPO. Analogously, each intersection of an iso-a curve with this horizontal line is a possible combination of the expected abnormal return and the payout ratio that is consistent with the market prices at the time of the IPO. Based on the figure, the expected abnormal return at IPO appears to be between 1% and 2%. This is somewhat higher than the expected abnormal return extracted from the average post-IPO discount. While explaining the initial pattern of closed-end fund discounts is beyond the scope of this paper, one possibility is that the decline in the average discount reflects a change in expectations. Essentially, the IPO price only reflects the expectations of the investors at IPO, but the secondary market price eventually reflects aggregate market expectations. Even the premium of 4% at IPO is inconsistent with large positive expected abnormal returns greater than 3% because such expected abnormal performance would imply a counterfactually high premium of at least 15%. Thus, IPO prices are also inconsistent with many of the large positive estimates of expected abnormal performance in the existing literature.

V. Are Funds of Funds Representative

Our estimates indicate that the market expects FoFs investments in unlisted PE funds to earn abnormal returns of about 0.5% per year. This estimate may be biased relative to the entire PE fund universe if the FoFs have the ability to select PE funds that will perform better than average. Therefore, we analyze whether the performance of PE funds held by FoFs is different from the performance of the population of unlisted PE funds using the Preqin database. Since FoFs do not report the performance of the underlying funds directly, we match these underlying funds to the performance data in Preqin by fund name. If FoFs are better than average selectors of PE funds, then there should be a difference between the performance of the matched sample of PE funds held by FoFs and the performance of the average fund in Preqin.

We use the performance of buyout and VC funds in the Preqin PE fund performance database to create our benchmark indices.[20] The Preqin database contains 2,581 buyout and VC funds with vintages from 1990 until 2008. [21] The internal rate of return (IRR) and fund size are available for 1,823 of these funds. Preqin gathers performance data "using a number of different methods, including collecting performance information directly from fund managers themselves, and also from requesting information from public institutional investors."[22] Therefore, Preqin may suffer from the same selection bias concerns affecting other databases. The requirement to match performance data in Preqin to the PE funds held by FoFs implies that the matched sample of underlying funds inherits whatever selection bias exists in Preqin. The question we pose in this section is whether FoF managers have selection skill. If they do have selection skill, then it should be evident within the sample of PE funds held by FoFs that overlap with the Preqin database. Since our objective is only to examine relative performance within Preqin, the fact that Preqin does not contain performance data for all PE funds is not a limitation.

We obtain data on the PE funds that FoFs held from their first annual report. Of the 24 FoFs in our sample, 17 report all their unlisted PE fund holdings. These 17 FoFs list a total of

[20] We exclude funds in categories such as distressed debt and timber from the benchmarks.
[21] While the VentureXpert database contains a broader sample of PE funds, the part of this database that reports fund performance does not list the funds by name. Hence, we cannot match the funds held by FoFs with VentureXpert.
[22] See http://www.preqin.com/type/private-equity-performance/1/4.

628 unique PE fund investments. Of these 628 investments, 312 funds are not in the Preqin database. Of the remaining 316 funds, 102 funds are excluded from the sample because the observations are missing performance, fund size, or classification data, or because the fund is not classified as a buyout or VC fund. The aggregate NAV of the remaining 214 funds held by the FoFs represents 29% of the total value of buyout and VC funds with vintages between 1990 and 2008 in Preqin. Therefore, the PE funds held by our sample of FoFs represent a substantial fraction of the Preqin PE fund universe.

We compare the ex-post performance of the PE funds held by FoFs with the ex-post performance of the funds in Preqin based on the IRR reported in Preqin. We construct several different benchmark portfolios for each fund held by the FoFs. Our first benchmark portfolio includes all funds of the same vintage, that is, all funds that were started in the same calendar year as the particular PE fund held by a FoF. The second benchmark portfolio includes all funds of the same vintage and fund type (buyout or VC). The third benchmark portfolio includes all funds matched by vintage, type, and size. To identify a size match, we partition funds into three size groups within each vintage and type. The size group within which a particular fund falls is the benchmark for that fund.

Table 11 reports the average benchmark-adjusted IRRs for the PE funds held by FoFs. This average is the difference between the IRRs of PE funds held by FoFs and the IRRs of benchmark portfolios in the Preqin database. We report the results with both equal-weighted and value-weighted benchmarks. The value-weighted benchmark uses the commitment amount of each fund to weight IRRs within the relevant control group. The benchmark-adjusted IRRs range from -0.076% to 0.495% for the six different benchmark portfolios. None of the benchmark-adjusted IRRs are significantly different from zero. Thus, the statistical evidence indicates that managers of FoFs do not exhibit selection skill. The evidence provided by Harris, Jenkinson, and Kaplan (2013) indicates that the average IRRs for the databases from Burgiss, Preqin, and Cambridge Associates are not significantly different from each other. Thus, it is unlikely that using another database to analyze the selection skill of FoF managers will lead to qualitatively different results.

Our finding in this context is consistent with the performance of asset managers in general. For example, Fung and Hsieh (2000) find that the performance of hedge fund FoFs and the performance of the general population of hedge funds are approximately equal after accounting for selection bias in hedge fund databases and for the second layer of fees charged by hedge fund FoFs. There is also ample evidence in the literature that managers of other asset classes, including open-end equity funds, bond funds, and closed-end equity funds, do not exhibit significant selection skill on average.

VI. Conclusion

We estimate the risk characteristics for private equity using the market prices of FoFs that hold unlisted PE funds and LPEs that make PE investments. Since we observe market prices directly, we estimate market risk loadings for PE using the standard approach for public equity. This procedure allows us to avoid many of the difficulties in the existing literature associated with the use of some combination of cash flows to investors and intermediate valuations of investments. The market risk loadings for private equity are close to one and the loadings for the Fama-French SMB factor are significantly positive. The performance of PE funds is also negatively related to the credit spread in addition to the substantial exposure to stock market risk. This relation to the credit spread suggests that pension funds selecting portfolio allocations involving private equity should take into account that PE funds contain considerable exposure to credit risk.

We find that market returns of listed FoFs and LPEs predict future self-reported returns of unlisted private equity funds in the PEPI index. This evidence indicates that these listed securities contain crucial information about unlisted private equity. Our results support the hypothesis that NAVs reported by PE funds do not reflect market valuation changes in a timely manner. However, by using these listed PE vehicles, investors are able to predict subsequently reported NAVs and take this information of future performance into account when making investment decisions.

We also estimate ex-ante expected abnormal returns embedded in market prices of these listed PE vehicles. This approach addresses several selection biases encountered in the PE literature. Our sample contains all publicly traded eligible FoFs and LPEs, including those that delist. The estimation procedure uses market prices instead of voluntary reports of NAVs and cash flows. The estimator is equal to the true expectation of abnormal performance if the market is efficient on average, that is, if the average FoF discount properly reflects expectations of fees and the performance of the underlying investments. Our results indicate that the market expects unlisted private equity funds to earn abnormal returns in a relatively narrow range between -0.5% and 2%. Earlier studies find ex-post estimates of abnormal returns for PE funds that range from -6% to 32%. Our results indicate that the market does not expect PE funds to earn such extreme abnormal returns in the long run. In fact, we show that the market does not expect significant negative abnormal returns or large positive abnormal returns.

A. Appendix

The price of the investor's claim P_t is equal to the value of the distributions to investors, V_t:

$$V_t = E_t\left[\sum_{s=1}^{\infty}\frac{(1-\theta-\pi)\left(R_{u,t+s}-\lambda\right)NAV_{t+s-1}}{\left(1+E_t\left[R_e\right]\right)^s}\right]. \quad \text{(A1)}$$

We assume that the value of the distributions to investors is proportional to the amount invested in the underlying portfolio, i.e., $V_t = \eta NAV_t$:

$$\eta NAV_t = E_t\left[\frac{(1-\theta-\pi)\left(R_{u,t+1}-\lambda\right)NAV_t + \eta NAV_{t+1}}{1+E_t\left[R_e\right]}\right]. \quad \text{(A2)}$$

We use the evolution equation for NAV_{t+1} and rearrange:

$$\eta = E_t\left[\frac{(1-\theta-\pi)\left(R_{u,t+1}-\lambda\right)+\eta\left(1+\theta\left(R_{u,t+1}-\lambda\right)\right)}{1+E_t\left[R_e\right]}\right]. \quad \text{(A3)}$$

Substituting the expected return for the underlying portfolio using the *K*-factor model and rearranging again yields an expression for $E_t\left[R_e\right]$ in terms of η:

$$E_t\left[R_e\right] = \left(\frac{1-\theta-\pi}{\eta}+\theta\right)\left(\alpha-\lambda+R_f+\sum_{k=1}^{K}\beta_{k,u}\left(E_t\left[R_k\right]-R_f\right)\right). \quad \text{(A4)}$$

Similarly, we write the actual return, $R_{e,t+1}$, as

$$R_{e,t+1} = \left(\frac{1-\theta-\pi}{\eta}+\theta\right)\left(\alpha-\lambda+R_f+\sum_{k=1}^{K}\beta_{k,u}\left(R_{k,t+1}-R_f\right)+\varepsilon_{u,t+1}\right). \quad \text{(A5)}$$

For the investor's claim, the systematic factor loading for each factor *k*, $\beta_{k,e}$, must be equal to $\left(\left(1-\theta-\pi/\eta\right)+\theta\right)\beta_{k,u}$. Hence, the factor model dictates that

$$\left(\frac{1-\theta-\pi}{\eta}+\theta\right)\left(\alpha-\lambda+R_f\right) = R_f. \quad \text{(A6)}$$

We rearrange to solve for η as a function of the parameters:

$$\eta = \frac{(1-\theta-\pi)\left(\alpha-\lambda+R_f\right)}{R_f-\theta\left(\alpha-\lambda+R_f\right)}. \quad \text{(A7)}$$

Therefore, the present value of the distributions to investors relative to net asset value and the associated discount rate are given by the following two equations:

$$\frac{V_t}{NAV_t} = \frac{(1-\theta-\pi)\left(\alpha-\lambda+R_f\right)}{R_f-\theta\left(\alpha-\lambda+R_f\right)} \quad \text{(A8), and}$$

$$E_t\left[R_e\right] = R_f + \left(\frac{R_f}{\alpha-\lambda+R_f}\right)\sum_{k=1}^{K}\beta_{k,u}\left(E_t\left[R_k\right]-R_f\right). \quad \text{(A9)}$$

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Table 1. Summary Statistics – Funds of Funds

This table presents descriptive statistics for our sample of 24 private equity funds of funds (FoFs). The characteristics of each FoF are from the first available annual report or from Datastream one year after the initial public offering (IPO). The sample period of IPOs for FoFs is from 1994 to 2008.

			Panel A: Exchange				
	N	%	% USD	Active	%	Dead	%
US	0	0.0%	0.0%	0	0.0%	0	0.0%
Europe	13	54.2%	62.9%	13	54.2%	0	0.0%
London	9	37.5%	35.8%	9	37.5%	0	0.0%
Australia	2	8.3%	1.3%	1	4.2%	1	4.2%
Total	24	100%	100%	23	95.8%	1	4.2%
			Panel B: Size				
	N	%	% USD	Active	%	Dead	%
<20m	2	8.3%	5.4%	2	8.3%	0	0.0%
20m - 100m	6	25.0%	7.1%	5	20.8%	1	4.2%
100m - 500m	10	41.7%	35.6%	10	41.7%	0	0.0%
500m - 1000m	4	16.7%	22.9%	4	16.7%	0	0.0%
1000m – 2500m	2	8.3%	29.0%	2	8.3%	0	0.0%
>2500m	0	0.0%	0.0%	0	0.0%	0	0.0%
Total	24	100%	100%	23	95.8%	1	4.2%
			Panel C: Geographical Focus				
	N	%	% USD	Active	%	Dead	%
North America	10	41.7%	58.8%	10	41.7%	0	0.0%
Europe	9	37.5%	28.9%	9	37.5%	0	0.0%
Rest of the World	5	20.8%	12.3%	4	16.7%	1	4.2%
Total	24	100.0%	100.0%	23	95.8%	1	4.2%

Table 2: Compensation and Ownership Characteristics – Funds of Funds

This table reports descriptive statistics for the 24 funds of funds (FoFs) in our sample. The components of the fee structure for each fund are from the IPO prospectus of the fund. The other fund characteristics are from the first available annual report or from Datastream one year after the IPO. The first column for every variable shows the median while the second shows the average. Market capitalization is the number of shares outstanding multiplied by price per share in millions of USD. The base management fee is specified as a fixed percentage of the assets managed by the FoF. The incentive fee is specified as a percentage of profits or excess profits above the hurdle rate that the FoF earns. The hurdle rate is the minimum return that must be earned before the proportional incentive fee begins to accrue. Ownership of Large Shareholders is obtained from the first annual report after IPO and is reported as a percentage of outstanding shares. The ownership information is available for 17 FoFs that report holdings of shareholders with more than 5% ownership. Underlying PE Funds is the number of distinct unlisted PE funds held by the FoF.

	Median	**Average**
Market Capitalization	187.5	385.6
Base Management Fee	1.00%	1.00%
Incentive Fee	8.75%	6.88%
Hurdle rate	7.75%	6.41%
Underlying PE Funds	19	55
Ownership of Large Shareholders	38.7%	34.9%

Table 3. Summary Statistics – Listed Private Equity Funds

This table presents the descriptive statistics for our sample of 129 listed private equity funds (LPEs). The characteristics of each LPE are from the first available annual report and from Datastream, one year after the IPO. The sample period is from 1994 to 2008.

	N	%	% USD	Active	%	Dead	%
Panel A: Exchange							
	N	%	% USD	Active	%	Dead	%
US	6	4.7%	34.5%	6	4.7%	0	0.0%
Europe	14	10.9%	0.8%	7	5.4%	7	5.4%
London	99	76.7%	62.2%	87	67.4%	12	9.3%
Rest of the World	10	7.8%	2.5%	5	3.9%	5	3.9%
Total	129	100%	100%	105	81.4%	24	18.6%
Panel B: Size							
	N	%	% USD	Active	%	Dead	%
<20m	42	32.6%	11.0%	34	26.4%	8	6.2%
20m - 100m	63	48.8%	24.9%	51	39.5%	12	9.3%
100m - 500m	17	13.2%	28.9%	15	11.6%	2	1.6%
500m - 1000m	0	0.0%	0.0%	0	0.0%	0	0.0%
1000m – 2500	1	0.8%	0.0%	0	0.0%	1	0.8%
>2500m	1	0.8%	27.0%	1	0.8%	0	0.0%
NA	5	3.9%	8.2%	4	3.1%	1	0.8%
Total	129	100%	100%	105	81.4%	24	18.6%
Panel C: Geographical Focus							
	N	%	% USD	Active	%	Dead	%
North America	13	10.1%	35.8%	12	9.3%	1	0.8%
Europe	87	67.4%	51.7%	73	56.6%	14	10.9%
Rest of the World	22	17.1%	9.6%	13	10.1%	9	7.0%
NA	7	5.4%	2.9%	7	5.4%	0	0.0%
Total	129	100%	100%	105	81.4%	24	18.6%

Table 4. Risk Characteristics of Private Equity

This table reports systematic risk estimates for the value-weighted FoF and LPE indices based on the 1-factor model (CAPM) and the 4-factor model. We use MSCI World or S&P 500 indices as market proxies. The one-month Treasury bill rate is the risk-free rate. SMB and HML are the Fama-French size and book-to-market factors, respectively, and UMD is the momentum factor. The dependent variables are excess returns on the indices reported in the column headings. The standard errors are based on the Newey-West estimator with six lags (in parentheses). The sample period is from January 1994 to December 2008. The asterisks *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

	FoF	**FoF**	**FoF**	**FoF**	**LPE**	**LPE**	**LPE**	**LPE**
Intercept	-0.0026	-0.0032	-0.0048	-0.0061	-0.0008	-0.0018	-0.0013	-0.0034
	(0.0057)	(0.0064)	(0.0067)	(0.0076)	(0.0038)	(0.0044)	(0.0043)	(0.0050)
MSCI World	0.8206***		0.8736***		1.1375***		1.1276***	
	(0.2063)		(0.2412)		(0.1244)		(0.1327)	
S&P 500		0.7070***		0.8121***		1.0493***		1.0993***
		(0.2226)		(0.2637)		(0.1597)		(0.1725)
SMB			0.4863***	0.5801***			0.4785***	0.6065***
			(0.1432)	(0.1607)			(0.0881)	(0.1010)
HML			0.3453	0.3790			0.2659	0.3371*
			(0.2201)	(0.2462)			(0.1793)	(0.1975)
MOM			0.0213	0.0449			-0.0223	-0.0779
			(0.1039)	(0.1156)			(0.0551)	(0.0626)
R^2	0.2948	0.2283	0.3639	0.3229	0.5451	0.4839	0.6113	0.5821
Observations	180	180	180	180	180	180	180	180

Table 5. The Impact of the Macroeconomic Environment on the Performance of Private Equity

This table reports the regression estimates of the excess return for the value-weighted FoF and LPE indices on the MSCI World index, GDP growth, and the credit spread. The one-month Treasury bill rate is the risk-free rate. Both GDP growth and the credit spread are demeaned and scaled by their standard deviations. The dependent variables are excess returns on the indices reported in the column headings. The standard errors are based on the Newey-West estimator with six lags (in parentheses). The sample period is from January 1994 to December 2008. The asterisks *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

	FoF	**FoF**	**FoF**	**FoF**	**LPE**	**LPE**	**LPE**	**LPE**
Intercept	-0.0022	-0.0023	-0.0022	-0.0035	-0.0006	-0.0005	-0.0006	-0.0004
	(0.0042)	(0.0048)	(0.0044)	(0.0048)	(0.0030)	(0.0031)	(0.0031)	(0.0030)
MSCI World	0.6245***	0.6841***	0.6261***	0.6288***	1.0344***	1.0283***	1.0362***	0.9752***
	(0.1052)	(0.1204)	(0.1049)	(0.1348)	(0.0898)	(0.0984)	(0.0861)	(0.0799)
SMB				0.5037***				0.4906***
				(0.1239)				(0.0796)
HML				0.2385**				0.1547
				(0.1154)				(0.1088)
MOM				-0.0314				-0.1528***
				(0.1000)				(0.0519)
GDP growth	0.0221**		0.0103	0.0105*	0.0116		-0.0014	-0.0007
	(0.0092)		(0.0063)	(0.0064)	(0.0087)		(0.0041)	(0.0040)
Credit spread		-0.0246***	-0.0186***	-0.0189***		-0.0196***	-0.0205***	-0.0211***
		(0.0067)	(0.0069)	(0.0063)		(0.0054)	(0.0058)	(0.0048)
R^2	0.3987	0.4352	0.4494	0.5195	0.5727	0.6316	0.6318	0.7040
Observations	180	180	180	180	180	180	180	180

Table 6. Comparison of Buyout and Venture Capital Risk Characteristics for Funds of Funds

This table reports the regression estimates of the excess return for the value-weighted FoF indices for FoFs with predominantly buyout or venture capital (VC) focus on various combinations of risk factors. Every year we reclassify each FoF as focused on buyout or VC based on the annual report filed for the previous year. We consider three specifications: 1-factor, 4-factor, and macroeconomic risk. The MSCI World index is the market proxy. The one-month Treasury bill rate is the risk-free rate. SMB and HML are the Fama-French size and book-to-market factors, respectively, and UMD is the momentum factor. The macroeconomic factors are GDP growth and the credit spread. Both GDP growth and the credit spread are demeaned and scaled by their standard deviations. The dependent variables are excess returns on the indices reported in the column headings. The standard errors are based on the Newey-West estimator with six lags (in parentheses). Since the index for FoFs with a buyout focus does not have any constituents from January 1994 until June 1997, the sample period for this comparison is from June 1997 to December 2008. The asterisks *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

	Buyout	**VC**	**Buyout**	**VC**	**Buyout**	**VC**	**Buyout**	**VC**
Intercept	0.0015	-0.0033	-0.0022	-0.0040	0.0066	-0.0030	0.0041	-0.0034
	(0.0069)	(0.0060)	(0.0083)	(0.0072)	(0.0053)	(0.0062)	(0.0058)	(0.0076)
MSCI World	0.6832***	0.9813***	0.7479***	0.9226***	0.4655***	0.9724***	0.4630***	0.8934***
	(0.2380)	(0.2011)	(0.2715)	(0.2070)	(0.1355)	(0.2148)	(0.1346)	(0.2013)
SMB			0.5336***	0.4918*			0.5362***	0.4923*
			(0.1739)	(0.2666)			(0.1504)	(0.2632)
HML			0.4016	0.1838			0.2498*	0.1707
			(0.2541)	(0.2545)			(0.1396)	(0.2631)
MOM			0.0484	-0.1668			-0.0211	-0.1740
			(0.0860)	(0.2461)			(0.0784)	(0.2435)
GDP growth					0.0109*	-0.0003	0.0115*	0.0015
					(0.0060)	(0.0107)	(0.0060)	(0.0103)
Credit spread					-0.0212***	-0.0021	-0.0207***	-0.0017
					(0.0060)	(0.0105)	(0.0052)	(0.0112)
R^2	0.2007	0.2681	0.2862	0.3161	0.3862	0.2686	0.4641	0.3172
Observations	139	139	139	139	139	139	139	139

Table 7. Comparison of Buyout and Venture Capital Risk Characteristics for Listed Private Equity Funds

This table reports the regression estimates of the excess return for the value-weighted LPE indices with predominantly buyout or venture capital (VC) focus on various combinations of risk factors. The LPE funds are classified as focused on buyout or VC based on data from VentureXpert. We consider three specifications: 1-factor, 4-factor, and macroeconomic risk. The MSCI World index is the market proxy. The one-month Treasury bill rate is the risk-free rate. SMB and HML are the Fama-French size and book-to-market factors, respectively, and UMD is the momentum factor. The macroeconomic factors are GDP growth and the credit spread. Both GDP growth and the credit spread are demeaned and scaled by their standard deviations. The dependent variables are excess returns on the indices reported in the column headings. The standard errors are based on the Newey-West estimator with six lags (in parentheses). The sample period for this comparison is from January 1994 to December 2008. The asterisks *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

	Buyout	**VC**	**Buyout**	**VC**	**Buyout**	**VC**	**Buyout**	**VC**
Intercept	-0.0003	0.0001	-0.0030	0.0007	0.0000	0.0002	-0.0018	0.0014
	(0.0054)	(0.0035)	(0.0059)	(0.0037)	(0.0040)	(0.0033)	(0.0036)	(0.0032)
MSCI World	0.9474***	1.1879***	1.0723***	1.1100***	0.8084***	1.1177***	0.8732***	0.9973***
	(0.2355)	(0.1136)	(0.2424)	(0.0998)	(0.1219)	(0.1275)	(0.1125)	(0.1120)
SMB			0.5511***	0.4358***			0.5666***	0.4449***
			(0.1825)	(0.0830)			(0.1648)	(0.0892)
HML			0.7618**	0.0272			0.6270***	-0.0595
			(0.3285)	(0.1145)			(0.2109)	(0.1117)
MOM			-0.1106	-0.1415*			-0.1510*	-0.1639**
			(0.0796)	(0.0816)			(0.0874)	(0.0669)
GDP growth					-0.0021	-0.0010	0.0008	-0.0012
					(0.0050)	(0.0048)	(0.0039)	(0.0048)
Credit spread					-0.0284***	-0.0143***	-0.0254***	-0.0166**
					(0.0091)	(0.0053)	(0.0062)	(0.0052)
R^2	0.3104	0.5225	0.4321	0.5780	0.4466	0.5564	0.5507	0.6257
Observations	180	180	180	180	180	180	180	180

Table 8. Systematic Risk and Lead-Lag Relations for the Private Equity Performance Index (PEPI)

This table examines the lead-lag relation between the performance for the PEPI in the US and the returns for the FoF and MSCI World indices. The dependent variable is the quarterly return on PEPI for the US, reported by Thomson Reuters and the National Venture Capital Association (NVCA) from Q1 of 1994 until Q4 of 2008. MSCI World is the return to the MSCI World index. The FoFs is the value-weight return on the index of listed funds of funds that invest in unlisted private equity funds. The standard errors are based on the Newey-West estimator with six lags (in parentheses). The asterisks *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

Dependent Variable: Private Equity Performance Index (PEPI) Return

	1	2	3	4	5	6	7
Intercept	0.0121**	0.0162**	0.0127**	0.0140**	0.0096**	0.0082**	0.0054
	(0.0059)	(0.0067)	(0.0050)	(0.0066)	(0.0046)	(0.0039)	(0.0038)
MSCI World	0.4532***		0.3065***		0.3440***	0.3631***	0.3575***
	(0.0703)		(0.0420)		(0.0399)	(0.0391)	(0.0420)
MSCI World (Lag)						0.1059*	0.0474
						(0.0599)	(0.0591)
FoF		0.2514***	0.1357***	0.2300***	0.0913***	0.0691***	0.0610**
		(0.0325)	(0.0246)	(0.0288)	(0.0210)	(0.0257)	(0.0285)
FoF (Lag)				0.0956**	0.1309***	0.0972***	0.0765**
				(0.0394)	(0.0330)	(0.0368)	(0.0359)
PEPI Return (Lag)							0.2062*
							(0.1219)
R^2	0.5665	0.4992	0.6526	0.5435	0.7305	0.7476	0.7602
Observations	60	60	60	59	59	59	59

Table 9. Average Discount for Funds of Funds and Listed Private Equity Funds in Event Time

This table reports the average fund discount in event time for the 24 exchange-traded funds of funds (FoFs) that invest in unlisted private equity funds and the average fund discount in event time for the 129 listed private equity funds (LPEs). We calculate the discount for each fund as one minus the price of the fund divided by the net asset value (NAV) of the fund. Price data are from Datastream and NAV data are from the IPO prospectuses and Bloomberg. Event month 1 is the first month after the IPO; event month 2 is the second month after the IPO, and so on.

Event Month	FoF Average Discount	FoF Standard Error	LPE Average Discount	LPE Standard Error
1	1.34%	1.99%	-3.10%	1.79%
2	3.59%	2.12%	-2.91%	1.97%
3	5.99%	2.06%	-1.01%	1.96%
4	7.04%	2.19%	-1.42%	2.14%
5	6.52%	2.33%	0.23%	2.43%
6	7.50%	2.44%	1.41%	2.23%
7	8.20%	2.74%	1.30%	2.38%
8	9.49%	2.61%	1.47%	2.42%
9	10.73%	2.73%	4.53%	2.46%
10	11.93%	2.76%	3.45%	2.42%
11	11.16%	3.27%	2.51%	2.50%
12	11.77%	3.49%	3.11%	2.51%

Table 10. The Average Simulated Discount for Funds of Funds

This table reports the average (across funds) of the simulated FoF discount for each particular combination of the payout ratio and the expected abnormal return. We calculate the discount for each fund as one minus the average simulated present value of all distributions to investors divided by the initial net asset value (NAV) of the fund. We compute the relevant present value of FoF equity separately for each FoF using the fund-specific fee structure. These calculations are based on 5,000 fund-specific Monte Carlo simulations. The simulation methodology is discussed in greater detail in the text. The expected abnormal return, α, is the market's expectation of the abnormal return for the underlying portfolio of PE investments. The payout ratio is the fraction of a fund's earnings that is paid out to equity holders.

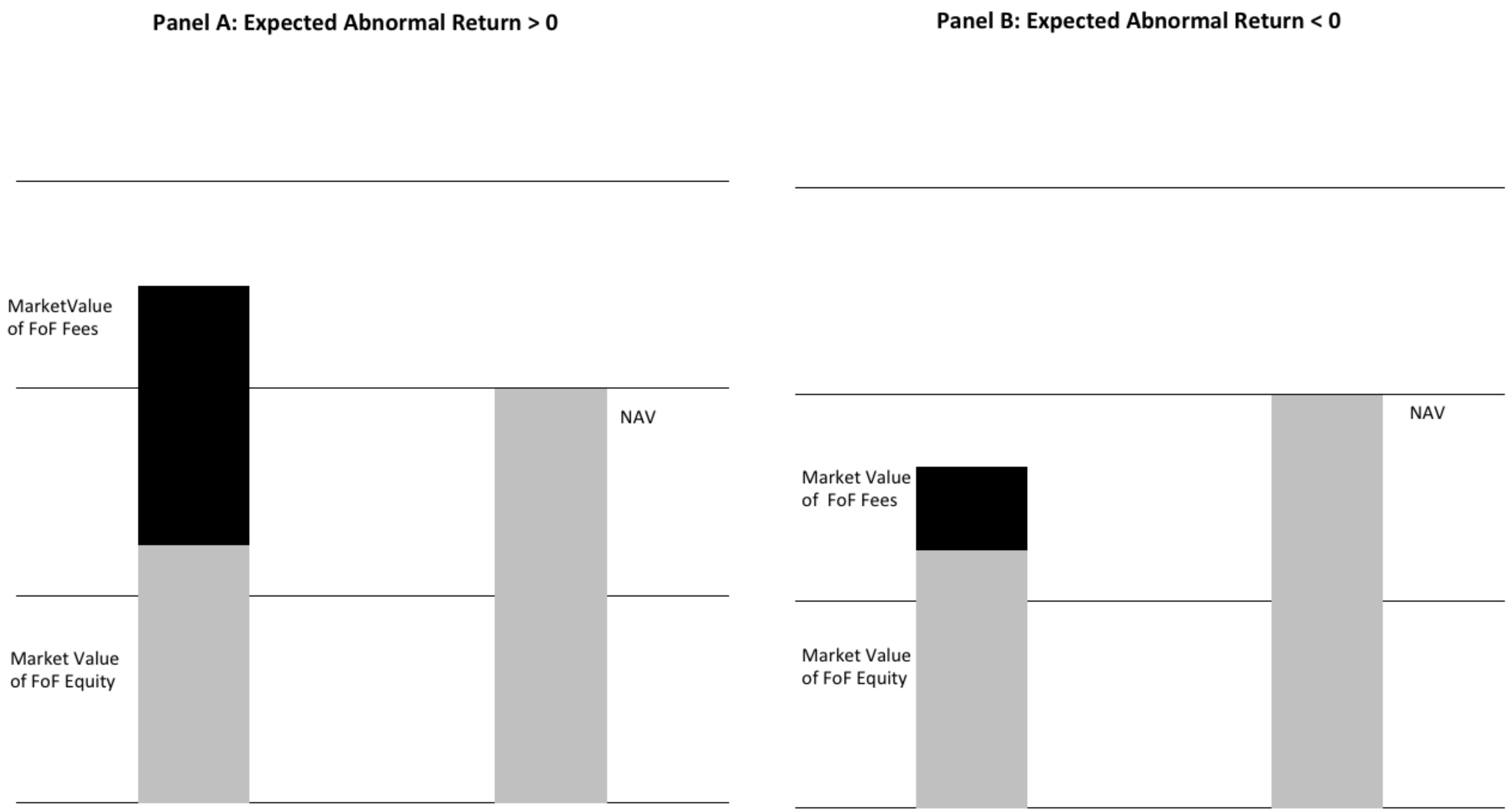
Payout Ratio (1-θ)	**Expected Abnormal Return (α)**				
	-2%	**-1%**	**0**	**1%**	**2%**
0.25	59.39%	48.79%	33.35%	7.11%	-43.77%
0.50	41.11%	31.85%	20.21%	5.90%	-12.50%
0.75	31.69%	23.48%	14.68%	4.27%	-7.29%
1	25.62%	18.69%	11.46%	3.61%	-4.83%

Table 11. Representativeness of the Private Equity Funds Held By Funds of Funds

This table reports the average difference between the IRRs of PE funds held by FoFs with performance data in Preqin and the IRRs of benchmark portfolios from Preqin. Each fund held by the a FoF in our sample is matched to a particular benchmark portfolio using various fund-specific characteristics. The benchmark portfolios are constructed for fund vintages from 1990 to 2008: in the first row the benchmark is all funds of the same vintage, in the second row the benchmark is all funds of the same vintage and type (buyout or venture capital), and in the third row the benchmark is all funds of the same vintage, type, and size tercile within each vintage and type. The column titled "EW Benchmark" uses equal-weighted benchmark portfolios and the column titled "VW Benchmark" uses value-weighted benchmarks. The standard errors are reported in parentheses. The asterisks *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

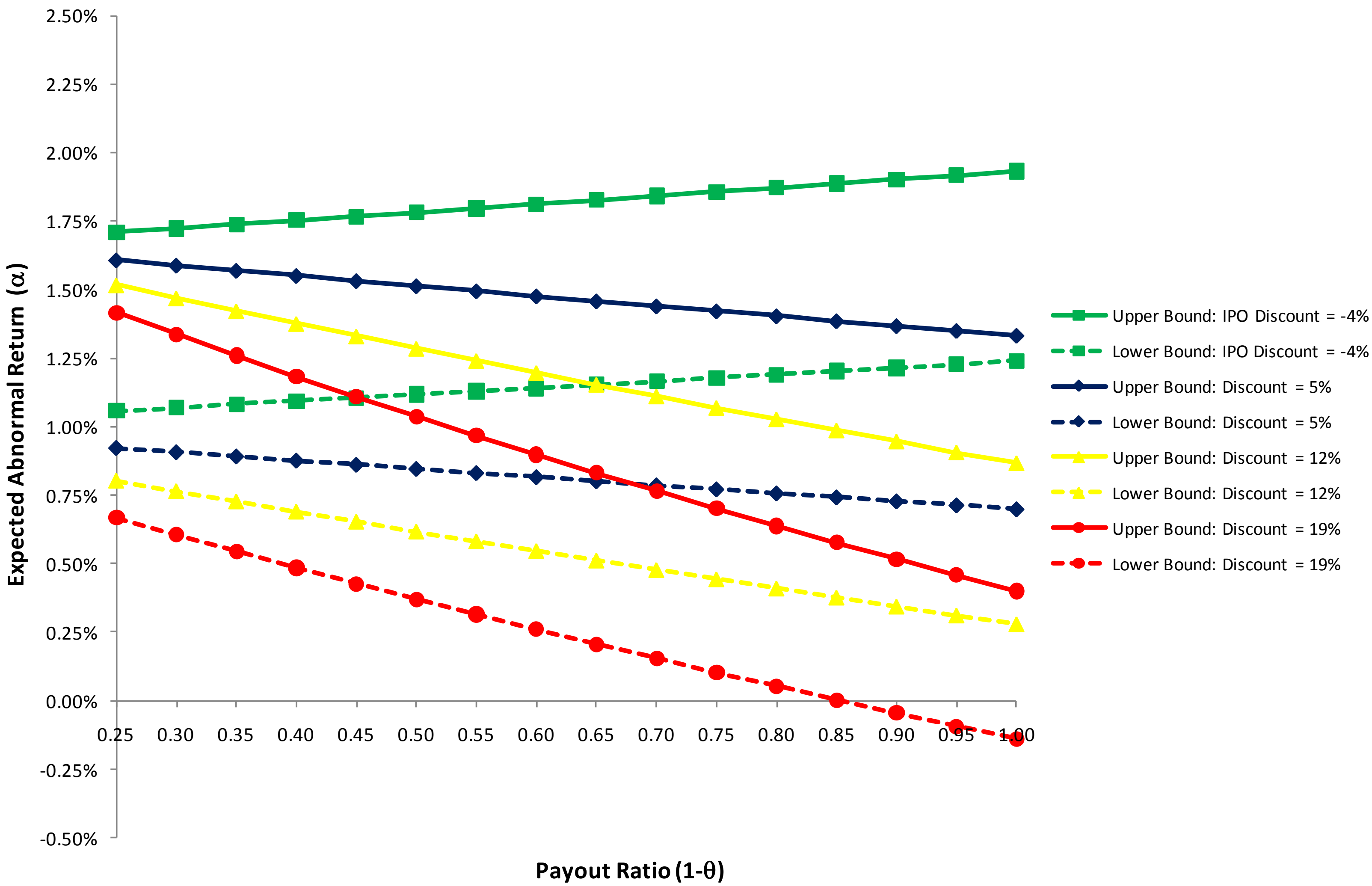
Benchmark-adjusted IRRs		
Match Criteria	EW Benchmark	VW Benchmark
Vintage	-0.004%	-0.076%
	(1.323)	(1.268)
Vintage and Type	-0.363%	0.311%
	(1.108)	(1.111)
Vintage, Type, and Size	0.464%	0.495%
	(1.097)	(1.092)

Figure 1. Market Value Versus Reported NAV of PE Investments Given the Expected Abnormal Return



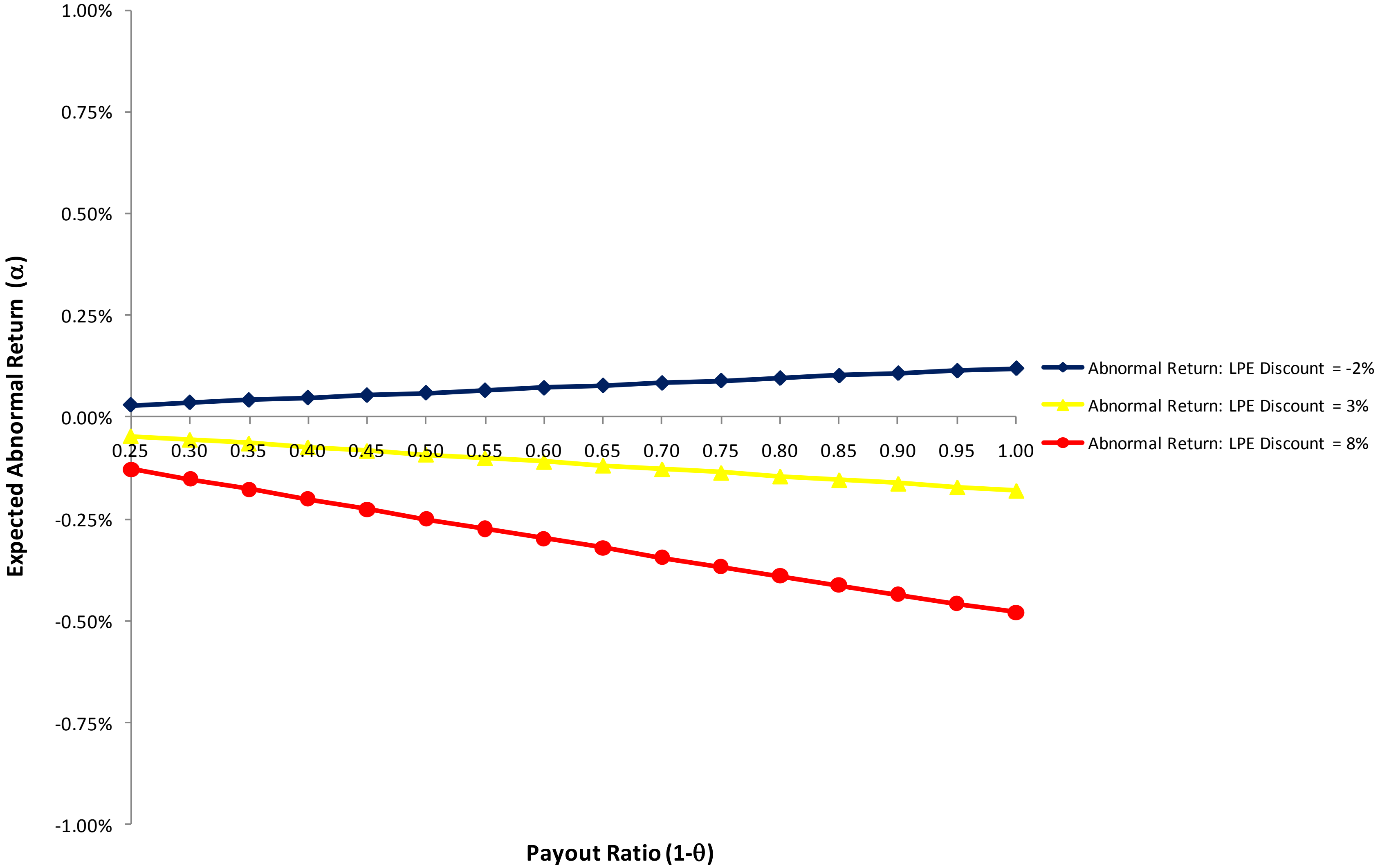
Each panel of this figure compares the unobserved market value of the unlisted PE funds held by the FoF (column 1) to the NAV or book value of the initial investment in PE (column 2). The market value of the underlying portfolio contains two components: 1) the aggregate value of the investors' shares and 2) the present value of FoF fees. If the market expects positive abnormal returns from PE investments then the total market value is greater than the book value (Panel A). However, if the market expects PE investments to earn negative abnormal returns then the total market value is less than the book value (Panel B).

Figure 2. Upper and Lower Bounds for the Abnormal Return Based on the FoF Discount



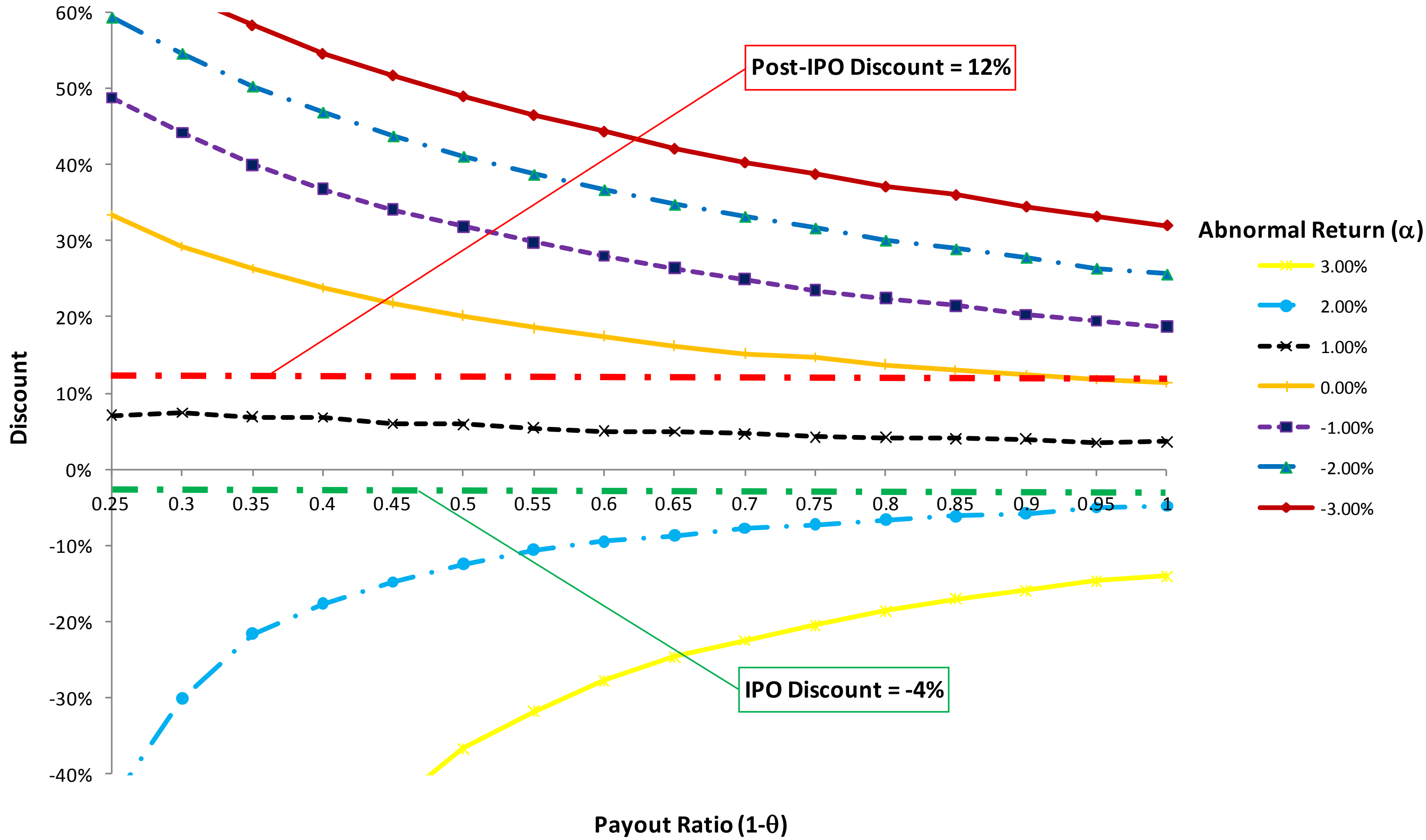
This figure presents the upper and lower bounds for the expected abnormal return, α, for four different levels of the FoF discount: -4%, 5%, 12%, and 19%. The average FoF discount at the time of the IPO is -4%, that is, a 4% premium. The other three levels reflect the point estimate and the end points of the 95% confidence interval for the average FoF discount twelve months after the IPO (see Table 10). The bounds for the expected abnormal return are determined by Equation (9) and Equation (11) using a base management fee of 1% and an incentive fee of 10%.

Figure 3. Abnormal Return Based on the LPE Discount



This figure presents the upper and lower bounds for the expected abnormal return, α, for three different levels of the LPE discount: -2%, 3%, and 8%. These three discount levels reflect the point estimate and the end points of the 95% confidence interval for the average LPE discount twelve months after the IPO (see Table 9). The bounds for the expected abnormal return are determined by Equation (10) using a base management fee of 0% and an incentive fee of 0% since these entities do not have a second layer of fees.

Figure 4. Simulated Relation for the Discount and the Abnormal Return



This figure presents the simulated relation between the average (across funds) FoF discount and payout ratio for different levels of the expected abnormal return, α. The line labeled "IPO Discount = -4%" plots the average FoF discount at the IPO. The line labeled "Post-IPO Discount = 12%" plots the average FoF discount one year after the IPO. We calculate the discount for each fund as unity minus the average simulated present value of all distributions to investors divided by the initial NAV of the fund. The expected abnormal return is the market's expectation of the abnormal return for the underlying portfolio of PE investments. The simulation methodology is discussed in greater detail in the text.