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JEL Classification: G12, C15

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SKEWNESS TERM STRUCTURE TESTS

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ABSTRACT

In this paper, we conduct skewness term structure tests to check whether the temporal structure of risk-neutral skewness is consistent with rational expectations. Because risk-neutral skewness is substantially mean reverting, skewness shocks should decay quickly and risk-neutral skewness of more distant option should display the rationally expected smoothing behavior. Using an equilibrium asset and option-pricing model in a production economy under jump diffusion with stochastic jump intensity, we derive this elasticity analytically. In an empirical application of the model using more than 20 years of data on S&P500 index options, we find that this elasticity turns out to be different than suggested under rational expectations - smaller on the short end (undereaction) and larger on the long end (overreaction) of the 'skewness curve'.

1. Introduction

Proponents of the efficient markets hypothesis would claim that investors correctly incorporate new information into asset prices. Bayesian rationality is assumed to be a good description of investor behavior. Empirical studies are challenging this view. One interesting and robust stylized fact that emerges from the index options literature is the overreaction puzzle of Stein (1989), which was further investigated by Poteshman (2001) and more recently by Christoffersen et al. (2013). Stein (1989) derives and empirically tests a model that describes the relationship between implied volatilities of options of different maturities. Assuming that volatility evolves according to a continuous-time mean-reverting AR1 process, with a constant long-run mean and a constant coefficient of mean-reversion, theoretically, the implied volatility of longer maturity (two-months) options should move in a responsive, but smoothing manner to changes in implied volatility of shorter maturity (one-month) options. However, the empirical values of this elasticity exceeded the theoretical upper bound of normal-reaction. Stein interprets his findings as overreaction, which is caused by market inefficiencies, claiming that this contradicts the rational expectations hypothesis for the term structure of implied volatilities.

Other studies challenge the simple mean-variance asset pricing framework and suggest to include higher moments. Among others, Kraus and Litzenberger (1976) derive a three-moment CAPM and show that systematic skewness is a priced risk factor. Harvey and Siddique (1999, 2000a, 2000b) use conditional skewness to mitigate the shortcomings of mean-variance asset pricing models in explaining cross-sectional variations in expected returns. Their findings suggest that conditional skewness is important and helps explaining the ex-ante market risk premiums. Among others, Conrad et al (2013) use options market data to extract estimates of higher moments of individual securities' probability density function. They find a significant negative relation between firm`s risk-neutral skewness and subsequent stock returns. In a related study, Chang et al. (2013) show that the risk-neutral market skewness is a priced risk factor in the cross section of stock returns, which cannot be explained by traditional four-factor models.

Variance and skewness in asset returns represent different types of risks. Using a behavioral paradigm, research in neurology shows that individuals' choice behavior is sensitive to both, dispersion (variance) and asymmetry (skewness) of outcomes (Symmonds et al (2011)). By scanning subjects with functional magnetic resonance imaging (fMRI), they find that individuals encode variance and skewness separately in the brain, the former being associated with parietal cortex and the latter with prefrontal cortex and ventral striatum. Participants are exposed to choices among a range of orthogonalized risk factors. The authors argue that risk is neither monolithic from a behavioral nor from a neural perspective. Their findings support the argument of dissociable components of risk factors and suggest separable effects of variance and skewness on asset market returns.

In contrast to the number of studies investigating the term structure of volatility, the term structure of skewness is not well understood. In this paper, we conduct skewness term structure tests to check whether the temporal structure of risk-neutral skewness is consistent with rational expectations. We develop a testing framework for the skewness term-structure in a simple production economy with a representative investor with CRRA utility. The stock index is assumed to follow a jump diffusion model with stochastic jump intensity. Because risk-neutral skewness is substantially mean reverting, skewness shocks should decay quickly and risk-neutral skewness of more distant option should display the rationally expected smoothing behavior. We derive this elasticity analytically and empirically test it using more than 20 years of data on S&P500 index options.

The paper proceeds as follows. Section 2 describes the theoretical model. Section 3 discusses the data and section 4 presents the empirical analysis. Section 5 concludes.

2. Theoretical model

We derive the equilibrium stock market risk premium in a simple economy with a representative investor with CRRA utility. The investor chooses a portfolio among the risk-free asset and capital market assets to maximize his life time utility. We model the capital market as having two sources of risk: diffusive risk and jump risk. We assume the jump intensity follows a stochastic process and the jump size follows a continuous distribution. To parameterize, the stock market index follows the process as

(1) $\frac{dS(t)}{S(t)} = \left(r + \phi - \lambda(t)E(e^x - 1)\right)dt + \sigma dB_s + (e^x - 1)dN$

(2) $d\lambda(t) = \kappa(\theta - \lambda(t))dt + \sigma_\lambda dB_\lambda$

where r is risk-free rate, ϕ represents the stock premium, σ denotes volatility, B_s and B_λ are standard Brownian motions in $\mathbb{R}$ (and dB_λ, dB_s the increments), N is Poisson process with intensity $\lambda(t)$ (and dN the increment), $(e^x - 1)$ is the percentage jump size. This guarantees that percentage jump size is at least larger than -1 and therefore the stock price due to jumps remains positive. We assume the jump size and jump intensity are independent. The jump size x follows a normal distribution independently over time with mean μ_x and variance σ_x^2. Combining the effects of random jump intensity and jump size, the term $\lambda(t)E(e^x - 1)dt$ is a compensation for the instantaneous change in expected stock returns introduced by the Poisson process N. Therefore, the term $(e^x - 1)dN - \lambda(t)E(e^x - 1)dt$ is an increment of the compensated compound Poisson process. The jump intensity follows a mean-reverting process, such that the process tends to drift towards its long-term mean θ, with the reverting speed $\kappa > 0$. For simplicity, we assume the two Brownian motions, B_s and B_λ, are independent[1].

The risk-free asset is represented by money market account $M(t)$ where investor can borrow and lend instantaneously at a rate r.

[1] Some paper assumes constant correlations between Brownian motions. The correlations would affect the value of central moments, but those effects are minor in our term-term structure test. We follow the discrete-time calculation of central moments in Christoffersen, Jacobs and Ornthanalai (2012) to assume the Poisson jump process to be independent of the Brownian motions, which indicates the independence between Brownian motion in jump intensity process and Brownian motion in stock price process.

(3) $\frac{dM(t)}{M(t)} = rdt$

Suppose there is a representative investor who has wealth W(t) and allocates it to money market account and stock market index. We assume the investor has a constant relative risk aversion utility function as $U = W(t)^{1-\gamma}/(1-\gamma)$, where $\gamma > 1$ is a measure of the magnitude of relative risk aversion. The investor chooses at each time t to invest a fraction ω of his wealth in stock $S(t)$ and a fraction $(1-\omega)$ of his wealth in money market account, in order to maximize the utility of terminal wealth.

(4) $max_{(\omega)} E_t[U(W(T), T)]$

Subject to his wealth constraint as

$$\frac{dW(t)}{W(t)} \equiv \omega\frac{dS(t)}{S(t)} + (1-\omega)\frac{dM(t)}{M(t)} = [r + \omega\phi - \omega\lambda(t)E(e^x - 1)]dt + \omega\sigma dB_s + \omega(e^x - 1)dN$$

Current research using jump-diffusion processes relies mostly on two kinds of specification. One is the affine jump-diffusion as specified in Duffie et al (2000), where the time-varying jump intensity depends on a linear function of the state variable. The other kind is the quadratic Gaussian class, for example, Ahn et al (2002), Chen et al (2004), where jump process depends on quadratic function of the state variables. In a recent research by Santa-Clara and Yan (2010), they explicitly write out a form of quadratic stochastic process for the jump intensity itself. We model the jump intensity to have a mean-reverting autoregressive stochastic process. One related paper is by Christoffersen, Jacobs and Ornthanalai (2012), who model the jump intensity in a similar way as Heston-Nandi type GARCH (1,1) dynamics, which is a discrete time version of mean-reverting stochastic process.

In this paper, we aim to capture the term-structure pattern of skewness through the lens of stochastic jump intensity. The intuition is simple. It has been documented in literature that large jump intensity is usually followed by large jump intensity, which exhibit mean-reverting at the same time; and an initial empirical test of the skewness time-series in this paper shows such kind of mean-reverting autoregressive pattern. If the market exhibit more downward jumps, the more frequently such jumps come, the more negative skewness would become. Capturing how jump intensity moves will help us understand better how skewness moves.

To solve (4), we follow Merton (1973) and define the optimal indirect utility $J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})$ to be

(5) $J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t}) \equiv max_{(\omega)} \mathrm{E_t}[\mathrm{U(W(T), T)}]$

The condition of optimality is given by the Bellman equation

(6) $\mathrm{E_t}[\mathrm{d}J] = 0$

Applying Ito's Lemma with jumps gives

$$\begin{aligned}\mathrm{d}J = {} & J_{\mathrm{t}}\mathrm{dt} + J_{\mathrm{w}}\mathrm{W(t)}\big(\mathrm{r} + \omega\phi - \omega\lambda(\mathrm{t})\mathrm{E}(\mathrm{e^x} - 1)\big)\mathrm{dt} + J_{\lambda}\kappa\big(\theta - \lambda(\mathrm{t})\big)\mathrm{dt} + \frac{1}{2}J_{\mathrm{ww}}\omega^2\mathrm{W(t)}^2\sigma^2\mathrm{dt} \\ & + \frac{1}{2}J_{\lambda\lambda}\sigma_{\lambda}^2\mathrm{dt} + J_{\mathrm{w}}\omega\mathrm{W(t)}\sigma\mathrm{dB_s} + J_{\lambda}\sigma_{\lambda}\mathrm{dB_{\lambda}} + [J\big(\mathrm{W(t)}\big(1 + \omega(\mathrm{e^x} - 1)\big), \lambda(\mathrm{t}), \mathrm{t}\big) \\ & - J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})]\mathrm{dN}\end{aligned}$$

where the subscripts of J denote the partial derivatives. The term $\Delta J \equiv [J\big(\mathrm{W(t)}\big(1 + \omega(\mathrm{e^x} - 1)\big), \lambda(\mathrm{t}), \mathrm{t}\big) - J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})]$ captures jumps in the optimal indirect function. Take conditional expectation $\mathrm{E_t}$ against $\mathrm{d}J$ to yield a more specific formula for (6) and it should hold for the optimal allocating fraction ω. Since market clears when the money market account is in zero net supply, we differentiate the formula (6) and substitute the condition $\omega = 1$ and the assumption that jump size is normal distributed as $\mathrm{x} \sim \mathrm{N}(\mu_{\mathrm{x}}, \sigma_{\mathrm{x}}^2)$ to get the equilibrium stock premium ϕ in terms of the optimal indirect function $J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})$ as follows

(7) $\phi = -\sigma^2 \frac{\mathrm{W(t)}J_{\mathrm{ww}}}{J_{\mathrm{w}}} - \lambda(\mathrm{t})\frac{1}{J_{\mathrm{w}}}(\mathrm{e}^{\mu_{\mathrm{x}} + \frac{1}{2}\sigma_{\mathrm{x}}^2} - 1)\mathrm{E}[J_{\mathrm{w}}(\mathrm{W(t)e^x}, \lambda(\mathrm{t}), \mathrm{t})] + (\mathrm{e}^{\mu_{\mathrm{x}} + \frac{1}{2}\sigma_{\mathrm{x}}^2} - 1)\lambda(\mathrm{t})$

The stock market index risk premium contains two components: the variance of the marginal utility of wealth, the covariance of the marginal utility of wealth with the compensated jump size[2] respectively.

Combining (6) and (7), we find a solution to the optimal indirect function and get the required risk premium formula.

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[2] Note that we specify a compensated compound Poisson process for the stock price dynamics. Therefore, the risk premium due to the jump size has the term $(\mathrm{e}^{\mu_{\mathrm{x}} + \frac{1}{2}\sigma_{\mathrm{x}}^2} - 1)\lambda(\mathrm{t})$, in addition to the one $[-\lambda(\mathrm{t})\frac{1}{J_{\mathrm{w}}}(\mathrm{e}^{\mu_{\mathrm{x}} + \frac{1}{2}\sigma_{\mathrm{x}}^2} - 1)\mathrm{E}[J_{\mathrm{w}}(\mathrm{W(t)e^x}, \lambda(\mathrm{t}), \mathrm{t})]]$ out of a compound Poisson process.

PROPOSITION 1 *In an economy with jump diffusion and one representative investor with CRRA utility function, the stock premium is a function of variance, jump size components and jump intensity:*

(8) $\phi = \gamma\sigma^2 - \lambda(t)e^{(1-\gamma)\mu_x+\frac{1}{2}(1-\gamma)^2\sigma_x^2} + \lambda(t)e^{-\gamma\mu_x+\frac{1}{2}\gamma^2\sigma_x^2} + \lambda(t)e^{\mu_x+\frac{1}{2}\sigma_x^2} - \lambda(t)$

Proof. See Appendix1.

Assume at time t there is a European option f with strike price K and it will mature at time T in this economy. The option price should be a function of the state variables and time, i.e. $f(S(t), \lambda^*(t), t)$. The state variables in risk-neutral measure are accordingly written as:

(9) $\frac{dS(t)}{S(t)} = \left(r - \lambda^*(t)E\left(e^{x^*} - 1\right)\right)dt + \sigma dB_s^* + \left(e^{x^*} - 1\right)dN^*$

(10) $d\lambda^*(t) = \kappa^*(\theta^* - \lambda^*(t))dt + \sigma_\lambda^* dB_\lambda^*$

In the presence of European option in this economy, the investor allocates a fraction ω_s of his wealth in stock $S(t)$, a fraction ω_f of his wealth in option f, and a fraction $(1 - \omega_s - \omega_f)$ in money market account, in order to maximize the utility of terminal wealth. We can therefore derive the risk-neutral measures for the jump-diffusion components of the stock returns under market clearing condition.

PROPOSITION 2 *In an economy with jump diffusion and one representative investor with CRRA utility function, the risk-neutral jump components are given by*

(11) $x^* \sim N(\mu_x - \gamma\sigma_x^2, \sigma_x^2)$

(12) $\lambda^*(t) = e^{-\gamma\mu_x+\frac{1}{2}\gamma^2\sigma_x^2}\lambda(t)$; $\sigma_\lambda^* = e^{-\gamma\mu_x+\frac{1}{2}\gamma^2\sigma_x^2}\sigma_\lambda$

(13) $\kappa^* = \kappa - 2\sigma_\lambda^2 C(\tau);\ \ \theta^* = \frac{e^{-\gamma\mu_x+\frac{1}{2}\gamma^2\sigma_x^2}}{\kappa-2\sigma_\lambda^2 C(\tau)}(\kappa\theta - \gamma\rho_{s\lambda}\sigma_\lambda\sigma + B(\tau)\sigma_\lambda^2)$

Where $\tau = T - t$*; B and C solve the following ODEs, with the subscripts denote the differentias and with initial conditions as* $B(0) = 0$*,* $C(0) = 0$*:*

$$B_\tau = 2\kappa\theta C(\tau) - \kappa B(\tau) + 2B(\tau)C(\tau)\sigma_\lambda^2 + (1-\gamma)e^{-\gamma\mu_x+\frac{1}{2}\gamma^2\sigma_x^2} + \gamma e^{(1-\gamma)\mu_x+\frac{1}{2}(1-\gamma)^2\sigma_x^2} - 1 \quad \text{(A.10)}$$

$C_\tau = -2\kappa C(\tau) + 2C(\tau)^2\sigma_\lambda^2$

Proof. See Appendix2.

Upon this point, we can write the conditional central moments of returns in both physical and risk-neutral measures. Define $R_{t+\tau} \equiv \ln\frac{S(t+\tau)}{S(t)}$. $R_{t+\tau}$ is the continuously compounded return viewed at time t for the future time horizon τ, i.e. the return during the time interval (t, $t+\tau$).

PROPOSITION 3 *In an economy with jump diffusion and one representative investor with CRRA utility function, the conditional central skewness in physical and risk-neutral measures are written as*

(14) $Skew_t(R_{t+\tau}) = \tau(\mu_x^3 + 3\mu_x\sigma_x^2)\lambda(t)$

(15) $Skew_t^*(R_{t+\tau}) = \tau((\mu_x - \gamma\sigma_x^2)^3 + 3(\mu_x - \gamma\sigma_x^2)\sigma_x^2)\lambda^*(t)$

Proof. See Appendix3.

Corollary*:*

1. If the jump size x follows normal distribution with negative mean μ_x and variance σ_x^2, the risk-neutral skewness is more negative than the physical one for the same time horizon; and the variance of risk-neutral skewness is larger than the variance of physical skewness.

2. For a given time-horizon τ, the conditional central skewness at time t is a linear function of jump intensity at time t.

Next we investigate the term structure of skewness dynamics. We define an instantaneous skewness at time t as $\psi(t)$. Instantaenous skewness should be equal to a horizon-free conditional skewness. To put it another way, instantaneous skewness considers the time horizon τ as an exogenously given constant in the conditional skewness viewed at time t, i.e. $\psi(t) = Skew_t(R_{t+\tau})\ with\ an\ exogenous\ \tau$. According to the Corollary, $\psi(t)$ is therefore a linear function of jump intensity $\lambda(t)$. We use the following denotations

$\overline{\psi^\tau} \equiv \tau(\mu_x^3 + 3\mu_x\sigma_x^2)$

$\overline{\psi^{*\tau}} \equiv \tau((\mu_x - \gamma\sigma_x^2)^3 + 3(\mu_x - \gamma\sigma_x^2)\sigma_x^2)$

to rewrite formula (14) and (15) as

(16) $\psi(t) = \overline{\psi^{\tau}}\lambda(t)$

(17) $\psi^{*}(t) = \overline{\psi^{*\tau}}\lambda^{*}(t)$

Since the increment of jump intensity is $d\lambda(t) = \kappa\big(\theta - \lambda(t)\big)dt + \sigma_{\lambda}dB_{\lambda}$, the dynamics of instantaneous skewness is accordingly written as:

$$d\psi(t) = \overline{\psi^{\tau}}d\lambda(t) = \kappa\big(\overline{\psi^{\tau}}\theta - \psi(t)\big)dt + \overline{\psi^{\tau}}\sigma_{\lambda}dB_{\lambda}$$

We notice the instantaneous skewness follows a continuous-time mean-reverting AR1 process, with the mean level of $\overline{\psi^{\tau}}\theta$ and a mean-reverting speed of κ. Using Ito's Lemma, the expectation of skewness as of time t+ i at time t will be given by

(18) $E_t(\psi(t+i)) = \overline{\psi^{\tau}}\theta + e^{-\kappa i}(\psi(t) - \overline{\psi^{\tau}}\theta)$

The conditional skewness should equal the averaged expected instantaneous skewness over the time interval $[t, t+\tau]$

(19a) $Skew_t(R_{t+\tau}) = \frac{1}{\tau}E_t\big[\int_0^{\tau}\psi(t+i)di\big] = \overline{\psi^{\tau}}\theta + \frac{1-e^{-\kappa\tau}}{\kappa\tau}(\psi(t) - \overline{\psi^{\tau}}\theta)$

The risk-neutral conditional skewness is similarly written as

(20a) $Skew_t^{*}(R_{t+\tau}) = \frac{1}{\tau}E_t\big[\int_0^{\tau}\psi^{*}(t+i)di\big] = \overline{\psi^{*\tau}}\theta^{*} + \frac{1-e^{-\kappa^{*}\tau}}{\kappa^{*}\tau}(\psi^{*}(t) - \overline{\psi^{*\tau}}\theta^{*})$

We learn from (19a) or (20a) that when instantaneous skewness is above its mean level, the conditional skewness should be decreasing with its time to maturity in a non-linear manner, and vise visa. Extending the formula (19a) and (20a) to include time lapse j to the time t, namely, let us view the conditional skewness over the time interval $[t+j, t+j+\tau]$, we get more general forms as

(19b) $Skew_{t+j}\big(R_{(t+j)+\tau}\big) = \frac{1}{\tau}E_t\big[\int_0^{\tau}\psi(t+j+i)di\big] = \overline{\psi^{\tau}}\theta + \frac{1-e^{-\kappa\tau}}{\kappa\tau}(\psi(t+j) - \overline{\psi^{\tau}}\theta)$

(20b) $Skew_{t+j}^{*}\big(R_{(t+j)+\tau}\big) = \frac{1}{\tau}E_t\big[\int_0^{\tau}\psi^{*}(t+j+i)di\big] = \overline{\psi^{*\tau}}\theta^{*} + \frac{1-e^{-\kappa^{*}\tau}}{\kappa^{*}\tau}(\psi^{*}(t+j) - \overline{\psi^{*\tau}}\theta^{*})$

Where $R_{(t+j)+\tau}$ is the continuously compounded return viewed at time (t+j) for the future time horizon τ, i.e. at the time interval (t+j, $t + j + \tau$). Since the risk-neutral skewness is extracted from options and therefore the time horizon τ could be considered to be in corresponding to a given time with a particular maturity. Suppose there are two options. One option has time to maturity τ_1 and the other τ_2, with $\tau_1 < \tau_2$. Using formula (20a), the following equation should hold:

(21) $$Skew_t^*\left(R_{t+\tau_2}\right) - \overline{\psi^{*\tau}}\theta^* = \frac{\tau_1\left(1-e^{-\kappa^*\tau_2}\right)}{\tau_2\left(1-e^{-\kappa^*\tau_1}\right)}\left(Skew_t^*\left(R_{t+\tau_1}\right) - \overline{\psi^{*\tau}}\theta^*\right)$$

Rearrange (21) and using (20b), we get a more general empirical test equation:

(22)

$$E_t\left[\left(Skew_{t+j}^*\left(R_{(t+j)+\tau_1}\right) - Skew_t^*\left(R_{t+\tau_1}\right)\right) - \frac{\tau_2\left(1-e^{-\kappa^*\tau_1}\right)\left(e^{-\kappa^* j}-1\right)}{\tau_1\left(1-e^{-\kappa^*\tau_2}\right) - \tau_2\left(1-e^{-\kappa^*\tau_1}\right)}\left(Skew_t^*\left(R_{t+\tau_2}\right) - Skew_t^*\left(R_{t+\tau_1}\right)\right)\right] = 0$$

Denote the parameter $\rho \equiv e^{-\kappa^*}$ and $\beta \equiv \frac{\tau_2\left(1-e^{-\kappa^*\tau_1}\right)\left(e^{-\kappa^* j}-1\right)}{\tau_1\left(1-e^{-\kappa^*\tau_2}\right)-\tau_2\left(1-e^{-\kappa^*\tau_1}\right)}$. In theory, the persistency of skewness time series, ρ, is geometrically decaying in mean-reverting speed κ^*, and the boundary elasticity of rational reaction, β, is a nonlinear function of short-term maturity, τ_1, long-term maturity, τ_2, time lapse, j, and the mean-reverting speed κ^*. In the spirit of Stein (1989), we derive a theoretical level for normal reaction in the form of an elasticity parameter, which can be empirically tested. Hence, any values below the theoretical elasticity parameter would be characterized as underreaction, while any values above would be interpreted as overreaction.

3. Data

We make use of data on S&P 500 index options, which are directly obtained from the Chicago Board Option Exchange (CBOE)[3]. The daily data covers the period January 1st 1990 until December 31th 2012, consisting of 5785 days, on which we obtain measures for the risk-neutral skewness for all traded maturities. The exchange uses a model-free approach applied to option prices to obtain a risk-neutral skewness measure (SKEW). This approach originates from Bakshi et al. (2003), and since then becames popular in empirical studies on options markets. To be specific, the risk-neutral skewness based on options with τ -month maturity can be computed as

$$Skew_t^*(R_{t+\tau}) = \frac{e^{r\tau}W_t(\tau) - 3\mu_t(\tau)e^{r\tau}V_t(\tau) + \mu_t^3(\tau)}{\left(\sqrt{e^{r\tau}V_t(\tau) - \mu_t^2(\tau)}\right)^3}$$

Where

$$\mu_t(\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V_t(\tau) - \frac{e^{r\tau}}{6}W_t(\tau) - \frac{e^{r\tau}}{24}X_t(\tau)$$

$$V_t(\tau) = \int_S^{+\infty} \frac{2\left(1 - \ln\left(\frac{K}{S_t}\right)\right)}{K^2} c_t(\tau, K)dK + \int_0^S \frac{2\left(1 + \ln\left(\frac{S_t}{K}\right)\right)}{K^2} p_t(\tau, K)dK$$

$$W_t(\tau) = \int_S^{+\infty} \frac{6\ln\left(\frac{K}{S_t}\right) - 3(\ln\left(\frac{K}{S_t}\right))^2}{K^2} c_t(\tau, K)dK - \int_0^S \frac{6\ln\left(\frac{S_t}{K}\right) - 3(\ln\left(\frac{S_t}{K}\right))^2}{K^2} p_t(\tau, K)dK$$

$$X_t(\tau) = \int_S^{+\infty} \frac{12\ln\left(\frac{K}{S_t}\right) - 4(\ln\left(\frac{K}{S_t}\right))^3}{K^2} c_t(\tau, K)dK + \int_0^S \frac{12\ln\left(\frac{S_t}{K}\right) - 4(\ln\left(\frac{S_t}{K}\right))^3}{K^2} p_t(\tau, K)dK$$

where S_t is the underlying S&P500 index level on day t, K is the exercise price of the option and r is the risk-free interest rate corresponding to the time to maturity (τ) of the option. c and p refer to call and put prices. In this way, one can obtain the risk-neutral skewness on a daily basis for all traded maturities. We interpolate the skewness measures between two maturities, one below and

[3] We thank the CBOE for making the data available to us.

one above the required time-to-maturity, in order to obtain fixed maturity risk-neutral skewness measures. The maturities considered are 1, 2, 3, 4, 6 and 8 months.

4. Empirical analysis

Summary statistics

Table I provides summary statistics of the skewness time series, with maturities ranging from 1 month to 8 months. The average skewness is negative for all maturities and, typically, skewness becomes less negative on average, but more volatile for longer maturities.

[Table I]

In Table II, we examine autocorrelations and partial correlations in the skewness time series over different horizons. We find positive autocorrelations for a 1-day lag ranging from 0.87 to 0.95, where the risk-neutral skewness process becomes more persistent for longer maturities. The partial correlations are greatly decreased to less than 0.22 after the first lag for all maturities, which is higher compared to results reported in previous studies on volatility. However, adding additional lags does not improve the explanatory power of the regression. The R^2 increases only slightly from 76.9% for a 1-day lag to 78.8%, when up to 5 daily lags are used.

[Table II]

Results suggest that the conclusions about the appropriateness of our specification are similar to those reached in volatility studies as early as in the 1980s (e.g. Poterba and Summers (1986) and Stein (1989)). It ever since becomes a convention to model variance process as an autoregressive process as ARCH, GARCH, etc., and their continuous time versions. However, to our knowledge, there is seldom research that has empirically investigated the structure of the risk-neutral skewness process.

Skewness Term Structure Tests

The theoretical model derived in the previous sections specifies a mean-reverting stochastic process for risk-neutral skewness. An explicit function of an elasticity parameter was proposed for the test of the skewness term-structure, as is shown in formula (22). In the following, we proceed with the empirical testing of the theoretically derived elasticity relationship. This section considers a specification on the values of the elasticity parameter. According to rational expectations, the prediction error given in the expectations operator on the left-hand side of formula (22) should be white noise. If the elasticity is smaller than expected, e,g, the prediction error is positively related with the short-term skewness time series itself, $Skew_t^*(\mathrm{R}_{t+\tau_1})$, there is evidence of underreaction. Similarly, if the elasticity is larger than expected, the prediction error is negatively related with short-term skewness, there is evidence of overreaction. Therefore, we can test for underreaction or overreaction by regression the prediction error on $Skew_t^*(\mathrm{R}_{t+\tau_1})$. However, the elasticity β depends on the time-to-maturity of the nearby and distant options, and, moreover, on the mean-reverting speed κ^*, which introduces some nonlinearity. Hence, in the following, we are investigating the impact of this non-linear relationship on the analysis.

[Table III]

Table III illustrates the values of elasticity parameter for various combinations of parameters. It shows that when the distant maturity is twice the nearby maturity, with the time lapse being equal to the time to maturity of the nearby options, β is equal to two and independent of κ^*. As a result, the regression equation can be generalized and simplified to

$$\left(Skew_{t+j}^*(\mathrm{R}_{(t+j)+\tau_1}) - Skew_t^*(\mathrm{R}_{t+\tau_1})\right) - 2\left(Skew_t^*(\mathrm{R}_{t+\tau_2}) - Skew_t^*(\mathrm{R}_{t+\tau_1})\right)$$

$$= \alpha + \beta Skew_t^*(\mathrm{R}_{t+\tau_1}) + \varepsilon_{t+j}$$

Where (j, τ_1, τ_2) = (1 month, 1 month, 2 months); (2, 2, 4); (3, 3, 6); (4, 4, 8).

As explained earlier, the prediction error should, according to rational expectations, be white noise, and, therefore not depend on the skewness of nearby options. Any significant estimate for β would bear out the under- or overreaction hypothesis. The regressions are OLS and the standard errors are corrected for serial correlations induced by the overlapping observations. Results are provided in Table IV. The coefficient for tests of the short end of the 'skewness curve', namely 1 month versus 2 months, is positive and statistically significant, which suggests underreaction. The coefficient for tests of the long end of the 'skewness curve', namely 4 month versus 8 months, is negative and statistically significant, which suggests overreaction. All other coefficients are not significantly different from zero.

[Table IV]

Furthermore, the subsample analysis in Table V reveals that the patterns that we observed previously also hold in the subsamples. On the short end of the 'skewness curve', we obtain underreaction, while the long end exhibits overreaction. For the middle part, we obtain mixed results, suggesting underreaction in the booming period of the 90's and overreaction in more recent years.

[Table V]

Robustness Checks

As a robustness check, we replicate the analysis with weekly data, the frequency that was used in previous studies on volatility. While the autocorrelation for a 1-week lag decreases only marginally, the partial correlations in the skewness time series are substantially reduced. For example, the partial correlation for the skewness time series of 1-month options decreases to less than 0.02 after the first lag for all maturities, which is a characteristic of a mean-reverting process[4]. We also run the same regressions over the complete term structure of risk-neutral skewness using weekly data. Results for the whole period are reported in Table VI and subsample results are shown in Table VII.

[Table VI and VII]

Overall, the results are slightly weaker, but still strongly suggest that our previous findings also hold for the lower sampling frequency. A similar picture arises from the subsample analysis.

[4] More results are not reported to save space.

5. Conclusion

In contrast to the number of studies investigating the term structure of volatility, the term structure of skewness is not well understood. In this paper, we conduct skewness term structure tests to check whether the temporal structure of skewness is consistent with rational expectations. We develop a testing framework in a simple production economy with a representative investor with CRRA utility. The stock index is assumed to follow a jump diffusion model with stochastic jump intensity. We derive the conditional moments of returns using the moment generating function of the Brownian and compensated compound Poisson processes, in both physical and risk-neutral measures. Stochastic jump intensity ensures that risk-neutral skewness follows a continuous-time mean-reverting process. Therefore, risk-neutral skewness shocks should decay quickly and risk-neutral skewness of more distant options should display the rationally expected smoothing behavior. We derive this elasticity analytically. In an empirical application of the model using more than 20 years of data on S&P500 index options, we find that this elasticity turns out to be different than suggested under rational expectations - smaller on the short end (undereaction) and larger on the long end (overreaction) of the 'skewness curve'.

Appendix1:

Consider any twice-differentiable function as the optimal indirect utility $J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})$ that is a conditional expectation of utility function of W and λ at a later date T, measured at time t.

$$J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t}) \equiv max_{(\omega)} \mathrm{E_t}[\mathrm{U(W(T), T)}] \quad \text{(A.1)}$$

To explore the linearity of the coefficients in subsequent partial differential equations (PDEs), we guess the functional form as:

$$J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t}) = g(\lambda(\mathrm{t}), \tau) \frac{\mathrm{W(t)}^{1-\gamma}}{(1-\gamma)} \quad \text{(A.2)}$$

Where $\tau \equiv \mathrm{T} - \mathrm{t}$, and $g(\lambda(\mathrm{t}), \tau)$ is a function independent of W(t).

Ito's Lemma with jump shows that:

$$\mathrm{d}J = J_{\mathrm{t}}\mathrm{dt} + J_{\mathrm{w}}\mathrm{W(t)}\big(\mathrm{r} + \omega\phi - \omega\lambda(\mathrm{t})\mathrm{E}(\mathrm{e^x} - 1)\big)\mathrm{dt} + J_{\lambda}\kappa\big(\theta - \lambda(\mathrm{t})\big)\mathrm{dt} + \frac{1}{2}J_{\mathrm{ww}}\omega^2\mathrm{W(t)}^2\sigma^2\mathrm{dt} + \frac{1}{2}J_{\lambda\lambda}\sigma_\lambda^2\mathrm{dt} + J_{\mathrm{w}}\omega\mathrm{W(t)}\sigma\mathrm{dB_s} + J_{\lambda}\sigma_\lambda\mathrm{dB_\lambda} + [J\big(\mathrm{W(t)}\big(1 + \omega(\mathrm{e^x} - 1)\big), \lambda(\mathrm{t}), \mathrm{t}\big) - J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})]\mathrm{dN} \quad \text{(A.3)}$$

By Bellman equation, we know that J must be a martingale:

$$\mathrm{E_t}[\mathrm{d}J] = 0 \quad \text{(A.4)}$$

Applying this to Equation (A.3) to get:

$$max_{(\omega)}\{ J_{\mathrm{t}} + J_{\mathrm{w}}\mathrm{W(t)}\big(\mathrm{r} + \omega\phi - \omega\lambda(\mathrm{t})\mathrm{E}(\mathrm{e^x} - 1)\big) + J_{\lambda}\kappa\big(\theta - \lambda(\mathrm{t})\big) + \frac{1}{2}J_{\mathrm{ww}}\omega^2\mathrm{W(t)}^2\sigma^2 + \frac{1}{2}J_{\lambda\lambda}\sigma_\lambda^2 + \lambda(\mathrm{t})\mathrm{E}[J\big(\mathrm{W(t)}\big(1 + \omega(\mathrm{e^x} - 1)\big), \lambda(\mathrm{t}), \mathrm{t}\big) - J(\mathrm{W(t)}, \lambda(\mathrm{t}), \mathrm{t})]\} = 0 \quad \text{(A.5)}$$

Taking partial derivative with respect to ω and using the market clearing condition $\omega = 1$ and using the assumption that jump size follows normal distribution as $\mathrm{x} \sim \mathrm{N}(\mu_\mathrm{x}, \sigma_\mathrm{x}^2)$, yields the equilibrium stock premium:

$$\phi = -\sigma^2 \frac{\mathrm{W(t)}J_{\mathrm{ww}}}{J_{\mathrm{w}}} - \lambda(\mathrm{t})\frac{1}{J_{\mathrm{w}}}(\mathrm{e}^{\mu_\mathrm{x} + \frac{1}{2}\sigma_\mathrm{x}^2} - 1)\mathrm{E}[J_{\mathrm{w}}(\mathrm{W(t)e^x}, \lambda(\mathrm{t}), \mathrm{t})] + (\mathrm{e}^{\mu_\mathrm{x} + \frac{1}{2}\sigma_\mathrm{x}^2} - 1)\lambda(\mathrm{t}) \quad \text{(A.6)}$$

Substituting (A.6) into (A.5) with $\omega = 1$ to get the following PDE satisfied by the optimal indirect function J:

$$J_t + rW(t)J_w - \frac{1}{2}W(t)^2\sigma^2 J_{ww} + J_\lambda \kappa\big(\theta - \lambda(t)\big) + \frac{1}{2}J_{\lambda\lambda}\sigma_\lambda^2 - \lambda(t)W(t)(e^{\mu_x + \frac{1}{2}\sigma_x^2} - 1)E[J_w(W(t)e^x, \lambda(t), t)] + \lambda(t)E[J(W(t)e^x, \lambda(t), t)] - \lambda(t)E[J(W(t), \lambda(t), t)] \quad = 0 \qquad \text{(A.7)}$$

Applying the functional form of optimal indirect function assumed in (A.2) and substitute it into (A.7) to get:

$$-g_\tau + r(1-\gamma)g + \frac{1}{2}\sigma^2\gamma(1-\gamma)g + \kappa\big(\theta - \lambda(t)\big)g_\lambda + \frac{1}{2}g_{\lambda\lambda}\sigma_\lambda^2 + \lambda(t)g(1-\gamma)e^{-\gamma\mu_x + \frac{1}{2}\gamma^2\sigma_x^2} + \lambda(t)g\gamma e^{(1-\gamma)\mu_x + \frac{1}{2}(1-\gamma)^2\sigma_x^2} - \lambda(t)g = 0 \quad \text{with the initial condition } g(\lambda(t), 0) = 1 \qquad \text{(A.8)}$$

To solve for the hyperbolic PDE whose coefficients are quadratic functions of $\lambda(t)$, we guess the functional form:

$$g(\lambda(t), \tau) = e^{A(\tau) + B(\tau)\lambda(t) + C(\tau)\lambda(t)^2} \qquad \text{(A.9)}$$

This guess exploits the linearity of the coefficients in the PDE (A.3). Substitute (A.9) into (A.8) and collect terms with the same powers of $\lambda(t)$ to reduce it to two ODEs:

$$B_\tau = 2\kappa\theta C(\tau) - \kappa B(\tau) + 2B(\tau)C(\tau)\sigma_\lambda^2 + (1-\gamma)e^{-\gamma\mu_x + \frac{1}{2}\gamma^2\sigma_x^2} + \gamma e^{(1-\gamma)\mu_x + \frac{1}{2}(1-\gamma)^2\sigma_x^2} - 1 \qquad \text{(A.10)}$$

$$C_\tau = -2\kappa C(\tau) + 2C(\tau)^2\sigma_\lambda^2 \qquad \text{(A.11)}$$

With the initial conditions $B(0) = 0; C(0) = 0$.

Meanwhile, by substituting the assumed functional form (A.2) into the equilibrium stock premium (A.6), we get:

$$\phi = \gamma\sigma^2 - \lambda(t)e^{(1-\gamma)\mu_x + \frac{1}{2}(1-\gamma)^2\sigma_x^2} + \lambda(t)e^{-\gamma\mu_x + \frac{1}{2}\gamma^2\sigma_x^2} + \lambda(t)e^{\mu_x + \frac{1}{2}\sigma_x^2} - \lambda(t) \qquad \text{(A.12)}$$

Appendix2:

In the presence of European option, $f(S(t), \lambda(t), t)$, in this economy, the investor allocates a fraction ω_s of his wealth in stock $S(t)$, a fraction ω_f in option f, and a fraction $(1-\omega_s-\omega_f)$ in money market account, in order to maximize the utility of terminal wealth.

In response to the jump-diffusion stock price process, we assume that under physical probability measure, the options price follows the process:

$$\frac{df}{f} = \left(r + \phi_f - \lambda(t)\mu_{\phi_f}\right)dt + \sigma_{fs}dB_s + \sigma_{f\lambda}dB_\lambda + Q_f dN \tag{B.1}$$

Where μ_{ϕ_f} is the mean jump size on the option; ϕ_f is the option risk premium; $Q_f \equiv [f(S(t)e^x, \lambda(t), t) - f(S(t), \lambda(t), t)]/f$ is the percentage jump size on the option. The Brownian motions and the jump process are independent. Under this process, we incorporate the same sources of risk in the underlying stock market into options market, but the magnitudes of the risk shocks in the options market, namely, $\sigma_{fs}, \sigma_{f\lambda}, Q_f$, are different from their counterparties specified in the stock market.

The investor aims at maximizing the utility of terminal wealth by choosing the fractions (ω_s, ω_f):

$$max_{(\omega_s,\omega_f)} E_t[U(W(T), T)]$$

Subject to his wealth constraint as

$$\frac{dW(t)}{W(t)} \equiv \omega_s \frac{dS(t)}{S(t)} + \omega_f \frac{df(S(t),\lambda(t),t)}{f(S(t),\lambda(t),t)} + (1-\omega_s-\omega_f)\frac{dM(t)}{M(t)} = \big[r + \omega_s\phi + \omega_f\phi_f - \omega_s\lambda(t)E(e^x-1) - \omega_f\lambda(t)\mu_{\phi_f}\big]dt + \omega_s\sigma dB_s + \omega_f\sigma_{fs}dB_s + \omega_f\sigma_{f\lambda}dB_\lambda + [\omega_s(e^x-1) + \omega_f Q_f]dN \tag{B.2}$$

Define the optimal indirect utility $J(W(t), \lambda(t), t)$ to be

$$J(W(t), \lambda(t), t) \equiv max_{(\omega_s,\omega_f)} E_t[U(W(T), T)]$$

Applying Ito's Lemma with jumps gives

$$dJ = J_t dt + J_w W(t)\big[r + \omega_s\phi + \omega_f\phi_f - \omega_s\lambda(t)E(e^x-1) - \omega_f\lambda(t)\mu_{\phi_f}\big]dt + J_\lambda\kappa\big(\theta-\lambda(t)\big)dt + \frac{1}{2}J_{ww}W(t)^2\big[\omega_s^2\sigma^2 + 2\omega_s\omega_f\sigma\sigma_{fs} + \omega_f^2\big(\sigma_{fs}^2 + \sigma_{f\lambda}^2\big)\big]dt + \frac{1}{2}J_{\lambda\lambda}\sigma_\lambda^2 dt + J_{w\lambda}W(t)\sigma_\lambda\omega_f\sigma_{f\lambda}dt +$$

$J_w W(t)[\omega_s \sigma + \omega_f \sigma_{fs}]dB_s + J_w W(t)\omega_f \sigma_{f\lambda} dB_\lambda + J_\lambda \sigma_\lambda dB_\lambda + [J(W(t)(1 + (\omega_s(e^x - 1) + \omega_f Q_f)), \lambda(t), t) - J(W(t), \lambda(t), t)]dN$ (B.3)

By Bellman equation, we know that J must be a martingale:

$$E_t[dJ] = 0 \quad \text{(B.4)}$$

Applying this to Equation (B.3) to get:

$$max_{(\omega_s,\omega_f)}\{ J_t + J_w W(t)\left[r + \omega_s \phi + \omega_f \phi_f - \omega_s \lambda(t) E(e^x - 1) - \omega_f \lambda(t)\mu_{\phi_f}\right] + J_\lambda \kappa\left(\theta - \lambda(t)\right) + \frac{1}{2} J_{ww} W(t)^2\left[\omega_s^2 \sigma^2 + 2\omega_s \omega_f \sigma \sigma_{fs} + \omega_f^2\left(\sigma_{fs}^2 + \sigma_{f\lambda}^2\right)\right] + \frac{1}{2} J_{\lambda\lambda} \sigma_\lambda^2 + J_{w\lambda} W(t)\sigma_\lambda \omega_f \sigma_{f\lambda} + \lambda(t) E[J(W(t)(1 + (\omega_s(e^x - 1) + \omega_f Q_f)), \lambda(t), t) - J(W(t), \lambda(t), t)]\} = 0 \quad \text{(B.5)}$$

Taking partial derivative with respect to ω_f and using the market clearing conditions $\omega_s = 1$, $\omega_f = 0$, yields the equilibrium risk premium on the option:

$$\phi_f = -\frac{W(t)J_{ww}}{J_w} \sigma \sigma_{fs} - \frac{J_{w\lambda}}{J_w} \sigma_\lambda \sigma_{f\lambda} - \lambda(t) \frac{1}{J_w} E[Q_f J_w(W(t)e^x, \lambda(t), t)] + \lambda(t)\mu_{\phi_f} \quad \text{(B.6)}$$

In the meantime, we could also use Ito's Lemma with jumps to the option price $f(S(t), \lambda(t), t)$ as follows:

$$df = f_t dt + f_s\left(r + \phi - \lambda(t)E(e^x - 1)\right)S(t)dt + f_\lambda \kappa\left(\theta - \lambda(t)\right)dt + \frac{1}{2} f_{ss} S(t)^2 \sigma^2 dt + \frac{1}{2} f_{\lambda\lambda} \sigma_\lambda^2 dt + f_s \sigma S(t) dB_s + f_\lambda \sigma_\lambda dB_\lambda + [f(S(t)e^x, \lambda(t), t) - f(S(t), \lambda(t), t)]dN \quad \text{(B.7)}$$

Combining Equation (B.7) and Equation (B.1) and collecting terms with the same magnitude of drifts, diffusions, jumps, yields the following three equations:

$$rf + \phi_f f - \lambda(t)\mu_{\phi_f} f = f_t + f_s\left(r + \phi - \lambda(t)E(e^x - 1)\right)S(t) + f_\lambda \kappa\left(\theta - \lambda(t)\right) + \frac{1}{2} f_{ss} S(t)^2 \sigma^2 + \frac{1}{2} f_{\lambda\lambda} \sigma_\lambda^2 + \lambda(t)E[f(S(t)e^x, \lambda(t), t) - f(S(t), \lambda(t), t)] \quad \text{(B.8)}$$

$$\sigma_{fs} f = f_s \sigma S(t) \quad \text{(B.9)}$$

$$\sigma_{f\lambda} f = f_\lambda \sigma_\lambda \quad \text{(B.10)}$$

Substituting (B.9) and (B.10) into the equilibrium risk premium on the option (B.6), we get:

$$\phi_f = -\frac{W(t)J_{ww}}{J_w}\sigma^2 S(t)\frac{f_s}{f} - \frac{J_{w\lambda}}{J_w}\sigma_\lambda^2\frac{f_\lambda}{f} - \lambda(t)\frac{1}{J_w}E[Q_f J_w(W(t)e^x,\lambda(t),t)] + \lambda(t)\mu_{\phi_f} \quad \text{(B.11)}$$

Using (B.11), we know:

$$rf + \phi_f f - \lambda(t)\mu_{\phi_f} f = rf - \frac{W(t)J_{ww}}{J_w}\sigma^2 S(t)f_s - \frac{J_{w\lambda}}{J_w}\sigma_\lambda^2 f_\lambda - \lambda(t)\frac{1}{J_w}E[Q_f J_w(W(t)e^x,\lambda(t),t)]f \quad \text{(B.12)}$$

In the meantime, from the equilibrium risk premium on the stock (A.6) in Appendix1, we know:

$$\big(r + \phi - \lambda(t)E(e^x - 1)\big)S(t) = rS(t) - \sigma^2\frac{W(t)J_{ww}}{J_w}S(t) - \lambda(t)\frac{1}{J_w}(e^{\mu_x + \frac{1}{2}\sigma_x^2} - 1)E[J_w(W(t)e^x,\lambda(t),t)]S(t) \quad \text{(B.13)}$$

Combining Equations (B.8), (B.12), (B.13), and using the fact that jump size follows normal distribution as $x \sim N(\mu_x, \sigma_x^2)$, we have the following PDE:

$$-f_t = -rf + \left(r - \lambda(t)\frac{1}{J_w}(e^{\mu_x + \frac{1}{2}\sigma_x^2} - 1)E[J_w(W(t)e^x,\lambda(t),t)]\right)S(t)f_s + \left(\kappa\big(\theta - \lambda(t)\big) + \frac{J_{w\lambda}}{J_w}\sigma_\lambda^2\right)f_\lambda + \frac{1}{2}f_{ss}S(t)^2\sigma^2 + \frac{1}{2}f_{\lambda\lambda}\sigma_\lambda^2 + \lambda(t)\frac{1}{J_w}E[Q_f J_w(W(t)e^x,\lambda(t),t)]f + \lambda(t)E[f(S(t)e^x,\lambda(t),t) - f(S(t),\lambda(t),t)] \quad \text{(B.14)}$$

To solve the PDE, we know in Appendix1 the functional form of optimal indirect utility $J(W(t),\lambda(t),t)$. Substituting (A.2) and (A.9) into (B.14) yields:

$$-f_t = -rf + \left(r - \lambda(t)e^{-\gamma\mu_x + \frac{1}{2}\gamma^2\sigma_x^2}(e^{\mu_x + \frac{1}{2}\sigma_x^2 - \gamma\sigma_x^2} - 1)\right)S(t)f_s + \left(\kappa\big(\theta - \lambda(t)\big) + \big(B(\tau) + 2C(\tau)\lambda(t)\big)\sigma_\lambda^2\right)f_\lambda + \frac{1}{2}f_{ss}S(t)^2\sigma^2 + \frac{1}{2}f_{\lambda\lambda}\sigma_\lambda^2 + \lambda(t)E[e^{-\gamma x}Q_f]f + \lambda(t)E[f(S(t)e^x,\lambda(t),t) - f(S(t),\lambda(t),t)] \quad \text{(B.15)}$$

In the paper, we already defined the risk-neutral process for both the stock process and the jump-intensity process, as shown in formula (9) and (10). Next, we apply Ito's Lemma with jumps to options price under risk-neutral probability measure, namely $f(S(t),\lambda^*(t),t)$.

$$df = f_t dt + f_s\left(r - \lambda^*(t)E\big(e^{x^*} - 1\big)\right)S(t)dt + f_{\lambda^*}\kappa^*(\theta^* - \lambda^*(t))dt + \frac{1}{2}f_{ss}S(t)^2\sigma^2 dt + \frac{1}{2}f_{\lambda^*\lambda^*}{\sigma_\lambda^*}^2 dt + f_s\sigma S(t)dB_s^* + f_{\lambda^*}\sigma_\lambda^* dB_\lambda^* + f\big(S(t)e^{x^*},\lambda^*(t),t\big) - f(S(t),\lambda^*(t),t)dN^* \quad \text{(B.16)}$$

Under risk-neutral measure, the drift term of df/f should be equal to risk-free rate r. Therefore Equation (B.16) leads to:

$$-f_t = -rf + \left(r - \lambda^*(t)E\left(e^{x^*} - 1\right)\right)S(t)f_s + \kappa^*\left(\theta^* - \lambda^*(t)\right)f_{\lambda^*} + \frac{1}{2}f_{ss}S(t)^2\sigma^2 + \frac{1}{2}f_{\lambda^*\lambda^*}{\sigma_\lambda^*}^2 + \lambda^*(t)E[f\left(S(t)e^{x^*}, \lambda^*(t), t\right) - f(S(t), \lambda^*(t), t)] \quad \text{(B.17)}$$

Under the assumption that jump size follows normal distribution as $x \sim N(\mu_x, \sigma_x^2)$, the relations between risk-neutral jump components and their physical counterparties in equations (11), (12), (13) are verified by substituting them into (B.17) to get (B.15).

Appendix3:

Define the continuously compounded return viewed at time t over the time interval $(t, t+\tau)$ as $R_{t+\tau} \equiv \ln\frac{S(t+\tau)}{S(t)}$. The conditional central skewness on return $R_{t+\tau}$ is expressed as

$$Skew_t(R_{t+\tau}) \equiv E_t[R_{t+\tau} - E_t(R_{t+\tau})]^3 \tag{C.1}$$

According to the stock price process $\frac{dS(t)}{S(t)} = \big(r + \phi - \lambda(t)E(e^x - 1)\big)dt + \sigma dB_s + (e^x - 1)dN$ and the assumption that the jump size follows normal distribution as $x \sim N(\mu_x, \sigma_x^2)$, we derive the following equations:

$$R_{t+\tau} \equiv \ln\frac{S(t+\tau)}{S(t)} = \left(r + \phi - \frac{1}{2}\sigma^2 - \lambda(t)E(e^x - 1)\right)\tau + \sigma B_{s,\tau} + \sum_{i=1}^{N_\tau} x_i \tag{C.2}$$

$$R_{t+\tau} - E_t(R_{t+\tau}) = \sigma B_{s,\tau} + (N_\tau - \lambda(t)\tau)\mu_x + \sum_{i=1}^{N_\tau}(x_i - \mu_x) \tag{C.3}$$

Where $B_{s,\tau}$ is the standard Brownian motion due to increments of B_s in the stock price process over the time interval $(t, t+\tau)$. N_τ is the jump numbers over the time interval $(t, t+\tau)$.

Since the Brownian process and jump process are independent, moment-generating functions of a standard Brownian motion, i.e. $g_{B_{s,\tau}}(m) = e^{\frac{1}{2}m^2\tau}$ and of a Poisson process, i.e. $g_{N_\tau}(m) = e^{\lambda\tau(e^m - 1)}$, are applied to get the following properties that are needed in order to calculate the conditional central skewness on $R_{t+\tau}$.

$$E(B_{s,\tau}) = g'_{B_{s,\tau}}(m)|_{m=0} = 0$$

$$E(B_{s,\tau}^2) = g''_{B_{s,\tau}}(m)|_{m=0} = \tau$$

$$E(B_{s,\tau}^3) = g'''_{B_{s,\tau}}(m)|_{m=0} = 0$$

$$E(N_\tau) = g'_{N_\tau}(m)|_{m=0} = \lambda\tau$$

$$E(N_\tau^2) = g''_{N_\tau}(m)|_{m=0} = \lambda^2\tau^2 + \lambda\tau$$

$$E(N_\tau^3) = g'''_{N_\tau}(m)|_{m=0} = \lambda^3\tau^3 + 3\lambda^2\tau^2 + \lambda\tau$$

Repeatedly substituting the above formulas into (C.1), and using assumption that the jump size follows normal distribution independently as $x \sim N(\mu_x, \sigma_x^2)$, we get:

$$Skew_t(R_{t+\tau}) \equiv E_t[R_{t+\tau} - E_t(R_{t+\tau})]^3$$

$$= E_t\left[\sigma B_{s,\tau} + (N_\tau - \lambda(t)\tau)\mu_x + \sum_{i=1}^{N_\tau}(x_i - \mu_x)\right]^3$$

$$= E_t\left[(N_\tau - \lambda(t)\tau)\mu_x + \sum_{i=1}^{N_\tau}(x_i - \mu_x)\right]^3$$

$$= \mu_x^3 E_t(N_\tau - \lambda(t)\tau)^3 + 3E_t\left[\left((N_\tau - \lambda(t)\tau)\mu_x\right)^2 \times \sum_{i=1}^{N_\tau}(x_i - \mu_x)\right]$$

$$+ 3E_t\left[(N_\tau - \lambda(t)\tau)\mu_x \times \left(\sum_{i=1}^{N_\tau}(x_i - \mu_x)\right)^2\right] + E_t\left[\sum_{i=1}^{N_\tau}(x_i - \mu_x)\right]^3$$

$$= \mu_x^3 E_t(N_\tau - \lambda(t)\tau)^3 + 3E_t\left((N_\tau - \lambda(t)\tau)\mu_x N_\tau (x_i - \mu_x)^2\right) + E_t(N_\tau)E_t(x_i - \mu_x)^3$$

$$= \tau(\mu_x^3 + 3\mu_x\sigma_x^2)\lambda(t)$$

(C.4)

In a similar manner, and using the result in Proposition2 that the jump size in risk-neutral measure follows normal distribution as $x^* \sim N(\mu_x - \gamma\sigma_x^2, \sigma_x^2)$, the conditional central skewness in risk-neutral measure is as follows:

$$Skew_t^*(R_{t+\tau}) = \tau((\mu_x - \gamma\sigma_x^2)^3 + 3(\mu_x - \gamma\sigma_x^2)\sigma_x^2)\lambda^*(t) \quad \text{(C.5)}$$

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Table I
Options Skewness - Summary Statistics
Daily data from Jan 1st 1990 to Dec 31th 2012

Maturity (months)	Mean	Standard Deviation	Min	Max	N
1	-1.695	0.549	-4.679	-0.104	5785
2	-1.625	0.470	-3.455	0.132	5785
3	-1.606	0.490	-4.387	0.229	5785
4	-1.645	0.473	-3.974	-0.230	5785
6	-1.593	0.557	-4.857	1.520	5785
8	-1.489	0.656	-7.045	3.408	5785

Table II
Skewness Time Series – Autocorrelations and Partial Correlations
where the implied daily persistency ρ^* is the autocorrelation raised to the 1/n power, where n is the lag length in days. The standard errors are shown in parentheses.

Lag length (days)	Autocorrelation	Partial Correlation	Implied daily persistency ρ^*
Panel A: Maturity One Month			
1	0.876 (0.006)	0.623 (0.013)	0.876
2	0.823 (0.007)	0.134 (0.015)	0.907
3	0.788 (0.008)	0.082 (0.015)	0.923
4	0.758 (0.008)	0.067 (0.015)	0.933
5	0.724 (0.009)	0.017 (0.013)	0.937
Panel B: Maturity Two Months			
1	0.887 (0.006)	0.560 (0.013)	0.887
2	0.848 (0.006)	0.155 (0.014)	0.920
3	0.822 (0.007)	0.072 (0.015)	0.936
4	0.808 (0.007)	0.112 (0.014)	0.948
5	0.786 (0.008)	0.043 (0.013)	0.952
Panel C: Maturity Three Months			
1	0.913 (0.005)	0.652 (0.013)	0.913
2	0.874 (0.006)	0.141 (0.015)	0.934
3	0.846 (0.007)	0.073 (0.015)	0.945
4	0.821 (0.007)	0.060 (0.015)	0.951
5	0.795 (0.007)	0.020 (0.013)	0.955
Panel D: Maturity Four Months			
1	0.944 (0.004)	0.617 (0.013)	0.944
2	0.923 (0.005)	0.184 (0.015)	0.960
3	0.906 (0.005)	0.088 (0.015)	0.967
4	0.890 (0.005)	0.026 (0.015)	0.971
5	0.876 (0.006)	0.052 (0.013)	0.973
Panel E: Maturity Six Months			
1	0.951 (0.004)	0.583 (0.013)	0.951
2	0.935 (0.004)	0.197 (0.015)	0.966
3	0.923 (0.005)	0.136 (0.015)	0.973
4	0.908 (0.005)	0.005 (0.015)	0.976
5	0.897 (0.005)	0.051 (0.013)	0.978
Panel F: Maturity Eight Months			
1	0.941 (0.004)	0.606 (0.013)	0.941
2	0.921 (0.005)	0.213 (0.015)	0.959
3	0.904 (0.005)	0.108 (0.015)	0.966
4	0.882 (0.006)	-0.026 (0.015)	0.969
5	0.868 (0.006)	0.064 (0.013)	0.972

Table III

Values of elasticity parameter $\beta \equiv \frac{\tau_2\left(1-e^{-\kappa^*\tau_1}\right)(e^{-\kappa^* j}-1)}{\tau_1(1-e^{-\kappa^*\tau_2})-\tau_2(1-e^{-\kappa^*\tau_1})}$

where τ_1 is the maturity of nearby options ; τ_2 is the maturity of distant options; j is time lapse in days; $\rho^* = e^{-\kappa^*}$ is the persistency of skewness time series on a daily basis; κ^* is the mean-reverting speed of skewness time series on a daily basis.

j (days)	$e^{-\kappa^*} = 0.8$ $\kappa^* = 0.223$	$e^{-\kappa^*} = 0.86$ $\kappa^* = 0.150$	$e^{-\kappa^*} = 0.9$ $\kappa^* = 0.105$	$e^{-\kappa^*} = 0.96$ $\kappa^* = 0.040$
Panel A: Maturity $\tau_1 = 21$ days and $\tau_2 = 42$ days				
1	0.4037	0.2923	0.2245	0.1389
2	0.7267	0.5436	0.4266	0.2723
3	0.9850	0.7598	0.6085	0.4004
4	1.1917	0.9458	0.7723	0.5233
5	1.3571	1.1057	0.9196	0.6414
…	…	…	…	…
21	2	2	2	2
22	2.0037	2.0123	2.0245	2.0589
23	2.0067	2.0228	2.0466	2.1155
…	…	…	…	…
Panel B: Maturity $\tau_1 = 42$ days and $\tau_2 = 84$ days				
1	0.40003	0.2804	0.2024	0.0975
2	0.7200	0.5217	0.3846	0.1912
3	0.9760	0.7291	0.5485	0.2811
4	1.1809	0.9075	0.6961	0.3674
5	1.3447	1.0610	0.8289	0.4503
…	…	…	…	…
42	2	2	2	2
43	2.00003	2.00049	2.0024	2.0175
44	2.00006	2.00092	2.0046	2.0344
…	…	…	…	…
Panel C: Maturity $\tau_1 = 63$ days and $\tau_2 = 126$ days				
1	0.4000003	0.2800	0.2002	0.0866
2	0.7200	0.5208	0.3804	0.1697
3	0.9760	0.7279	0.5427	0.2495
4	1.1808	0.9060	0.6887	0.3262
5	1.3446	1.0592	0.8200	0.3997
…	…	…	…	…
63	2	2	2	2
64	2.0000003	2.000021	2.000262	2.006617
65	2.0000006	2.000039	2.000498	2.012970
…	…	…	…	…
Panel D: Maturity $\tau_1 = 84$ days and $\tau_2 = 168$ days				
1	0.4	0.28000088	0.20002867	0.08268031
2	0.72000001	0.52080164	0.38005448	0.16205342
3	0.97600001	0.72789029	0.54207770	0.23825159
4	1.18080001	0.90598653	0.68789860	0.31140184
5	1.34464001	1.05914930	0.81913742	0.38162608
…	…	…	…	…
84	2	2	2	2
85	2.00000000	2.00000088	2.00002867	2.00268031
86	2.00000001	2.00000164	2.00005448	2.00525342

Table IV

Prediction error against risk-neutral skewness

$$\left(Skew^{*}_{t+j}\left(\mathrm{R}_{(t+j)+\tau_1}\right) - Skew^{*}_{t}\left(\mathrm{R}_{t+\tau_1}\right)\right) - 2\left(Skew^{*}_{t}\left(\mathrm{R}_{t+\tau_2}\right) - Skew^{*}_{t}\left(\mathrm{R}_{t+\tau_1}\right)\right) = \alpha + \beta Skew^{*}_{t}\left(\mathrm{R}_{t+\tau_1}\right) + \varepsilon_{t+j}.$$

Where $Skew^{*}_{t}\left(\mathrm{R}_{t+\tau_1}\right)$ is the risk-neutral skewness of options with nearby maturity. $Skew^{*}_{t}\left(\mathrm{R}_{t+\tau_2}\right)$ is the risk-neutral skewness of options with distant maturity. The time lapse is set as $j = \tau_1$.

Panel A : Maturity $\tau_1 = 21$ days and $\tau_2 = 42$ days				
	Coefficient β	Standard Error	t-Statistic	N
1990-2012	0.293	0.017	16.79	5785
Panel B : Maturity $\tau_1 = 42$ days and $\tau_2 = 84$ days				
	Coefficient β	Standard Error	t-Statistic	N
1990-2012	0.011	0.018	0.64	5785
Panel C : Maturity $\tau_1 = 63$ days and $\tau_2 = 126$ days				
	Coefficient β	Standard Error	t-Statistic	N
1990-2012	-0.023	0.021	-1.101	5785
Panel D : Maturity $\tau_1 = 84$ days and $\tau_2 = 168$ days				
	Coefficient β	Standard Error	t-Statistic	N
1990-2012	-0.509	0.024	-21.14	5785

Table V

Prediction error against risk-neutral skewness – subsample analysis

$$\left(Skew^*_{t+j}\left(\mathrm{R}_{(t+j)+\tau_1}\right) - Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right)\right) - 2\left(Skew^*_t\left(\mathrm{R}_{t+\tau_2}\right) - Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right)\right) = \alpha + \beta Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right) + \varepsilon_{t+j}.$$

Where $Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right)$ is the risk-neutral skewness of options with nearby maturity. $Skew^*_t\left(\mathrm{R}_{t+\tau_2}\right)$ is the risk-neutral skewness of options with distant maturity. The time lapse is set as $j = \tau_1$.

Panel A : Maturity $\tau_1 = 21$ days and $\tau_2 = 42$ days

Subsample Periods	Coefficient β	Standard Error	t-Statistic	N
1990-1999	0.488	0.028	16.98	2527
2000-2009	0.290	0.024	11.87	2504
2010-2012	0.197	0.058	3.35	754

Panel B : Maturity $\tau_1 = 42$ days and $\tau_2 = 84$ days

Subsample Periods	Coefficient β	Standard Error	t-Statistic	N
1990-1999	0.520	0.030	17.10	2527
2000-2009	-0.064	0.026	-2.43	2504
2010-2012	-0.571	0.073	-7.73	754

Panel C : Maturity $\tau_1 = 63$ days and $\tau_2 = 126$ days

Subsample Periods	Coefficient β	Standard Error	t-Statistic	N
1990-1999	0.708	0.041	17.18	2527
2000-2009	0.022	0.026	0.82	2504
2010-2012	-0.376	0.081	-4.60	754

Panel D : Maturity $\tau_1 = 84$ days and $\tau_2 = 168$ days

Subsample Periods	Coefficient β	Standard Error	t-Statistic	N
1990-1999	-0.141	0.063	-2.23	2527
2000-2009	-0.228	0.022	-10.10	2504
2010-2012	-0.240	0.086	-2.77	754

Table VI
Prediction error against risk-neutral skewness – Robustness check with *weekly* data

$$\left(Skew^{*}_{t+j}\left(R_{(t+j)+\tau_1}\right) - Skew^{*}_{t}\left(R_{t+\tau_1}\right)\right) - 2\left(Skew^{*}_{t}\left(R_{t+\tau_2}\right) - Skew^{*}_{t}\left(R_{t+\tau_1}\right)\right) = \alpha + \beta Skew^{*}_{t}\left(R_{t+\tau_1}\right) + \varepsilon_{t+j}.$$

Where $Skew^{*}_{t}\left(R_{t+\tau_1}\right)$ is the risk-neutral skewness of options with nearby maturity. $Skew^{*}_{t}\left(R_{t+\tau_2}\right)$ is the risk-neutral skewness of options with distant maturity. The time lapse is set as $j = \tau_1$. Wednesdays are used to construct the weekly data set. If a Wednesday is a holiday day, we use the weekday following that Wednesday.

	Coefficient β	Standard Error	t-Statistic	N
Panel A : Maturity $\tau_1 = 4$ weeks and $\tau_2 = 8$ weeks				
1990-2012	0.263	0.037	6.99	1186
Panel B : Maturity $\tau_1 = 8$ weeks and $\tau_2 = 16$ weeks				
1990-2012	0.021	0.040	0.52	1186
Panel C : Maturity $\tau_1 = 12$ weeks and $\tau_2 = 24$ weeks				
1990-2012	-0.009	0.047	-0.19	1186
Panel D : Maturity $\tau_1 = 16$ weeks and $\tau_2 = 32$ weeks				
1990-2012	-0.480	0.054	-8.89	1186

Table VII

Prediction error against risk-neutral skewness – Robustness check of *weekly* data with subsample periods

$$\left(Skew^*_{t+j}\left(\mathrm{R}_{(t+j)+\tau_1}\right) - Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right)\right) - 2\left(Skew^*_t\left(\mathrm{R}_{t+\tau_2}\right) - Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right)\right) = \alpha + \beta Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right) + \varepsilon_{t+j}.$$

Where $Skew^*_t\left(\mathrm{R}_{t+\tau_1}\right)$ is the risk-neutral skewness of options with nearby maturity. $Skew^*_t\left(\mathrm{R}_{t+\tau_2}\right)$ is the risk-neutral skewness of options with distant maturity. The time lapse is set as $j = \tau_1$. Wednesdays are used to construct the weekly data set. If a Wednesday is a holiday day, we use the weekday following that Wednesday.

Subsample Periods	Coefficient β	Standard Error	t-Statistic	N
Panel A : Maturity $\tau_1 = 4$ weeks and $\tau_2 = 8$ weeks				
1990-1999	0.456	0.059	7.72	516
2000-2009	0.226	0.054	4.12	515
2010-2012	0.239	0.138	1.72	155
Panel B : Maturity $\tau_1 = 8$ weeks and $\tau_2 = 16$ weeks				
1990-1999	0.469	0.068	6.83	516
2000-2009	-0.027	0.059	-0.45	515
2010-2012	-0.623	0.155	-4.00	155
Panel C : Maturity $\tau_1 = 12$ weeks and $\tau_2 = 24$ weeks				
1990-1999	0.749	0.094	7.97	516
2000-2009	0.047	0.058	0.81	515
2010-2012	-0.533	0.189	-2.81	155
Panel D : Maturity $\tau_1 = 16$ weeks and $\tau_2 = 32$ weeks				
1990-1999	-0.107	0.142	-0.74	516
2000-2009	-0.194	0.051	-3.77	515
2010-2012	-0.366	0.185	-1.97	155