Sum Rate Maximizing Multigroup Multicast Beamforming under Per-antenna Power Constraints

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***Abstract*—A multi-antenna transmitter that conveys independent sets of common data to distinct groups of users is herein considered, a model known as physical layer multicasting to multiple co-channel groups. In the recently proposed context of per-antenna power constrained multigroup multicasting, the present work focuses on a novel system design that aims at maximizing the total achievable throughput. Towards increasing the system sum rate, the available power resources need to be allocated to well conditioned groups of users. A detailed solution to tackle the elaborate sum rate maximizing, multigroup multicast problem, under per-antenna power constraints is therefore derived. Numerical results are presented to quantify the gains of the proposed algorithm over heuristic solutions. The solution is applied to rayleigh as well as Vandermonde channels. The latter case is typically realised in uniform linear array transmitters operating in the far field, where line-of-sight conditions are realized. In this setting, a sensitivity analysis with respect to the angular separation of co-group users is included. Finally, a simple scenario providing important intuitions for the sum rate maximizing multigroup multicast solutions is elaborated.**

***Index Terms*—Sum Rate Maximization; Multicast Multigroup beamforming; Per Antenna Constraints; Power Allocation**

I. Introduction & Related Work

Advanced transmit signal processing techniques are currently employed to optimize the performance of multi-antenna transmitters without compromising the complexity of single antenna receivers. These beamforming (or equivalently precoding) techniques efficiently manage the co-channel interferences to achieve the targeted service requirements (Quality of Service–QoS targets). The optimal downlink transmission strategy, in the sense of minimizing the total transmit power under guaranteed per user QoS constraints, was given in [1], [2]. Therein, the powerful tool of Semi-Definite Relaxation (SDR) reduced the non-convex quadratically constrained quadratic problem (QCQP) into a relaxed semi-definite programming instance by changing the optimization variables and disregarding the unit-rank constraints over the new variable. The relaxed solution was proven to be optimal. In the same direction, the multiuser downlink beamforming problem that aims at maximizing the minimum over all users signal to interference plus noise ratio (SINR), was optimally solved in [3]. The goal of the later formulation is to increase the fairness of the system by boosting the SINR of the user that is further away from a targeted performance. Hence, the problem is commonly referred to as *max–min fair*.

In the contributions discussed so far, power flexibility amongst the transmit antennas is a fundamental assumption. Hence, in all the above optimization problems, a sum power constraint (SPC) at the transmitter is imposed. The more elaborate transmit beamforming problem under per-antenna power constraints (PACs) was formulated and solved in [4]. The motivation for the PACs originates from practical system implementation aspects. The lack of flexibility in sharing energy resources amongst the antennas of the transmitter is usually the case. Individual amplifiers per antenna are common practice. Although flexible amplifiers could be incorporated in multi-antenna transmitters, specific communication systems cannot afford this design. Examples of such systems can be found in satellite communications, where highly complex payloads are restrictive and in distributed antenna systems where the physical co-location of the transmitting elements is not a requisite.

In the new generation of multi-antenna communication standards, physical layer (PHY) multicasting has the potential to efficiently address the nature of traffic demand. An inherent consideration of the hitherto presented literature is that independent data is addressed to multiple users. When a symbol is addressed to more than one users, however, a more elaborate problem formulation is emanated. In this direction, the PHY multicasting problem was proposed, proven NP-hard and accurately approximated by SDR and Gaussian randomization techniques in [5]. Following this, a unified framework for physical layer multicasting to multiple interfering groups, where independent sets of common data are transmitted to multiple interfering groups of users by the multiple antennas, was presented in [6]. Therein, the QoS and the *max–min fair* problems were formulated, proven NP-hard and accurately approximated for the SPC multicast multigroup case. Extending these works, a consolidated solution for the weighted max–min fair multigroup multicast beamforming under PACs has been derived in [7], [8]. To this end, the well established tools of SDR and Gaussian randomization where combined with bisection to obtain highly accurate and efficient solutions.

The fundamental consideration of multicasting, that is a single transmission addressing a group of users, constrains the system performance according to the worst user. Therefore, the maximization of the minimum SINR is the most relevant problem and the fairness criterion is imperative. When advancing to multigroup multicast systems, however, the service levels between different groups can be adjusted towards achieving some other optimization goal. The consideration to maximize the total system sum rate in a multigroup multicast context was initially considered in [9]. In these works, only SPCs where considered. In more detail, a heuristic iterative algorithm was developed based on the principle of decoupling the beamforming design and the power allocation problem.

In the present contribution, in contrast to existing works, PACs are imposed on the problem of maximizing the total throughput of the multigroup multicast system. To this end, the *max sum rate* ($\max \mathrm{SR}$) multigroup multicast problem under PACs is formulated and solved.

Notation: In the remainder of this paper, bold face lower case and upper case characters denote column vectors and matrices, respectively. The operators $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\dagger}$, $|\cdot|$ and $||\cdot||_2^2$ correspond to the transpose, the conjugate transpose, the absolute value and the Frobenius norm of matrices and vectors, while $[\cdot]_{ij}$ denotes the i,j-th element of a matrix. $\mathrm{Tr}(\cdot)$ denotes the trace operator over square matrices and $\mathrm{diag}(\cdot)$ denotes a square diagonal matrix with elements that of the input vector. Calligraphic indexed characters denote sets. Finally, $\mathbb{R}^+$ denotes the set of real positive numbers.

II. System Model

Herein, the focus is on a multi-user (MU) multiple input single output (MISO) multicast system. Assuming a single transmitter, let N_t denote the number of transmitting elements and N_u the total number of users served. The input-output analytical expression will read as $y_i = \mathbf{h}_i^{\dagger}\mathbf{x} + n_i$, where $\mathbf{h}_i^{\dagger}$ is a $1 \times N_t$ vector composed of the channel coefficients (i.e. channel gains and phases) between the i-th user and the N_t antennas of the transmitter, $\mathbf{x}$ is the $N_t \times 1$ vector of the transmitted symbols and n_i is the independent complex circular symmetric (c.c.s.) independent identically distributed (i.i.d) zero mean Additive White Gaussian Noise (AWGN), measured at the i-th user's receive antenna.

Focusing in a multigroup multicasting scenario, let there be a total of $1 \leq G \leq N_u$ multicast groups with $\mathcal{I} = \{\mathcal{G}_1, \mathcal{G}_2, \dots \mathcal{G}_G\}$ the collection of index sets and $\mathcal{G}_k$ the set of users that belong to the k-th multicast group, $k \in \{1 \dots G\}$. Each user belongs to only one group, thus $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset, \forall i, j \in \{1 \cdots G\}$. Let $\mathbf{w}_k \in \mathbb{C}^{N_t \times 1}$ denote the precoding weight vector applied to the transmit antennas to beamform towards the k-th group. By collecting all user channels in one channel matrix, the general linear signal model in vector form reads as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{H}\mathbf{W}\mathbf{s} + \mathbf{n}$, where $\mathbf{y}$ and $\mathbf{n} \in \mathbb{C}^{N_u}$, $\mathbf{x} \in \mathbb{C}^{N_t}$ and $\mathbf{H} \in \mathbb{C}^{N_u \times N_t}$. The multigroup multicast scenario imposes a precoding matrix $\mathbf{W} \in \mathbb{C}^{N_t \times G}$ that includes as many precoding vectors (i.e columns) as the number of groups. This is the number of independent symbols transmitted, i.e. $\mathbf{s} \in \mathbb{C}^{G}$. The power radiated by each antenna element is a linear combination of all precoders and reads as [4]

$$P_n = \left[\sum_{k=1}^{G} \mathbf{w}_k \mathbf{w}_k^{\dagger}\right]_{nn} = \left[\mathbf{W}\mathbf{W}^{\dagger}\right]_{nn}, \tag{1}$$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots \mathbf{w}_G]$ is the precoding matrix and $n \in \{1 \dots N_t\}$ is the antenna index. The fundamental difference between the SPC of [6] and the proposed PAC is clear in (1), where instead of one, N_t constraints are realized, each one involving all the precoding vectors.

III. Sum Rate Maximization

In a multicast scenario, the performance of all the receivers listening to the same multicast is dictated by the worst rate in the group. A multigroup multicasting scenario, however, entails the flexibility to maximize the total system rate by providing different service levels amongst groups. The multigroup multicast $\max \mathrm{SR}$ optimization aims at maximizing the minimum SINR only within each group while in parallel maximize the sum of the rates of all groups. Intuitively, this can be achieved by reducing the power of the users that achieve higher SINR than the minimum achieved in the group they belong. Additionally, groups that contain compromised users are turned off and their users driven to service unavailability. Subsequently, power is not consumed in order to mitigate the channel conditions. Any remaining power budget is then reallocated to well conditioned and balanced in term of channel conditions groups. In [9], the SPC max sum rate problem was solved using a two step heuristic iterative optimization algorithm based on the methods of [6] and [10]. Therein, the SPC multicast beamforming problem of [6] is iteratively solved with input QoS targets defined by the worst user per group of the previous iteration. The derived precoders push all the users of the group closer to the worst user thus saving power. Following that, a power redistribution takes place via the sub-gradient method [10] towards maximising the total system rate.

A. Per-antenna Power Constrained Optimization

The present work focuses on the per-antenna power constrained sum rate maximization problem, formally defined as

$$\mathcal{SR}: \quad \max_{\{\mathbf{w}_k\}_{k=1}^{G}} \sum_{i=1}^{N_u} \log_2(1+\gamma_i)$$

$$\text{subject to: } \gamma_i = \min_{m \in G_k} \frac{|\mathbf{w}_k^{\dagger}\mathbf{h}_m|^2}{\sum_{l \neq k}^{G} |\mathbf{w}_l^{\dagger}\mathbf{h}_m|^2 + \sigma_m^2}, \quad \forall i \in \mathcal{G}_k, k, l \in \{1 \dots G\}, \tag{2}$$

$$\text{and to: } \left[\sum_{k=1}^{G} \mathbf{w}_k \mathbf{w}_k^{\dagger}\right]_{nn} \leq P_n, \quad \forall n \in \{1 \dots N_t\}, \tag{3}$$

Problem $\mathcal{SR}$ receives as input the per-antenna power constraint vector $\mathbf{p}_{ant} = [P_1, P_2 \dots P_{N_t}]$. Following the common in the literature notation for ease of reference, the optimal objective value of $\mathcal{SR}$ will be denoted as $c^* = \mathcal{SR}(\mathbf{p}_{ant})$ and the associated optimal point as $\{\mathbf{w}_k^{\mathcal{SR}}\}_{k=1}^{G}$. The novelty of the $\mathcal{SR}$ lies in the PACs, i.e. (3) instead of the conventional SPCs proposed in [9]. To the end of solving this problem, a heuristic algorithm is proposed. By utilizing recent results [7], the new algorithm calculates the per-antenna power constrained precoders. More specifically, instead of solving the QoS sum power minimization problem of [6], the proposed algorithm calculates the PAC precoding vectors by solving the per-antenna power minimization problem [7]:

$$\begin{aligned}\mathcal{Q}:\quad &\min_{r,\ \{\mathbf{w}_k\}_{k=1}^{G}} \quad r\\ &\text{subject to } \frac{|\mathbf{w}_k^\dagger \mathbf{h}_i|^2}{\sum_{l\neq k}^{G}|\mathbf{w}_l^\dagger \mathbf{h}_i|^2+\sigma_i^2}\geq \gamma_i, &(4)\\ &\qquad \forall i\in\mathcal{G}_k, k,l\in\{1\ldots G\},\\ &\text{and to } \frac{1}{P_n}\left[\sum_{k=1}^{G}\mathbf{w}_k\mathbf{w}_k^\dagger\right]_{nn}\leq r, &(5)\\ &\qquad \forall n\in\{1\ldots N_t\},\end{aligned}$$

where $r\in\mathbb{R}^+$. Problem $\mathcal{Q}$ receives as input SINR the target vector $\mathbf{g}=[\gamma_1,\gamma_2,\ldots\gamma_{N_u}]$, that is the individual QoS constraints of each user, as well as the per-antenna power constraint vector $\mathbf{p}_{ant}$. Let the optimal objective value of $\mathcal{Q}$ be denoted as $r^*=\mathcal{Q}(\mathbf{g},\mathbf{p}_{ant})$ and the associated optimal point as $\{\mathbf{w}_k^{\mathcal{Q}}\}_{k=1}^{G}$. This problem is solved using the well established methods of SDR and Gaussian randomisation [11]. A more detailed description of the solution of $\mathcal{Q}$ can be found in [7], [8] and is herein omitted for shortness.

Let us rewrite the precoding vectors calculated from $\mathcal{Q}$ as $\{\mathbf{w}_k^{\mathcal{Q}}\}_{k=1}^{G}=\{\sqrt{p_k}\mathbf{v}_k\}_{k=1}^{G}$ with $||v_k||_2^2=1$ and $\mathbf{p}=[p_1\ldots p_k]$. By this normalization, the beamforming problem can be decoupled into two problems. The calculation of the beamforming directions, i.e. the normalized $\{\mathbf{v}_k\}_{k=1}^{G}$, and the power allocation over the existing groups, i.e. the calculation of $\mathbf{p}_k$. Since the exact solution of $\mathcal{SR}$ is not straightforwardly obtained, this decoupling allows for a two step optimization. Under general unicasting assumptions, the SR maximizing power allocation under fixed beamforming direction is a convex optimization problem [10]. However, when multigroup multicasting is considered, the cost function $F_e=\sum_{k=1}^{G}\log\left(1+\min_{i\in\mathcal{G}_k}\{\text{SINR}_i\}\right)$ is no longer differentiable due to the $\min_{i\in\mathcal{G}_k}$ operation and one has to adhere to sub-gradient solutions [9].[1]

In the present contribution, the calculation of the beamforming directions is based on $\mathcal{Q}$. Following this, the power reallocation is achieved via the sub-gradient method [10] under specific modifications that are hereafter described. The proposed algorithm, presented in Alg. 1, is an iterative two step algorithm. In each step of the process, the QoS targets $\mathbf{g}$ are calculated as the minimum target per group of the previous iteration, i.e. $\gamma_i=\min_{i\in\mathcal{G}_k}\{\text{SINR}_i\},\forall i\in\mathcal{G}_k,k\in\{1\ldots G\}$. Therefore, the new precoders require equal or less power to achieve the same system SR. Any remaining power is then redistributed amongst the groups to the end of maximizing the total system throughput, via the sub-gradient method [10].

Focusing of the latter method, let us denote $\mathbf{s}=\{s_k\}_{k=1}^{G}=\{\log p_k\}_{k=1}^{G}$, as the logarithmic power vector, the sub-gradient search method reads as

$$\mathbf{s}(l+1)=\prod_{\mathcal{P}_a}\left[\mathbf{s}(l)-\delta(l)\cdot\mathbf{r}(l)\right], \qquad (6)$$

where $\prod_{\mathcal{P}_a}[\mathbf{x}]$ denotes the projection operation of point $\mathbf{x}\in\mathbb{R}_G$ onto the set $\mathcal{P}_a$.The parameters $\delta(l)$ and $\mathbf{r}(l)$ are the step of the search and the sub-gradient of the $\mathcal{SR}$ cost function at the point $\mathbf{s}(l)$, respectively. The analytic calculation of $\mathbf{r}(l)$ is given in [9], [10] and is omitted herein for shortness.

In order to account for the more complicated PACs, a the following consideration is substantiated. The projection operation, i.e. $\prod_{\mathcal{P}_a}[\cdot]$, constrains each iteration of the sub-gradient to the feasibility set of the $\mathcal{SR}$ problem. The present investigation necessitates the projection over a PAC set rather than a conventional SPC set proposed in [9]. Formally, the herein considered set of PACs is defined as

$$\mathcal{P}_a=\left\{\mathbf{p}\in\mathbb{R}_G^+ \middle| \left[\sum_{k=1}^{G}\mathbf{v}_k\text{diag}(\mathbf{p})\mathbf{v}_k^\dagger\right]_{nn}\leq P_n\right\}, \qquad (7)$$

where the element of the power vector $\mathbf{p}=\exp(\mathbf{s})$ with $\mathbf{p},\mathbf{s}\in\mathbb{R}_G$, represent the power allocated to the corresponding group. It should be stressed that this power is inherently different that the power transmitted by each antenna $\mathbf{p}_{ant}\in\mathbb{R}_{N_t}$. The connection between $\mathbf{p}_{ant}$ and $\mathbf{p}$ is given by the normalized beamforming vectors as easily observed in (7). The per-antenna constrained projection is formally defined as

$$\begin{aligned}&\mathcal{P}:\min_{\mathbf{p}}||\mathbf{p}-\mathbf{x}||_2^2\\ &\text{subject to } \left[\sum_{k=1}^{G}\mathbf{v}_k\text{diag}(\mathbf{p})\mathbf{v}_k^\dagger\right]_{nn}\leq P_n, &(8)\\ &\qquad \forall n\in\{1\ldots N_t\},\end{aligned}$$

where $\mathbf{p}\in\mathbb{R}^G$ and $\mathbf{x}=\exp\left(\mathbf{s}(l)-\delta(l)\cdot\mathbf{r}(l)\right)$. Problem $\mathcal{P}$ is a convex optimization problem and can thus be solved to arbitrary accuracy using standard numerical methods [12].

Subsequently, the solution of (6) is given as $\mathbf{s}(l+1)=\log\left(\mathbf{p}^*\right)$, where $\mathbf{p}^*=\mathcal{P}\left(\mathbf{p}_{ant},\mathbf{x}\right)$ is the optimal point of convex problem $\mathcal{P}$. To summarize the solution process, the per-antenna power constrained sum rate maximizing algorithm is presented in Alg. 1.

B. Complexity & Convergence Analysis

An important discussion involves the complexity of the proposed algorithm. The complexity of the techniques employed to approximate a solution of the highly complex, NP-hard multigroup multicast problem under PACs is presented in [7], [8]. Therein, the computational burden for an accurate approximate solution of the per-antenna power minimization problem $\mathcal{Q}$ has been calculated. In summary, the relaxed power minimization is an SDP instance with G matrix variables of $N_t\times N_t$ dimensions and N_u+N_t linear constraints. The present work relies on the CVX tool [12] which calls numerical solvers such as SeDuMi to solve semi-definite programs. The interior point methods employed to solve this SDP require at most $\mathcal{O}(\sqrt{GN_t}\log(1/\epsilon)$ iterations, where ϵ

[1]The direct use of the logarithmic rate function is not possible in the sub-gradient solution process. However, mild approximations allow for accurate heuristic solutions, as in detail explained in [9].

Input: (see Tab.I) $\{\mathbf{w}_k^{(0)}\}_{k=1}^G = \sqrt{P_{tot}/(G\cdot N_t)}\cdot \mathbf{1}_{N_t}$
Output: $\{\mathbf{w}_k^{\mathcal{SR}}\}_{k=1}^G$
begin
  while $\mathcal{SR}$ *does not converge* **do**
    $i = i+1$;
    Step 1: Solve $r^* = \mathcal{Q}(\mathbf{g}_{(i)}, \mathbf{p})$ to calculate $\{\mathbf{w}_k^{(i)}\}_{k=1}^G$. The input SINR targets $\mathbf{g}_{(i)}$ are given by the minimum SINR per group, i.e. $\gamma_i = \min_{i\in\mathcal{G}_k}\{\text{SINR}_i\}, \forall i \in \mathcal{G}_k, k \in \{1\dots G\}$.
    Step 2: Initialize the sub-gradient search algorithm as: $\mathbf{p}^{(i)} = \{p_k\}_{k=1}^G = \{||\mathbf{w}_k^{(i)}||_2^2\}_{k=1}^G$, $\mathbf{s}^{(i)} = \{s_k\}_{k=1}^G = \{\log p_k\}_{k=1}^G$, $\{\mathbf{v}_k^{(i)}\}_{k=1}^G = \{\mathbf{w}_k^{(i)}/p_k^{(i)}\}_{k=1}^G$.
    Step 3: Calculate one iteration of the sub-gradient power control algorithm $\mathbf{s}^{(i+1)} = \prod_{\mathcal{P}_a}\left[\mathbf{s}^{(i)} - \delta\cdot\mathbf{r}(i)\right]$ where $\mathbf{s} = \log(\mathbf{p})$, $\mathcal{P}_a = \left\{\mathbf{p}\in\mathbb{R}_G^+ | \left[\sum_{k=1}^G \mathbf{v}_k \text{diag}(\mathbf{p})\mathbf{v}_k^\dagger\right]_{nn} \leq P_n\right\}$
    Step 4: Calculate the current throughput: $c^* = \mathcal{SR}(\mathbf{p}_{ant})$ with $\{\mathbf{w}_k^{\mathcal{SR}}\}_{k=1}^G = \{\mathbf{w}_k^{(i+1)}\}_{k=1}^G = \{\mathbf{v}_k^{(i)}\exp(s_k^{(i+1)})\}_{k=1}^G$
  end
end

Algorithm 1: $\max$ SR multigroup multicasting under PACs.

TABLE I
INPUT PARAMETERS

Parameter	Symbol	Value
Sub-gradient Iterations	l_{max}	1
Sub-gradient step	δ	0.4
Gaussian Randomizations	N_{rand}	100
Total Power at the T_x	P_{tot}	$[-20:20]$ dBW
Per-antenna constraints	$\mathbf{p}_{ant}$	P_{tot}/N_t
User Noise variance	σ_i^2	$1,\ \forall i\in\{1\dots N_u\}$

is the desired numerical accuracy of the solver. Moreover, in each iteration not more than $\mathcal{O}(G^3N_t^6 + GN_t^3 + N_uGN_t^2)$ arithmetic operations will be performed. The solver used [12] also exploits the specific structure of matrices hence the actual running time is reduced. Next, a fixed number of iterations of the Gaussian randomization method is performed [11]. In each randomization, a linear problem (LP) is solved with a worst case complexity of $\mathcal{O}(G^{3.5}\log(1/\epsilon))$ for an $\epsilon-$optimal solution. The accuracy of the solution increases with the number of randomizations [5], [6], [11].

Focusing on the proposed algorithm, the main complexity burden originates from the solution of a SDP. The remaining three steps of Alg. 1 involve a closed form sub-gradient calculation as given in [10] and the projection operation, which is a real valued least square problem under N_t quadratic inequality PACs. Consequently, the asymptotic complexity of the derived algorithm is polynomial, dominated by the complexity of the QoS multigroup multicast problem under PACs. The convergence of Alg. 1 is guaranteed given that the chosen step size satisfies the conditions given in [9], [10].

IV. NUMERICAL RESULTS

In the present section, numerical results are presented to quantify the performance gains of the proposed SR maximization problem under various channel assumptions. As benchmark, the original SPC solutions are re-scaled to respect the PACs, if and only if a constraint is over satisfied.[2] Rescaling is achieved by multiplying each line of the precoding matrix with the square root of the inverse level of power over satisfaction of the corresponding antenna, i.e.

$$\alpha = \sqrt{\max_n\{[\mathbf{p}_{ant}]_n\}/\left[\mathbf{W}\mathbf{W}^\dagger\right]_{nn}} \quad (9)$$

A. Multigroup multicasting over Rayleigh Channels

The performance of $\mathcal{SR}$ in terms of SR is compared to the performance of the solutions of [9] in a per-antenna constrained transmitter operating over Rayleigh channels in this paragraph. A system with $N_t = 4$ transmit antennas and $N_u = 8$ users uniformly allocated to $G = 4$ groups is assumed, while the channels are generated as Gaussian complex variable instances with unit variance and zero mean. For every channel instance, the solutions of the SPC [9] and the proposed PAC $\max$ SR are evaluated and compared to the weighted fair solutions of [7], [8]. The exact input parameters employed for the algorithmic solution are presented in Tab. I. For fair comparison, the total power constraint P_{tot} [Watts] is equally distributed amongst the transmit antennas when PACs are considered, hence each antenna can radiate at most P_{tot}/N_t [Watts]. The results are averaged over one hundred channel realizations, while the noise variance is normalized to one for all receivers. The achievable SR is plotted in Fig. 1 with respect to the total transmit power P_{tot} in dBW. Clearly, in a practical PAC scenario, the proposed optimization problem outperforms existing solutions over the whole SNR range. More significantly, the gains of the derived solution are more apparent in the high power region. In the low power noise limited region, interferences are not the issue and the fair solutions perform close to the throughput maximizing solution. On the contrary, in the high power regime, the interference limited fairness solutions saturate in terms of SR performance. For $P_{tot} = 20$ dBW, the $\max$ SR solutions attain gains of more than 30% in terms of SR over the fair approaches. Interestingly, for the same available transmit power, the PAC optimization proposed herein, attains 20% gains over re-scaled to respect the per-antenna constraints $\max$ SR solutions. Finally, it clearly noted in Fig. 1 that the reported gains increase with respect to the transmit power.

A significant issue for the multicast applications is the scaling of the solution versus an increasing number of receivers per multicast. The increasing number of users per group degrades the performance for the weighted fair problems, as shown in [7], [8]. For the case tackled herein, the $\max$ SR solutions are

[2]Such a rescaling is originally proposed in [7], in order to quantify the gains of the proposed approach. As expected, if rescaling is not applied, the SPC solutions will achieve higher rates since the transmitter is designed under less constraints. This performance, however, is attained at the cost of not respecting the per-antenna constraints. This it is not considered in the present work.

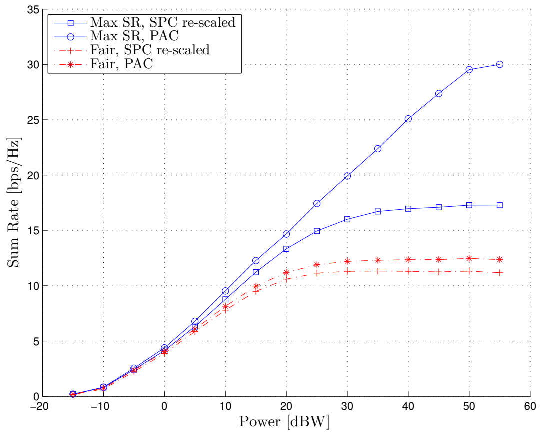

Fig. 1. SR with SPC and PAC versus increasing total power P_{tot} [dBW].

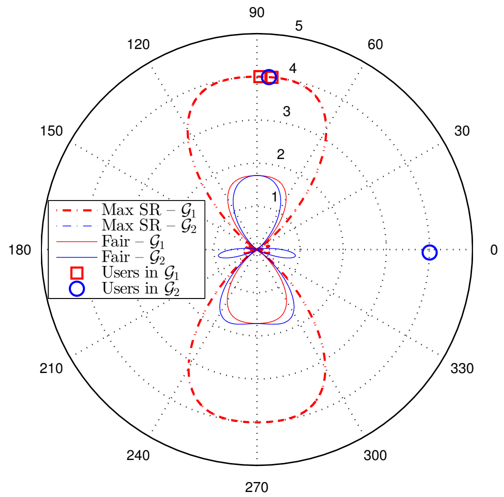

Fig. 3. Antenna radiation patterns of aULA transmitter, optimised for $\max$ SR and fairness (User positions given with colored markers).

compared to the fairness solutions as depicted in Fig. 2 with respect to an increasing ratio of users per group $\rho = N_u/G$. According to these curves, the $\mathcal{SR}$ solution is exhibiting a higher resilience to the increasing number of users per group, compared to the fair solutions. The re-scaled solutions remain suboptimal in terms of sum rate when compared to proposed solution for any user per group ratio.

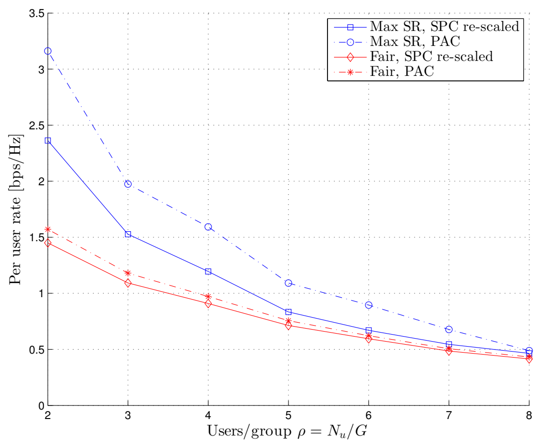

Fig. 2. Sum rate with SPC and PAC versus an increasing ratio of users per group $\rho = N_u/G$.

B. Uniform Linear Arrays

To the end of investigating the sensitivity of the proposed algorithm with respect to the angular separation of co-group users, a uniform linear array (ULA) transmitter is considered. Assuming far-field, line-of-sight conditions, the user channels can be modeled using Vandermonde matrices. Let us consider a ULA with $N_t = 4$ antennas, serving 4 users allocated to 2 distinct groups. The co-group angular separation is $\theta_1 = 5^\circ$ and $\theta_2 = 45^\circ$ for $\mathcal{G}_1$ and $\mathcal{G}_2$ respectively. In Fig. 3, the user positions and the optimized radiation pattern for this is transmitter plotted. The symmetry due to the inherent ambiguity of the ULA is apparent. Clearly, the fair beamforming design optimizes the lobes to provide equal service levels to all users. The three upper users (close to the 90° angle) receive higher power but also receive adjacent group interference. The fourth user, despite being in a more favorable in terms of interference position, is not allocated much power since its performance is constrained by the performance of the almost orthogonal, compromised user. Remembering that the noise level is equal to one and that the beam pattern is plotted in linear scale, all users achieve a SINR equal to 0.6, thus leading to a total SR of 1.2 [bps/Hz]. On the contrary, the $\max$ SR optimization, shuts down the compromised group (i.e. $\mathcal{G}_2$) and allocates the saved power to the well conditioned users of $\mathcal{G}_1$. This way, the system is interference free and each active user attains a higher service level. The achievable SNR is equal to 4 assuming normalized noise, (but only for the two active user of $\mathcal{G}_1$) and leads to a SR of more than 4.6 [bps/Hz]. Consequently, the proposed solution attains a 33% of increase in sum rate for the specific scenario, at the expense of sacrificing service availability to the ill conditioned users.

In Fig. 4, the performance in terms of the SR optimization is investigated versus an increasing angular separation. When co-group users are collocated, i.e. $\theta = 0^\circ$, the highest performance is attained. As the separation increases, the performance is reduced reaching the minimum when users from different groups are placed in the same position, i.e. $\theta = 45^\circ$. The proposed solution outperforms a re-scaled to respect the per-antenna constraints, SPC solution, over the span of the angular separations. Also, the $\max$ SR solution performs equivalently to the fair solution under good channel conditions. However, when the angular separation of co-group users increases, the SR optimization exploits the deteriorating channel conditions and gleans gains of more than 25% over all other solutions.

C. Sum Rate Maximization Paradigm

Towards exhibiting the differences between the weighted fair and the $\max$ SR designs in the multigroup multicasting

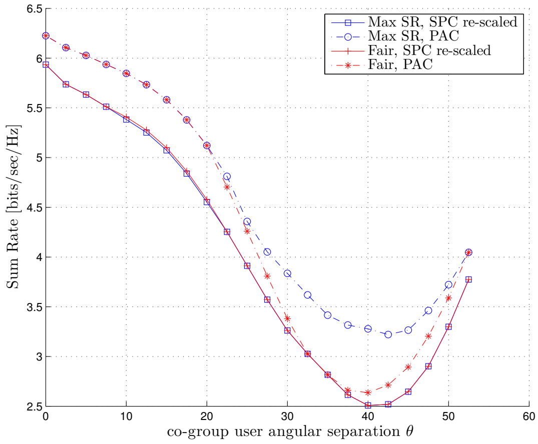

Fig. 4. Achievable sum rates for ULA transmitter with respect to increasing co-group user angular separation.

context, a small scale paradigm is presented. Let there be a ULA transmitter that serves eight users allocated into four groups, as depicted in Fig. 5. The attributes of the specific channel instance depict one possible instance of the system where one group, namely $\mathcal{G}_3$, has users with large angular separation while $\mathcal{G}_4$ has users with similar channels. The rate of each user is plotted in Fig. 5 for the case of a weighted fair optimization (equal weights are assumed) and for the case of a SR maximizing optimization. Considering that each user is constrained by the minimum group rate, the sum rates are given in the legend of the figure. In the weighted fair case, the common rate at which all users will receive data is 0.83 [bps/Hz] leading to a sum rate of 6.64 [bps/Hz]. The minimum SINRs and hence the minimum rates are balanced between the groups since the fair optimization considers equal weights. The SR maximizing optimization, however, reduces the group that contains the compromised users in order to reallocate this power to the well conditioned group and therefore increase the system throughput to 9.9 [bps/Hz]. Consequently, a gain of almost 40% is realized in terms of total system rate. This gain is traded-off by driving users in $\mathcal{G}_3$ to the unavailability region.

V. Conclusions & Future Work

In the present work, optimum linear precoding vectors are derived when independent sets of common information are transmitted by a per-antenna power constrained array of antennas to distinct co-channel sets of users. In this context, a novel sum rate maximization multigroup multicast problem under PACs is formulated. A detailed solution for this elaborate problem is presented based on the well established methods of semidefinite relaxation, Gaussian randomization and sub-gradient power optimization. The performance of the SR maximizing multigroup multicast optimization is examined under various system parameters and important insights on the system design are gained.

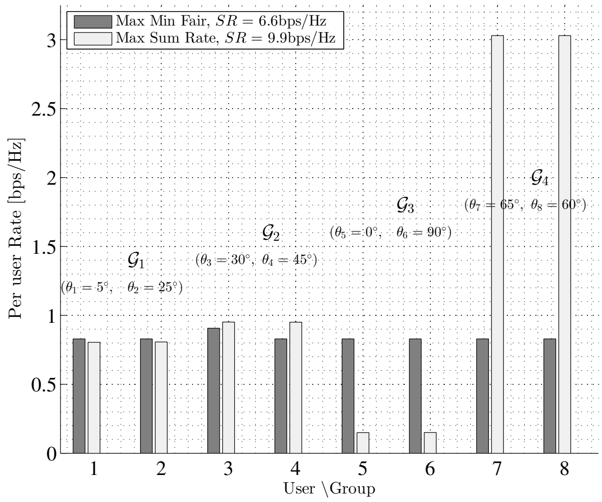

Fig. 5. Achievable per user rates of multigroup multicast users under weighted fair and max sum rate optimization.

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