# Optimal mix of funded and unfunded pension systems:

## The case of Luxembourg

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joint work with
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& Bruno LOVAT (University of Lorraine)

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# Outline

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  - Parameter adjustment in the Pay-as-you-go system
  - and/or development complementary systems (mix of funded and unfunded system)

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- a theoretical model based on a diversification principle

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Finite mixture model (Daniel S. Nagin (Carnegie Mellon University))

- mixture : population composed of a mixture of unobserved groups
- finite : sums across a finite number of groups

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# The finite mixture model

If the data are (censored) normally distributed

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$$L = \frac{1}{\sigma} \prod_{i=1}^{N} \sum_{j=1}^{r} \pi_j \prod_{t=1}^{T} \phi \left( \frac{y_{i_t} - \beta^j x_{i_t}}{\sigma} \right). \tag{1}$$

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**Software:**

SAS-based Proc Traj procedure
by Bobby L. Jones (Carnegie Mellon University).

$\Rightarrow$ quasi-Newton procedure maximum research routine

# Outline



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# Proc Traj procedure

20 years of work for workers beginning their carrier between 1982 and 1987

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DATA TEST;
      INPUT ID O1-O20 T1-T20;
      CARDS;
data
RUN;
```

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```
DATA TEST;
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RUN;

PROC TRAJ DATA=TEST OUTPLOT=OP OUTSTAT=OS OUT=OF
OUTEST=OE ITDETAIL;
      ID ID; VAR O1-O20; INDEP T1-T20;
      MODEL CNORM; MAX 8000; NGROUPS 6; ORDER 4 4 4 4 4 4;
RUN;
```

# Results for 9 groups

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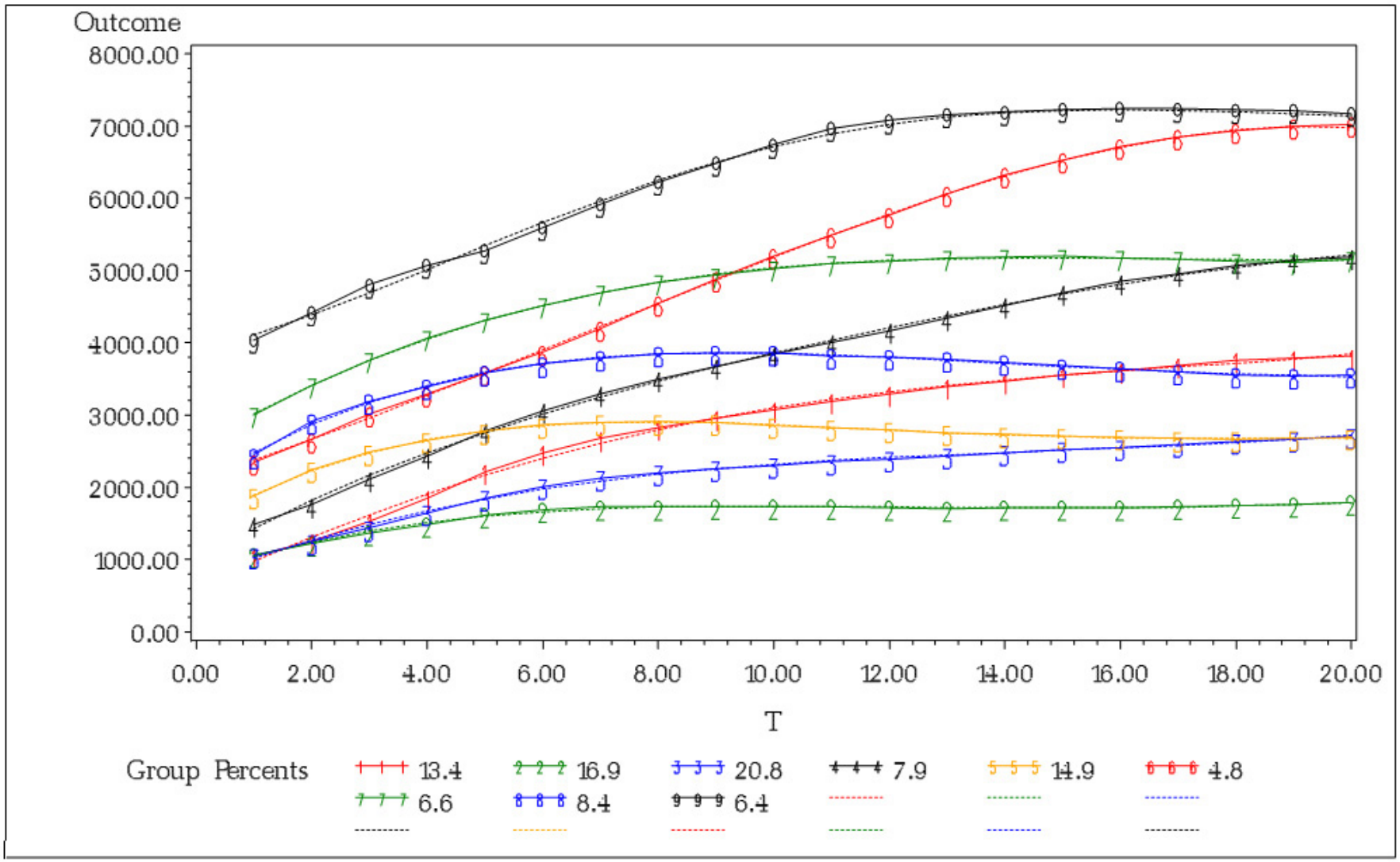

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# Working hypotheses

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- Hypothesis 2. Let $d$ denotes the intergenerational demographical rate, i.e. at time $t$, if $N_0$ denotes the number of people beginning to work and $N_t$ the number of people working for $t$ years, then

$$N_t = \frac{N_0}{(1+d)^t}.$$

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$$\tau_1 = \frac{S_0 + ... + \frac{S_T}{(1+d)^T}}{\frac{k}{(1+d)^{T+1}} P_{T+1} + ... + \frac{k}{(1+d)^{T+T^*}} P_{T+T^*}}.$$

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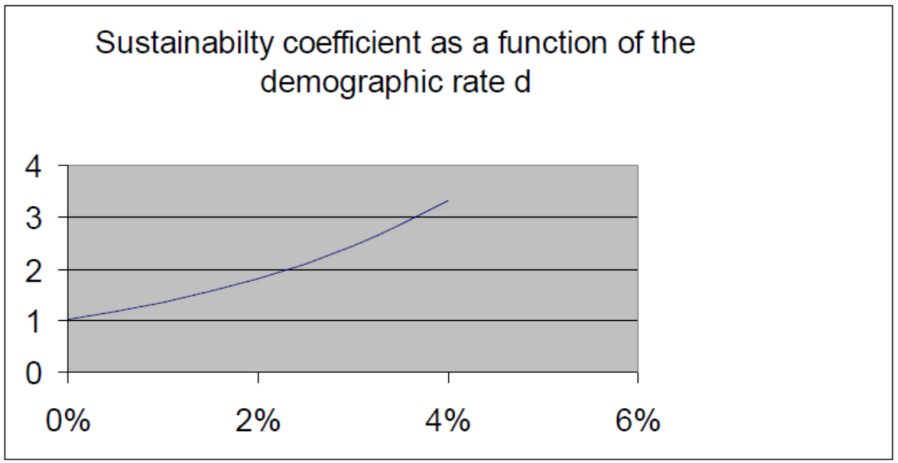

# Analysis of the current pension system

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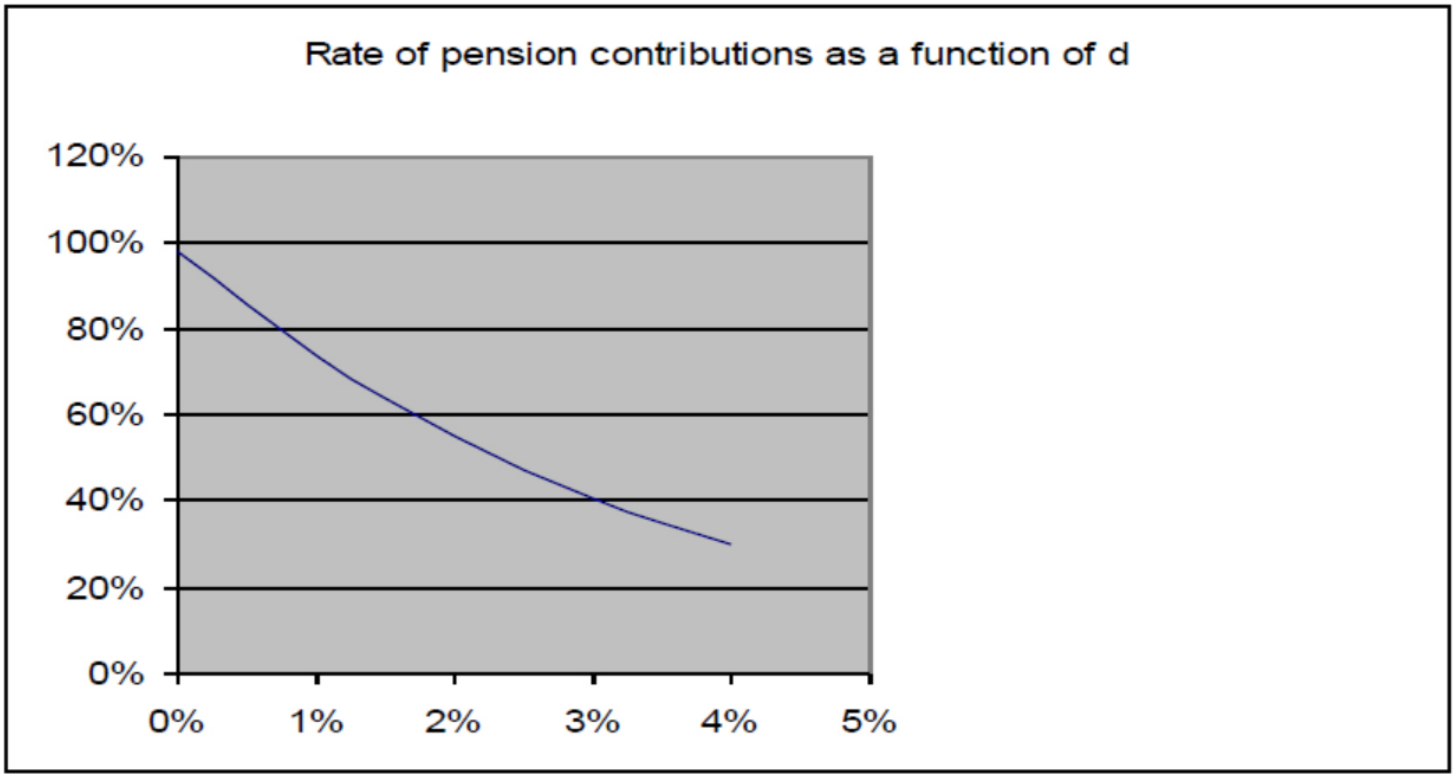
Rate of necessary pension contributions to keep the system sustainable at the long run:



# Working hypotheses (2)

- Hypotheses 3. We suppose that every individual of group number $j$ invests every year of his activity a fixed amount $a_j$ which generates savings according to the market rate $i$.

# Sustainability coefficient of the funded system

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$$\tau_2 = \frac{S_j}{a_j(i-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}.$$

# Systemic risk

Modelisation based on portfolio type risk management principles

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| | Market risk | Demographic risk |
|---|---|---|
| Repartition | Negligeable | Extreme |
| Capitalization | Extreme | Negligeable |

# Global sustainability coefficient

$$\tau = x\tau_1 + (1 - x)\tau_2$$

is the number of euros necessary to pay 1 euro for the pension.

Here $x$ euros come from the PAYG system and $1 - x$ euros from capitalization.

# Global sustainability coefficient

$$\tau = x\tau_1 + (1-x)\tau_2$$

is the number of euros necessary to pay 1 euro for the pension.

Here $x$ euros come from the PAYG system and $1-x$ euros from capitalization.

We want to limit the risk of the hybrid system without reducing the pension and in the same time minimize the capitalization effort.

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# Gain of sustainability and optimal saving amount

$$G(x) = \frac{var(\tau_1) - var[\tau(x)]}{var(\tau_1)}$$

measures the gain of sustainability of the mixed system with respect of the PAYG system.

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We suppose that the utility function $U = U(a)$ of an active worker is decreasing in $a$.

# Gain of sustainability and optimal saving amount

Theorem. The value $x = x^*$ for which the utility function $U$ attains its maximum under the sustainability constraint

$$G(x) \leq G^*$$

is given by $x^* = 1 - G^*$.

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Theorem. The value $x = x^*$ for which the utility function $U$ attains its maximum under the sustainability constraint

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is given by $x^* = 1 - G^*$.

Moreover the individual needs a constant annual saving amount

$$a^* = \sqrt{\frac{G^* K}{var(\tau 1)(1 - G^*)}},$$

where $K = \frac{S_j}{a_j(i-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}$ depends on the salary trajectory.

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# Example

An individual worker wants to divide by 2 the variability of his PAYG sustainability constraint needs to save annually at least the following amount (depending on his salary evolution subgroup):

| Group | G1 | G2 | G3 | G4 | G5 | G6 | G7 | G8 | G9 |
|---|---|---|---|---|---|---|---|---|---|
| Annuity | 4466€ | 713€ | 1448€ | 5231€ | 220€ | 6364€ | 2809€ | 743€ | 3140€ |

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| Curve 1 | Curve 2 | Curve 3 | Curve 4 | Curve 5 |
|---|---|---|---|---|
| $\lambda_1 = 3.07\%$ | $\lambda_2 = 0.96\%$ | $\lambda_3 = 1.45\%$ | $\lambda_4 = 2.82\%$ | $\lambda_5 = 0.19\%$ |
| Curve 6 | Curve 7 | Curve 8 | Curve 9 | |
| $\lambda_6 = 2.58\%$ | $\lambda_7 = 1.28\%$ | $\lambda_8 = 0.48\%$ | $\lambda_9 = 1.09\%$ | |

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We propose the following model:

$$y_{i_t} = \left(\beta_0^j + \sum_{l=1}^{L} \alpha_{0l}^j x_l + \gamma_0^j z_{i_t}\right) + \left(\beta_1^j + \sum_{l=1}^{L} \alpha_{1l}^j x_l + \gamma_1^j z_{i_t}\right) Age_{i_t}$$
$$+ \left(\beta_2^j + \sum_{l=1}^{L} \alpha_{2l}^j x_l + \gamma_2^j z_{i_t}\right) Age_{i_t}^2 + \left(\beta_3^j + \sum_{l=1}^{L} \alpha_{3l}^j x_l + \gamma_3^j z_{i_t}\right) Age_{i_t}^3$$
$$+ \left(\beta_4^j + \sum_{l=1}^{L} \alpha_{4l}^j x_l + \gamma_4^j z_{i_t}\right) Age_{i_t}^4 + \varepsilon_{i_t},$$

where $\varepsilon_{i_t} \sim \mathcal{N}(0, \sigma)$, $\sigma$ being a constant standard deviation.

# The new Database

**The data : second dataset** Salaries of all workers in Luxembourg which began to work in Luxembourg between 1980 and 1990 at an age less than 30 years.

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- Salary trajectories depending on socioeconomic and macroeconomic covariates.
- More realistic hypotheses for the economic modeling (time dependent demographical and market rates).
- More precise use of the group trajectories.

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# Bibliography


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