MODEL REDUCTION FOR FRACTURE: ALGEBRAIC REDUCTION AND HOMOGENIZATION

Ahmad Akbari[1], **Pierre Kerfriden**[1], **Olivier Goury**[1]
Stéphane P.A. Bordas[1,2]

[1] Cardiff University School of Engineering, Cardiff, UK
[2]Université du Luxembourg, Research Unit in Engineering, Computational Mechanics Group, Luxembourg `stephane.bordas@uni.lu,bordasS@cardiff.ac.uk`

Key words: *Multiscale, homogenisation, adaptivity, cohesive crack, polycrystalline materials.*

In this work, we discuss two classes of methods to reduce the complexity of (multi scale) fracture simulations. In a first part, we discuss algebraic model reduction. We show that algebraic model reduction such as the proper orthogonal decomposition cannot be used directly because of the lack of correlation introduced by the damage or cracks. We demonstrate the use of proper orthogonal decompositions by subdomains as a candidate to reduce computational expenses in non-linear fracture simulations whilst controlling the error level.

We then consider algebraic model reduction, namely the proper orthogonal decomposition(POD) to drastically reduce the computational time associated with computing the response of representative volume elements (RVEs) used in homogenization, e.g. by the FE2 method. The snapshots are obtained by solving the RVE boundary value problem for various loading paths. To speed-up the computations, system approximation through the discrete empirical interpolation (DEIM) is used and allows the evaluation of the internal forces for only a small subset of the elements making the RVE structure.

In a second part, we propose an adaptive hybrid multiscale method for modelling fracture in a heterogeneous material that is composed of orthotropic grains with cohesive interfaces between the grains. Instead of a direct solver, FE^2 method [1] based on homogenisation is employed in order to compute the effective behaviour of the heterogeneous microscopic material on the coarser scale. At this scale the modelling error due to the homogenisation is still low [3]. The coarse scale is discretized with unstructured triangular finite elements, and adaptive mesh refinement is used to control the discretizsation error. While the mesh refinement keeps the discretisation error with in a certain range, the modelling error increases due to the fact that by refining the coarse elements, the scale separation assumption which is a key issue for homogenisation may no longer be fulfilled[4]. Whereas the modelling error is inversely proportional to the size of the coarse elements, a critical element size can be found that corresponds to the critical value of the modelling error. A critical zone emerges when the size of a coarse element reaches the critical size, or if the underlying representative volume element of the microstructure loses stability due to localisation (lack of scale separation). Thereafter, a zoom-in process is triggered that replaces the corresponding coarse elements of the critical zone with high resolution microscale mesh to which it glues the coarse scale mesh through a strong coupling technique using Lagrange multipliers [5]. The high resolution region can gradually be extended to include the newly emerging critical zones. A local arc-length technique is adopted to trace the highly non-linear curve of the global load-displacement by controlling the opening of microscopic cohesive cracks in the fully resolved regions.

The proposed adaptive multiscale method allows us to introduce progressive discrete micro cracks at the macroscale. The unstructured mesh enables us to model problems with non-regular shapes, and the arc-length method, defined over multiple scales, allows the regularisation of softening problems that are treated in quasi-statics. We exercise this method on the simulation of polycrystalline fracture, where each grain is considered orthotropic and compare results to direct numerical simulation.

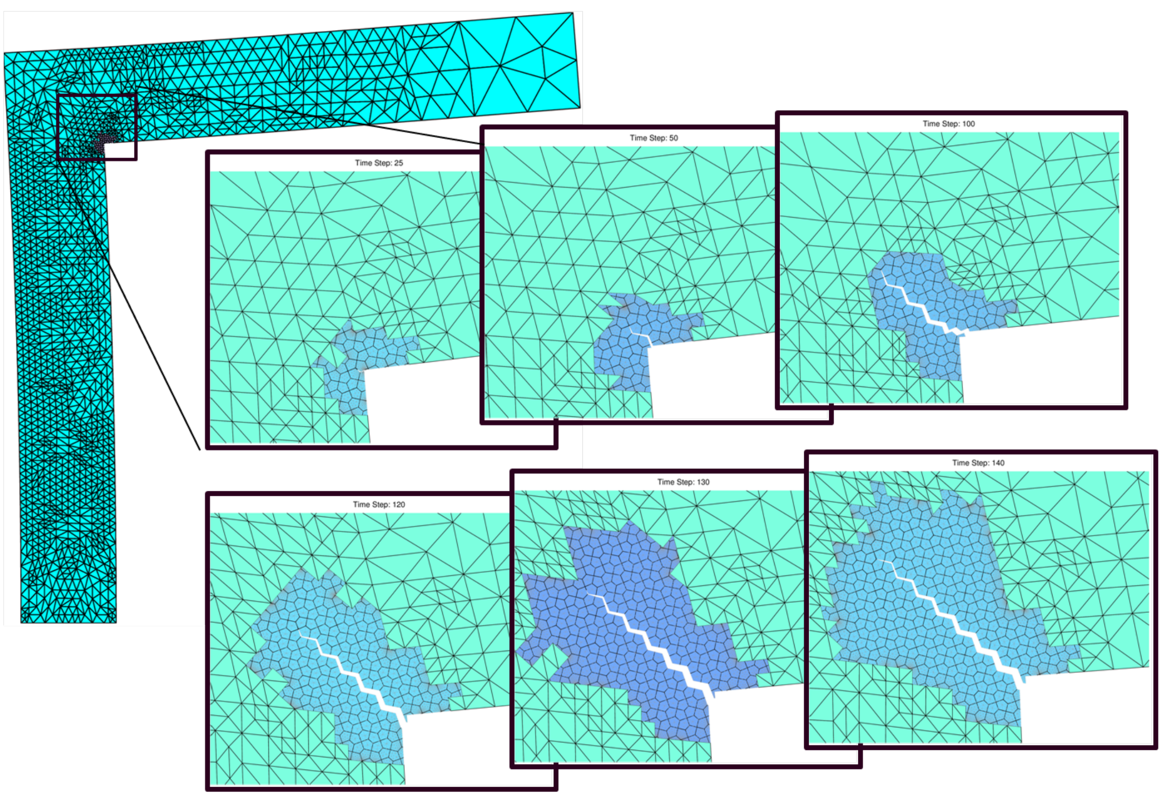

Figure 1: Adaptive multiscale modelling of fracture

REFERENCES

[1] F. Feyel and J. Chaboche. FE^2 multiscale approach for modelling the elastoviscoplastic behaviour of long fibre SiC/Ti composite materials. *Comp. Meth. Appl. Mech. Engrg.*, **183**: 309–330, 2000.

[2] P. Kerfriden, O. Allix, and P. Gosselet. A threescale domain decomposition method for the 3D analysis of debonding in laminates. *Computational Mechanics*, **44** (3):343–362, 2009.

[3] F. Larsson and K. Runesson. On two-scale adaptive FE analysis of micro-heterogeneous media with seamless scale-bridging. *Comp. Meth. Appl. Mech. Engrg.*, **200**(37-40):2662–2674, 2011.

[4] A. Romkes, J. T. Oden, and K. Vemaganti. Multi-scale goal-oriented adaptive modeling of random heterogeneous materials. *Mechanics of Materials*, **38** (8-10): 859–872, 2006.

[5] O. Lloberas-Valls, D. J. Rixen, A. Simone, and L. J. Sluys. Multiscale domain decomposition analysis of quasi-brittle heterogeneous materials. *Int. J. Num. Meth. Engrg*, **89**: 1337–1366, 2012.

[6] F. J. Vernerey and M. Kabiri. An adaptive concurrent multiscale method for microstructured elastic solids, *Comp. Meth. Appl. Mech. Engrg.*, **241-244**: 52–64, 2012.