

# Multiscale modelling of fracture...

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**Saarland University, 20140131**  
**Workshop Multiscale Simulations**



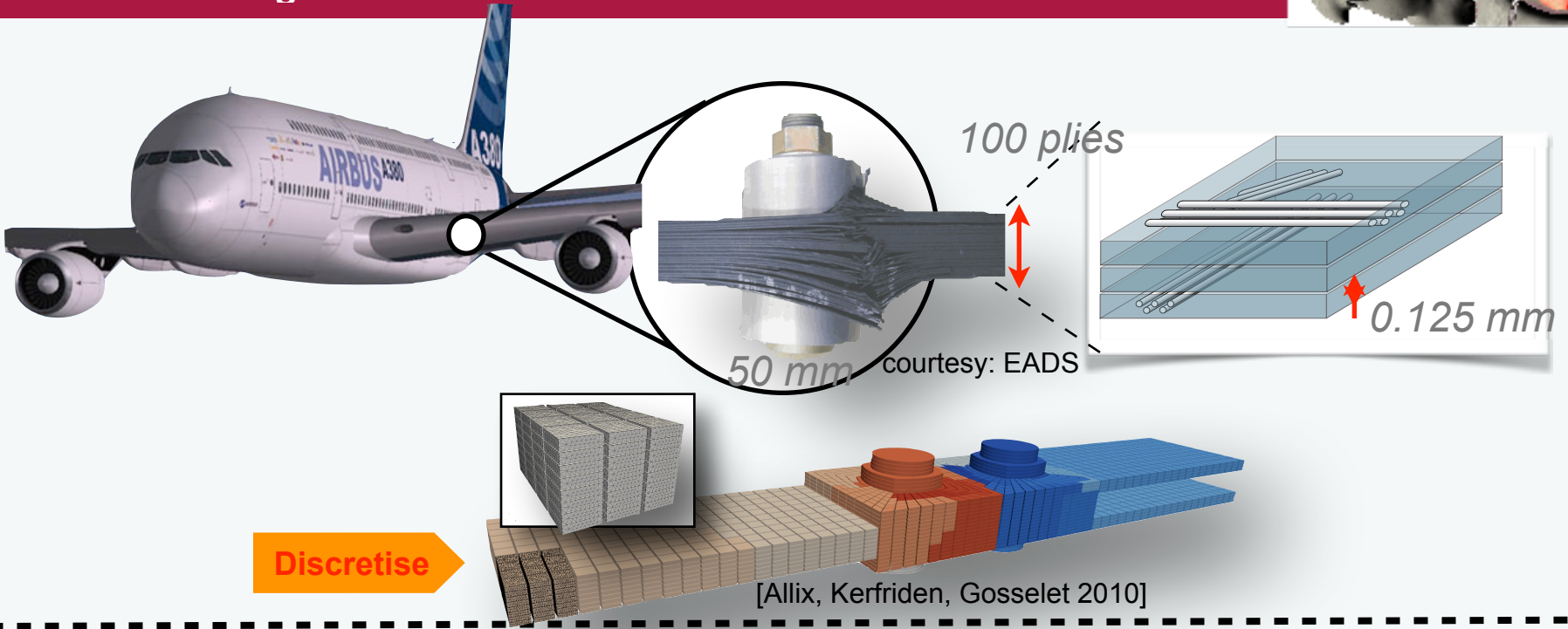
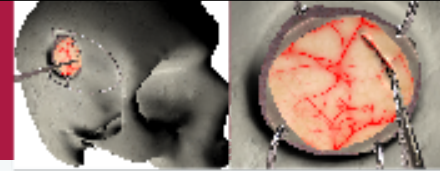
the group...  
November 2012

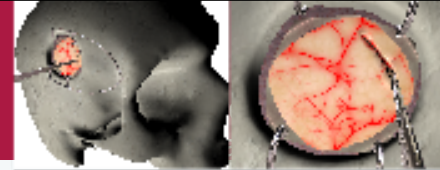


thank you

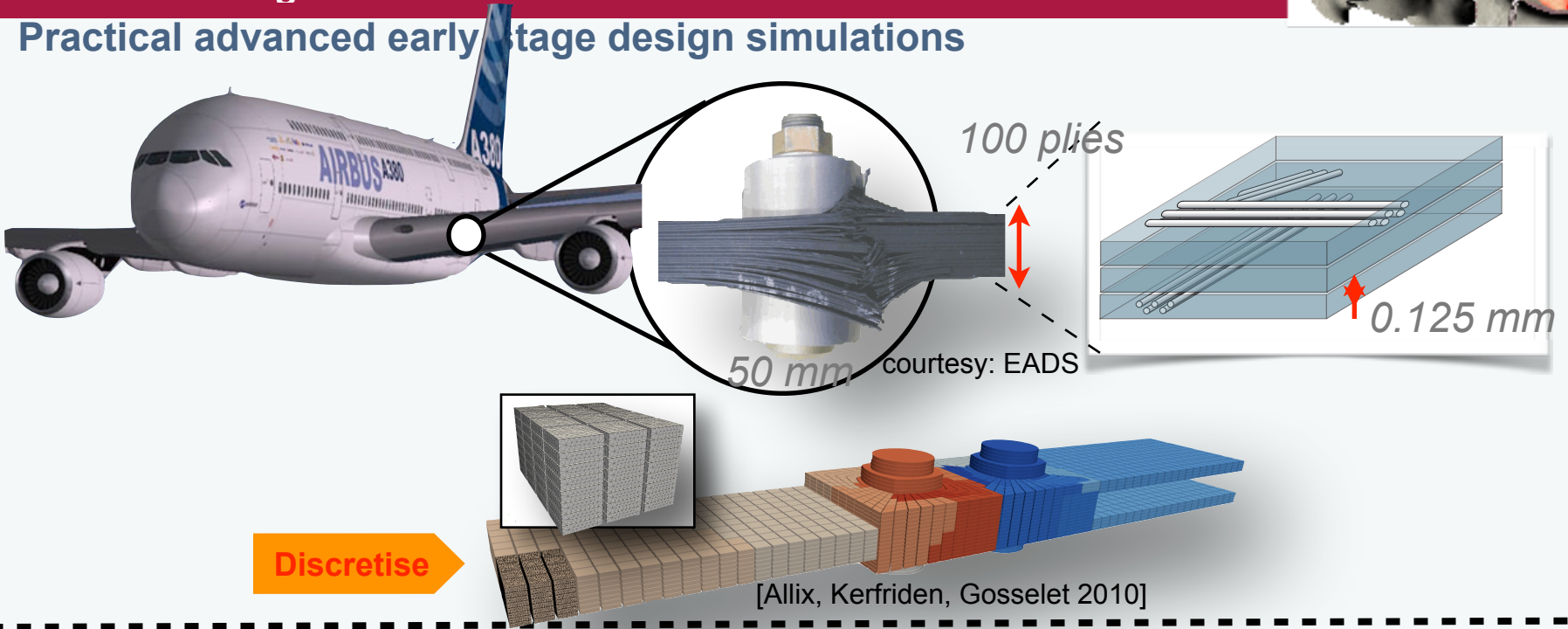
# Research directions

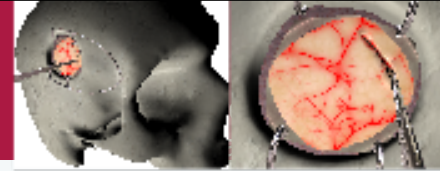
- Model reduction methods for fracture/cutting
  - homogenisation (FE<sup>2</sup>, ...)
  - algebraic model reduction (e.g. POD)
- Advanced discretization methods
  - XFEM
  - Meshfree
  - IGA (FEM & BEM)



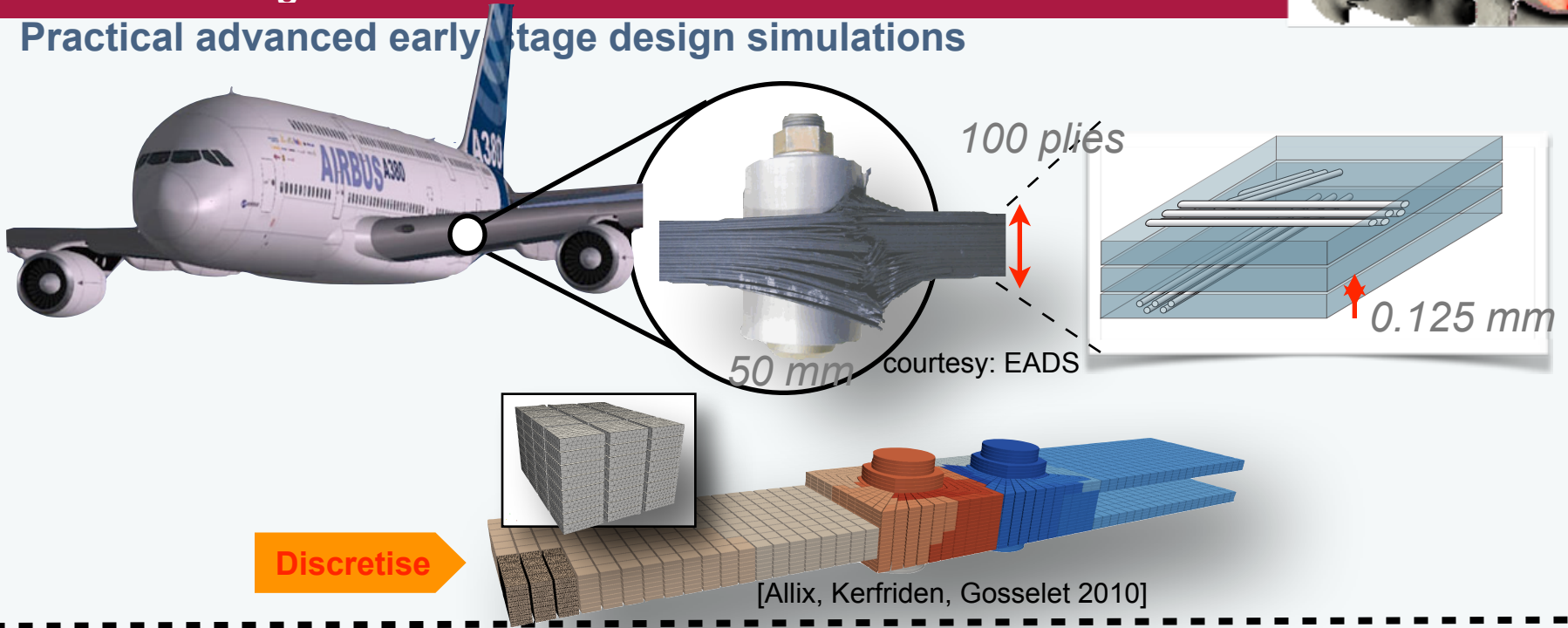


## Practical advanced early stage design simulations

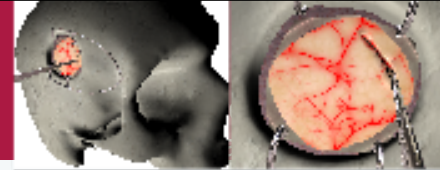




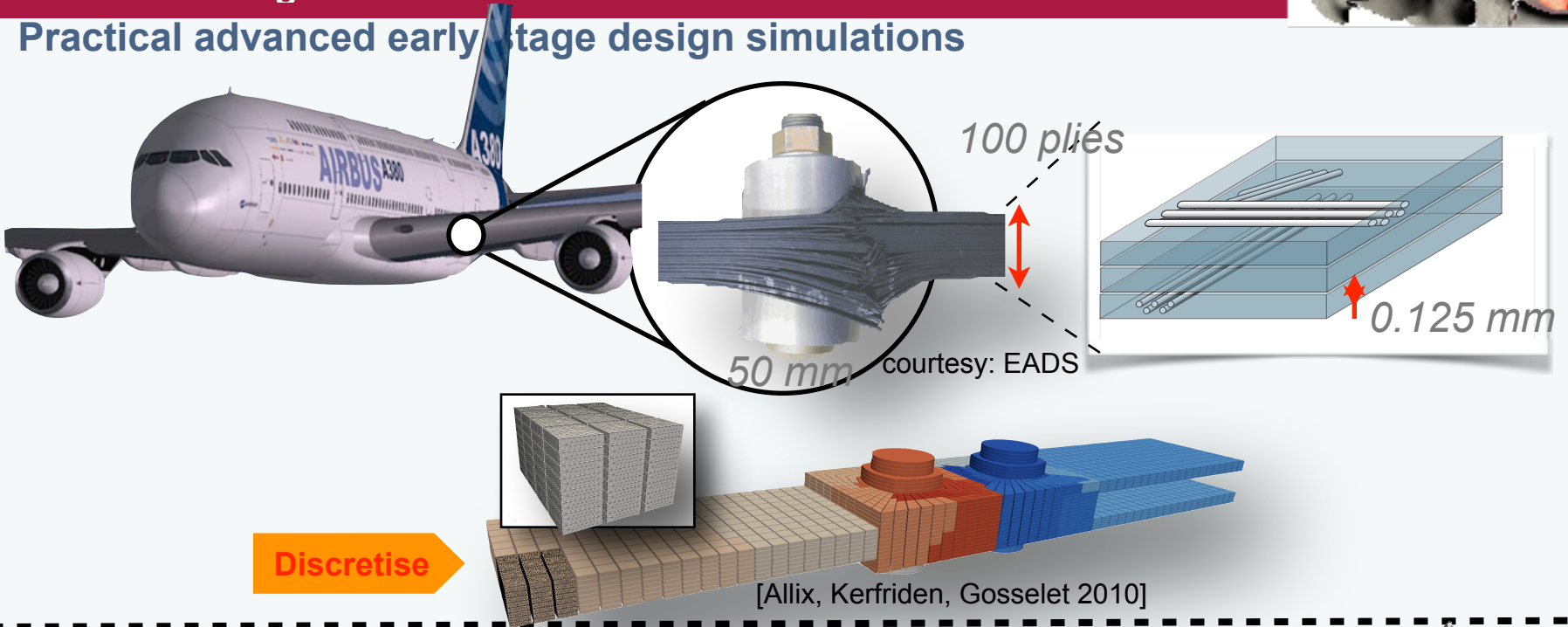
## Practical advanced early stage design simulations



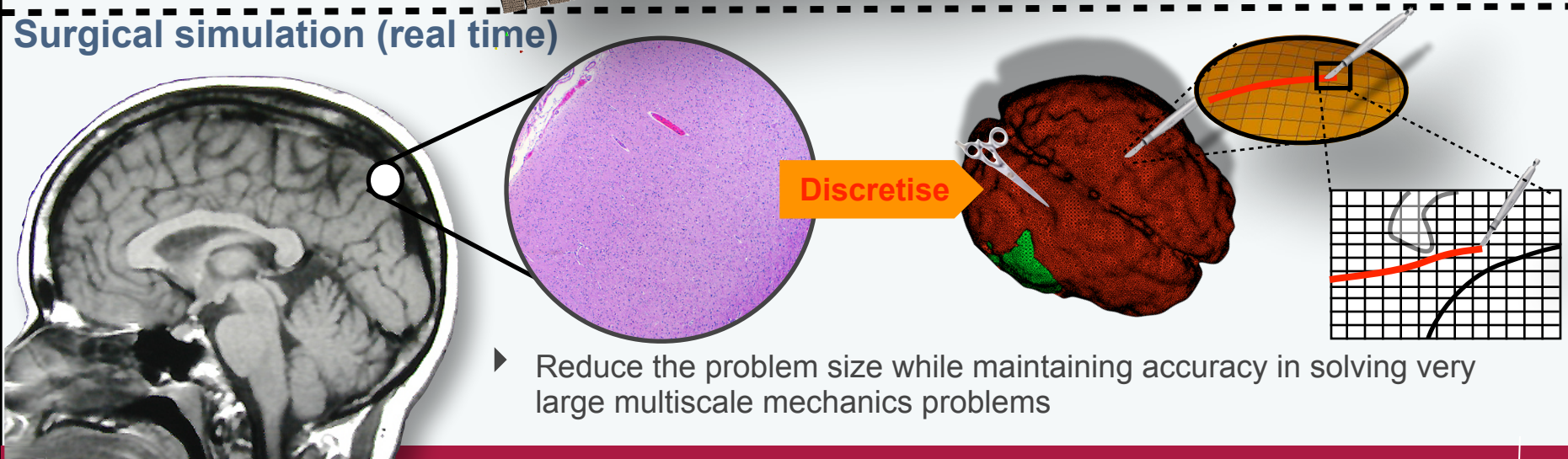
## Surgical simulation (real time)



## Practical advanced early stage design simulations



## Surgical simulation (real time)



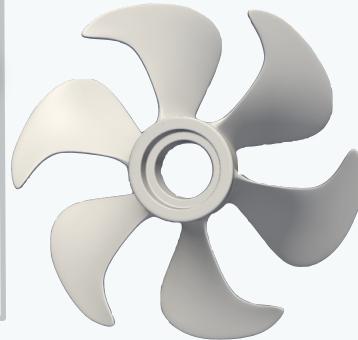
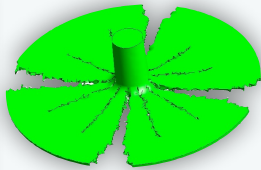
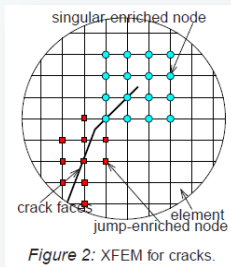
## Advanced discretisations to alleviate the mesh burden

eXtended  
FEM/BEM

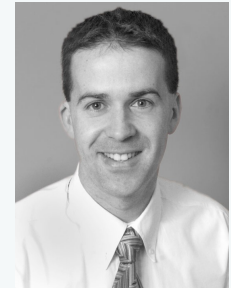
Meshless

CAD /  
Analysis

Element  
technology



ITN  
INSIST



Timon Rabczuk  
Weimar

Isogeometric  
Analysis

Boundary  
Elements

Ambient  
Space FEM

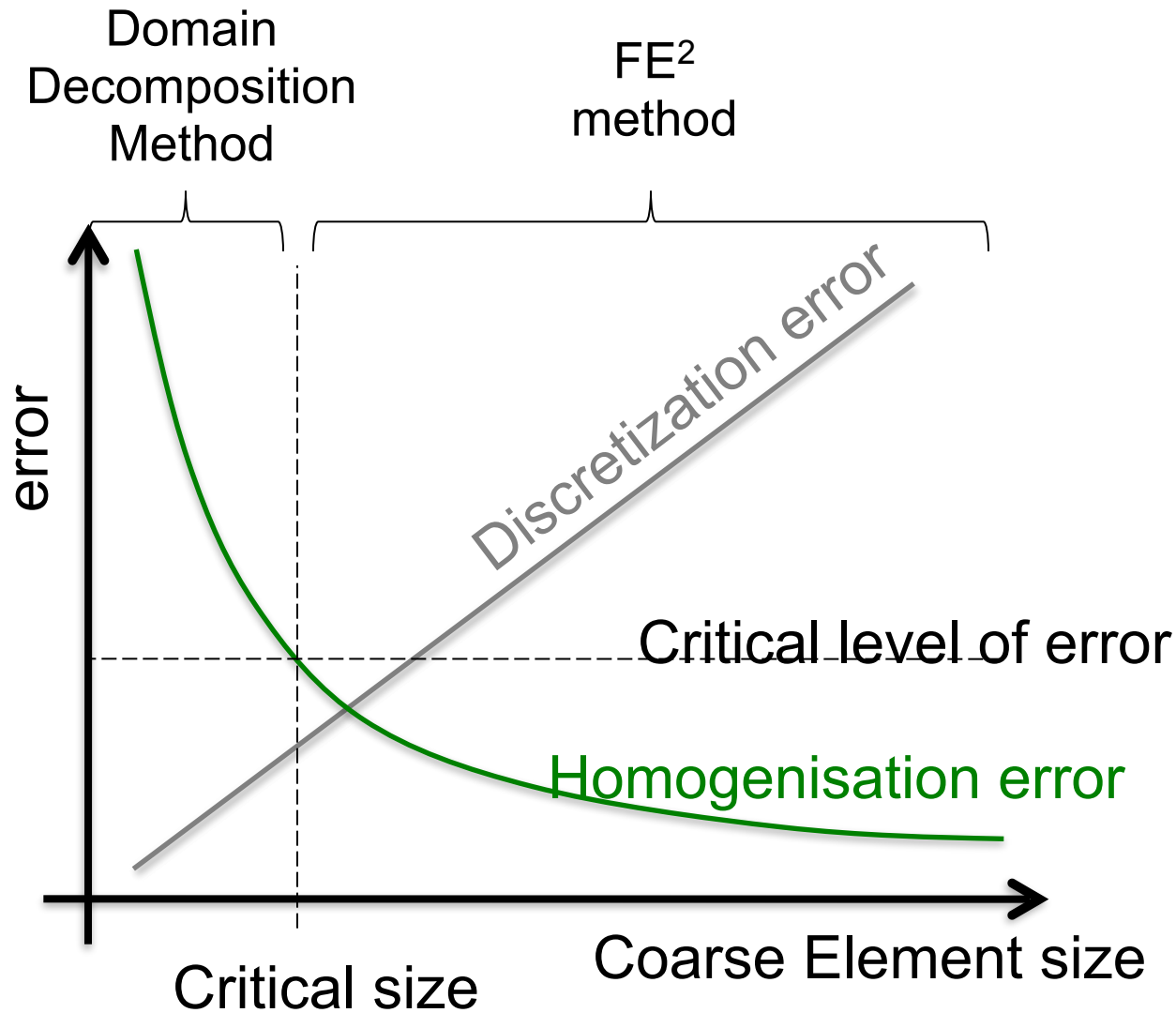
Isogeometric Boundary Element  
Analysis



# Outline

- Introduction: Multiscale methods for Fracture
- Adaptive multiscale method
  - Strategy
  - Fine scale problem
  - Coarse scale problem
    - FE<sup>2</sup> method
    - Adaptive mesh refinement
  - Coupling fine and coarse discretisations
  - Results
    - L-shape problem
    - Notched bar under Uni-axial tension

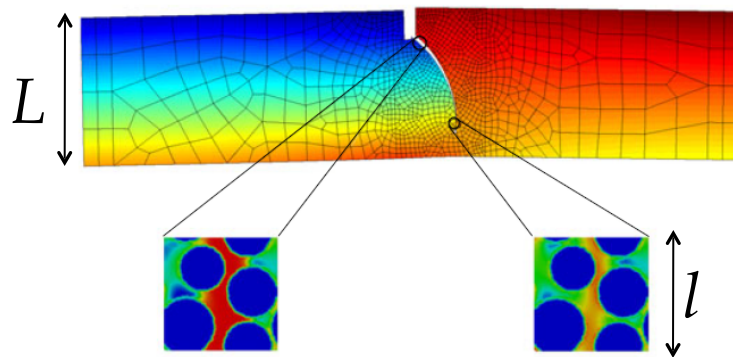
# Error control in multiscale modelling



# Multiscale methods for Fracture

## ■ Non-concurrent

Damage zone is modelled by a macroscopic cohesive crack that homogenises the failure zone.

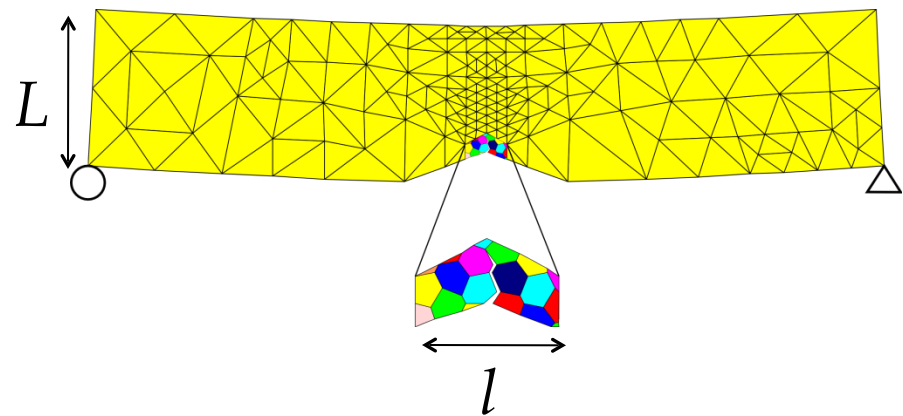


V.P. Nguyen 2012

$$L/l \gg 1$$

## ■ Concurrent

Damage zone is modelled directly at the microscale and coupled to the coarse scale.

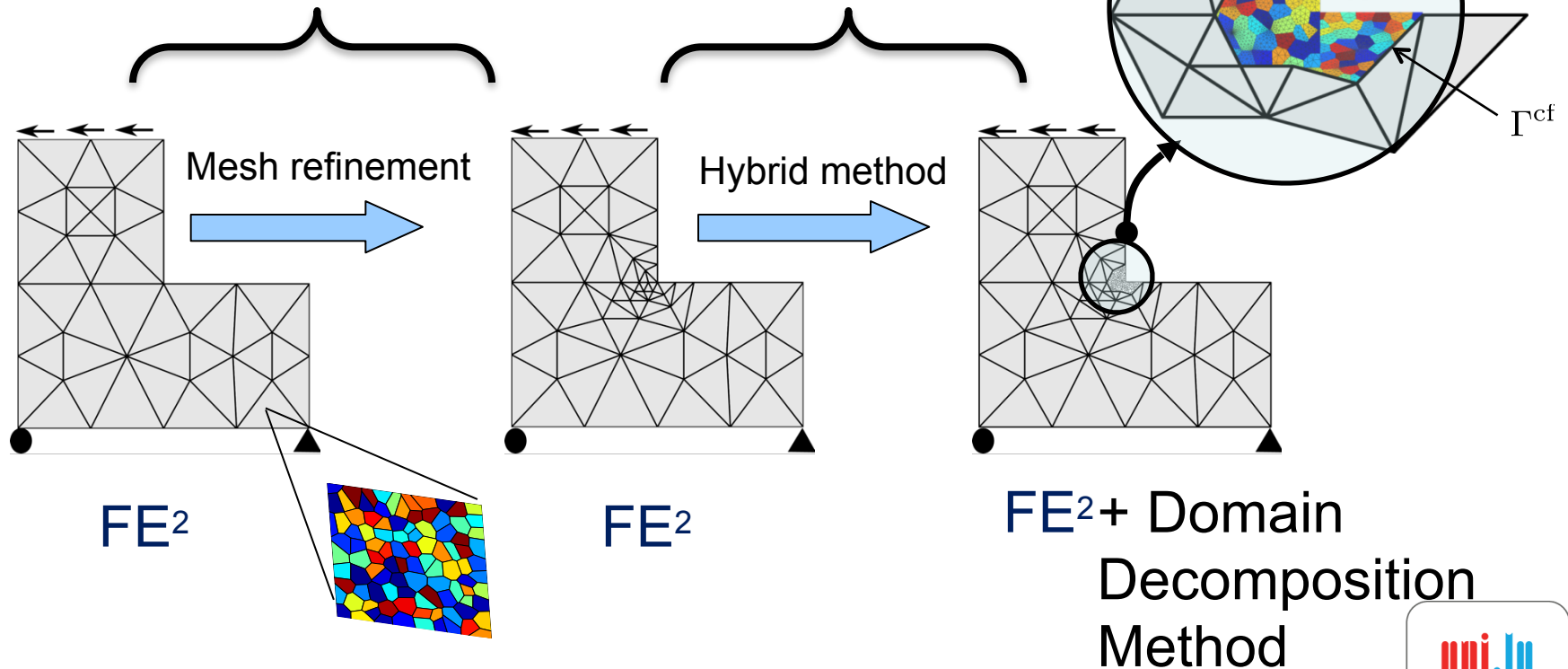


$$L/l > 1$$

# Adaptive multiscale method: A Concurrent approach

➤ Strategy:

- control the coarse scale discretization error
- control the modelling error



# Fine Scale: micro-structure

➤ Microscale problem:

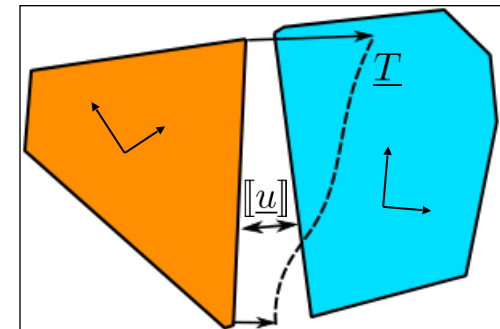
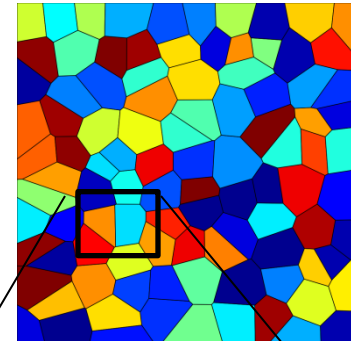
$$\int_{\Omega/\Gamma_c} \boldsymbol{\sigma}(\mathbf{u}) : \delta \boldsymbol{\varepsilon} \, d\Omega + \int_{\Gamma_c} \mathbf{T} \cdot [[\delta \mathbf{u}]] \, d\Omega = \int_{\partial\Omega} \mathbf{f} \cdot \delta \mathbf{u} \, d\Gamma$$

- Orthotropic grains

$$\forall \mathbf{x} \in \Omega/\Gamma_c, \quad \boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon}$$

- Cohesive interface

$$\forall \mathbf{x} \in \Gamma_c, \quad \mathbf{T}|_t = T \left( ([\mathbf{u}]|_T)_{T \leq t} \right)$$



# Coarse Scale

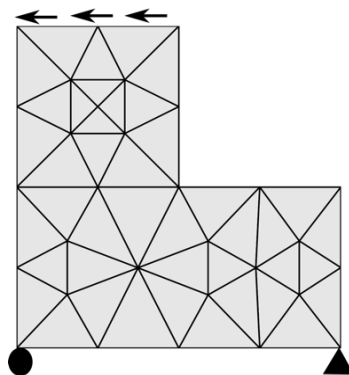
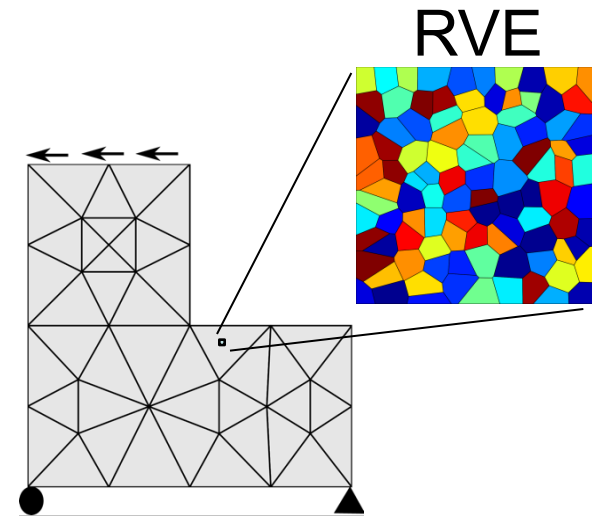
➤ Macroscale problem:

- FE<sup>2</sup> Method

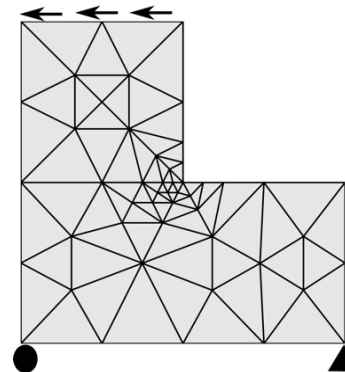
Based on averaging theorem  
(computational homogenisation)

- Adaptive mesh refinement

Error estimation by Zienkiewicz-Zhu-type recovery technique

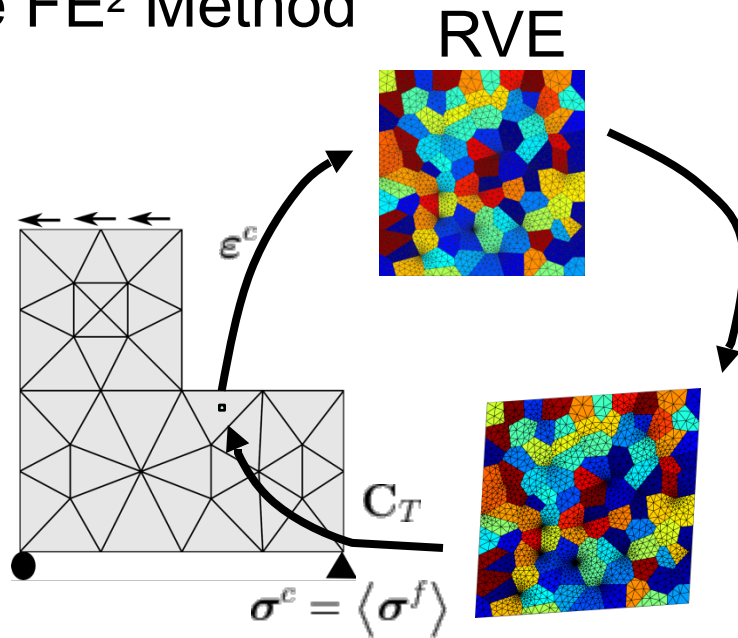


Mesh refinement

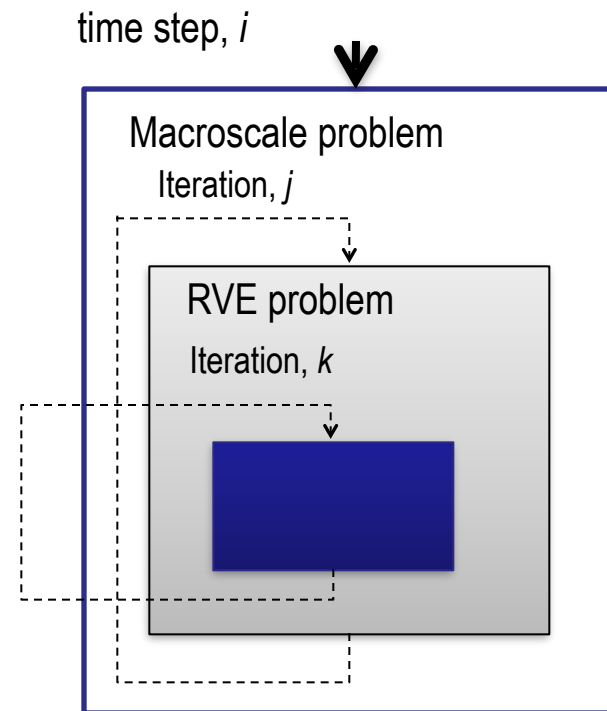


# Coarse Scale: FE2

- The FE<sup>2</sup> Method



❖ Shortcoming of the FE<sup>2</sup> Method :



Lack of scale separation  
RVE cannot be found in the **softening regime**

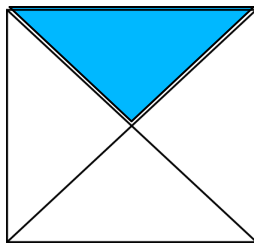
# Coarse Scale: Adaptive mesh refinement

## ➤ Coarse scale Adaptive mesh refinement

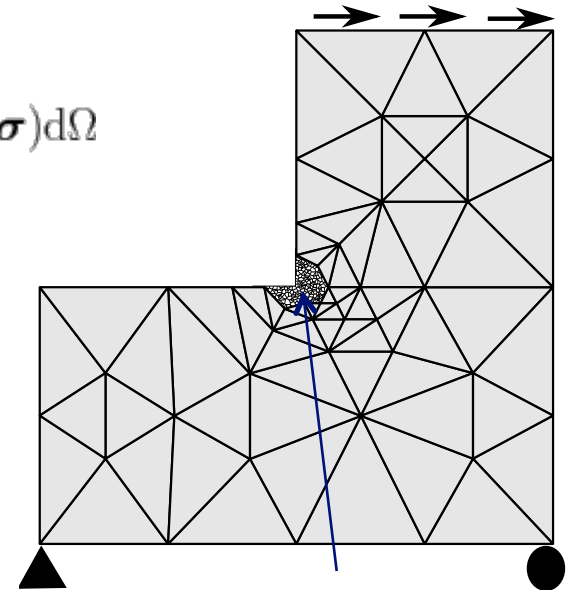
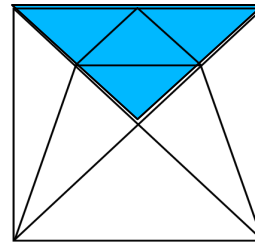
- Error estimation by Zienkiewicz-Zhu-type recovery technique

$$\|e\| = \int_{\Omega_c} (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}) : \left( \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}} \Big|_{\mathbf{u}^c} \right)^{-1} : (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}) d\Omega$$

Element to refine



Refined mesh



Error due to the discretisation of  $\Omega^f$  neglected

- Convergence criterion:  $\frac{\|e\|}{\|\boldsymbol{\sigma}\|} < Tol$

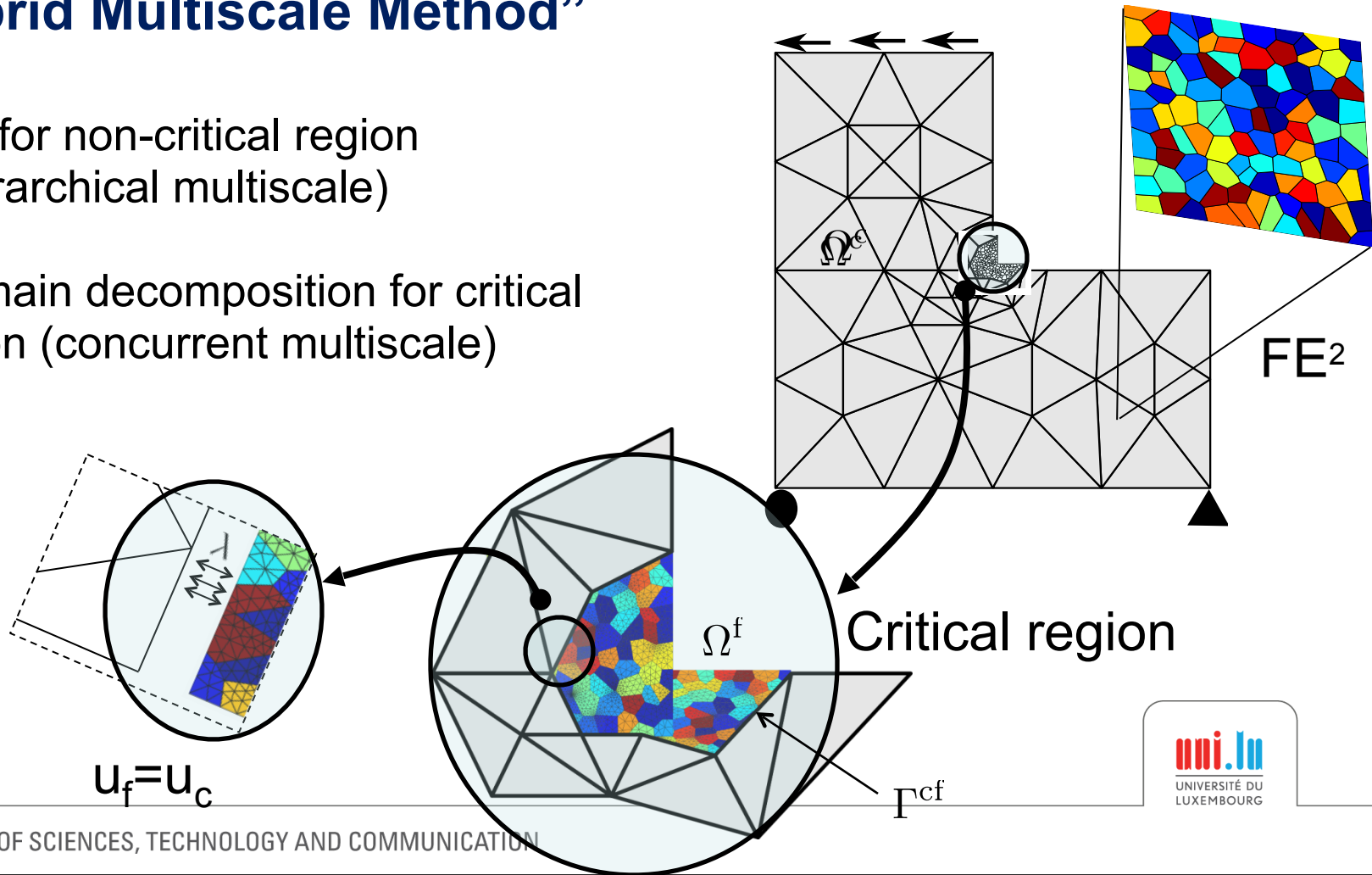


# Fine-Coarse scales Coupling

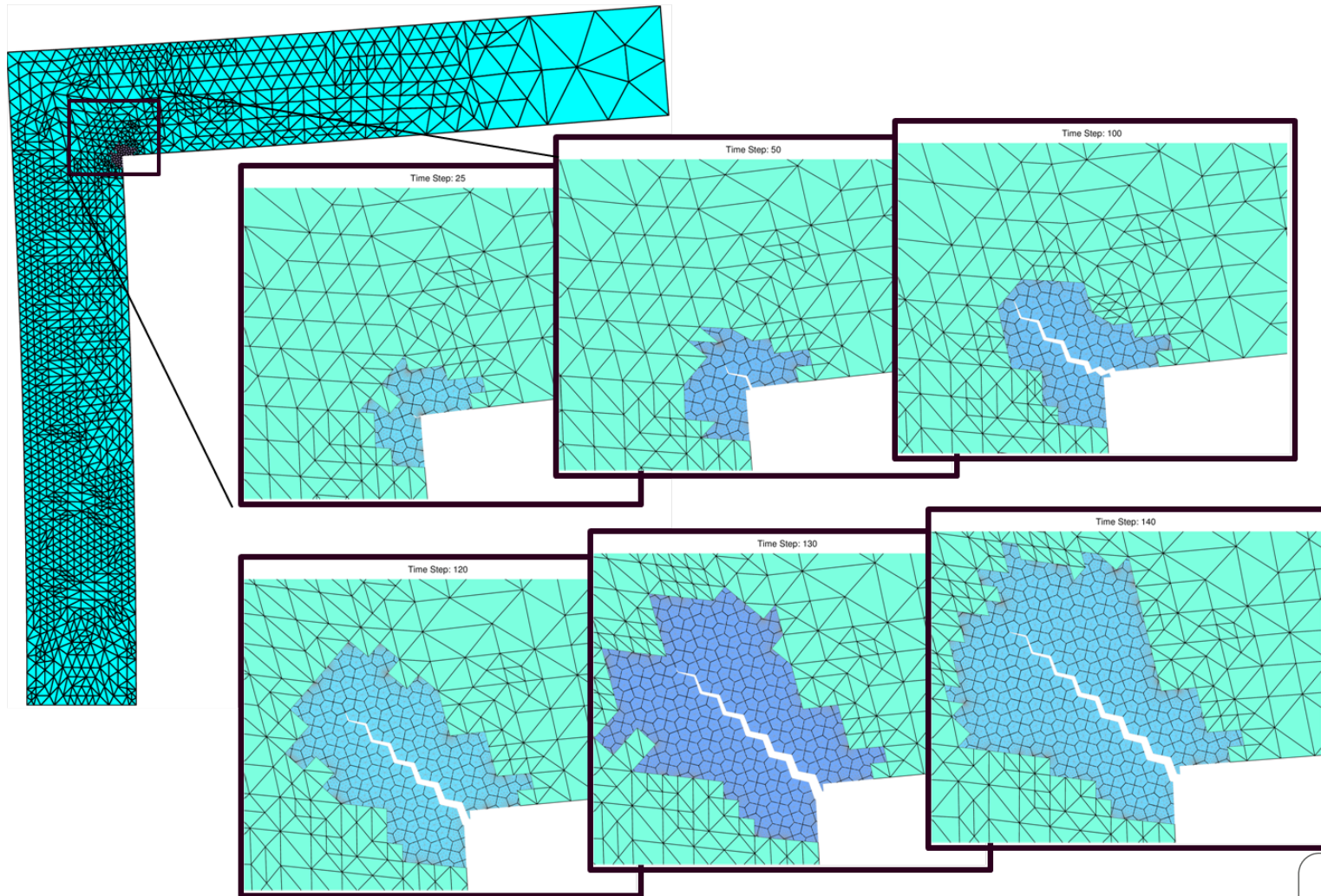
What is the solution for the FE<sup>2</sup> shortcoming:

“Hybrid Multiscale Method”

- FE<sup>2</sup> for non-critical region (hierarchical multiscale)
- Domain decomposition for critical region (concurrent multiscale)

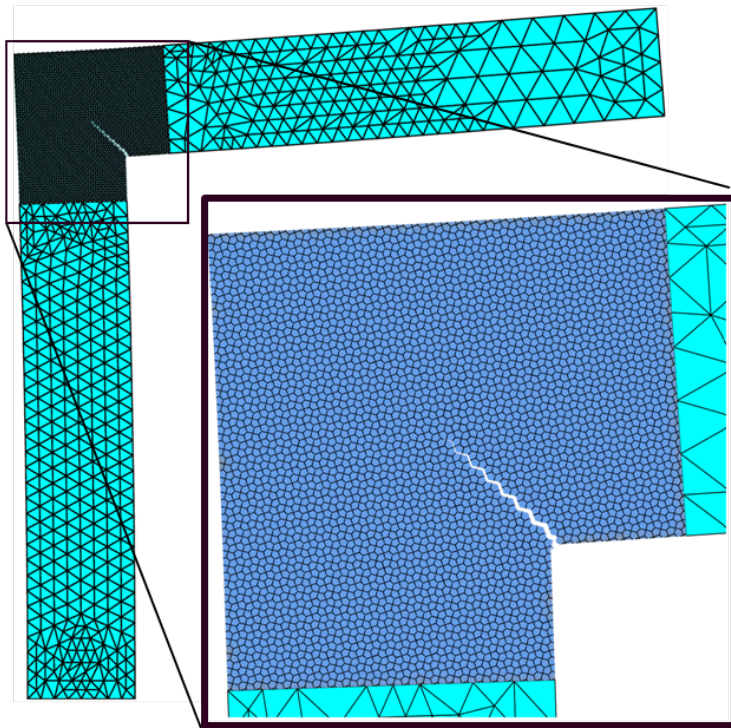


# Results: L-shape

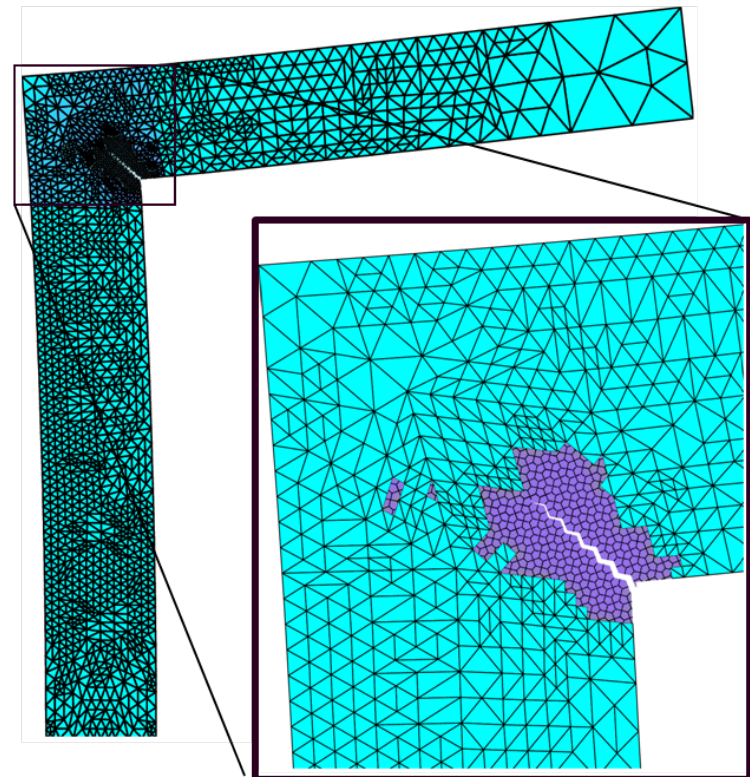


# Results: L-shape

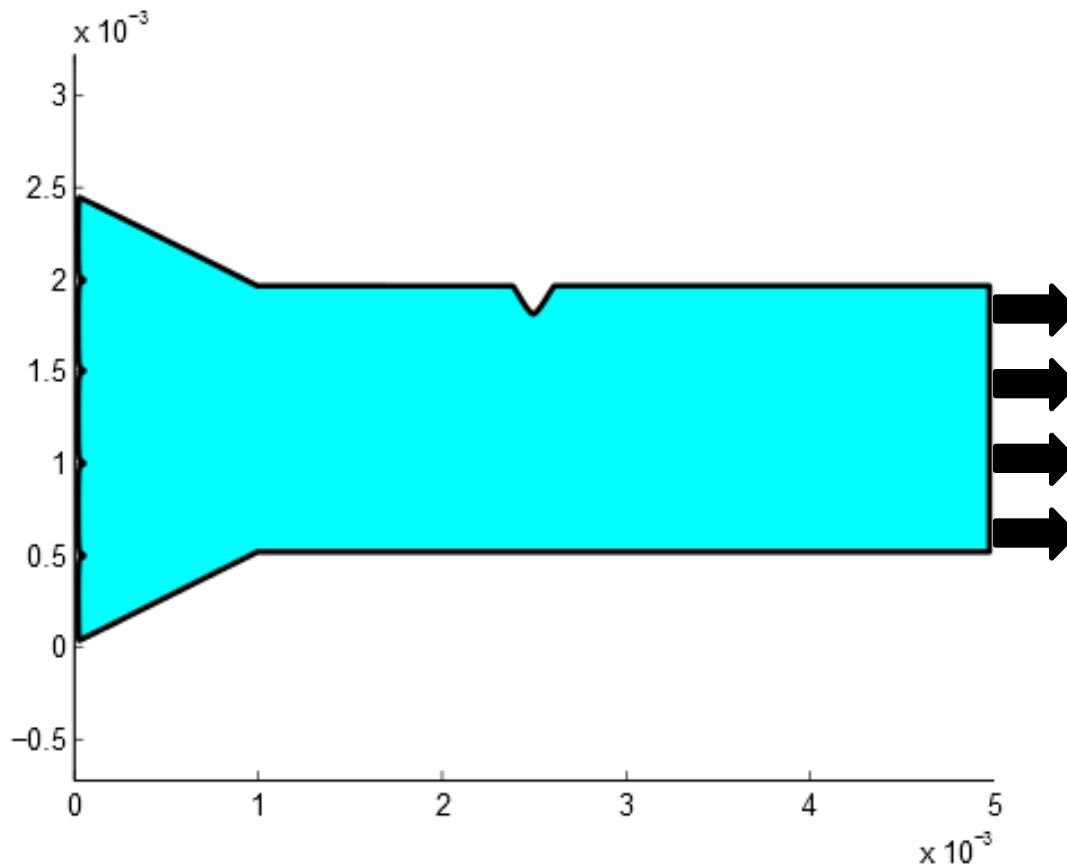
Direct Numerical Solution



Adaptive Multiscale method



# Results: uni-axial tension

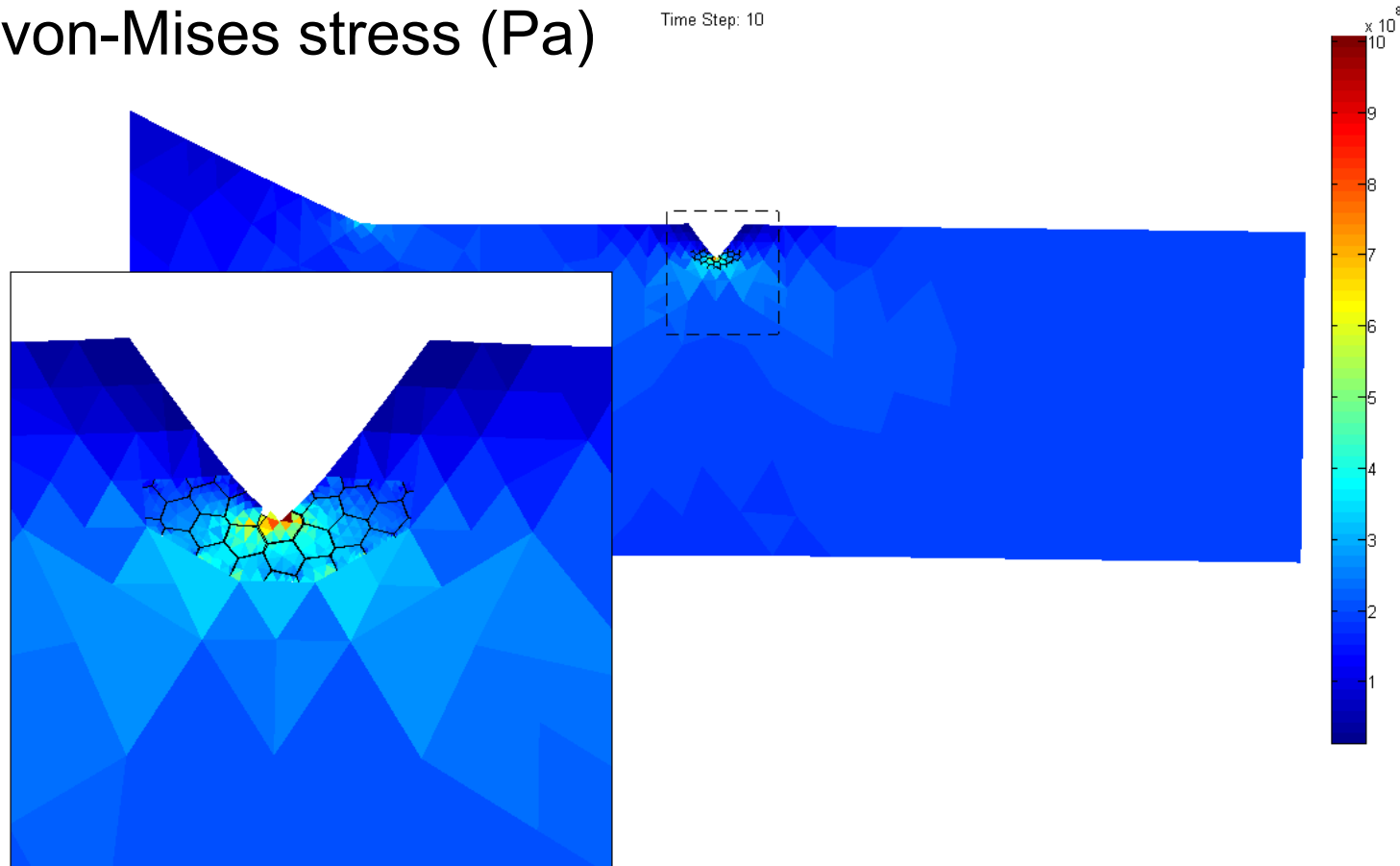


❖ Sizes are in mm

# Results: uni-axial tension

von-Mises stress (Pa)

Time Step: 10

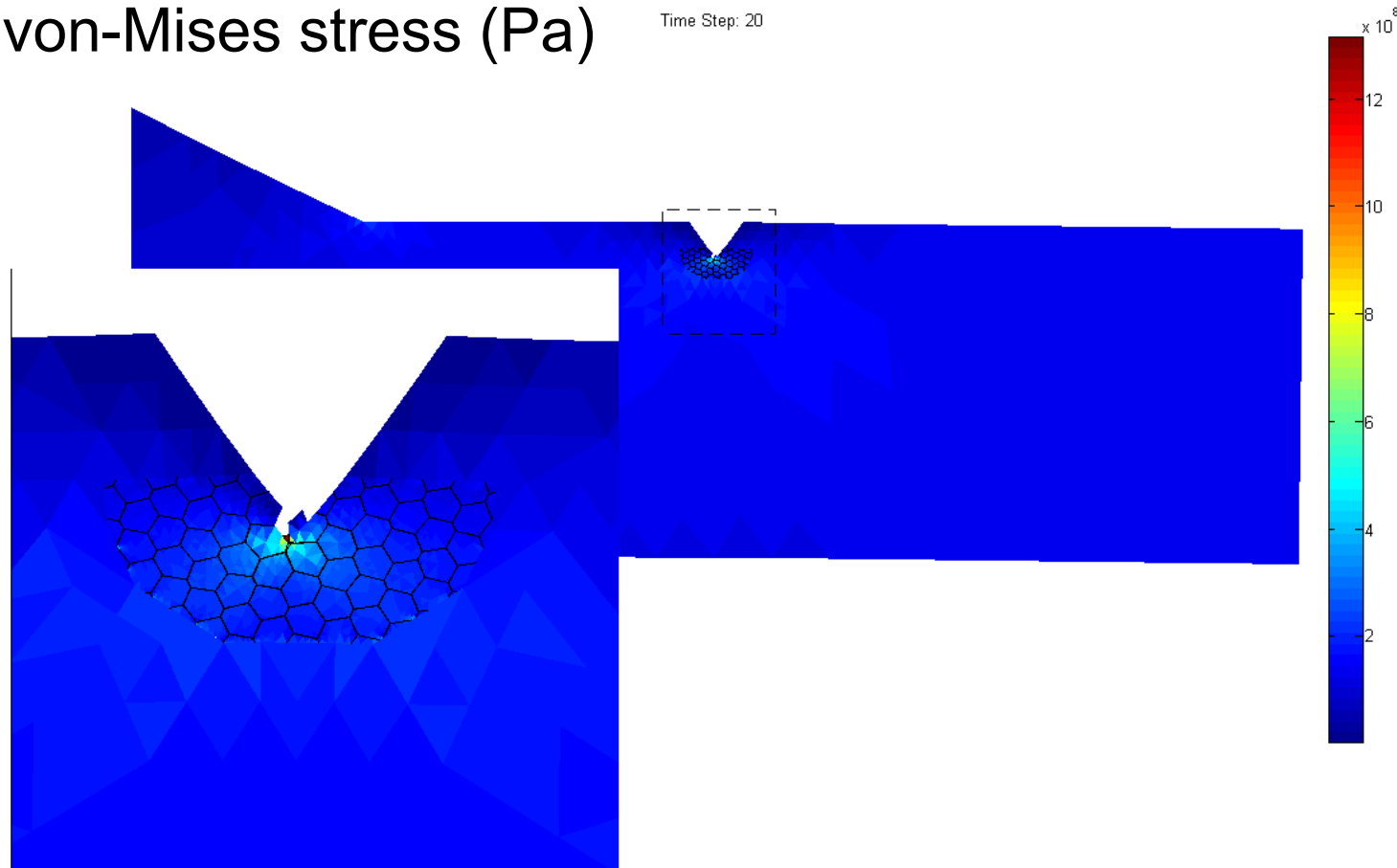


❖ 100X (magnification of displacement)

# Results: uni-axial tension

von-Mises stress (Pa)

Time Step: 20

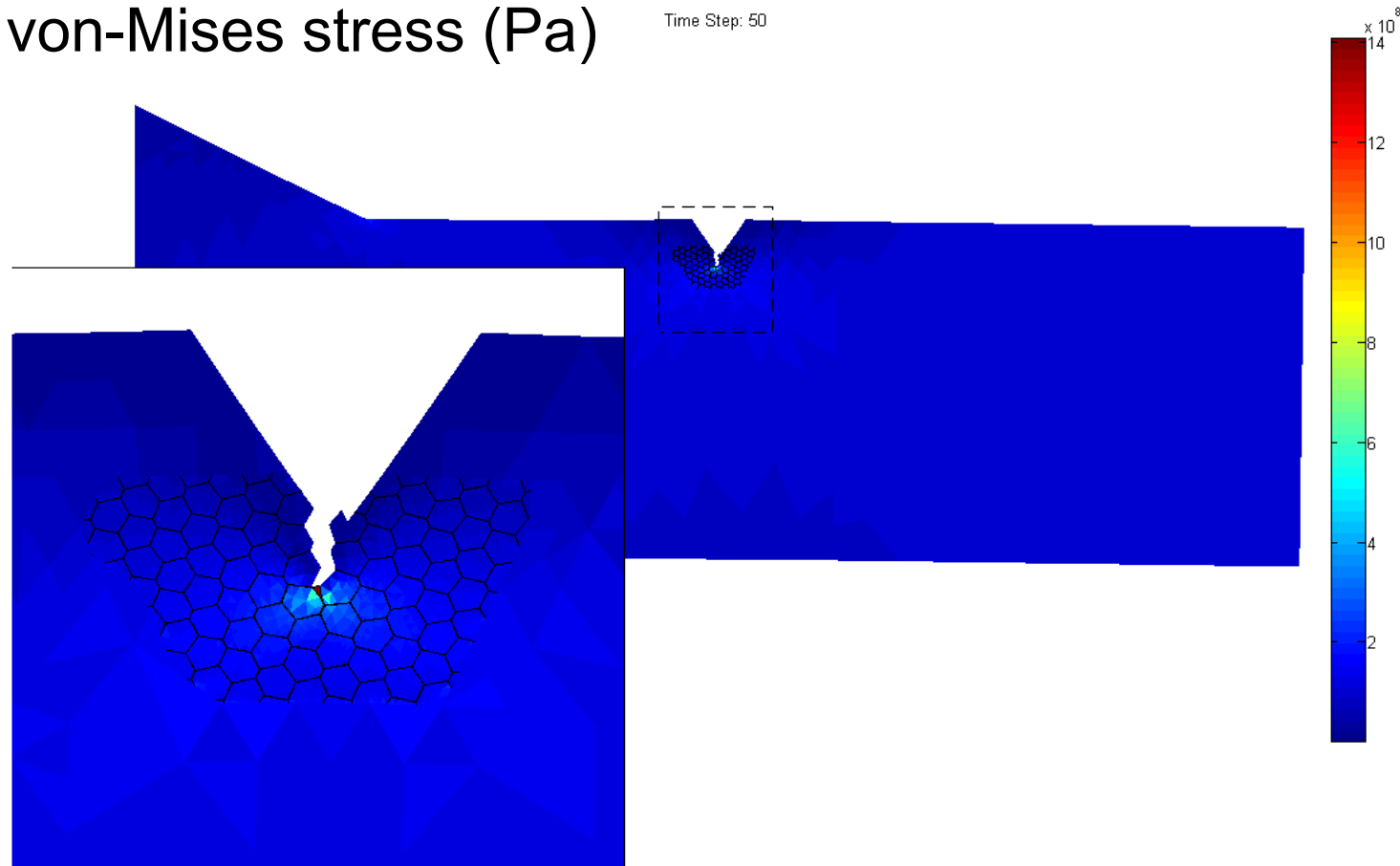


❖ 100X (magnification of displacement)

# Results: uni-axial tension

von-Mises stress (Pa)

Time Step: 50

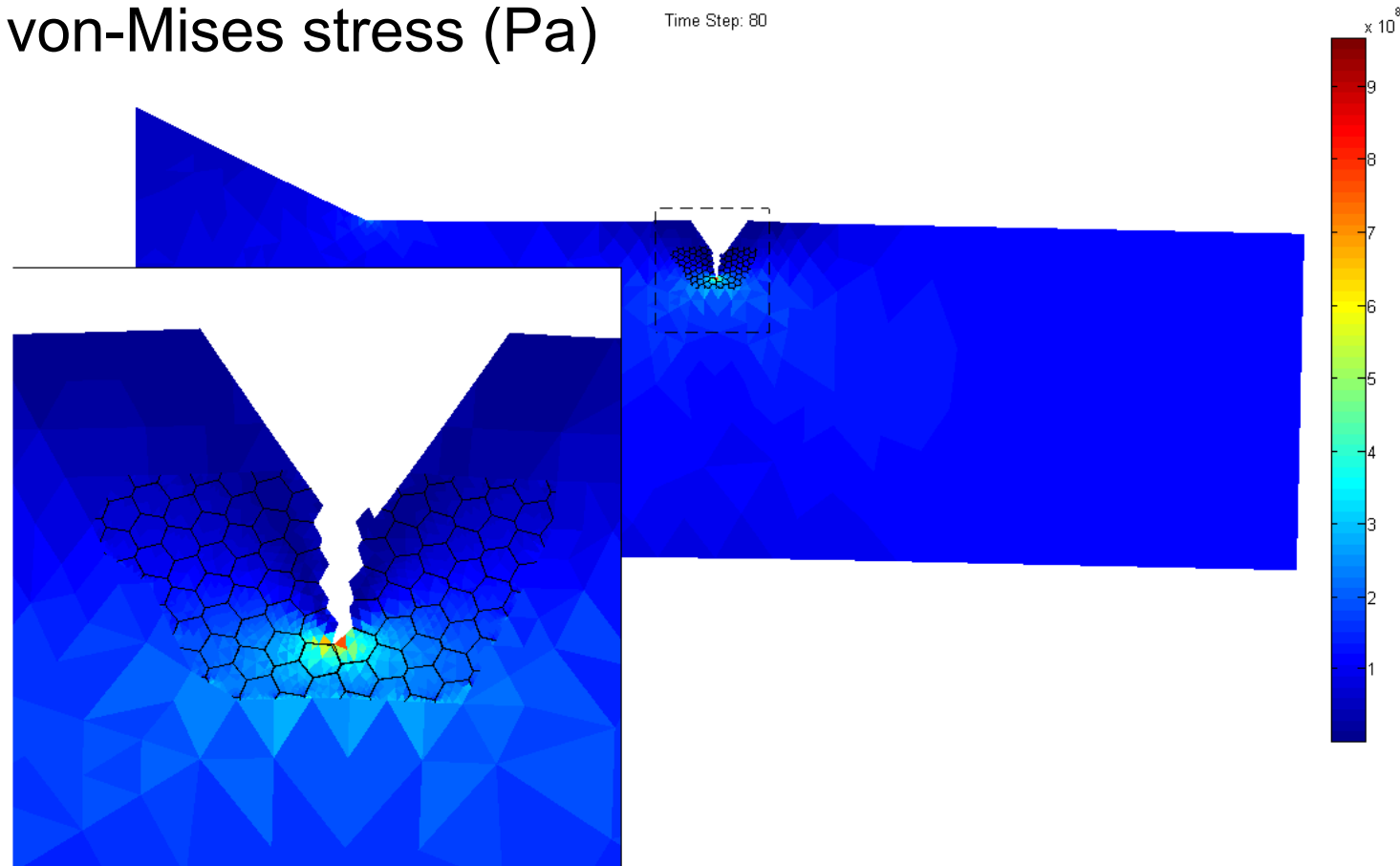


❖ 100X (magnification of displacement)

# Results: uni-axial tension

von-Mises stress (Pa)

Time Step: 80



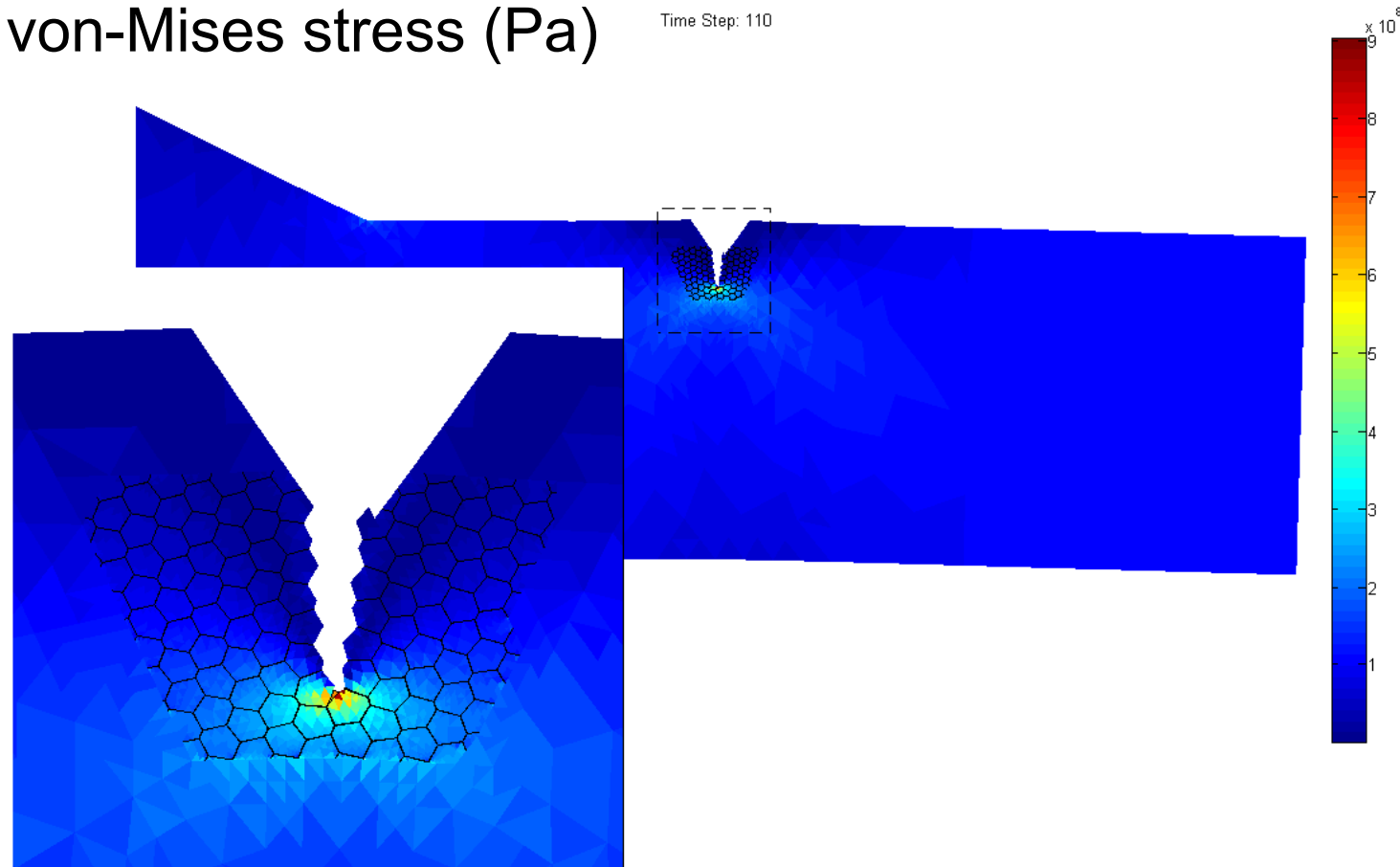
❖ 100X (magnification of displacement)



# Results: uni-axial tension

von-Mises stress (Pa)

Time Step: 110



❖ 100X (magnification of displacement)

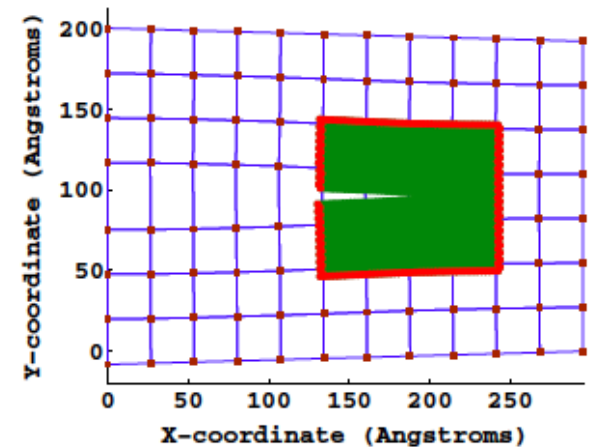
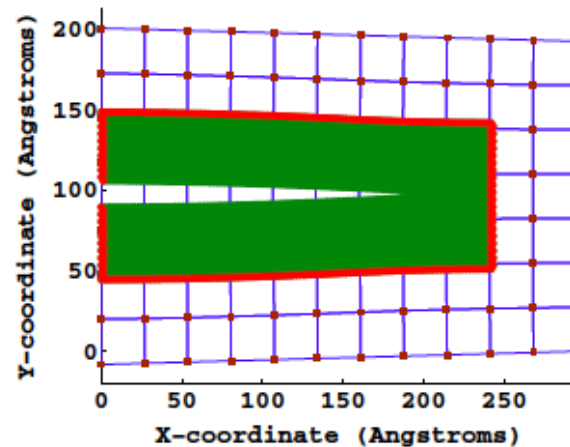
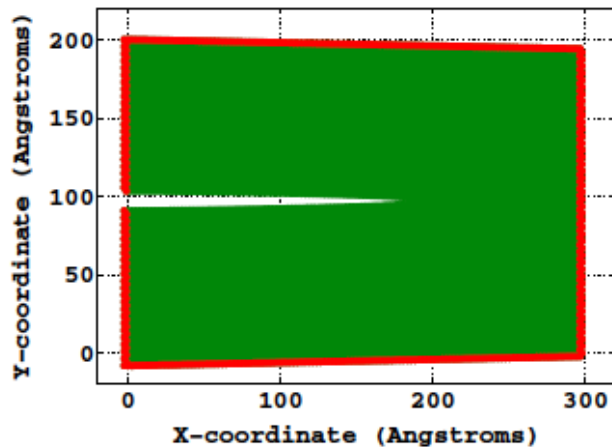
# Adaptive Multiscale Method

An adaptive multiscale method was developed for discrete fracture in polycrystalline materials:

- An unstructured mesh is used for the coarse scale problem
- A local arc-length was used to control crack speed in the fully resolved region.
- A recovery based error indicator was employed to limit discretization error at each time step.

# Perspectives

- coarsening once the crack is open
- molecular dynamics at the fine scale



- real-life problems! :)
- coupling with algebraic model reduction (POD)



## **TWO POST DOCS TWO FACULTY POSITIONS AVAILABLE**

### **OPEN SOURCE CODES**

**PERMIX:** Multiscale, XFEM, large deformation, coupled 2 LAMMPS, ABAQUS, OpenMP -  
Fortran 2003, C++

**MATLAB Codes:** XFEM, 3D ISOGEOMETRIC XFEM, 2D ISOGEOMETRIC BEM, 2D MESHLESS  
**DOWNLOAD @ <http://cmechanicsos.users.sourceforge.net/>**

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Request membership @

[http://groups.google.com/group/computational\\_mechanics\\_discussion/about](http://groups.google.com/group/computational_mechanics_discussion/about)

# Publications - model reduction

- <http://orbilu.uni.lu/handle/10993/12024>
- <http://orbilu.uni.lu/handle/10993/12012>
- <http://orbilu.uni.lu/handle/10993/10207>
- <http://orbilu.uni.lu/handle/10993/12454>
- <http://orbilu.uni.lu/handle/10993/12453>
- <http://orbilu.uni.lu/handle/10993/14475>
- <http://orbilu.uni.lu/handle/10993/10206>

# Mesh-burden reduction

- <http://orbilu.uni.lu/handle/10993/12159>
- <http://orbilu.uni.lu/handle/10993/14135>
- <http://orbilu.uni.lu/handle/10993/13847>
- <http://orbilu.uni.lu/handle/10993/12157>
- <http://orbilu.uni.lu/handle/10993/11850>

# Demos

- Surgical simulation
  - <http://www.youtube.com/watch?v=KqM7rh6sE8s>
  - <http://www.youtube.com/watch?v=DYBRKbEiHj8>
- Multi-crack growth
  - <http://www.youtube.com/watch?v=6yPb6NXnex8>
  - <http://www.youtube.com/watch?v=7U2o5bFvj8E>

# Demos

- <http://www.youtube.com/watch?v=90NAq76mVmQ>
- Solder joint durability
  - <http://www.youtube.com/watch?v=Ri96Wv6zBNU>
  - [http://www.youtube.com/watch?v=1g3Pe\\_9XN9I](http://www.youtube.com/watch?v=1g3Pe_9XN9I)



# Damage tolerance assessment directly from CAD

- <http://www.youtube.com/watch?v=RV0gidOT0-U>
- <http://www.youtube.com/watch?v=cYhaj6SPLTE>
- <http://orbilu.uni.lu/handle/10993/12159>
- <http://orbilu.uni.lu/handle/10993/14135>
- <http://orbilu.uni.lu/handle/10993/13847>
- <http://orbilu.uni.lu/handle/10993/12157>

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- <http://orbilu.uni.lu/handle/10993/11850>