HIGHER LIE THEORY

University of Luxembourg

December 9 – 11, 2013

Mini-course

Integration of Leibniz algebras

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Invited speakers

Camilo Arias Abad
Simon Covez
Michał Jóźwikowski
Yvette Kosmann-Schwarzbach
Olga Kravchenko
Chris Rogers
Dmitry Roytenberg
Christopher Schommer-Pries
Urs Schreiber
Pavol Ševera
Zoran Škoda
Marco Zambon
Chenchang Zhu

Abstracts

Workshop "Higher Lie Theory"
December 9–11, 2013, Luxembourg

Mini-course

Friedrich Wagemann Integration of Leibniz algebras[1]

Some methods have been developed recently to investigate smooth objects (Lie racks) which have a natural Leibniz algebra structure on their tangent spaces, and to integrate a given Leibniz algebra into such a Lie rack. We will present preliminary material due to Kinyon and Weinstein, and then the four methods we know of to integrate Leibniz algebras. The first known method was developed by S. Covez in his 2010 thesis. Covez regards a Leibniz algebra as an abelian extension of a Lie algebra by some representation, and the main point is to integrate the Leibniz 2-cocycle associated to this extension. Simon will explain the details in his lecture. Kinyon and Kinyon-Weinstein developed two integration procedures closer to the BCH formula for Lie algebras. Both are based on the formula $X * Y = \exp(\mathrm{ad}_X)(Y)$. A fourth procedure is developed by Mostovoy in terms of formal "group" laws. The last part of our lectures concerns recent work by Dherin–Wagemann, where the Kinyon–Weinstein procedure is used to deformation quantize the dual of a Leibniz algebra. In this context, we state the quantization problem for generalized Poisson manifolds, i.e. manifolds with a non necessarily skew-symmetric Poisson bracket.

Talks

Camilo Arias Abad Higher dimensional analogues of braid representations

Given a complex semisimple Lie algebra and representations of it, the Khono–Drinfeld construction produces representations of the braid group B_n, which is the fundamental group of the configuration space of n points in the plane. In this talk we will discuss a higher dimensional analogue of this construction that arises from flat connections in the configuration spaces of points in $\mathbb{R}^n$. This is based on joint work with F. Schaetz.

Simon Covez On the conjectural Leibniz (co)homology for groups

The goal of this talk is to present results being consistent with conjectures of J.-L. Loday about the existence of a Leibniz (co)homology for groups. During this talk we will see that most of the conjectural properties of this Leibniz (co)homology are satisfied by the rack (co)homology.

Michał Jóźwikowski Higher Lie algebroids

Natural examples of Lie algebroids are obtained by reducing the tangent bundle of a Lie groupoid. During the talk we will study geometric objects originating form the reduction of higher tangent bundles of a Lie groupoid. Such structures are natural examples of what we shall call "higher Lie algebroids". The rest of the talk will be devoted to the study of their geometric properties.

Yvette Kosmann-Schwarzbach Double Poisson brackets. A survey

Motivated by the search for a non-commutative analogue of Poisson algebras, Michel Van den Bergh developped the theory of double Poisson algebras in a comprehensive article, published in 2008 in the Transactions of the AMS, in which he defined the double Schouten-Nijenhuis bracket, double quasi-Poisson algebras and the induced brackets on the path algebras of quivers and on representation spaces. Anne Pichereau and Geert Van de Weyer initiated the study of double Poisson cohomology (2008). I aim to describe the main ingredients and some of the results of this theory. If time permits, I shall mention further

[1] lecture notes available on the workshop website

work of Gwénaël Massuyeau and Vladimir Turaev (2012, August 2013) who proved that there is a canonical quasi-Poisson structure on the represention space of an oriented surface with boundary, induced by a double quasi-Poisson bracket, and then extended their work to the study of brackets on loop algebras of higher-dimensional manifolds. Turaev (June 2013) generalized the theory of double brackets to the graded set-up, giving a general definition of double Gerstenhaber algebras, and to the study of representation algebras associated with co-algebras equipped with a cyclic bilinear form. Alexander Odesski, Vladimir Rubtsov and Vladimir Sokolov (2012) have studied quadratic double brackets on free algebras and their relation to the solutions of the associative Yang-Baxter equation, and they recently (November 2013) extended their results to the case of parameter-dependent associative Yang-Baxter equations. Thus the theory of double brackets is being developped in relation with both topology and integrable systems. Higher-order analogues of double brackets remain to be explored.

Olga Kravchenko Knot invariants and cluster algebras

Sergey Fomin and Andrei Zelevinsky have introduced a notion of cluster algebras in 2001 creating an algebraic framework for the study of canonical bases in quantum groups. It soon turned out that the combinatorics of cluster algebras also appear in many other subjects. These ubiquitous appearances continue to manifest themselves. In particular, with Vladimir Fock and Misha Polyak we encountered cluster algebras in knot theory. In this talk I will start with a definition of a cluster algebra, which is a commutative algebra with sets of generators called clusters and maps from cluster to cluster called seed mutations given by certain rules encoded in the structure. Cluster varieties are varieties with charts being clusters and transition maps given by mutations. Certain version of them turns out to have a Poisson structure. I will show how Reidemeister moves of a knot diagram could be interpreted in the language of the corresponding cluster variety. In particular, the second Reidemeister move corresponds to a Poisson reduction, while the third Reidemeister move is given by a sequence of four mutations. There is a beautiful interpretation of it in terms of the Stasheff associahedron.

Christopher Rogers L_∞-algebras and geometric prequantization

I will describe a "homotopical analog" of the prequantization procedure developed by Kostant, Kirillov, and Souriau in which symplectic geometry is used to produce central extensions of Lie algebras. Analogously, our construction geometrically produces L_∞-extensions using higher-degree closed differential forms. Such a form canonically gives an L_∞-cocycle whose homotopy fiber acts as the L_∞ analog of the Poisson algebra. When the form represents an integral cohomology class, this L_∞-algebra is homotopy equivalent to a DGLA corresponding to the infinitesimal autoequivalences of a higher gerbe, in analogy with the prequantization of the Poisson algebra as vector fields on a principal circle bundle. Applications of this procedure include constructing Heisenberg-like L_∞-algebras such as the "string Lie 2-algebra". This is joint work with Domenico Fiorenza and Urs Schreiber.

Dmitry Roytenberg A Dold–Kan-type correspondence for superalgebras of differentiable functions and a "differential graded" approach to derived differential geometry

The commutative algebra appropriate for differential geometry is provided by the algebraic theory of C^∞-algebras – an enhancement of the theory of commutative algebras which admits all C^∞ functions of n variables (rather than just the polynomials) as its n-ary operations. Derived differential geometry requires a homotopy version of these algebras for its underlying commutative algebra. We present a model for the latter based on the notion of a "differential graded structure" on a superalgebra of differentiable functions, understood – following Ševera – as a (co)action of the monoid of endomorphisms of the odd line. This view of a differential graded structure enables us to construct, in a conceptually transparent way, a Dold–Kan-type correspondence relating our approach with models based on simplicial C^∞–algebras, generalizing a classical result of Quillen for commutative and Lie algebras. It may also shed new light on Dold–Kan-type correspondences in other contexts (e.g. operads and algebras over them). A similar differential graded approach exists for every geometry whose ground ring contains the rationals, such as real analytic or holomorphic. This talk is partly based on joint work with David Carchedi (arXiv:1211.6134 and arXiv:1212.3745).

Christopher Schommer-Pries String connections and torsion

Pavol Ševera Moduli spaces of flat connections and quantization of Lie bialgebras

Many interesting Poisson manifolds, in particular Poisson-Lie groups, arise as moduli spaces of flat connections on surfaces with decorated boundary. Moduli spaces admit a rather straightforward deformation quantization in terms of a Drinfeld associator. As an application we get a new quantization of Lie bialgebras, technically simpler than the quantization of Etingof and Kazhdan.

Zoran Škoda Toward integration of Lie algebras in Loday–Pirashvili category

It is well known that the category of Leibniz algebras embeds into the category of Lie algebras in the symmetric monoidal category of linear maps with so called infinitesimal tensor product, so called Loday-Pirashvili (LP) category. In characteristics zero, an analogue of Ado's theorem holds for Lie algebras in LP category, hence a program of integration of such Lie algebras mainly depends on the resolution of the case of (an LP generalization of) matrix Lie algebras, hence the quest for matrix LP groups and the study of their infinitesimal geometry. I came across candidate internal bialgebras in LP category for the matrix LP groups; while the study of invariant differential operators in this context is started with toy examples, and the calculation of Matija Bašić of an internal analogue of Weyl algebra, which shows combinatorially interesting new analogues of derivations.

Thomas Strobl Mathematics around Lie 2-algebroids and the tensor hierarchy in gauged supergravity

This talk has essentially two (related) parts. In the first part I present two theorems providing a description of Q2-manifolds, one close to crossed modules of Lie algebroids and another one close to Courant algebroids. In the second part we unravel the underlying mathematical structure of the celebrated embedding tensor in gauged supergravity theories. This will lead us to Leibniz algebras, the Loday differential and associated L_∞-algebras that give rise to the higher gauge theories. The first part is joint work with M. Grützmann from 2005 and 2008. The second part ongoing joint work with A. Kotov, S. Lavau and H. Samtleben, but also relying on previous work on higher gauge theories and Q-bundles with Grützmann and Kotov, respectively.

Marco Zambon Singular foliations and subalgebroids

We consider singular foliations, meant as a suitable submodule of vector fields on a manifold. We will review the ingenious construction by Androulidakis–Skandalis of a groupoid H, called holonomy groupoid, associated to any singular foliation. The groupoid H is only a topological groupoid, but we will show that the restriction of H to any leaf is a smooth Lie groupoid. Further, we will relate H to holonomy transformations – meant as equivalence classes of diffeomorphisms between slices transverse to the leaves – and raise the question of whether there is a higher groupoid encoding honest diffeomorphisms. Finally, we will introduce singular subalgebroids of an arbitrary Lie algebroid, and sketch how to extend to this case the construction of the holonomy groupoid.

Chenchang Zhu Integration of Courant algebroids

Classical Lie theory gives a pair of adjoint functors, differentiation and integration, between the category of (finite dimensional) Lie algebras and that of Lie groups. Categorification of Lie algebras and Lie groups gives us Lie 2-algebras and Lie 2-groups respectively. Even higher Lie algebra(oid)s and Lie group(oid)s can be achieved through the homotopy Lie operad and Kan complexes respectively. Courant algebroids, showing up in Poisson geometry, higher symplectic geometry, and generalized complex geometry, give rise to an important example of Lie 2-algebroids. In this talk, we will show that under a higher representation theory, representation up to homotopy, Courant algebroids can be realized as semidirect products and thus their integration can be performed easily.

List of participants

1. Camilo Arias Abad, University of Zurich
2. Giuseppe Bonavolonta, University of Luxembourg
3. Olivier Brahic, Universidade Federal do Parana
4. Martin Callies, University of Göttingen
5. Simon Covez, University of Strasbourg
6. Tiffany Covolo, University of Luxembourg
7. Martin Doubek, Charles University, Praha
8. Janusz Grabowski, Polish Academy of Sciences, Warsaw
9. Eduardo Hoefel, Universidade Federal do Parana
10. John Huerta, Instituto Superior Técnico, Lisboa
11. Michał Jóźzwikowski, Polish Academy of Sciences, Warsaw
12. Benoît Jubin, University of Luxembourg
13. Yvette Kosmann-Schwarzbach, Ecole Polytechnique, Palaiseau
14. Alexei Kotov, University of Tromso
15. Olga Kravchenko, Université Lyon 1
16. Camille Laurent-Gengoux, Université de Lorraine
17. Karl Leicht, Université Lille 1
18. Damjan Pištalo, University of Luxembourg
19. Norbert Poncin, University of Luxembourg
20. Jian Qiu, University of Luxembourg
21. Salim Rivière, University of Luxembourg
22. Chris Rogers, University of Göttingen
23. Mikolaj Rotkiewicz, Polish Academy of Sciences, Warsaw
24. Dmitry Roytenberg, University of Utrecht
25. Andrew Russhard, University of Aberdeen
26. Vincent Schlegel, Universität Zürich
27. Jean-Marc Schlenker, University of Luxembourg
28. Martin Schlichenmaier, University of Luxembourg
29. Christopher Schommer-Pries, Max Planck Institute, Bonn
30. Urs Schreiber, University of Nijmegen
31. Pavol Ševera, University of Geneva
32. Zoran Škoda, University of Zagreb
33. Thomas Strobl, Université Lyon 1
34. Matteo Tommasini, University of Luxembourg
35. Yannick Voglaire, University of Luxembourg
36. Friedrich Wagemann, University of Nantes
37. Christoph Wockel, University of Hamburg
38. Sinan Yalin, Université Lille 1
39. Marco Zambon, Autonomous University of Madrid
40. Alessandro Zampini, University of Luxembourg
41. Krzysztof Zawisza, University of Warsaw
42. Chenchang Zhu, University of Göttingen