An R-package for finite mixture models

Jang SCHILTZ (University of Luxembourg)

joint work with
Jean-Daniel GUIGOU (University of Luxembourg),
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December 7, 2013

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Outline

1. The Basic Finite Mixture Model of Nagin

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2. Generalizations of the basic model

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3. Muthén's model

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General description of Nagin's model

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Hence, this model can be interpreted as functional fuzzy cluster analysis.

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Let $Y_i = y_{i_1}, y_{i_2}, ..., y_{i_T}$ be T measures of the variable, taken at times $t_1, ...t_T$ for subject number i.

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Aim of the analysis: Find r groups of trajectories of a given kind (for instance polynomials of degree 4, $P(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4$.

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- finite : sums across a finite number of groups
- mixture : population composed of a mixture of unobserved groups

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$$y_{i_t} = \beta_0^j + \beta_1^j Age_{i_t} + \beta_2^j Age_{i_t}^2 + \beta_3^j Age_{i_t}^3 + \beta_4^j Age_{i_t}^4 + \varepsilon_{i_t}, \tag{4}$$

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Hence,

$$p^j(y_{i_t}) = \frac{1}{\sigma}\phi\left(\frac{y_{i_t} - \beta^j t_{it}}{\sigma}.\right) \qquad (5)$$

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It is difficult to force this constraint in model estimation.

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It is too complicated to get closed-forms equations.

Available Software

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SAS-based Proc Traj procedure

By Bobby L. Jones (Carnegie Mellon University).

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R-package crimCV

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Just implements a zero-inflation Poission model.

Future Software

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Will take us probably another year before completion.

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Bayesian Information Criterion:

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Rule:

The bigger the BIC, the better the model!

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$$CVE = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{T} \sum_{t=1}^{T} \left| y_{i_t} - \hat{y}_{i_t}^{[-i]} \right|. \tag{10}$$

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Rule:

The smaller the CVE, the better the model!

Their "proof" that CVE is better than BIC

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TO1:

ngr	llike	AIC	BIC	CVE
1	−13967.63	27945.26	27982.26	1.0902792
2	−11929.40	23880.81	23962.22	0.9128347
3	−11424.68	22883.37	23009.18	0.9592355
4	−11191.28	22428.55	22598.77	0.9052791
5	−11016.19	22090.37	22304.99	0.8535441
6	−10886.30	21842.61	22101.63	0.8334242
7	−10805.59	21693.18	21996.60	0.8261734
8	−10732.58	21559.16	21906.99	**0.8123785**
9	−10684.54	**21475.08**	**21867.31**	0.8240060

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To be classified into a small group, an individual really needs to be strongly consistent with it.

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OCC_j should be greater than 5 for all groups.

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Diagonostic 3: Comparing $\hat{\pi}_j$ to the Proportion of the Sample Assigned to Group j

The ratio of the two should be close to 1.

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Diagonostic 4: Confidence Intervals for Group Membership Probabilities

The confidence intervals for group membership probabilities estimates should be narrow, i.e. standard deviation of $\hat{\pi}_j$ should be small.

An application example

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The data : first dataset Salaries of workers in the private sector in Luxembourg from 1940 to 2006.

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- age in the first year of professional activity

Result for 9 groups (dataset 1)

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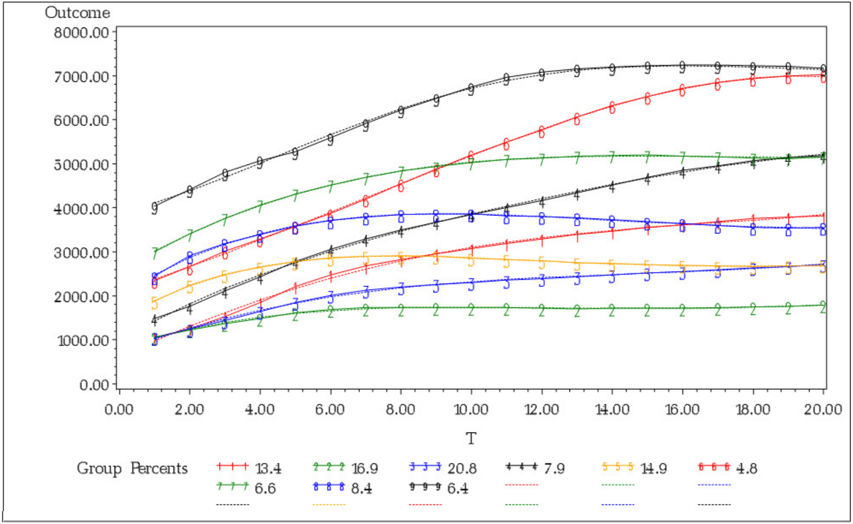

Results for 9 groups (dataset 1)

Maximum Likelihood Estimates
Model: Censored Normal (CNORM)

Group	Parameter	Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
1	Intercept	589.03067	18.46813	31.894	0.0000
	Linear	387.72145	11.31617	34.263	0.0000
	Quadratic	-14.36621	2.12997	-6.745	0.0000
	Cubic	-0.01563	0.15109	-0.103	0.9176
	Quartic	0.00856	0.00358	2.395	0.0166
2	Intercept	784.79156	15.75939	49.798	0.0000
	Linear	277.63602	9.78078	28.386	0.0000
	Quadratic	-28.36731	1.83236	-15.481	0.0000
	Cubic	1.17739	0.12972	9.076	0.0000
	Quartic	-0.01635	0.00307	-5.330	0.0000
3	Intercept	709.28728	15.90545	44.594	0.0000
	Linear	318.88029	8.97949	35.512	0.0000
	Quadratic	-21.54540	1.69611	-12.703	0.0000
	Cubic	0.62010	0.12002	5.167	0.0000
	Quartic	-0.00440	0.00284	-1.554	0.1203

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Multinomial logit model:

$$\pi_j(x_i) = \frac{e^{x_i\theta_j}}{\sum_{k=1}^{r} e^{x_i\theta_k}}, \tag{13}$$

where θ_j denotes the effect of x_i on the probability of group membership.

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Multinomial logit model:

$$\pi_j(x_i) = \frac{e^{x_i\theta_j}}{\sum_{k=1}^{r} e^{x_i\theta_k}}, \tag{13}$$

where θ_j denotes the effect of x_i on the probability of group membership.

$$L = \frac{1}{\sigma}\prod_{i=1}^{N}\sum_{j=1}^{r}\frac{e^{x_i\theta_j}}{\sum_{k=1}^{r} e^{x_i\theta_k}}\prod_{t=1}^{T}\phi\left(\frac{y_{i_t} - \beta^j t_{i_t}}{\sigma}\right). \tag{14}$$

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Confidence intervals for the probabilities of group membership can be computed by a parametric bootstrap technique.

Adding covariates to the trajectories (1)

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$$y_{i_t} = \beta_0^j + \beta_1^j Age_{i_t} + \beta_2^j Age_{i_t}^2 + \beta_3^j Age_{i_t}^3 + \beta_4^j Age_{i_t}^4 + \alpha_1^j z_1 + ... + \alpha_L^j z_L + \varepsilon_{i_t}, \quad (15)$$

where $\varepsilon_{i_t} \sim \mathcal{N}(0, \sigma)$, σ being a constant standard deviation and z_l are covariates that may depend or not upon time t.

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Moreover, the influence of the covariates is limited to the intercept of the trajectory.

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An application example

The data : second dataset Salaries of all workers in Luxembourg which began to work in Luxembourg between 1980 and 1990 at an age less than 30 years.

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- year of birth of children

Adding covariates to the trajectories (3)

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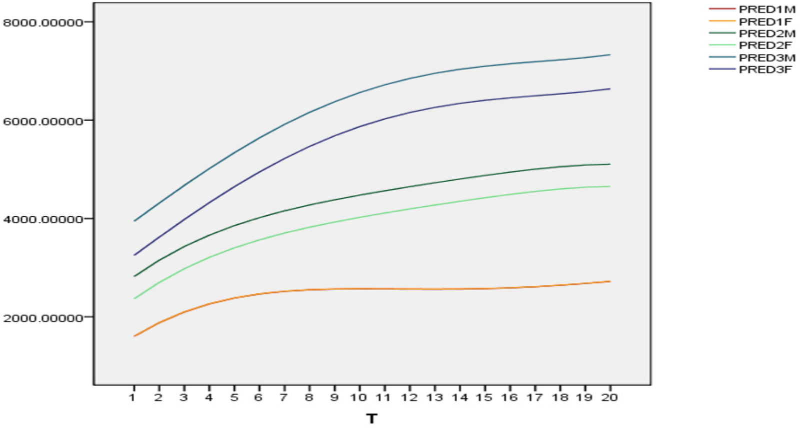
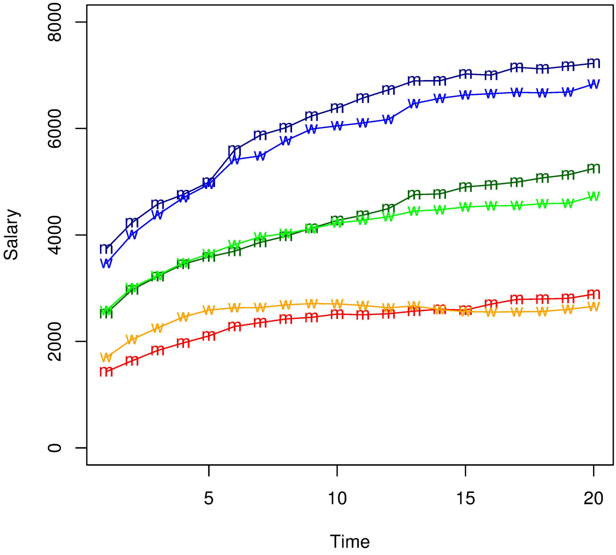


What's really going on



Our model

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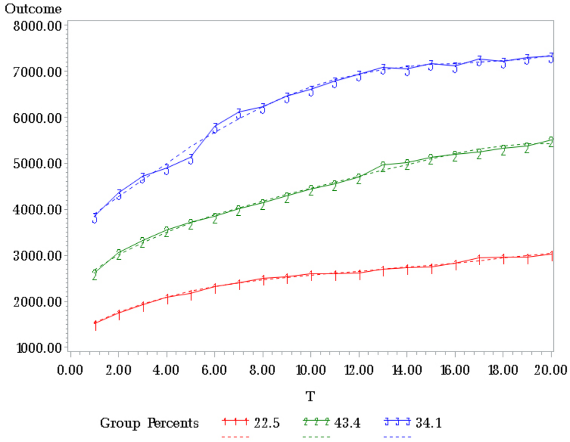
We propose the following model:

$$
\begin{aligned}
y_{i_t} = \left(\beta_0^j + \sum_{l=1}^{L} \alpha_{0l}^j x_l + \gamma_0^j z_{i_t}\right) + \left(\beta_1^j + \sum_{l=1}^{L} \alpha_{1l}^j x_l + \gamma_1^j z_{i_t}\right) Age_{i_t} \\
+ \left(\beta_2^j + \sum_{l=1}^{L} \alpha_{2l}^j x_l + \gamma_2^j z_{i_t}\right) Age_{i_t}^2 + \left(\beta_3^j + \sum_{l=1}^{L} \alpha_{3l}^j x_l + \gamma_3^j z_{i_t}\right) Age_{i_t}^3 \\
+ \left(\beta_4^j + \sum_{l=1}^{L} \alpha_{4l}^j x_l + \gamma_4^j z_{i_t}\right) Age_{i_t}^4 + \varepsilon_{i_t},
\end{aligned}
$$

where $\varepsilon_{i_t} \sim \mathcal{N}(0, \sigma)$, σ being a constant standard deviation.

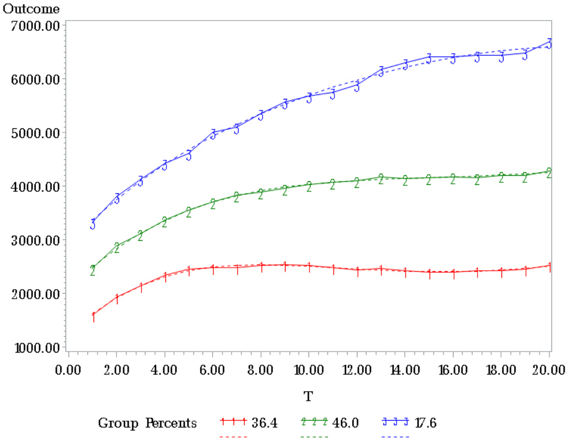
An alternative analysis (1)

Salary trajectories of the men



An alternative analysis (2)

Salary trajectories of the women



Outline

Muthén's model (1)

Muthén and Shedden (1999): Generalized growth curve model

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Software:

Mplus package by L.K. Muthén and B.O Muthén.

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Muthén's model (2)

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Fewer groups are required to specify a satisfactory model.

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3. Can create the illusion of non-existing groups.

Outline

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Outlook

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- Handling missing data.

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